

•  $\int \sqrt{1-x^2} dx$  ; Utilizar C.V.

Prueba de C.V.

$$1-x^2 = t^2$$

$$x = \sqrt{1-t^2}$$

$$dx = \frac{-2t}{2\sqrt{1-t^2}} dt$$

Más complicado

X

C.V:

$$\begin{cases} x = \text{sen } t \\ dx = \text{cos } t \, dt \end{cases}$$

$$\int \sqrt{1-x^2} \, dx =$$

$$= \int \sqrt{1-\text{sen}^2 t} \cdot \text{cos } t \, dt = \int \text{cos } t \cdot \text{cos } t \, dt = \textcircled{*}$$

$\text{sen}^2 t + \text{cos}^2 t = 1 \Rightarrow \text{cos } t = \sqrt{1-\text{sen}^2 t}$

$$\textcircled{*} = \int \text{cos}^2 t \, dt = \int \left( \frac{1+\text{cos } 2t}{2} \right) dt = \int \left( \frac{1}{2} + \frac{1}{2} \text{cos } 2t \right) dt = \textcircled{+}$$

$$\text{cos}^2 t = \frac{1+\text{cos } 2t}{2}$$

$$\textcircled{+} = \frac{1}{2}t + \frac{1}{2} \int \text{cos } 2t \, dt = \frac{1}{2}t + \frac{1}{2} \cdot \frac{1}{2} \int \text{cos } 2t \, dt =$$

$$= \frac{1}{2}t + \frac{1}{4} \text{sen } 2t + C =$$

$$\int f'(x) \cdot \text{cos } f(x) \, dx = \text{sen } f(x) + C$$

$$x = \text{sen } t \Rightarrow t = \text{arcsen } x$$

$$\Rightarrow \frac{1}{2} \text{arcsen } x + \frac{1}{4} \text{sen}(2 \text{arcsen } x) + C$$