Calcula:

$$\lim_{x\to 0}\left(\frac{x^2+1}{2x+1}\right)^{\frac{1}{x}} \qquad \lim_{x\to 2}\left(\frac{2x^2-x-1}{7-x}\right)^{\frac{1}{x-2}}$$

a) $\lim\limits_{x\to 0}\left(\frac{x^2+1}{2x+1}\right)^{\frac{1}{x}} = \{1^\infty\} = \lim\limits_{x\to 0} e^{\frac{1}{x}\left[\frac{x^2+1}{2x+1}-1\right]} =$

$= \lim\limits_{x\to 0} e^{\frac{1}{x}\left[\frac{x^2+1-2x-1}{2x+1}\right]} = \lim\limits_{x\to 0} e^{\frac{1}{x}\left[\frac{x^2-2x}{2x+1}\right]} =$

$= \lim\limits_{x\to 0} e^{\frac{1}{x}\frac{x(x-2)}{2x+1}} = \lim\limits_{x\to 0} e^{\frac{x-2}{2x+1}} = e^{\frac{-2}{1}} = e^{-2} = \frac{1}{e^2}$

b) $\lim\limits_{x\to 2}\left(\frac{2x^2-x-1}{7-x}\right)^{\frac{1}{x-2}} = \left(\frac{2\cdot 4-2-1}{5}\right)^{\frac{1}{0}} = \{1^\infty\} =$

$= \lim\limits_{x\to 2} e^{\frac{1}{x-2}\cdot\left[\frac{2x^2-x-1}{7-x}-1\right]} = \lim\limits_{x\to 2} e^{\frac{1}{x-2}\frac{2x^2-x-1-7+x}{7-x}} =$

$= \lim\limits_{x\to 2} e^{\frac{1}{x-2}\frac{2x^2-8}{7-x}} = e^{\frac{1}{0}\cdot\frac{8-8}{5}} = e^{\left\{\frac{0}{0}\right\}} = ⊛$

Factorizamos: $2x^2-8=0 \Rightarrow x^2=4 \Rightarrow x=\pm 2$

$⊛ = \lim\limits_{x\to 2} e^{\frac{1}{x-2}\cdot\frac{(x-2)(x+2)}{7-x}} = \lim\limits_{x\to 2} e^{\frac{x+2}{7-x}} = e^{\frac{4}{5}}$