Calcula

$$\lim_{x\to 2}\left[\frac{3}{x^2-5x+6}-\frac{4}{x-2}\right] \qquad \lim_{x\to 2}\left(\frac{1-\sqrt{3-x}}{x-2}\right)$$

$$\lim_{x\to 0}\left(\frac{\sqrt{x+9}-3}{x^2}\right) \qquad \lim_{x\to 0}\left[\frac{\sqrt{1+x}-\sqrt{1-x}}{3x}\right]$$

a) $\lim_{x\to 2}\left[\frac{3}{x^2-5x+6}-\frac{4}{x-2}\right] = \frac{3}{4-10+6}-\frac{4}{0} = \{\infty-\infty\} =$

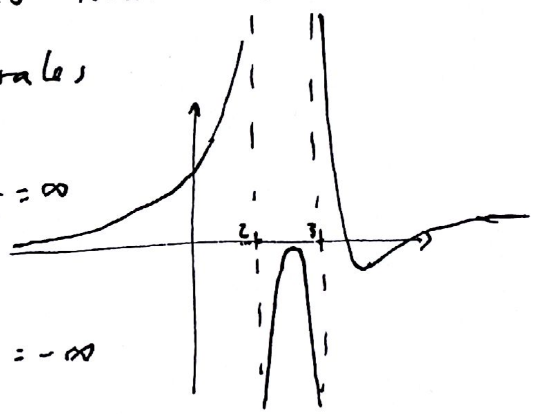
Factorizamos: $x^2-5x+6=0 \Rightarrow x=\frac{+5\pm\sqrt{25-24}}{2} = \left\langle\begin{matrix}3\\2\end{matrix}\right. \Rightarrow (x-3)(x-2)=0$

$= \lim_{x\to 2}\left[\frac{3}{(x-2)(x-3)}-\frac{4}{(x-2)}\right] = \lim_{x\to 2}\frac{3-4(x-3)}{(x-2)(x-3)} =$

$= \lim_{x\to 2}\frac{-4x+15}{(x-2)(x-3)} = \frac{-8+15}{0\cdot(-1)} = \infty$

OJO: si $\lim_{x\to x_0} f(x) = \pm\infty$ con x_0 finito $\Rightarrow$

$\Rightarrow$ comprobar límites laterales



$x\to 2^-$: $\lim_{x\to 2^-}\frac{-4x+15}{(x-2)(x-3)} = \frac{-8+15}{0^-\cdot(-1)} = \frac{7}{0^+} = \infty$

$x\to 2^+$: $\lim_{x\to 2^+}\frac{-4x+15}{(x-2)(x-3)} = \frac{-8+15}{0^+\cdot(-1)} = \frac{7}{0^-} = -\infty$

como $\lim_{x\to 2^-} f(x) \neq \lim_{x\to 2^+} f(x) \Rightarrow$ No existe límite

b) $\lim_{x\to 2}\frac{1-\sqrt{3-x}}{x-2} = \frac{1-1}{0} = \left\{\frac{0}{0}\right\} = \lim_{x\to 2}\frac{(1-\sqrt{3-x})(1+\sqrt{3-x})}{(x-2)(1+\sqrt{3-x})} =$

$= \lim_{x\to 2}\frac{1-(3-x)}{(x-2)(1+\sqrt{3-x})} = \lim_{x\to 2}\frac{-2+x}{(x-2)(1+\sqrt{3-x})} =$

$= \lim_{x\to 2}\frac{x-2}{(x-2)(1+\sqrt{3-x})} = \lim_{x\to 2}\frac{1}{1+\sqrt{3-x}} = \frac{1}{1+\sqrt{1}} = \frac{1}{2}$

c) $$\lim_{x\to 0} \frac{\sqrt{x+9}-3}{x^2} = \frac{3-3}{0} = \left\{\frac{0}{0}\right\} = \lim_{x\to 0} \frac{(\sqrt{x+9}-3)(\sqrt{x+9}+3)}{x^2(\sqrt{x+9}+3)} =$$

$$= \lim_{x\to 0} \frac{x+9-9}{x^2(\sqrt{x+9}+3)} = \lim_{x\to 0} \frac{1}{x(\sqrt{x+9}+3)} = \frac{1}{0\cdot 6} = \infty$$

Tenemos que estudiar límites laterales

$$x\to 0^- : \lim_{x\to 0^-} \frac{1}{x(\sqrt{x+9}+3)} = \frac{1}{0^-\cdot 6} = -\infty$$

$$x\to 0^+ : \lim_{x\to 0^+} \frac{1}{x(\sqrt{x+9}+3)} = \frac{1}{0^+\cdot 6} = +\infty$$

Como $\lim_{x\to 0^-} f(x) \neq \lim_{x\to 0^+} f(x) \Rightarrow$ No existe límite

d) $$\lim_{x\to 0} \frac{\sqrt{1+x}-\sqrt{1-x}}{3x} = \frac{1-1}{0} = \left\{\frac{0}{0}\right\} =$$

$$= \lim_{x\to 0} \frac{(\sqrt{1+x}-\sqrt{1-x})\cdot(\sqrt{1+x}+\sqrt{1-x})}{3x(\sqrt{1+x}+\sqrt{1-x})} = \lim_{x\to 0} \frac{1+x-(1-x)}{3x(\sqrt{1+x}+\sqrt{1-x})} =$$

$$= \lim_{x\to 0} \frac{2x}{3x(\sqrt{1+x}+\sqrt{1-x})} = \lim_{x\to 0} \frac{2}{3(\sqrt{1+x}+\sqrt{1-x})} = \frac{2}{3\cdot(1+1)} = \frac{1}{3}$$