

Calcula el valor que debe tener k para que las siguientes funciones sean continuas:

$$f(x) = \begin{cases} x+1, & x \leq 2 \\ k-x, & x > 2 \end{cases} \quad g(x) = \begin{cases} x+k, & x \leq 0 \\ x^2-1, & x > 0 \end{cases} \quad h(x) = \begin{cases} e^{kx}, & x \leq 0 \\ x+2k, & x > 0 \end{cases}$$

$$a) \quad f(x) = \begin{cases} x+1 & x \leq 2 \\ k-x & x > 2 \end{cases} \quad \xrightarrow{k=5} \quad f(x) = \begin{cases} x+1 & x \leq 2 \\ 5-x & x > 2 \end{cases}$$

Para que sea continua: $\lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^+} f(x)$

$$\left. \begin{aligned} \lim_{x \rightarrow 2^-} f(x) &= \lim_{x \rightarrow 2^-} x+1 = 2+1 = 3 \\ \lim_{x \rightarrow 2^+} f(x) &= \lim_{x \rightarrow 2^+} k-x = k-2 \end{aligned} \right\} \quad 3 = k-2 \Rightarrow k = 5$$

$$b) \quad g(x) = \begin{cases} x+k & x \leq 0 \\ x^2-1 & x > 0 \end{cases} \quad \xrightarrow{k=-1} \quad g(x) = \begin{cases} x-1 & x \leq 0 \\ x^2-1 & x > 0 \end{cases}$$

Para que sea continua $\lim_{x \rightarrow 0^-} g(x) = \lim_{x \rightarrow 0^+} g(x)$

$$\left. \begin{aligned} \lim_{x \rightarrow 0^-} g(x) &= \lim_{x \rightarrow 0^-} x+k = k \\ \lim_{x \rightarrow 0^+} g(x) &= \lim_{x \rightarrow 0^+} x^2-1 = -1 \end{aligned} \right\} \quad k = -1$$

$$c) \quad h(x) = \begin{cases} e^{kx} & x \leq 0 \\ x+2k & x > 0 \end{cases} \quad \xrightarrow{k=\frac{1}{2}} \quad h(x) = \begin{cases} e^{\frac{x}{2}} & x \leq 0 \\ x+1 & x > 0 \end{cases}$$

Para que sea continua $\lim_{x \rightarrow 0^-} h(x) = \lim_{x \rightarrow 0^+} h(x)$

$$\left. \begin{aligned} \lim_{x \rightarrow 0^-} h(x) &= \lim_{x \rightarrow 0^-} e^{kx} = e^{k \cdot 0} = e^0 = 1 \\ \lim_{x \rightarrow 0^+} h(x) &= \lim_{x \rightarrow 0^+} x+2k = 2k \end{aligned} \right\} \quad 1 = 2k \Rightarrow k = \frac{1}{2}$$