Calcula los siguientes límites:

$$\lim_{x\to+\infty}\left(\frac{x^2+1}{x^2-1}\right)^{x^2} \qquad \lim_{x\to+\infty}\left(\frac{x+1}{x-2}\right)^{2x-1} \qquad \lim_{x\to+\infty}\left(\frac{x-1}{x+3}\right)^{x+2}$$

$$\lim_{x\to+\infty}\left(\frac{3x-4}{3x-2}\right)^{\frac{x+1}{3}} \qquad \lim_{x\to+\infty}\left(1-\frac{1}{x^2}\right)^{3x-2} \qquad \lim_{x\to+\infty}\left(\frac{x-3}{x+2}\right)^{x^2-5}$$

Recordar: $\lim_{x\to x_0} f(x)^{g(x)} = \{1^\infty\} = \lim_{x\to x_0} e^{g(x)\cdot[f(x)-1]}$

a) $\lim_{x\to\infty}\left(\frac{x^2+1}{x^2-1}\right)^{x^2} = \{1^\infty\} = \lim_{x\to\infty} e^{x^2\left[\frac{x^2+x}{x^2-1}-1\right]} =$

$= \lim_{x\to\infty} e^{x^2\left[\frac{x^2+x-x^2+1}{x^2-1}\right]} = \lim_{x\to\infty} e^{x^2\frac{x+1}{x^2-1}} = \lim_{x\to\infty} e^{\frac{x^3+x^2}{x^2-1}} = e^\infty = \infty$

b) $\lim_{x\to\infty}\left(\frac{x+1}{x-2}\right)^{2x-1} = \{1^\infty\} = \lim_{x\to\infty} e^{(2x-1)\left[\frac{x+1}{x-2}-1\right]} =$

$= \lim_{x\to\infty} e^{(2x-1)\frac{x+1-x+2}{x-2}} = \lim_{x\to\infty} e^{(2x-1)\frac{3}{x-2}} = \lim_{x\to\infty} e^{\frac{6x-3}{x-2}} = e^6$

c) $\lim_{x\to\infty}\left(\frac{x-1}{x+3}\right)^{x+2} = \{1^\infty\} = \lim_{x\to\infty} e^{(x+2)\left[\frac{x-1}{x+3}-1\right]} =$

$= \lim_{x\to\infty} e^{(x+2)\cdot\frac{x-1-x-3}{x+3}} = \lim_{x\to\infty} e^{(x+2)\frac{-4}{x+3}} = \lim_{x\to\infty} e^{\frac{-4x-8}{x+3}} = e^{-4}$

d) $\lim_{x\to\infty}\left(\frac{3x-4}{3x-2}\right)^{\frac{x+1}{3}} = \{1^\infty\} = \lim_{x\to\infty} e^{\frac{x+1}{3}\cdot\left[\frac{3x-4}{3x-2}-1\right]} =$

$= \lim_{x\to\infty} e^{\frac{x+1}{3}\cdot\left[\frac{3x-4-3x+2}{3x-2}\right]} = \lim_{x\to\infty} e^{\frac{x+1}{3}\cdot\frac{-2}{3x-2}} =$

$= \lim_{x\to\infty} e^{\frac{-2x-2}{9x-6}} = e^{\frac{-2}{9}}$

e) $\lim_{x\to\infty} \left(1 - \frac{1}{x^2}\right)^{3x-2} = \{1^\infty\} = \lim_{x\to\infty} e^{(3x-2)\cdot\left[1 - \frac{1}{x^2} - 1\right]} =$

$= \lim_{x\to\infty} e^{3x-2 \cdot \frac{-1}{x^2}} = \lim_{x\to\infty} e^{\frac{-3x+2}{x^2}} = e^0 = 1$

f) $\lim_{x\to\infty} \left(\frac{x-3}{x+2}\right)^{x^2-5} = \{1^\infty\} = \lim_{x\to\infty} e^{(x^2-5)\left[\frac{x-3}{x+2} - 1\right]} =$

$= \lim_{x\to\infty} e^{(x^2-5)\left[\frac{x-3-x-2}{x+2}\right]} = \lim_{x\to\infty} e^{(x^2-5)\left(\frac{-5}{x+2}\right)} =$

$= \lim_{x\to\infty} e^{\frac{-5x^2+25}{x+2}} = e^{-\infty} = \frac{1}{e^\infty} = \frac{1}{\infty} = 0$