Halla:

$$\lim_{x\to-\infty} \sqrt{x^2+2x}-\sqrt{x^2-4} \qquad\qquad \lim_{x\to-\infty} \sqrt{x^2+1}+x$$

Recordar $\lim\limits_{x\to-\infty} f(x) = \lim\limits_{x\to+\infty} f(-x)$

$(-x)^2 = x^2$

a) $\lim\limits_{x\to-\infty} \sqrt{x^2+2x}-\sqrt{x^2-4} = \lim\limits_{x\to+\infty} \sqrt{x^2-2x}-\sqrt{x^2-4} =$

$$= \{\infty-\infty\} = \lim_{x\to\infty} \frac{\left(\sqrt{x^2-2x}-\sqrt{x^2-4}\right)\cdot\left(\sqrt{x^2-2x}+\sqrt{x^2-4}\right)}{\sqrt{x^2-2x}+\sqrt{x^2-4}} =$$

$(a+b)(a-b) = a^2-b^2$

$$= \lim_{x\to\infty} \frac{x^2-2x-(x^2-4)}{\sqrt{x^2-2x}+\sqrt{x^2-4}} = \lim_{x\to\infty} \frac{-2x+4}{\sqrt{x^2-2x}+\sqrt{x^2-4}} = \left\{\frac{-\infty}{\infty}\right\} =$$

$$= \lim_{x\to\infty} \frac{x\left(-2+\frac{4}{x}\right)}{x\sqrt{1-\frac{2}{x}}+x\sqrt{1-\frac{4}{x^2}}} = \lim_{x\to\infty} \frac{-2+\frac{4}{x}}{\sqrt{1-\frac{2}{x}}+\sqrt{1-\frac{4}{x^2}}} = \frac{-2+0}{\sqrt{1-0}+\sqrt{1-0}} = \frac{-2}{\sqrt{1}+\sqrt{1}} = -1$$

b) $\lim\limits_{x\to-\infty} \sqrt{x^2+1}+x = \lim\limits_{x\to+\infty} \sqrt{x^2+1}-x = \{\infty-\infty\} =$

$$= \lim_{x\to+\infty} \frac{\left(\sqrt{x^2+1}-x\right)\left(\sqrt{x^2+1}+x\right)}{\sqrt{x^2+1}+x} = \lim_{x\to+\infty} \frac{x^2+1-x^2}{\sqrt{x^2+1}+x} =$$

$$= \lim_{x\to\infty} \frac{1}{\sqrt{x^2+1}+x} = \frac{1}{\infty+\infty} = \frac{1}{\infty} = 0$$