Calcula el límite cuando $x \to -\infty$ de las siguientes expresiones:

$$\frac{3x^3+5}{x+2} - \frac{4x^3-x}{x-2} \qquad \frac{x^3}{2x^2+1} - \frac{x}{2} \qquad \sqrt{x^2+x} - \sqrt{x^2+1} \qquad 2x + \sqrt{x^2+x}$$

$$\sqrt{x^2+2x} + x \qquad \left(1+\frac{3}{x}\right)^{2x} \qquad \left(1-\frac{1}{x}\right)^{5x+3} \qquad \left(\frac{x^2+x-1}{x^2+2}\right)^{3x-1}$$

Recordar: $\lim\limits_{x\to-\infty} f(x) = \lim\limits_{x\to+\infty} f(-x)$

$$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = e \quad ; \quad \lim_{x\to\infty}\left(1+\frac{k}{x}\right)^x = e^k$$

$$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{ax+b} = e^a$$

a) $$\lim_{x\to-\infty} \frac{3x^3+5}{x+2} - \frac{4x^3-x}{x-2} = \lim_{x\to+\infty} \frac{-3x^3+5}{-x+2} - \frac{-4x^3+x}{-x-2} =$$

$$= \lim_{x\to\infty} \frac{3x^3-5}{x-2} - \frac{4x^3-x}{x+2} = \{\infty - \infty\} =$$

$$= \lim_{x\to\infty} \frac{(x+2)(3x^3-5)-(x-2)(4x^3-x)}{(x+2)(x-2)} = \lim_{x\to\infty} \frac{3x^4-5x+6x^3-10-[4x^4-x^2-8x^3+2x]}{x^2-4}$$

$$= \lim_{x\to\infty} \frac{3x^4-5x+6x^3-10-4x^4+x^2+8x^3-2x}{x^2-4} =$$

$$= \lim_{x\to\infty} \frac{-x^4+14x^3+x^2-7x-10}{x^2-4} = -\infty$$

b) $$\lim_{x\to-\infty} \frac{x^3}{2x^2+1} - \frac{x}{2} = \lim_{x\to+\infty} \frac{-x^3}{2x^2+1} + \frac{x}{2} = \{-\infty + \infty\} =$$

$$= \lim_{x\to+\infty} \frac{-2x^3+x(2x^2+1)}{2(2x^2+1)} = \lim_{x\to\infty} \frac{x}{4x^2+2} = 0$$

c) $\lim\limits_{x\to-\infty} \sqrt{x^2+x} - \sqrt{x^2+1} = \lim\limits_{x\to\infty} \sqrt{x^2-x} - \sqrt{x^2+1} = \{\infty-\infty\} =$

$= \lim\limits_{x\to\infty} \frac{\left(\sqrt{x^2-x} - \sqrt{x^2+1}\right)\left(\sqrt{x^2-x} + \sqrt{x^2+1}\right)}{\sqrt{x^2-x} + \sqrt{x^2+1}} = \lim\limits_{x\to\infty} \frac{x^2-x-(x^2+1)}{\sqrt{x^2-x}+\sqrt{x^2+1}} =$

$= \lim\limits_{x\to\infty} \frac{-x-1}{\sqrt{x^2-x}+\sqrt{x^2+1}} = \lim\limits_{x\to\infty} \frac{x\left(-1-\frac{1}{x}\right)}{x\sqrt{1-\frac{1}{x}} + x\sqrt{1+\frac{1}{x^2}}} =$

$= \lim\limits_{x\to\infty} \frac{-1-\frac{1}{x}}{\sqrt{1-\frac{1}{x}}+\sqrt{1+\frac{1}{x^2}}} = \frac{-1+0}{\sqrt{1-0}+\sqrt{1+0}} = \frac{-1}{\sqrt{1}+\sqrt{1}} = -\frac{1}{2}$

d) $\lim\limits_{x\to-\infty} 2x + \sqrt{x^2+x} = \lim\limits_{x\to+\infty} -2x + \sqrt{x^2-x} = \{-\infty+\infty\} =$

$= \lim\limits_{x\to\infty} \sqrt{x^2-x} - 2x = \lim\limits_{x\to\infty} \frac{\left(\sqrt{x^2-x}-2x\right)\left(\sqrt{x^2-x}+2x\right)}{\sqrt{x^2-x}+2x} =$

$= \lim\limits_{x\to\infty} \frac{x^2-x-4x^2}{\sqrt{x^2-x}+2x} = \lim\limits_{x\to\infty} \frac{-3x^2-x}{\sqrt{x^2-x}+2x} = -\infty$

e) $\lim\limits_{x\to-\infty} \sqrt{x^2+2x} + x = \lim\limits_{x\to+\infty} \sqrt{x^2-2x} - x = \{\infty-\infty\} =$

$= \lim\limits_{x\to+\infty} \frac{\left(\sqrt{x^2-2x}-x\right)\left(\sqrt{x^2-2x}+x\right)}{\sqrt{x^2-2x}+x} = \lim\limits_{x\to\infty} \frac{x^2-2x-x^2}{\sqrt{x^2-2x}+x} =$

$= \lim\limits_{x\to\infty} \frac{-2x}{\sqrt{x^2-2x}+x} = \frac{-2}{\sqrt{1}+1} = \frac{-2}{2} = -1$

f) $\lim\limits_{x\to-\infty}\left(1+\frac{3}{x}\right)^{2x} = \lim\limits_{x\to+\infty}\left(1+\frac{3}{-x}\right)^{-x} = 1^{-\infty} = \frac{1}{\{1^{\infty}\}} =$

$= \lim\limits_{x\to+\infty}\left(1+\frac{-3}{x}\right)^{-x} = \lim\limits_{x\to+\infty}\left[\left(1+\frac{-3}{x}\right)^{x}\right]^{-1} = \left[e^{-3}\right]^{-1} = e^{3}$

g) $\lim\limits_{x\to-\infty}\left(1-\frac{1}{x}\right)^{5x+3} = \lim\limits_{x\to+\infty}\left(1-\frac{1}{-x}\right)^{-5x+3} =$

$= \lim\limits_{x\to\infty}\left(1+\frac{1}{x}\right)^{-5x+3} = e^{-5}$

$\cancel{\lim\limits_{x\to\infty} f(x)^{g(x)} = \{1^{\infty}\} = \lim\limits_{x\to\infty} e^{g(x)[f(x)-1]}}$

h) $\lim\limits_{x\to-\infty}\left(\frac{x^2+x-1}{x^2+2}\right)^{3x-1} = \lim\limits_{x\to+\infty}\left(\frac{x^2-x-1}{x^2+2}\right)^{-3x-1} =$

$= 1^{-\infty} = \frac{1}{\{1^{\infty}\}} \overset{(*)}{=} \lim\limits_{x\to\infty} e^{(-3x-1)\left[\frac{x^2-x-1}{x^2+2}-1\right]} =$

$= \lim\limits_{x\to\infty} e^{(-3x-1)\left[\frac{x^2-x-1-x^2-2}{x^2+2}\right]} = \lim\limits_{x\to\infty} e^{(-3x-1)\left[\frac{-x-2}{x^2+2}\right]} =$

$= \lim\limits_{x\to\infty} e^{\frac{3x^2+6x+x+2}{x^2+2}} = \lim\limits_{x\to\infty} e^{\frac{3x^2+7x+2}{x^2+2}} = e^{\frac{3}{1}} = e^{3}$

(*)

$\lim\limits_{x\to x_0} f(x)^{g(x)} = \{1^{\infty}\} = \lim\limits_{x\to x_0} e^{g(x)\cdot[f(x)-1]}$