

Calcula los siguientes límites:

$$\lim_{x \rightarrow 1} \frac{x^2 - 7x + 6}{x^3 + 4x^2 + 5x + 2}$$

$$\lim_{h \rightarrow 0} \frac{(x-1)^3}{\frac{1-x^2}{(x+h)^2 - x^2}}$$

a) $\lim_{x \rightarrow 1} \frac{x^2 - 7x + 6}{1-x} = \frac{1-7+6}{0} = \left\{ \frac{0}{0} \right\} = (*)$

Factorizamos: $x^2 - 7x + 6 = 0 \Rightarrow x = \frac{7 \pm \sqrt{49-24}}{2} = \frac{7 \pm 5}{2} = \begin{matrix} 6 \\ 1 \end{matrix}$

$$(*) = \lim_{x \rightarrow 1} \frac{(x-6) \cdot (x-1)}{1-x} = \lim_{x \rightarrow 1} \frac{(x-6)(x-1)}{-(x-1)} =$$

$$= \lim_{x \rightarrow 1} -(x-6) = -(1-6) = 5$$

L'Hôp

con L'Hôp: $\lim_{x \rightarrow 1} \frac{x^2 - 7x + 6}{1-x} = \left\{ \frac{0}{0} \right\} \xrightarrow{\text{L'Hôp}} \lim_{x \rightarrow 1} \frac{2x-7}{-1} = \frac{2-7}{-1} = 5$

b) $\lim_{x \rightarrow 1} \frac{(x-1)^3}{1-x^2} = \frac{(1-1)^3}{1-1^2} = \left\{ \frac{0}{0} \right\} \xrightarrow{\text{L'Hôp}} \lim_{x \rightarrow 1} \frac{(x-1)^3}{-(x-1)(x+1)} = (*)$

Factorizamos: $1-x^2 = 0 \Rightarrow x = \pm 1 \Rightarrow -(x-1)(x+1) = 0$

$$(*) = \lim_{x \rightarrow 1} \frac{(x-1)^2}{-(x+1)} = \frac{(1-1)^2}{-(1+1)} = \frac{0}{-2} = 0$$

L'Hôp

con L'Hôp: $\lim_{x \rightarrow 1} \frac{(x-1)^3}{1-x^2} = \left\{ \frac{0}{0} \right\} \xrightarrow{\text{L'Hôp}} \lim_{x \rightarrow 1} \frac{3(x-1)^2}{-2x} = \frac{3(1-1)^2}{-2} = \frac{0}{-2} = 0$

$$c) \lim_{x \rightarrow -1} \frac{x^3 + 4x^2 + 5x + 2}{x^2 - x - 2} = \frac{-1 + 4 - 5 + 2}{1 + 1 - 2} = \left\{ \frac{0}{0} \right\} = (*)$$

Factorizar:

$$x^2 - x - 2 = 0$$

$$x = \frac{1 \pm \sqrt{1+8}}{2} = \begin{matrix} 2 \\ -1 \end{matrix}$$

$$(x-2)(x+1) = 0$$

$$x^3 + 4x^2 + 5x + 2 = 0$$

$$x^3 + 4x^2 + 5x + 2 = (x+1) \cdot (\quad)$$

$$\begin{array}{r} x^3 + 4x^2 + 5x + 2 \quad \overline{) x+1} \\ x^3 + x^2 \\ \hline 0 + 3x^2 + 5x + 2 \\ 3x^2 + 3x \\ \hline 0 \quad 2x + 2 \\ 2x + 2 \\ \hline 0 \quad 0 \end{array}$$

$$\frac{\text{Dividendo}}{\text{divisor}} = \text{cociente} + \frac{\text{resto}}{\text{divisor}}$$

$$\frac{x^3 + 4x^2 + 5x + 2}{x+1} = x^2 + 3x + 2$$

$$x^3 + 4x^2 + 5x + 2 = (x+1)(x^2 + 3x + 2)$$

$$(*) = \lim_{x \rightarrow -1} \frac{(x+1)(x^2 + 3x + 2)}{(x-2)(x+1)} = \lim_{x \rightarrow -1} \frac{x^2 + 3x + 2}{x-2} = \frac{1 - 3 + 2}{-1 - 2} = \frac{0}{-3} = 0$$

con L'Hôpital: $\lim_{x \rightarrow -1} \frac{x^3 + 4x^2 + 5x + 2}{x^2 - x - 2} = \left\{ \frac{0}{0} \right\} \xrightarrow{\text{L'Hôpital}} \lim_{x \rightarrow -1} \frac{3x^2 + 8x + 5}{2x - 1} =$

$$= \frac{3 - 8 + 5}{-2 - 1} = \frac{0}{-3} = 0$$

$$d) \lim_{h \rightarrow 0} \frac{(x+h)^2 - x^2}{h} = \frac{(x+0)^2 - x^2}{0} = \left\{ \frac{0}{0} \right\} =$$

$$= \lim_{h \rightarrow 0} \frac{x^2 + h^2 + 2x \cdot h - x^2}{h} = \lim_{h \rightarrow 0} \frac{h^2 + 2x \cdot h}{h} =$$

$$= \lim_{h \rightarrow 0} \frac{h(h + 2x)}{h} = \lim_{h \rightarrow 0} h + 2x = 0 + 2x = 2x$$