

$$A01 := \{ \{ \text{Cos} [\theta_1] , -\text{Sin} [\theta_1] \text{Cos} [\alpha_1] , \text{Sin} [\theta_1] \text{Sin} [\alpha_1] , a_1 \text{Cos} [\theta_1] \} , \{ \text{Sin} [\theta_1] , \text{Cos} [\theta_1] \text{Cos} [\alpha_1] , -\text{Cos} [\theta_1] \text{Sin} [\alpha_1] , a_1 \text{Sin} [\theta_1] \} \}$$

$$\theta_1 = -q_1;$$

$$a_1 = -658.07;$$

$$\alpha_1 = -\pi/2;$$

$$d_1 = 1134;$$

$$A12 := \{ \{ \text{Cos} [\theta_2] , -\text{Sin} [\theta_2] \text{Cos} [\alpha_2] , \text{Sin} [\theta_2] \text{Sin} [\alpha_2] , a_2 \text{Cos} [\theta_2] \} , \{ \text{Sin} [\theta_2] , \text{Cos} [\theta_2] \text{Cos} [\alpha_2] , -\text{Cos} [\theta_2] \text{Sin} [\alpha_2] , a_2 \text{Sin} [\theta_2] \} \}$$

$$\theta_2 = -q_2;$$

$$a_2 = -1689.31;$$

$$\alpha_2 = 0;$$

$$d_2 = 0;$$

$$A23 := \{ \{ \text{Cos} [\theta_3] , -\text{Sin} [\theta_3] \text{Cos} [\alpha_3] , \text{Sin} [\theta_3] \text{Sin} [\alpha_3] , a_3 \text{Cos} [\theta_3] \} , \{ \text{Sin} [\theta_3] , \text{Cos} [\theta_3] \text{Cos} [\alpha_3] , -\text{Cos} [\theta_3] \text{Sin} [\alpha_3] , a_3 \text{Sin} [\theta_3] \} \}$$

$$\theta_3 = \pi/2 + q_3;$$

$$a_3 = -2128.98;$$

$$\alpha_3 = \pi/2;$$

$$d_3 = 367.5;$$

$$A34 := \{ \{ \text{Cos} [\theta_4] , -\text{Sin} [\theta_4] \text{Cos} [\alpha_4] , \text{Sin} [\theta_4] \text{Sin} [\alpha_4] , a_4 \text{Cos} [\theta_4] \} , \{ \text{Sin} [\theta_4] , \text{Cos} [\theta_4] \text{Cos} [\alpha_4] , -\text{Cos} [\theta_4] \text{Sin} [\alpha_4] , a_4 \text{Sin} [\theta_4] \} \}$$

$$\theta_4 = \pi/2 + q_4;$$

$$a_4 = 353.87;$$

$$\alpha_4 = \pi/2;$$

$$d_4 = 0;$$

$$A45 := \{ \{ \text{Cos} [\theta_5] , -\text{Sin} [\theta_5] \text{Cos} [\alpha_5] , \text{Sin} [\theta_5] \text{Sin} [\alpha_5] , a_5 \text{Cos} [\theta_5] \} , \{ \text{Sin} [\theta_5] , \text{Cos} [\theta_5] \text{Cos} [\alpha_5] , -\text{Cos} [\theta_5] \text{Sin} [\alpha_5] , a_5 \text{Sin} [\theta_5] \} \}$$

$$\theta_5 = q_5;$$

$$a_5 = -452;$$

$$\alpha_5 = -\pi/2;$$

$$d_5 = 0;$$

A23//MatrixForm

A34//MatrixForm

A45//MatrixForm

$$\begin{pmatrix} -\sin[q_3] & 0 & \cos[q_3] & 2128.98\sin[q_3] \\ \cos[q_3] & 0 & \sin[q_3] & -2128.98\cos[q_3] \\ 0 & 1 & 0 & 367.5 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

A34

$$\begin{pmatrix} \cos[q_5] & 0 & -\sin[q_5] & -452\cos[q_5] \\ \sin[q_5] & 0 & \cos[q_5] & -452\sin[q_5] \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

A01//MatrixForm

A12//MatrixForm

A02 = A01 * A12//MatrixForm

A02 = A01.A12//MatrixForm

$$\begin{pmatrix} \cos[q_1] & 0 & \sin[q_1] & -658.07\cos[q_1] \\ -\sin[q_1] & 0 & \cos[q_1] & 658.07\sin[q_1] \\ 0 & -1 & 0 & 1134 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} \cos[q_2] & \sin[q_2] & 0 & -1689.31\cos[q_2] \\ -\sin[q_2] & \cos[q_2] & 0 & 1689.31\sin[q_2] \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} \cos [q_1] \cos [q_2] & 0 & 0 & 1.11168 \times 10^6 \cos [q_1] \cos [q_2] \\ \sin [q_1] \sin [q_2] & 0 & 0 & 1.11168 \times 10^6 \sin [q_1] \sin [q_2] \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 0. + \cos [q_1] \cos [q_2] & 0. + \cos [q_1] \sin [q_2] & 0. + \sin [q_1] & 0. - 658.07 \cos [q_1] - 1689.31 \cos [q_1] \cos [q_2] \\ 0. - \cos [q_2] \sin [q_1] & 0. - \sin [q_1] \sin [q_2] & 0. + \cos [q_1] & 0. + 658.07 \sin [q_1] + 1689.31 \cos [q_2] \sin [q_1] \\ \sin [q_2] & -\cos [q_2] & 0 & 1134. - 1689.31 \sin [q_2] \\ 0 & 0 & 0 & 1. \end{pmatrix}$$

A05//MatrixForm

$$\left(\begin{aligned} & \cos [q_5] (\cos [q_4] (0. + \sin [q_1]) - (\cos [q_3] (0. + \cos [q_1] \sin [q_2]) - (0. + \cos [q_1] \cos [q_2]) \sin [q_3]) \sin [q_4]) + (\cos [q_5] ((0. + \cos [q_1]) \cos [q_4] - (\cos [q_3] (0. - \sin [q_1] \sin [q_2]) - (0. - \cos [q_2] \sin [q_1]) \sin [q_3]) \sin [q_4]) + (\cos [q_5] (-\cos [q_2] \cos [q_3] - \sin [q_2] \sin [q_3]) \sin [q_4] + (\cos [q_3] \sin [q_4] \\ & 0. \end{aligned} \right.$$

```
Py = Simplify[{ {0, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 0, 0} }.A05.{0, 0, 0, 1}]
```

$$\{-452.(-0.813053 + \cos [q_4](-0.782898 + \cos [q_5])) \sin [q_1] + 353.87 \cos [q_1](-1.85964 + \cos [q_3] \sin [q_2](-6.0$$
$$\{0., -452.\text{Cos}[q_1](-0.813053 + \text{Cos}[q_4](-0.782898 + \text{Cos}[q_5])) - 353.87\text{Sin}[q_1](-1.85964 + \text{Cos}[q_3]\text{Sin}[q_2](-6$$
$$\{0., 0., -452. (-2.50885 + \text{Sin}[q_2] (3.73741 + \text{Sin}[q_3] (-4.71013 + (-0.782898 + \text{Cos}[q_5]) \text{Sin}[q_4]) + \text{Cos}[q_3] \text{Sin}[q_4])$$

$$\mathbf{Jv} := \{ \{ D[\mathbf{Px}, q_1], D[\mathbf{Px}, q_2], D[\mathbf{Px}, q_3], D[\mathbf{Px}, q_4], D[\mathbf{Px}, q_5] \}, \{ D[\mathbf{Py}, q_1], D[\mathbf{Py}, q_2], D[\mathbf{Py}, q_3], D[\mathbf{Py}, q_4], D[\mathbf{Py}, q_5] \}, \{ D[\mathbf{Pz}, q_1], D[\mathbf{Pz}, q_2], D[\mathbf{Py}, q_3], D[\mathbf{Py}, q_4], D[\mathbf{Py}, q_5] \} \}$$

J //MatrixForm

Transpose[J //MatrixForm

J

Transpose[J]

$$R = \{ \{1, 0, 0, 0\}, \{0, 1, 0, 0\}, \{0, 0, 1, 0\} \} \cdot \mathbf{A05} \cdot \{ \{1, 0, 0\}, \{0, 1, 0\}, \{0, 0, 1\}, \{0, 0, 0\} \}$$

$$\{ \{ 0. + \cos[q_5] (\cos[q_4] (0. + \sin[q_1]) - (\cos[q_3] (0. + \cos[q_1] \sin[q_2]) - (0. + \cos[q_1] \cos[q_2]) \sin[q_3]) \sin[q_4]) \}$$

$$S_1 = D[R, q_1] \cdot \text{Transpose}[R];$$

$$\mathbf{wx}_1 = S_1[[3, 2]];$$

$$\mathbf{wy}_1 = S_1[[1, 3]];$$

$$\mathbf{wz}_1 = S_1[[2, 1]];$$

$$S_2 = D[R, q_2] \cdot \text{Transpose}[R];$$

$$\mathbf{wx}_2 = S_2[[3, 2]];$$

$$\mathbf{wy}_2 = S_2[[1, 3]];$$

$$\mathbf{wz}_2 = S_2[[2, 1]];$$

$$S_3 = D[R, q_3] \cdot \text{Transpose}[R];$$

$$\mathbf{wx}_3 = S_3[[3, 2]];$$

$$\mathbf{wy}_3 = S_3[[1, 3]];$$

$$\mathbf{wz}_3 = S_3[[2, 1]];$$

$$S_4 = D[R, q_4].\text{Transpose}[R];$$

$$\mathbf{wx}_4 = S_4[[3, 2]];$$

$$\mathbf{wy}_4 = S_4[[1, 3]];$$

$$\mathbf{wz}_4 = S_4[[2, 1]];$$

$$S_5 = D[R, q_5].\text{Transpose}[R];$$

$$\mathbf{wx}_5 = S_5[[3, 2]];$$

$$\mathbf{wy}_5 = S_5[[1, 3]];$$

$$\mathbf{wz}_5 = S_5[[2, 1]];$$

$$J = \text{FullSimplify}[\{\{D[\mathbf{Px}, q_1], D[\mathbf{Px}, q_2], D[\mathbf{Px}, q_3], D[\mathbf{Px}, q_4], D[\mathbf{Px}, q_5]\}, \{D[\mathbf{Py}, q_1], D[\mathbf{Py}, q_2], D[\mathbf{Py}, q_3], D[\mathbf{Py}, q_4], D[\mathbf{Py}, q_5]\}, \{D[\mathbf{Pz}, q_1], D[\mathbf{Pz}, q_2], D[\mathbf{Py}, q_3], D[\mathbf{Py}, q_4], D[\mathbf{Py}, q_5]\}, \{\mathbf{wx}_1, \mathbf{wx}_2, \mathbf{wx}_3, \mathbf{wx}_4, \mathbf{wx}_5\}, \{\mathbf{wy}_1, \mathbf{wy}_2, \mathbf{wy}_3, \mathbf{wy}_4, \mathbf{wy}_5\}, \{\mathbf{wz}_1, \mathbf{wz}_2, \mathbf{wz}_3, \mathbf{wz}_4, \mathbf{wz}_5\}\}, \text{Dimensions}[J]$$

$$\{\{\{\cos[q_1](367.5 + \cos[q_4](353.87 - 452.\cos[q_5])) + \sin[q_1](658.07 + \cos[q_2](1689.31 + \sin[q_3](-2128.98 - \sin[q_4](1689.31 + \sin[q_5](-2128.98 - \sin[q_2](367.5 + \cos[q_1](353.87 - 452.\cos[q_5]))))\}]\}]\}$$

$$\text{Simplify}[A34]//\text{MatrixForm}$$

$$\text{Simplify}[A45]//\text{MatrixForm}$$

$$\text{Simplify}[A05]//\text{MatrixForm}$$

$$\text{FullSimplify}[\mathbf{Px}]//\text{MatrixForm}$$

$$\text{FullSimplify}[\mathbf{Py}]//\text{MatrixForm}$$

$$\text{FullSimplify}[\mathbf{Pz}]//\text{MatrixForm}$$

$$\text{FullSimplify}[R]//\text{MatrixForm}$$

$$\begin{pmatrix} -\sin[q_4] & 0 & \cos[q_4] & -353.87\sin[q_4] \\ \cos[q_4] & 0 & \sin[q_4] & 353.87\cos[q_4] \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} \cos [q_5] & 0 & -\sin [q_5] & -452 \cos [q_5] \\ \sin [q_5] & 0 & \cos [q_5] & -452 \sin [q_5] \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} \cos [q_4] \cos [q_5] \sin [q_1] + \cos [q_1] (-\cos [q_5] \sin [q_2 - q_3] \sin [q_4] + \cos [q_2 - q_3] \sin [q_5]) & -\cos [q_1] \cos [q_4] \cos [q_5] + \sin [q_1] (\cos [q_5] \sin [q_2 - q_3] \sin [q_4] - \cos [q_2 - q_3] \sin [q_5]) & \cos [q_3] \cos [q_4] \sin [q_5] \\ \cos [q_2 - q_3] \cos [q_5] \sin [q_4] + \sin [q_2 - q_3] \sin [q_5] & 0. & 0. \end{pmatrix}$$

FullSimplify [*D* [**Px**, *q*₁]]

FullSimplify [*D* [**Px**, *q*₂]]

FullSimplify [*D* [**Px**, *q*₃]]

FullSimplify [*D* [**Px**, *q*₄]]

FullSimplify [*D* [**Px**, *q*₅]]

FullSimplify [*D* [**Py**, *q*₁]]

FullSimplify [*D* [**Py**, *q*₂]]

FullSimplify [*D* [**Py**, *q*₃]]

FullSimplify [*D* [**Py**, *q*₄]]

FullSimplify [*D* [**Py**, *q*₅]]

FullSimplify [*D* [**Pz**, *q*₁]]

FullSimplify [*D* [**Pz**, *q*₂]]

FullSimplify [*D* [**Pz**, *q*₃]]

FullSimplify [*D* [**Pz**, *q*₄]]

FullSimplify [*D* [**Pz**, *q*₅]]

Simplify [**wx**₁]

Simplify [**wy**₁]

Simplify [**wz**₁]

Simplify [wx₂]

Simplify [wy₂]

Simplify [wz₂]

Simplify [wx₃]

Simplify [wy₃]

Simplify [wz₃]

Simplify [wx₄]

Simplify [wy₄]

Simplify [wz₄]

Simplify [wx₅]

Simplify [wy₅]

Simplify [wz₅]

$$\begin{aligned} & \{ \cos [q_1] (367.5 + \cos [q_4] (353.87 - 452 \cos [q_5])) + \sin [q_1] (658.07 + \cos [q_2] (1689.31 + \sin [q_3] (-2128.98 - 3 \\ & \{ 353.87 \cos [q_1] (\sin [q_2] (4.77382 + \sin [q_3] (-6.01628 - 1 \sin [q_4])) + \cos [q_2] \cos [q_3] (-6.01628 - 1 \sin [q_4]) + 1 \\ & \{ 353.87 \cos [q_1] (-1.27731 \cos [q_2 - q_3] \cos [q_5] \sin [q_4] + \cos [q_2] \cos [q_3] (6.01628 + 1 \sin [q_4]) + \sin [q_2] \sin [q_3] (6 \\ & \{ \cos [q_1] \cos [q_4] (-353.87 \cos [q_3] \sin [q_2] + 452 \cos [q_5] \sin [q_2 - q_3] + 353.87 \cos [q_2] \sin [q_3]) + (-353.87 + 452 \cos [q_4] \sin [q_1] \sin [q_5] + \cos [q_1] (-452 \cos [q_2 - q_3] \cos [q_5] - 452 \sin [q_2 - q_3] \sin [q_4] \sin [q_5]) , 0, 0 \} \\ & \{ 0, (-367.5 + \cos [q_4] (-353.87 + 452 \cos [q_5])) \sin [q_1] + \cos [q_1] (658.07 + \cos [q_2] (1689.31 + \sin [q_3] (-2128.98 - 3 \\ & \{ 0, -353.87 \sin [q_1] (\sin [q_2] (4.77382 + \sin [q_3] (-6.01628 - 1 \sin [q_4])) + \cos [q_2] \cos [q_3] (-6.01628 - 1 \sin [q_4]) + 1 \\ & \{ 0, -353.87 \sin [q_1] (-1.27731 \cos [q_2 - q_3] \cos [q_5] \sin [q_4] + \cos [q_2] \cos [q_3] (6.01628 + 1 \sin [q_4]) + \sin [q_2] \sin [q_3] (6 \\ & \{ 0, \cos [q_4] \sin [q_1] (353.87 \cos [q_3] \sin [q_2] - 452 \cos [q_5] \sin [q_2 - q_3] - 353.87 \cos [q_2] \sin [q_3]) + 452 \cos [q_1] (-0.7 \\ & \{ 0, 452 \cos [q_2 - q_3] \cos [q_5] \sin [q_1] + (452 \cos [q_1] \cos [q_4] + 452 \sin [q_1] \sin [q_2 - q_3] \sin [q_4]) \sin [q_5] , 0 \} \\ & \{ 0, 0, 0 \} \end{aligned}$$

$$\begin{aligned}
& \{0, 0, \text{Cos}[q_3] \text{Sin}[q_2] (-2128.98 + (-353.87 + 452.\text{Cos}[q_5]) \text{Sin}[q_4]) + \text{Cos}[q_2] (-1689.31 + \text{Sin}[q_3] (2128.98 + (353.87 - 452.\text{Cos}[q_5]) \text{Sin}[q_4]) - 1.\text{Cos}[q_2] \text{Sin}[q_5]) + 452.\text{Sin}[q_3] (\text{Cos}[q_2] \text{Sin}[q_5] - \text{Cos}[q_2] \text{Sin}[q_4] \text{Sin}[q_5]) + \text{Sin}[q_3] (-1.\text{Cos}[q_2] \text{Cos}[q_5] - \text{Sin}[q_2] \text{Sin}[q_4] \text{Sin}[q_5])\} \\
& \{0, 0, -452.\text{Cos}[q_3] (\text{Sin}[q_2] (-4.71013 + (-0.782898 + \text{Cos}[q_5]) \text{Sin}[q_4]) - 1.\text{Cos}[q_2] \text{Sin}[q_5]) + 452.\text{Sin}[q_3] (\text{Cos}[q_2] \text{Sin}[q_5] - \text{Cos}[q_2] \text{Sin}[q_4] \text{Sin}[q_5]) + \text{Sin}[q_3] (-1.\text{Cos}[q_2] \text{Cos}[q_5] - \text{Sin}[q_2] \text{Sin}[q_4] \text{Sin}[q_5])\} \\
& \{0, 0, -452.\text{Cos}[q_2 - q_3] \text{Cos}[q_4] (-0.782898 + \text{Cos}[q_5])\} \\
& \{0, 0, -452. (\text{Cos}[q_3] (\text{Cos}[q_5] \text{Sin}[q_2] - \text{Cos}[q_2] \text{Sin}[q_4] \text{Sin}[q_5]) + \text{Sin}[q_3] (-1.\text{Cos}[q_2] \text{Cos}[q_5] - \text{Sin}[q_2] \text{Sin}[q_4] \text{Sin}[q_5])\} \\
& 0 \\
& 0 \\
& -1 \\
& -\text{Sin}[q_1] \\
& -\text{Cos}[q_1] \\
& 0 \\
& \text{Sin}[q_1] \\
& \text{Cos}[q_1] \\
& 0 \\
& \text{Cos}[q_1] \text{Cos}[q_2 - q_3] \\
& -\text{Cos}[q_2 - q_3] \text{Sin}[q_1] \\
& \text{Sin}[q_2 - q_3] \\
& \text{Cos}[q_1] \text{Cos}[q_4] \text{Sin}[q_2 - q_3] + \text{Sin}[q_1] \text{Sin}[q_4] \\
& -\text{Cos}[q_3] \text{Cos}[q_4] \text{Sin}[q_1] \text{Sin}[q_2] + \text{Cos}[q_2] \text{Cos}[q_4] \text{Sin}[q_1] \text{Sin}[q_3] + \text{Cos}[q_1] \text{Sin}[q_4] \\
& -\text{Cos}[q_2 - q_3] \text{Cos}[q_4]
\end{aligned}$$

FullSimplify [{**{wx₁, wx₂, wx₃, wx₄, wx₅**}, **{wy₁, wy₂, wy₃, wy₄, wy₅**}, **{wz₁, wz₂, wz₃, wz₄, wz₅**}]}]//**MatrixForm**

$$\begin{pmatrix} 0 & -\sin[q_1] & \sin[q_1] & \cos[q_1] \cos[q_2 - q_3] & \cos[q_1] \cos[q_4] \sin[q_2 - q_3] + \sin[q_1] \sin[q_4] \\ 0 & -\cos[q_1] & \cos[q_1] & -\cos[q_2 - q_3] \sin[q_1] & -\cos[q_4] \sin[q_1] \sin[q_2 - q_3] + \cos[q_1] \sin[q_4] \\ -1 & 0 & 0 & \sin[q_2 - q_3] & -\cos[q_2 - q_3] \cos[q_4] \end{pmatrix}$$