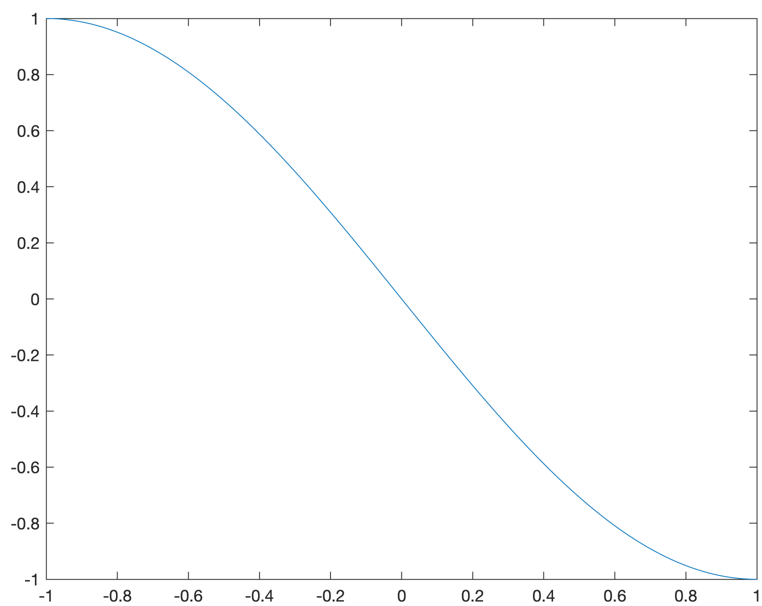
We wish to find a function with infinite number of saddle points:

$$\begin{aligned} x+1 &= 2\cdot k + v \\ k &= \mathsf{quotient}(x+1,2) \\ v &= \mathsf{remainder}(x+1,2) \\ g(x) &= -2\cdot k - \sin\left(\frac{\pi}{2}(v-1)\right) \end{aligned}$$

or equivalently,

$$g(x) = -\sin\left(\frac{\pi}{2}(x-2k)\right) - 2k, \qquad x \in (2k-1, 2k+1]$$

- In $(-1,1]$ $(k=0)$,
  - $x=-1, g=0$,
  - $x=0, g=0$,
  - $x=1, g=-1, g'(x)=0, g''(x)>0$



- Then $g(x)$ repeats the patten of $[-1,1]$ where $g(x) = -\sin\left(\frac{\pi}{2}x\right)$.
- Suppose we start from very far $x>0, y>0$ and **`maximize`** a 2-D function:

$$f(x,y) = -g(x)\cdot g(y)$$

at k-th piece,

$$f_x = g(y) \cdot \frac{\pi}{2} \cos\left(\frac{\pi}{2}(x-2k)\right) = 0,$$

we have two cases,

$$\begin{cases} g(y) = 0 \\ \cos(\cdot) = 0 \end{cases} \quad \Rightarrow \begin{cases} y = 0 \\ x = 2k+1 \end{cases}$$

similarly, consider,

$$f_y = 0 \Rightarrow \begin{cases} x = 0 \\ y = 2k+1 \end{cases}$$

and second-order derivatives,

$$\begin{aligned} f_{xx} &= -g(y) \cdot \frac{\pi^2}{4} \sin\left(\frac{\pi}{2}(x-2k)\right) \\ f_{xy} &= +\frac{\pi^2}{4} \cos(\cdot)\cos(\cdot) \equiv 0 \end{aligned}$$

- at $(x, y) = (2k+1, 2k+1)$,

$$\begin{aligned} &f_{xx} = -\frac{\pi^2}{4} g(y) = 2k+1 > 0 \\ &f_{xy} = \frac{\pi^2}{4} \cos(\cdot)\cos(\cdot) = 0 \\ &H \succeq 0 \end{aligned}$$

- at $(x, y) = (2k+1, 0)$,

$$\begin{aligned} &f_{xx} = -\frac{\pi^2}{4} g(y) = 0 \\ &f_{xy} = \frac{\pi^2}{4} \cos(\cdot)\cos(\cdot) = 0 \\ &H = 0 \end{aligned}$$

- at $(x, y) = (0, 0)$,

$$\begin{aligned} &f_{xx} = -\frac{\pi^2}{4} g(y) = 0 \\ &f_{xy} = \frac{\pi^2}{4} \cos(\cdot)\cos(\cdot) = \frac{\pi^2}{4} > 0 \\ &H \preceq 0 \end{aligned}$$

So, $(x, y) = (0, 0)$ is the only maximum and $(x, y) = (2k+1, 2k+1), \forall k$ are saddle points (remember we are maximizing)