Calcula el valor de k para que cada una de las siguientes funciones sea continua:

$$f(x) = \begin{cases} \dfrac{x^4 - 1}{x - 1}, & x \neq 1 \\ k, & x = 1 \end{cases} \qquad g(x) = \begin{cases} \dfrac{\sqrt{x} - 1}{x - 1}, & x \neq 1 \\ k, & x = 1 \end{cases}$$

a) $f(x) = \begin{cases} \dfrac{x^4-1}{x-1} & x \neq 1 \\ k & x = 1 \end{cases}$

Para que $f(x)$ sea continua $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) = f(1)$

$\lim_{x \to 1^-} f(x) = \lim_{x \to 1} \dfrac{x^4-1}{x-1} = \left\{ \dfrac{0}{0} \right\} = (*)$

Factorizamos $x^4 - 1 = 0$

$x^4 - 1 \;\big|\; x - 1$

$x^4 - x^3 \quad\quad x^3 + x^2 + x + 1$

$0 + x^3 - 1$

$x^3 - x^2$

$0 + x^2 - 1$

$x^2 - x$

$0 + x - 1$

$x - 1$

$0 \quad 0$

$\dfrac{\text{Dividendo}}{\text{Divisor}} = \text{cociente} + \dfrac{\text{resto}}{\text{divisor}}$

$\dfrac{x^4-1}{x-1} = x^3 + x^2 + x + 1$

$(*) = \lim_{x \to 1} x^3 + x^2 + x + 1 = 1 + 1 + 1 + 1 = 4$

$\lim_{x \to 1^+} f(x) = \lim_{x \to 1} \dfrac{x^4-1}{x-1} = 4 \left\{ \begin{array}{l} 4 = k \Rightarrow \quad k = 4 \end{array} \right.$

$f(1) = k$

$f(x) = \begin{cases} \dfrac{x^4-1}{x-1} & x \neq 1 \\ 4 & x = 1 \end{cases}$

b) $g(x) = \begin{cases} \dfrac{\sqrt{x}-1}{x-1} & x \neq 1 \\ k & x = 1 \end{cases}$

Para que $g(x)$ sea continua $\lim_{x \to 1^-} g(x) = \lim_{x \to 1^+} g(x) = g(1)$

$$\lim_{x \to 1^-} g(x) = \lim_{x \to 1} \frac{\sqrt{x}-1}{x-1} = \frac{1-1}{1-1} = \left\{\frac{0}{0}\right\} =$$

$$= \lim_{x \to 1} \frac{(\sqrt{x}-1)\cdot(\sqrt{x}+1)}{(x-1)\cdot(\sqrt{x}+1)} \overset{(*)}{=} \lim_{x \to 1} \frac{x-1}{(x-1)\cdot(\sqrt{x}+1)} =$$

$$= \lim_{x \to 1} \frac{1}{\sqrt{x}+1} = \frac{1}{\sqrt{1}+1} = \frac{1}{2}$$

$$\lim_{x \to 1^+} g(x) = \lim_{x \to 1} \frac{\sqrt{x}-1}{x-1} = \frac{1}{2} \Bigg\} \quad \frac{1}{2} = k \Rightarrow k = \frac{1}{2}$$

$g(1) = k$

$$g(x) = \begin{cases} \dfrac{\sqrt{x}-1}{x-1} & x \neq -1 \\ 1/2 & x = 1 \end{cases}$$

(*)
$(a+b)(a-b) = a^2 - b^2$