• $\int \frac{dx}{e^{2x}-3e^{x}}$; Utilizar c.v.

$\int \frac{dx}{e^{2x}-3e^{x}} = ⊛$ Nota: $e^{2x} = (e^{x})^{2}$

$e^{x} = t$

$\ln e^{x} = \ln t \Rightarrow x = \ln t$

$dx = \frac{1}{t}\,dt$

$⊛ = \int \frac{\frac{1}{t}\,dt}{t^{2}-3t} = \int \frac{1}{t}\cdot\frac{1}{t^{2}-3t}\,dt =$

$= \int \frac{1}{t^{3}-3t^{2}}\,dt$ $t^{3}-3t^{2}=0$, $t^{2}(t-3)=0$ $t_1 = 0$, $t_2 = 0$, $t_3 = 3$

$\frac{1}{t^{3}-3t^{2}} = \frac{A}{t} + \frac{B}{t^{2}} + \frac{C}{t-3} = \frac{At(t-3)+B(t-3)+Ct^{2}}{t^{2}(t-3)}$

$1 = At(t-3)+B(t-3)+Ct^{2}$

$t=0 \Rightarrow 1 = -3B \Rightarrow B = -1/3$

$t=3 \Rightarrow 1 = 9C \Rightarrow C = 1/9$

$t=1 \Rightarrow 1 = -2A - 2B + C = -2A + \frac{2}{3} + \frac{1}{9}$

$+2A = -\frac{9}{9} + \frac{6}{9} + \frac{1}{9} \Rightarrow 2A = -\frac{2}{9} \Rightarrow A = -\frac{1}{9}$

$$\int \frac{1}{t^3-3t^2}\,dt = \int\left(\frac{-1/9}{t} + \frac{-1/3}{t^2} + \frac{1/9}{t-3}\right)dt =$$

$$= -\frac{1}{9}\ln t - \frac{1}{3}\,\frac{t^{-1}}{-1} + \frac{1}{9}\ln|t-3| + C' = \qquad t = e^x$$

$$= -\frac{1}{9}\ln e^x + \frac{1}{3}(e^x)^{-1} + \frac{1}{9}\ln|e^x-3| + C'$$

Comprobación:

$$\frac{d}{dx}\left(-\frac{1}{9}x + \frac{1}{3}e^{-x} + \frac{1}{9}\ln|e^x-3| + C'\right) =$$

$$= -\frac{1}{9} + \frac{1}{3}e^{-x}\cdot(-1) + \frac{1}{9}\,\frac{e^x}{e^x-3} = \frac{1}{9}\left(-1 - 3\,\frac{1}{e^x} + \frac{e^x}{e^x-3}\right) =$$

$$= \frac{1}{9}\left(\frac{-e^x(e^x-3) - 3(e^x-3) + e^{2x}}{e^x(e^x-3)}\right) = \frac{1}{9}\left(\frac{-e^{2x}+3e^x-3e^x+9+e^{2x}}{e^{2x}-3e^x}\right) =$$

$$= \frac{1}{9}\left(\frac{9}{e^{2x}-3e^x}\right) = \frac{1}{e^{2x}-3e^x} \qquad \checkmark$$