Calcula los siguientes límites:

$$\lim_{x\to 1}\frac{x^2-7x+6}{1-x} \qquad \lim_{x\to 1}\frac{(x-1)^3}{1-x^2}$$

$$\lim_{x\to -1}\frac{x^3+4x^2+5x+2}{x^2-x-2} \qquad \lim_{h\to 0}\frac{(x+h)^2-x^2}{h}$$

a) $\lim_{x\to 1}\frac{x^2-7x+6}{1-x} = \frac{1-7+6}{0} = \left\{\frac{0}{0}\right\} = ⊛$

Factorizamos: $x^2-7x+6=0 \Rightarrow x=\frac{7\pm\sqrt{49-24}}{2} = \frac{7\pm 5}{2} = \left\langle {6 \atop 1} \right.$

$⊛ = \lim_{x\to 1}\frac{(x-6)\cdot(x-1)}{1-x} = \lim_{x\to 1}\frac{(x-6)(x-1)}{-(x-1)} =$

$= \lim_{x\to 1} -(x-6) = -(1-6) = 5$

Con LHôp: $\lim_{x\to 1}\frac{x^2-7x+6}{1-x} = \left\{\frac{0}{0}\right\} \overset{\text{LHôp}}{=} \lim_{x\to 1}\frac{2x-7}{-1} = \frac{2-7}{-1} = 5$

b) $\lim_{x\to 1}\frac{(x-1)^3}{1-x^2} = \frac{(1-1)^3}{1-1^2} = \left\{\frac{0}{0}\right\} = \lim_{x\to 1}\frac{(x-1)^3}{(x-1)(x+1)} = ⊛$

Factorizamos: $1-x^2=0 \Rightarrow x=\pm 1$

$⊛ = \lim_{x\to 1}\frac{(x-1)^2}{(x+1)} = \frac{(1-1)^2}{1+1} = \frac{0}{2} = 0$

Con LHôp: $\lim_{x\to 1}\frac{(x-1)^3}{1-x^2} = \left\{\frac{0}{0}\right\} \overset{\text{LHôp}}{=} \lim_{x\to 1}\frac{3(x-1)^2}{-2x} = \frac{3(1-1)^2}{-2} = \frac{0}{-2} = 0$

c) $\lim_{x\to -1} \frac{x^3+4x^2+5x+2}{x^2-x-2} = \frac{-1+4-5+2}{1+1-2} = \left\{\frac{0}{0}\right\} = (*)$

Factorizar:

$x^2-x-2=0$

$x = \frac{1\pm\sqrt{1+8}}{2} = \begin{cases} 2 \\ -1 \end{cases}$

$(x-2)(x+1)=0$

$x^3+4x^2+5x+2=0$

$x^3+4x^2+5x+2=(x+1)\cdot(\qquad)$

$$\begin{array}{l|l} x^3+4x^2+5x+2 & x+1 \\ \hline x^3+x^2 & x^2+3x+2 \\ 0+3x^2+5x+2 & \\ \quad 3x^2+3x & \\ \quad 0 \quad 2x+2 & \\ \qquad 2x+2 & \\ \qquad 0 \quad 0 & \end{array}$$

$\frac{\text{Dividendo}}{\text{divisor}} = \text{cociente} + \frac{\text{resto}}{\text{divisor}}$

$\frac{x^3+4x^2+5x+2}{x+1} = x^2+3x+2$

$x^3+4x^2+5x+2 = (x+1)(x^2+3x+2)$

$(*) = \lim_{x\to -1} \frac{(x+1)(x^2+3x+2)}{(x-2)(x+1)} = \lim_{x\to -1} \frac{x^2+3x+2}{x-2} = \frac{1-3+2}{-1-2} = \frac{0}{-3} = 0$

Con LHôp: $\lim_{x\to -1} \frac{x^3+4x^2+5x+2}{x^2-x-2} = \left\{\frac{0}{0}\right\} \overset{\text{L'Hôp}}{=} \lim_{x\to -1} \frac{3x^2+8x+5}{2x-1} =$

$= \frac{3-8+5}{-2-1} = \frac{0}{-3} = 0$

d) $$\lim_{h \to 0} \frac{(x+h)^2 - x^2}{h} = \frac{(x+0)^2 - x^2}{0} = \left\{\frac{0}{0}\right\} =$$

$$= \lim_{h \to 0} \frac{x^2 + h^2 + 2x \cdot h - x^2}{h} = \lim_{h \to 0} \frac{h^2 + 2x \cdot h}{h} =$$

$$= \lim_{h \to 0} \frac{h(h + 2x)}{h} = \lim_{h \to 0} h + 2x = 0 + 2x = 2x$$