MMO 2020 Shortlist Problems

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§1 Introduction

Hello all, we are very happy that we have finally conducted our Olympiad exam MMO(MetriX Mathematical Olympiad) successfully which has first time occurred as the 1st MMO and has gained a nice attribute and popularity in 2020 competitions as far appreciated by the participants. Many have stated that the difficulty was the very same as IMO with problems being quite tricky, also there were some problems which were even more nice and difficult. Due to encouragement and request of many participants we have decided to conduct it every half year that is twice a year.

§1.1 Origin of MMO

The competition was found by me(Aritra aka Aritra12), and we finally gave it into shape with my 5 friends Functional_Equation , MNJ2357 , TLP.39 , Amar_04 and Mr.C . Jointly this was our first successful Olympiad. Without the effort of our all members it would not have been possible to do this. For more information visit this thread on AoPS on here `MMO~AoPS~thread`

§1.2 Comments

If you have any further comments pls dont forget to say it to us in order we can improve it more next time. Here are our 6 members of our MMO committee this time , you can pm anyone or all at a time

- `Aritra12`
- `MNJ2357`
- `TLP.39`
- `Functional_equation`
- `Amar_04`
- `Mr.C`

§2 Algebra

A1. If we take four integers $a < b < c < d$ from the segeunce $1, 2, 3....2n$ then prove that there does not exists such integers a, b, c, d for which

$$\frac{bc - ad}{-(b+c) + (a+d)} = n^2$$

`(Proposed By Tarry122)`

A2. Find all polynomials p such that $deg(p) \in Z$ and its not 1 and there exists two non-constant, functions f, g such that $f - g = p$ and we have $f(x + g(y)) = f(x)$ for every real x, y

`(Proposed by Mr.C)`

A3. For positive integers a, b, c, d, e we have that $an^4 + bn^3 + cn^2 + dn + e$ is a fourth power of an integer for infinite natural numbers n , prove that there exist integer x, y such that

$$a = x^4, b = 4x^3y, c = 6x^2y^2, d = 4xy^3, e = y^4$$

`(Proposed by Mr.C)`

A4. Prove the expression when $abc = 1$, $a, b, c > 0$,

$$\sum_{cyc} \frac{ac(a + c^5) + 1}{b^{12} + a(a^{13}c + 1)} \le a^6 + b^6 + c^6$$

`(Proposed by Aritra12)`

A5. Let $\mathcal{G}$ be highest integer lower bound of the expression where x, y, z are natural numbers, find $\mathcal{A}^2$

$$\left(\sum_{cyc} x^2(y - z)\right)\left(\sum_{cyc} \frac{e^x}{yz(z - y)}\right) > \mathcal{A}$$

`(Proposed by Aritra12)`

A6. Find the maximum value for non-negative reals x, y, z which are less than 1 and also satisfies the condition $x + y + z = 1$

$$\frac{2}{7}\sqrt{\frac{2xy}{7(1 - z)}} + \sqrt{\frac{yz}{1 - x}} + \sqrt{\frac{zx}{1 - y}}$$

`(Proposed By Aritra12)`

§3 Number Theory

N1. Find the largest nonnegative integer N satisfying the following: There exist N consecutive integers such that $9n^2\pi(n)+1$ is a square.

`(Proposed by MNJ2357 )`

N2. Does there exist a nonconstant polynomial P with integer coefficients such that

$$p \text{ is prime, } p|P(n) \text{ for some integer n} \Longrightarrow p \equiv 1 (\text{mod } 4) \text{ ?}$$

§4 Combinatorics

C1. In the city of $'$greeds$'$, Mr.greed has committed a crime and is running from the police , police is informed that Mr.greed is hiding in a cell of a m, n greed, the police want to play a game, and give him a chance to run , so they take 2 arbitrary integers a, b where $ab|(m, n), (m, n) > a^b$ and then check every single cell with (i, j) coordinates if either i or j (perhaps even both) are in the form of $ax + by$, $(x, y$ are non-negative integers) , what is the chance that M.greed can escape from the police depending on a, b

`(Proposed by Mr.C )`

C2. A and Z decided to play a match of chess on 9×9 chess board as they were feeling bored, but suddenly two mischievous boys namely K and J comes and snatch our all chess pieces because they wanted to play chess with our chess pieces , but they were very determined to play as we had nothing else to do, in the mean time N gave us some cuboidal dominoes each of which takes up space of three unit square of a chessboard ,but N gave the condition that we have to follow some rules to get his dominoes that is we can always place one more cuboidal dominoes on the board without moving the other dominoes and we can't place it on a diagonal manner. Can you determine the maximum number of dominoes which N gave A and Z for playing the game.

`(Proposed by Aritra12)`

§5 Geometry

G1. Let ABC be a triangle with centroid G. GD, GE, GF are symmedians of triangles GBC, GCA, GAB, respectively. L is symmedian point of ABC. L^* is inversion of L in circle (ABC). Prove that AD, BE, CF and GL^* are concurrent.

(Proposed by Buratinogiggle)

G2. Let the points H and N_9 to be the orthocenter and the Nine-Point center of $\triangle ABC$ respectively and let R to be the circumradius of $\triangle ABC$. If $\angle BAC = \alpha, \angle ABC = \beta, \angle ACB = \gamma$. Prove that

$$\left(\frac{2HN_9}{R}\right)^2 \leqslant 9 - 8\sqrt{3} \cdot \sin\alpha \cdot \sin\beta \cdot \sin\gamma.$$

(Proposed by Functional_equation)

G3. Let D, E, and F be the respective feet of the A, B, and C altitudes in $\triangle ABC$, and let M and N be the respecive midpoints of $\overline{AC}$ and $\overline{AB}$. Lines DF and DE intersect the line through A parallel to BC at X and Y, respectively. Lines MX and YN intersect at Z. Prove that the circumcircles of $\triangle EFZ$ and $\triangle XYZ$ are tangent.

(Proposed by mathman3880)

G4. In triangle ABC denote I_A as the Excircle WRT to A let the Excircle touch the sides AB, BC, CA at M, N, P respectivly , let T be the midpoint of MP if TC and MN hit it T' and if BC and PM hit at T'' then prove I_A is the orthocenter of T, T', T'' .

(Proposed by Mr.C)

§6 Remarks

Thank you all for participating in the contest this was our first time so we did not have many problem in shortlist. We hope we can do it better next time.Thank You all for participation.

Thank You