



Robotic arm

1 Jacobian for 3 Degrees

The Denavit -Hartenberg parameters

θ_i	d_i	a_i	α_i
$\theta_1 = -q_1$	$d_1 = 1134mm$	$a_1 = -658.07mm$	$\alpha_1 = -\frac{\pi}{2}$
$\theta_2 = -q_2$	$d_2 = 0mm$	$a_2 = -1689.31mm$	$\alpha_2 = 0$
$\theta_3 = \frac{\pi}{2} + q_3$	$d_3 = 367.5mm$	$a_3 = 2128.98mm$	$\alpha_3 = \frac{\pi}{2}$
$\theta_4 = \frac{\pi}{2} + q_4$	$d_4 = 0$	$a_4 = 353.87mm$	$\alpha_4 = \frac{\pi}{2}$
$\theta_5 = q_5$	$d_5 = 0$	$a_5 = 452mm$	$\alpha_5 = \frac{\pi}{2}$

The transformation between each framework

$$A_{01} = \begin{pmatrix} \cos[q_1] & 0 & \sin[q_1] & -658.07\cos[q_1] \\ -\sin[q_1] & 0 & \cos[q_1] & 658.07\sin[q_1] \\ 0 & -1 & 0 & 1134 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$A_{12} = \begin{pmatrix} \cos [q_2] & \sin [q_2] & 0 & -1689.31 \cos [q_2] \\ -\sin [q_2] & \cos [q_2] & 0 & 1689.31 \sin [q_2] \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$A_{23} = \begin{pmatrix} -\sin [q_3] & 0 & \cos [q_3] & 2128.98 \sin [q_3] \\ \cos [q_3] & 0 & \sin [q_3] & -2128.98 \cos [q_3] \\ 0 & 1 & 0 & 367.5 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

The complete transformation is given by

$$A_{03} = \begin{pmatrix} & Px \\ R & Py \\ & Pz \\ 0. & 0. & 0. & 1. \end{pmatrix}$$

$$Px = 367.5 \sin [q_1] + \cos [q_1] (-658.07 - 2128.98 \cos [q_3] \sin [q_2] + \cos [q_2] (-1689.31 + 2128.98 \sin [q_3]))$$

$$Py = 367.5 \cos [q_1] + \sin [q_1] (658.07 + 2128.98 \cos [q_3] \sin [q_2] + \cos [q_2] (1689.31 - 2128.98 \sin [q_3]))$$

$$Pz = 1134. + 2128.98 \cos [q_2] \cos [q_3] + \sin [q_2] (-1689.31 + 2128.98 \sin [q_3])$$

$$R = \begin{pmatrix} \cos [q_1] \sin [q_2 - q_3] & \sin [q_1] & \cos [q_1] \cos [q_2 - q_3] \\ -\sin [q_1] \sin [q_2 - q_3] & \cos [q_1] & -\cos [q_2 - q_3] \sin [q_1] \\ -\cos [q_2 - q_3] & 0. & \sin [q_2 - q_3] \end{pmatrix}$$

The obtained Jacobian is given by

$$J = \begin{bmatrix} J_v \\ J_\omega \end{bmatrix} \quad (1)$$

$$J_v = \begin{bmatrix} \frac{\partial P_x}{\partial q_1} & \frac{\partial P_x}{\partial q_2} & \frac{\partial P_x}{\partial q_3} \\ \frac{\partial P_y}{\partial q_1} & \frac{\partial P_y}{\partial q_2} & \frac{\partial P_y}{\partial q_3} \\ \frac{\partial P_z}{\partial q_1} & \frac{\partial P_z}{\partial q_2} & \frac{\partial P_z}{\partial q_3} \end{bmatrix} \quad (2)$$

$$J_\omega = \begin{bmatrix} \omega_{x1} & \omega_{x2} & \omega_{x3} \\ \omega_{y1} & \omega_{y2} & \omega_{y3} \\ \omega_{z1} & \omega_{z2} & \omega_{z3} \end{bmatrix} \quad (3)$$

$$\frac{\partial P_x}{\partial q_1} = 367.5 \cos[q_1] + \sin[q_1] (658.07 + 1689.31 \cos[q_2] + 2128.98 \sin[q_2 - q_3])$$

$$\frac{\partial P_x}{\partial q_2} = \cos[q_1] (-2128.98 \cos[q_2 - q_3] + 1689.31 \sin[q_2])$$

$$\frac{\partial P_x}{\partial q_3} = 2128.98 \cos[q_1] \cos[q_2 - q_3]$$

$$\frac{\partial P_y}{\partial q_1} = -367.5 \sin[q_1] + \cos[q_1] (658.07 + 1689.31 \cos[q_2] + 2128.98 \sin[q_2 - q_3])$$

$$\frac{\partial P_y}{\partial q_2} = \sin[q_1] (2128.98 \cos[q_2 - q_3] - 1689.31 \sin[q_2])$$

$$\frac{\partial P_y}{\partial q_3} = -2128.98 \cos[q_2 - q_3] \sin[q_1]$$

$$\frac{\partial P_z}{\partial q_1} = 0$$

$$\frac{\partial P_z}{\partial q_2} = -1689.31 \cos[q_2] - 2128.98 \sin[q_2 - q_3]$$

$$\frac{\partial P_z}{\partial q_3} = 2128.98 \sin[q_2 - q_3]$$

$$J_\omega = \begin{bmatrix} 0 & -\sin[q_1] & \sin[q_1] \\ 0 & -\cos[q_1] & \cos[q_1] \\ -1 & 0 & 0 \end{bmatrix}$$

2 Jacobian for 6 degrees

For the Jacobian with the 6 degree of freedom the transformation between the rest of frameworks are

$$A_{34} = \begin{pmatrix} -\sin[q_4] & 0 & \cos[q_4] & -353.87 \sin[q_4] \\ \cos[q_4] & 0 & \sin[q_4] & 353.87 \cos[q_4] \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$A_{45} = \begin{pmatrix} \cos [q_5] & 0 & -\sin [q_5] & -452 \cos [q_5] \\ \sin [q_5] & 0 & \cos [q_5] & -452 \sin [q_5] \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$A_{03} = \begin{pmatrix} & Px \\ R & Py \\ & Pz \\ 0. & 0. & 0. & 1. \end{pmatrix}$$

where

$$\begin{aligned} Px &= (367.5 + \cos [q_4] (353.87 - 452 \cos [q_5])) \sin [q_1] \\ &\quad - 658.07 \cos [q_1] \\ &\quad + \cos [q_1] \cos [q_3] \sin [q_2] (-2128.98 - 353.87 \sin [q_4]) \\ &\quad + \cos [q_1] 452 \cos [q_5] \sin [q_2 - q_3] \sin [q_4] \\ &\quad + \cos [q_1] (\cos [q_2] (-1689.31 + \sin [q_3] (2128.98 + 353.87 \sin [q_4])) - 452 \cos [q_2 - q_3] \sin [q_5]) \end{aligned}$$

$$\begin{aligned} Py &= \cos [q_1] (367.5 + \cos [q_4] (353.87 - 452 \cos [q_5])) \\ &\quad + \sin [q_1] (658.07 + \cos [q_2] (1689.31 + \sin [q_3] (-2128.98 - 353.87 \sin [q_4]))) \\ &\quad - \sin [q_1] (452 \cos [q_5] \sin [q_2 - q_3] \sin [q_4]) \\ &\quad \sin [q_1] (+\cos [q_3] \sin [q_2] (2128.98 + 353.87 \sin [q_4]) + 452 \cos [q_2 - q_3] \sin [q_5]) \end{aligned}$$

$$\begin{aligned} Pz &= 1,134.0002 \\ &\quad - 452 \sin [q_2] (3.73741 + \sin [q_3] (-4.71013 + (-0.782898 + \cos [q_5]) \sin [q_4]) + \cos [q_3] \sin [q_5]) \\ &\quad - 452 \cdot (\cos [q_2] (\cos [q_3] (-4.71013 + (-0.782898 + \cos [q_5]) \sin [q_4]) - 1 \cdot \sin [q_3] \sin [q_5])) \end{aligned}$$

$$R = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$$

$$\begin{aligned}
r_{11} &= \cos [q_4] \cos [q_5] \sin [q_1] + \cos [q_1] (-\cos [q_5] \sin [q_2 - q_3] \sin [q_4] + \cos [q_2 - q_3] \sin [q_5]) \\
r_{12} &= -\cos [q_1] \cos [q_4] \sin [q_2 - q_3] - \sin [q_1] \sin [q_4] \\
r_{13} &= -\cos [q_4] \sin [q_1] \sin [q_5] + \cos [q_1] (\cos [q_2 - q_3] \cos [q_5] + \sin [q_2 - q_3] \sin [q_4] \sin [q_5]) \\
r_{21} &= \cos [q_1] \cos [q_4] \cos [q_5] + \sin [q_1] (\cos [q_5] \sin [q_2 - q_3] \sin [q_4] - \cos [q_2 - q_3] \sin [q_5]) \\
r_{22} &= \cos [q_4] \sin [q_1] \sin [q_2 - q_3] - \cos [q_1] \sin [q_4] \\
r_{23} &= -\cos [q_2 - q_3] \cos [q_5] \sin [q_1] - (\cos [q_1] \cos [q_4] + \sin [q_1] \sin [q_2 - q_3] \sin [q_4]) \sin [q_5] \\
r_{31} &= \cos [q_2 - q_3] \cos [q_5] \sin [q_4] + \sin [q_2 - q_3] \sin [q_5] \\
r_{32} &= \cos [q_2 - q_3] \cos [q_4] \\
r_{33} &= \cos [q_5] \sin [q_2 - q_3] - \cos [q_2 - q_3] \sin [q_4] \sin [q_5]
\end{aligned}$$

The obtained Jacobian is given by

$$J = \begin{bmatrix} J_v \\ J_\omega \end{bmatrix} \quad (4)$$

$$J_v = \begin{bmatrix} \frac{\partial P_x}{\partial q_1} & \frac{\partial P_x}{\partial q_2} & \frac{\partial P_x}{\partial q_3} \\ \frac{\partial P_y}{\partial q_1} & \frac{\partial P_y}{\partial q_2} & \frac{\partial P_y}{\partial q_3} \\ \frac{\partial P_z}{\partial q_1} & \frac{\partial P_z}{\partial q_2} & \frac{\partial P_z}{\partial q_3} \end{bmatrix} \quad (5)$$

$$J_\omega = \begin{bmatrix} \omega_{x1} & \omega_{x2} & \omega_{x3} \\ \omega_{y1} & \omega_{y2} & \omega_{y3} \\ \omega_{z1} & \omega_{z2} & \omega_{z3} \end{bmatrix} \quad (6)$$

$$\begin{aligned}
\frac{\partial P_x}{\partial q_1} &= \cos [q_1] (367.5 + \cos [q_4] (353.87 - 452 \cos [q_5])) \\
&\quad + \sin [q_1] (658.07 + \cos [q_2] (1689.31 + \sin [q_3] (-2128.98 - 353.87 \sin [q_4]))) \\
&\quad + \sin [q_1] (-452 \cos [q_5] \sin [q_2 - q_3] \sin [q_4] + \cos [q_3] \sin [q_2] (2128.98 + 353.87 \sin [q_4])) \\
&\quad + \sin [q_1] (452 \cos [q_2 - q_3] \sin [q_5])
\end{aligned}$$

$$\begin{aligned}
\frac{\partial P_x}{\partial q_2} &= 353.87 \cos [q_1] (\sin [q_2] (4.77382 + \sin [q_3] (-6.01628 - 1 \sin [q_4]))) \\
&\quad + 353.87 \cos [q_1] (\cos [q_2] \cos [q_3] (-6.01628 - 1 \sin [q_4])) \\
&\quad + 353.87 \cos [q_1] (+1.27731 \cos [q_2 - q_3] \cos [q_5] \sin [q_4] + 1.27731 \sin [q_2 - q_3] \sin [q_5])
\end{aligned}$$

$$\begin{aligned}
\frac{\partial P_x}{\partial q_3} &= 353.87 \cos [q_1] (-1.27731 \cos [q_2 - q_3] \cos [q_5] \sin [q_4] + \cos [q_2] \cos [q_3] (6.01628 + 1 \sin [q_4])) \\
&\quad + 353.87 \cos [q_1] (\sin [q_2] \sin [q_3] (6.01628 + 1 \sin [q_4]) - 1.27731 \sin [q_2 - q_3] \sin [q_5])
\end{aligned}$$

$$\begin{aligned}\frac{\partial P_x}{\partial q_4} &= \cos [q_1] \cos [q_4] (-353.87 \cos [q_3] \sin [q_2] + 452. \cos [q_5] \sin [q_2 - q_3] + 353.87 \cos [q_2] \sin [q_3]) + \\ &(-353.87 + 452. \cos [q_5]) \sin [q_1] \sin [q_4]\end{aligned}$$

$$\frac{\partial P_x}{\partial q_5} = 452. \cos [q_4] \sin [q_1] \sin [q_5] + \cos [q_1] (-452. \cos [q_2 - q_3] \cos [q_5] - 452. \sin [q_2 - q_3] \sin [q_4] \sin [q_5])$$

$$\begin{aligned}\frac{\partial P_y}{\partial q_1} &= (-367.5 + \cos [q_4] (-353.87 + 452. \cos [q_5])) \sin [q_1] \\ &+ \cos [q_1] (658.07 + \cos [q_2] (1689.31 + \sin [q_3] (-2128.98 - 353.87 \sin [q_4]))) \\ &+ \cos [q_1] (-452. \cos [q_5] \sin [q_2 - q_3] \sin [q_4] + \cos [q_3] \sin [q_2] (2128.98 + 353.87 \sin [q_4])) \\ &+ \cos [q_1] (+452. \cos [q_2 - q_3] \sin [q_5])\end{aligned}$$

$$\begin{aligned}\frac{\partial P_y}{\partial q_2} &= -353.87 \sin [q_1] (\sin [q_2] (4.77382 + \sin [q_3] (-6.01628 - 1. \sin [q_4]))) \\ &- 353.87 \sin [q_1] (\cos [q_2] \cos [q_3] (-6.01628 - 1. \sin [q_4]) + 1.27731 \cos [q_2 - q_3] \cos [q_5] \sin [q_4]) \\ &- 353.87 \sin [q_1] (+1.27731 \sin [q_2 - q_3] \sin [q_5])\end{aligned}$$

$$\begin{aligned}\frac{\partial P_y}{\partial q_3} &= -353.87 \sin [q_1] (-1.27731 \cos [q_2 - q_3] \cos [q_5] \sin [q_4] + \cos [q_2] \cos [q_3] (6.01628 + 1. \sin [q_4])) \\ &- 353.87 \sin [q_1] (+\sin [q_2] \sin [q_3] (6.01628 + 1. \sin [q_4]) - 1.27731 \sin [q_2 - q_3] \sin [q_5])\end{aligned}$$

$$\begin{aligned}\frac{\partial P_y}{\partial q_4} &= \cos [q_4] \sin [q_1] (353.87 \cos [q_3] \sin [q_2] - 452. \cos [q_5] \sin [q_2 - q_3] - 353.87 \cos [q_2] \sin [q_3]) + \\ &452. \cos [q_1] (-0.782898 + \cos [q_5]) \sin [q_4]\end{aligned}$$

$$\frac{\partial P_y}{\partial q_5} = 452. \cos [q_2 - q_3] \cos [q_5] \sin [q_1] + (452. \cos [q_1] \cos [q_4] + 452. \sin [q_1] \sin [q_2 - q_3] \sin [q_4]) \sin [q_5]$$

$$\frac{\partial P_z}{\partial q_1} = 0$$

$$\begin{aligned}\frac{\partial P_z}{\partial q_2} &= \cos [q_3] \sin [q_2] (-2128.98 + (-353.87 + 452. \cos [q_5]) \sin [q_4]) \\ &+ \cos [q_2] (-1689.31 + \sin [q_3] (2128.98 + (353.87 - 452. \cos [q_5]) \sin [q_4])) \\ &- 452. \cos [q_2 - q_3] \sin [q_5]\end{aligned}$$

$$\begin{aligned}\frac{\partial P_z}{\partial q_3} &= -452. \cos [q_3] (\sin [q_2] (-4.71013 + (-0.782898 + \cos [q_5]) \sin [q_4]) - 1. \cos [q_2] \sin [q_5]) + \\ &452. \sin [q_3] (\cos [q_2] (-4.71013 + (-0.782898 + \cos [q_5]) \sin [q_4]) + \sin [q_2] \sin [q_5])\end{aligned}$$

$$\frac{\partial P_z}{\partial q_4} = -452. \cos [q_2 - q_3] \cos [q_4] (-0.782898 + \cos [q_5])$$

$$\begin{aligned}
\frac{\partial Pz}{\partial q_5} &= -452 \text{Cos}[q_3] (\text{Cos}[q_5] \text{Sin}[q_2] - \text{Cos}[q_2] \text{Sin}[q_4] \text{Sin}[q_5]) \\
&\quad -452. (+\text{Sin}[q_3] (-1. \text{Cos}[q_2] \text{Cos}[q_5] - \text{Sin}[q_2] \text{Sin}[q_4] \text{Sin}[q_5]))
\end{aligned}$$

$$J_\omega = \begin{pmatrix} 0 & -\text{Sin}[q_1] & \text{Sin}[q_1] & \text{Cos}[q_1] \text{Cos}[q_2 - q_3] & \text{Cos}[q_1] \text{Cos}[q_4] \text{Sin}[q_2 - q_3] + \text{Sin}[q_1] \text{Sin}[q_4] \\ 0 & -\text{Cos}[q_1] & \text{Cos}[q_1] & -\text{Cos}[q_2 - q_3] \text{Sin}[q_1] & -\text{Cos}[q_4] \text{Sin}[q_1] \text{Sin}[q_2 - q_3] + \text{Cos}[q_1] \text{Sin}[q_4] \\ -1 & 0 & 0 & \text{Sin}[q_2 - q_3] & -\text{Cos}[q_2 - q_3] \text{Cos}[q_4] \end{pmatrix}$$