$$\frac{d}{dx}\left(\ln(x^2+1)\right) = \frac{2x}{x^2+1}$$

$\bullet \int \ln(x^2+1)\,dx$

$$I = \int \ln(x^2+1)\,dx = \left\| \begin{array}{ll} u = \ln(x^2+1) ; & du = \frac{2x}{x^2+1}dx \\ dv = dx ; & v = x \end{array} \right\| = \circledast$$

$\int u\,dv = u\cdot v - \int v\,du$

$$\circledast = x \cdot \ln(x^2+1) - \underbrace{\int x \cdot \frac{2x}{x^2+1}\,dx}_{I_1}$$

$$I_1 = \int \frac{2x^2}{x^2+1}\,dx = 2\int \frac{x^2}{x^2+1}\,dx = 2\int \frac{x^2+1-1}{x^2+1}\,dx =$$

$$= 2\int\left(\frac{x^2+1}{x^2+1} + \frac{-1}{x^2+1}\right)dx = 2\int\left(1 - \frac{1}{x^2+1}\right)dx$$

comprobación:

$$\begin{array}{l|l} x^2 & x^2+1 \\ \hline \underline{x^2+1} & 1 \\ 0 \quad -1 & \end{array} \Rightarrow \frac{x^2}{x^2+1} = 1 + \frac{-1}{x^2+1} \quad \checkmark$$

$$I = x\ln(x^2+1) - I_1 = x\ln(x^2+1) - 2\int\left(1 - \frac{1}{x^2+1}\right)dx =$$

$$= x\ln(x^2+1) - \left[2\left(x - \operatorname{arctg} x\right)\right] + C$$