Calcula el límite de las siguientes funciones cunado $x \to +\infty$

$$f(x) = \frac{5x^2 - 2x + 1}{(2x-1)^2} \qquad g(x) = \frac{x + logx}{logx} \qquad h(x) = \frac{3 + 2\sqrt{x}}{\sqrt{2x+1}} \qquad i(x) = \frac{3 \cdot 2^x}{2^x + 1}$$

a) $\displaystyle \lim_{x \to \infty} \frac{5x^2 - 2x + 1}{(2x-1)^2} = \lim_{x \to \infty} \frac{5x^2 - 2x + 1}{4x^2 - 4x + 1} = \left\{ \frac{\infty}{\infty} \right\} =$

$\displaystyle = \lim_{x \to \infty} \frac{x^2\left(5 - \frac{2}{x} + \frac{1}{x^2}\right)}{x^2\left(4 - \frac{4}{x} + \frac{1}{x^2}\right)} = \lim_{x \to \infty} \frac{5 - \frac{2}{x} + \frac{1}{x^2}}{4 - \frac{4}{x} + \frac{1}{x^2}} = \frac{5}{4}$

con L'Hôp: $\displaystyle \lim_{x \to \infty} \frac{5x^2 - 2x + 1}{(2x-1)^2} = \left\{ \frac{\infty}{\infty} \right\} \underset{\text{L'Hôp}}{=} \lim_{x \to \infty} \frac{10x - 2}{2(2x-1)\cdot 2} =$

$\displaystyle = \left\{ \frac{\infty}{\infty} \right\} \overset{\text{LHôp}}{=} \lim_{x \to \infty} \frac{10}{2 \cdot 2 \cdot 2} = \frac{10}{8} = \frac{5}{4}$

b) $\displaystyle \lim_{x \to \infty} \frac{x + \log x}{\log x} = \left\{ \frac{\infty}{\infty} \right\} = \infty$ (orden x > orden $\log x$)

con L'Hôp: $\displaystyle \lim_{x \to \infty} \frac{x + \log x}{\log x} = \left\{ \frac{\infty}{\infty} \right\} \overset{\text{LHôp}}{=} \lim_{x \to \infty} \frac{1 + \frac{1}{x} \cdot \frac{1}{\ln 10}}{\frac{1}{x} \cdot \frac{1}{\ln 10}} =$

$\displaystyle = \frac{1 + \frac{1}{\infty}}{\frac{1}{\infty}} = \frac{1}{0} = \infty$

c) $\lim\limits_{x\to\infty} \dfrac{3+2\sqrt{x}}{\sqrt{2x+1}} = \left\{\dfrac{\infty}{\infty}\right\} = \lim\limits_{x\to\infty} \dfrac{\sqrt{x}\left(\frac{3}{\sqrt{x}}+2\right)}{\sqrt{x\left(2+\frac{1}{x}\right)}} =$

$= \lim\limits_{x\to\infty} \dfrac{\sqrt{x}\left(\frac{3}{\sqrt{x}}+2\right)}{\sqrt{x}\left(\sqrt{2+\frac{1}{x}}\right)} = \lim\limits_{x\to\infty} \dfrac{\frac{3}{\sqrt{x}}+2}{\sqrt{2+\frac{1}{x}}} = \dfrac{\frac{3}{\infty}+2}{\sqrt{2+\frac{1}{\infty}}} = \dfrac{2}{\sqrt{2}}$

d) $\lim\limits_{x\to\infty} \dfrac{3\cdot 2^x}{2^x+1} = \left\{\dfrac{\infty}{\infty}\right\} = \lim\limits_{x\to\infty} \dfrac{2^x(3)}{2^x\left(1+\frac{1}{2^x}\right)} =$

$= \lim\limits_{x\to\infty} \dfrac{3}{1+\frac{1}{2^x}} = \dfrac{3}{1+\frac{1}{\infty}} = \dfrac{3}{1} = 3$

Con LHop: $\lim\limits_{x\to\infty} \dfrac{3\cdot 2^x}{2^x+1} = \left\{\dfrac{\infty}{\infty}\right\} \overset{\text{LHop}}{=} \lim\limits_{x\to\infty} \dfrac{3\cdot \cancel{2^x}\cdot \cancel{\ln 2}}{\cancel{2^x}\cdot\cancel{\ln 2}} =$

$= \lim\limits_{x\to\infty} 3 = 3$