UNCONSTRAINED OPTIMIZATION

AMPL's MINOS solver vs ME

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1st Function (282.mod)

$$f(x) = (x_1 - 1)^2 + (x_{10} - 1)^2 + 10\sum_{i=1}^{9}(10 - i)(x_i{}^2 - x_{i+1})^2$$

	AMPL	Modified Newton	DFP*	BFGS
# of iterations	95	55	313	116
# of function evaluations	239	350	6320	905
# of gradient evaluations	238	56	314	117

* Stucked because of Armijo at $f(x) = 0.997964$.

- Optimum $x^* = [1 \ldots 1]$ and $f(x^*) = 0$.

2nd Function (283.mod)

$$f(x) = \left(\sum_{i=1}^{10} i^3 (x_i - 1)^2 \right)^3$$

	AMPL	Modified Newton	DFP	BFGS
# of iterations	39	11	29,209	48
# of function evaluations	140	85	380,197	400
# of gradient evaluations	139	12	29,210	49

- Optimum $x^* = [1 \ldots 1]$ and $f(x^*) = 0$.

3rd Function (287.mod)

$$f(x) = \sum_{i=1}^{9} \begin{array}{c} 100({x_i}^2 - x_{i+5})^2 + (x_i - 1)^2 + 90({x_{i+10}}^2 - x_{i+15})^2 \\ +(x_{i+10} - 1)^2 + 10.1[(x_{i+5} - 1)^2 + (x_{i+15} - 1)^2] \\ +19.8(x_{i+5} - 1)(x_{i+15} - 1) \end{array}$$

	AMPL	Modified Newton	DFP	BFGS
# of iterations	82	46	364	128
# of function evaluations	174	205	2,239	1,016
# of gradient evaluations	173	47	365	129

- Optimum $x^* = [1 \dots 1]$ and $f(x^*) = 0$.

4th Function (288.mod)

$$f(x) = \sum_{i=1}^{5} \begin{matrix} (x_i + 10x_{i+5})^2 + 5(x_{i+10} - x_{i+15})^2 + \\ +(x_{i+5} - 2x_{i+10})^4 + 10(x_i - x_{i+15})^4 \end{matrix}$$

	AMPL	Modified Newton	DFP	BFGS
# of iterations	88	8	195	48
# of function evaluations	261	36	1093	323
# of gradient evaluations	260	9	196	49

- Optimum $x^* = [0 \ldots 0]$ and $f(x^*) = 0$.

5^{th} Function (289.mod)

$$f(x) = 1 - e^{-\sum_{i=1}^{30} \frac{{x_i}^2}{60}}$$

	AMPL	Modified Newton	DFP	BFGS
# of iterations	79	3	11	8
# of function evaluations	186	14	106	64
# of gradient evaluations	185	4	12	9

- Optimum $x^* = [0 \ldots 0]$ and $f(x^*) = 0$.

6^{th} Function (Freudenstein and Roth)

$$f(x) = \left[x_1 - x_2\left(2 - x_2(5 - x_2)\right) - 13\right]^2 + \left[x_1 - x_2\left(14 - x_2(1 + x_2)\right) - 29\right]^2$$

	AMPL	Modified Newton*	DFP*	BFGS*
# of iterations	7	6	27	23
# of function evaluations	19	24	196	168
# of gradient evaluations	18	7	28	24

- Optimum found by me: $x^* = [11.41278 \quad -0.89681]$ and $f(x^*) = 24.4921$.
- Optimum found by AMPL: $x^* = [5 \quad 4]$ and $f(x^*) = 0$.

7^{th} Function (Bard)

	AMPL	Modified Newton	DFP	BFGS
# of iterations	13	19	28	18
# of function evaluations	34	142	204	135
# of gradient evaluations	33	20	29	19

- Optimum $x^* = [0.0824 \quad 1.13304 \quad 2.34269]$ and $f(x^*) = 0.008214877$.

General Unconstrained Optimization Results (1)

- **Tuning tolerances** is quite important for the convergence of the algorithms given that any computer has a finite precision.

- **Armijo** line search is a really good method for finding the step length at each iteration given that we have a "good" direction (Newton, BFGS etc.). Moreover, it was much **less computationally expensive** compared to other exact line searches (Golden Section search etc.)

- **Maintaining positive definiteness** at each iteration is of high importance in order to guarantee that we have a negative directional derivative and our algorithm is going to converge in a finite number of steps.

- **Updating factorizations** was important in order to improve the speed of our algorithms since we tried to approximate the actual Hessians when we did not have them in our disposal (BFGS, DFP methods).

General Unconstrained Optimization Results (2)

- **Gradient (Steepest) Descent** method was proved not to be such a "good" method when we are close to the actual optimum since the rate of convergence is much slower (zigzagging) compared to Newton (quadratic rate of convergence) and BFGS (superlinear rate of convergence). It can be considered as good if it is used only for the initial steps.

- **Newton** was proved to be the **fastest method** and it should be preferred whenever we have access to the actual Hessian of the function and the computational power is high enough for the given problem.

- To conclude with, if we had to **order the methods** for solving an Unconstrained Optimization problem, the order would be the following:

1) Newton/Modified Newton
2) BFGS
3) DFP
4) Gradient (Steepest) Descent

Thank you