Test of Hypothesis

- Statistical Hypothesis: A statement about the parameters of one or more population.
  - EXAMPLE:

    (i) The average height of an adult Indian male is 5’5”.

    (ii) Japanese are shorter than Indians

    (iii) The volume of liquid filling in station A is more than that of filling station B.

    (iv) If we reduce the price of a product by Rs 3, there will be increase in sales by 15%.

    (v) There is a significant improvement in the performance of the employees after imparting a skill-based training.
- Hypothesis Testing: Decision making procedure whether to accept or reject the statement.

Steps Involved in Hypothesis Testing

1. Formulation of the hypothesis
2. Choose a suitable significance level, α
3. State appropriate test statistic
4. Computation of the test statistic
5. Decide whether to accept or reject the hypothesis

1. Formulate a hypothesis

- Conventionally two hypotheses are used: Null hypothesis (H_0) and Alternative hypothesis (H_1)

  $H_0 : \mu = \mu_0$

  $H_1 : \mu \neq \mu_0, \mu > \mu_0, \mu < \mu_0$

  Where μ = population mean

  μ_0 = some hypothesized value

- Decision is accepting/rejecting the null hypothesis. Rejecting the H_0 means accepting H_1 hypothesis.

- Example 1: Suppose we want to test the hypothesis that "The average height of an adult Indian male is 165 cm."

  Null Hypothesis: $H_0 : \mu = 165$

  Alternate Hypothesis: $H_1 : \mu \neq 165$

- Example 2: Suppose we want to test the hypothesis that "The average height of an adult Indian male is more than 165 cm."

  Null Hypothesis: $H_0 : \mu = 165$

  Alternate Hypothesis: $H_1 : \mu > 165$

2. Set up a suitable significance level

- Confidence with which a null hypothesis is rejected depends upon the significance level used.
- A significance level, α = 5% or 0.05 means that the risk of making a wrong decision is 5%.
- Confidence level = $1 - \alpha$.

3. Select test criteria

- Various techniques or statistics are used for testing the hypothesis.
- The common ones are

  Z test (for variance known OR large sample > 30),

  t test (for small sample size ≤ 30),

  Chi-square (χ^2),

  F test.
- Selection of a test techniques depends on sample size, type of data used, etc.

Condition for using Z-test or t-test

I. Population Standard deviation, σ is known :

Use z-test

II. Population Standard deviation, σ is unknown

1. Sample size $n > 30$ **use z-test**
2. Sample size $n \leq 30$ **use t-test**

4. Computation

Computation of corresponding statistic value.

5. Decision

Now to see whether the computed value is in the region of acceptance/rejection as per the table value corresponding to significance level.

Test on Means (one-sample)

Case		Null Hypothesis	Test Statistic	Alternate Hypothesis	Rejection Criteria of H_0
Population standard deviation, σ known		$H_0 : \mu = \mu_0$	$Z_0 = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$	$H_1 : \mu \neq \mu_0$ $H_1 : \mu > \mu_0$ $H_1 : \mu < \mu_0$	$\lvert Z_0 \rvert > Z_{\alpha/2}$ $Z_0 > Z_\alpha$ $Z_0 < -Z_\alpha$
Population standard deviation, σ unknown	Sample size n > 30	$H_0 : \mu = \mu_0$	$Z_0 = \frac{\bar{x} - \mu_0}{s / \sqrt{n}}$	$H_1 : \mu \neq \mu_0$ $H_1 : \mu > \mu_0$ $H_1 : \mu < \mu_0$	$\lvert Z_0 \rvert > Z_{\alpha/2}$ $Z_0 > Z_\alpha$ $Z_0 < -Z_\alpha$
	Sample size n ≤ 30	$H_0 : \mu = \mu_0$	$t_0 = \frac{\bar{x} - \mu_0}{s / \sqrt{n}}$	$H_1 : \mu \neq \mu_0$ $H_1 : \mu > \mu_0$ $H_1 : \mu < \mu_0$	$\lvert t_0 \rvert > t_{\alpha/2, n-1}$ $t_0 > t_{\alpha, n-1}$ $t_0 < -t_{\alpha, n-1}$

Example 1

The mean breaking strength of the cables supplied by a manufacturer is 1800 with a standard deviation of 100. In order to test this claim a sample of 50 cables is tested. It is found that the mean breaking strength is 1850. Can we support the claim at a 0.01 level of significance?

H_0: $\mu = 1800$
H_1: $\mu \neq 1800$

- $\bar{x} = 1850$, $n = 50$, $\sigma = 100$,

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{1850 - 1800}{\frac{100}{\sqrt{50}}} = 3.54$$

- $Z_{0.005} = 2.575$ at $\alpha = 0.01$ level of significance.

- Since $Z_0 = 3.54 > Z_{\alpha/2} = 2.54$, Reject H_o.

- The breaking strength of the cables of 1800 does not support the claim.

Example 2:

Does an average box of cereal contain more than 368 grams of cereal? A random sample of 25 boxes showed its mean 372.5 grams. The company has specified σ to be 15 grams. Test at the $\alpha = 0.05$ level.

Solution

H_0: $\mu = 368$

H_1: $\mu > 368$

$\alpha = 0.05$

$n = 25$

- Test statistics:

$$Z = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}} = 1.50$$

- Critical Value: 1.645
- Decision: Since $Z_0 = 1.5 < Z_\alpha = 1.645$, we accept H_o at $\alpha = .05$
- Conclusion : No evidence that the boxes contain more than 368 gm of cereal.

Example 3:

The average breaking strength of steel of rods is specified to be 18.5 thousand lbs. For this a sample of 14 rods was tested. The mean and standard deviation from the sample were obtained as 17.85 and 1.955 respectively. Test the significance of the deviation of value at $\alpha = 5\%$.

Example 4

The mean weekly sales of chocolate bar in candy stores was 146.3 bars per store. After advertising campaign the mean weekly sales in 22 stores for a typical week increased to 153.7 and showed a standard deviation of 17.2. Was the advertising campaign successful?

Example 5

The Quality Control Department of a company manufacturing light bulbs passes a lot of bulbs if the average life of bulbs exceeds 1600 hours. Past experience indicates that a reasonable value for standard deviation of breaking strength is 120 hours. A sample of 40 was collected and upon inspection the average mean life is found to be1570 hours.

(a) State the hypothesis that should be tested.

(b) Test this hypothesis at 5% level of significance. What are your conclusions?

Test on Means (two samples)

Hypothesis Tests for a Difference in Means, Variances Known or large sample size

Null hypothesis: $H_0: \mu_1 - \mu_2 = \Delta_0$

Test statistic: $$Z_0 = \frac{\overline{X}_1 - \overline{X}_2 - \Delta_0}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}$$

Alternative Hypotheses	Rejection Criterion
$H_1: \mu_1 - \mu_2 \neq \Delta_0$	$z_0 > z_{\alpha/2}$ or $z_0 < -z_{\alpha/2}$
$H_1: \mu_1 - \mu_2 > \Delta_0$	$z_0 > z_\alpha$
$H_1: \mu_1 - \mu_2 < \Delta_0$	$z_0 < -z_\alpha$

Hypothesis Tests for a Difference in Means, Variances unknown and small sample size

The **pooled estimator** of σ^2, denoted by S_p^2, is defined by

$$S_p^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}$$

Null hypothesis: $H_0\colon \mu_1 - \mu_2 = \Delta_0$

Test statistic: $$T_0 = \frac{\overline{X}_1 - \overline{X}_2 - \Delta_0}{S_p\sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}}$$

Alternative Hypothesis	**Rejection Criterion**
$H_1\colon \mu_1 - \mu_2 \neq \Delta_0$	$t_0 > t_{\alpha/2,n_1+n_2-2}$ or $t_0 < -t_{\alpha/2,n_1+n_2-2}$
$H_1\colon \mu_1 - \mu_2 > \Delta_0$	$t_0 > t_{\alpha,n_1+n_2-2}$
$H_1\colon \mu_1 - \mu_2 < \Delta_0$	$t_0 < -t_{\alpha,n_1+n_2-2}$

Example 6

Two machines are used for filling glass bottles with a soft-drink beverage. The filling processes have known standard deviations of 0.01 litre and 0.015 litre, respectively. A random sample of 25 bottles from machine 1 and 20 bottles from machine 2 results in average net contents of 2.04 litres and 2.07 litres, respectively.

1. Test the hypothesis that both machines fill to the same net contents, using $\alpha = 0.05$. What are your conclusions?
2. Construct a 95% confidence interval on the difference in mean fill volume.

Example 7

The following table gives the closing price stocks of ten FMCGs companies as on Jan 31st, 2012. These share-prices are to be compared with share-prices of the same companies as in the last year. Analyze the data at 1% level of significance and comment whether there is a significant decrease as compared to the last year.

Company	Closing Price (January 31st, 2012)	Closing Price (January 31st, 2011)
XML Company	33.89	43.78
IWL Enterprices	26.32	22.56
TYE Company	72.67	90.91
NIR Company	18.53	20.45
SUP Enterprices	61.45	61.45
ANT Company	45.49	54.21
ZNN Enterprices	80.00	79.27
KYE Enterprices	58.12	58.75
PCU Company	41.38	40.86
NTR Company	39.06	40.12

What if there are more than Two Populations?

Analysis of Variance (ANOVA)

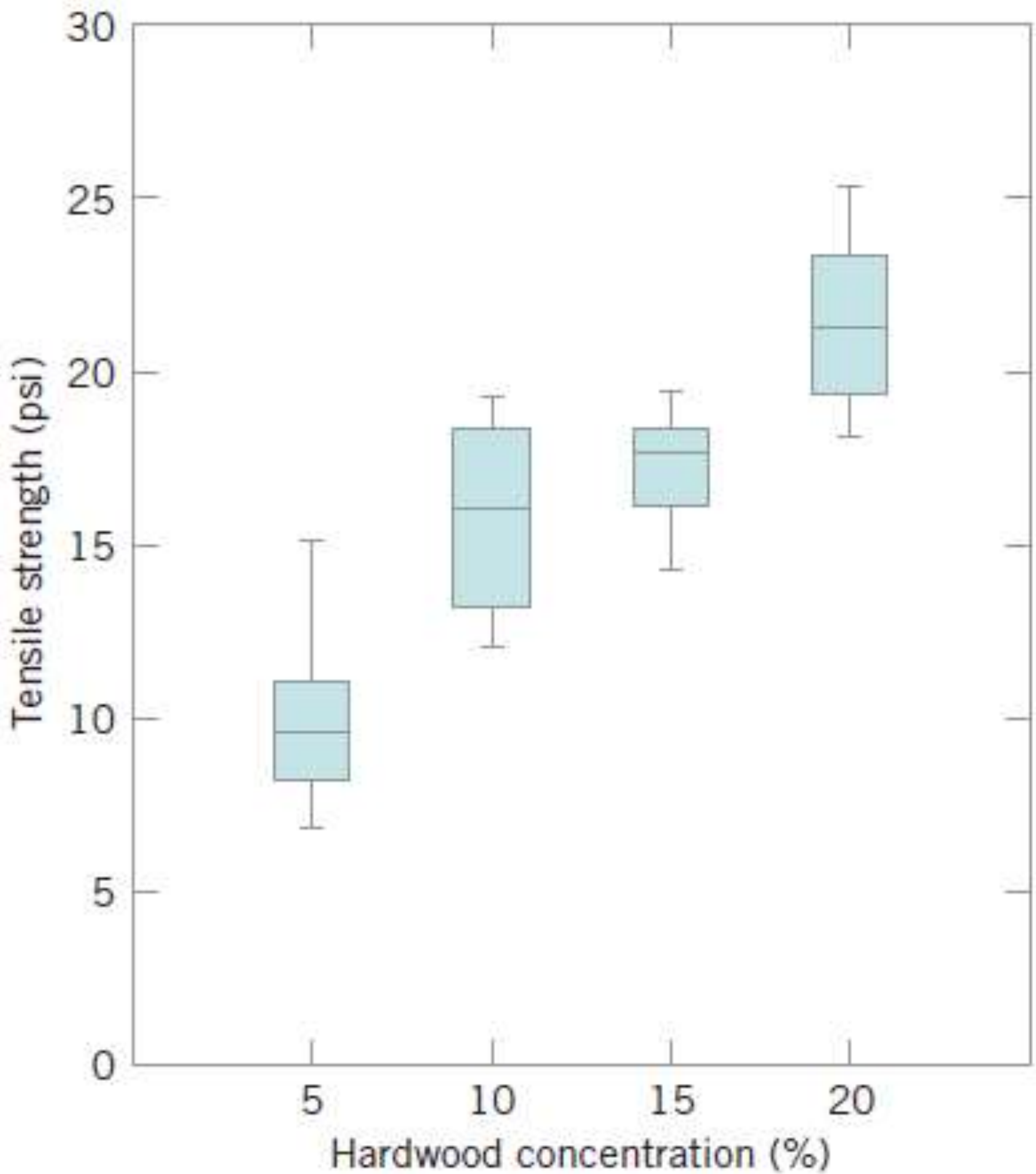
- A manufacturer of paper, used for making grocery bags, is interested in improving the tensile strength of the product.
- Product engineering thinks that tensile strength is a function of the hardwood concentration in the pulp and it range from 5% to 20%.
- They decides to investigate at four levels of hardwood concentration: 5%, 10%, 15%, and 20%, and make up six test specimens at each concentration level, using a pilot plant.
- All 24 specimens are tested on a laboratory tensile tester, in random order. The data from this experiment are shown in Table

Tensile Strength of Paper (psi)

Hardwood Concentration (%)	Observations						Totals	Averages
	1	2	3	4	5	6		
5	7	8	15	11	9	10	60	10.00
10	12	17	13	18	19	15	94	15.67
15	14	18	19	17	16	18	102	17.00
20	19	25	22	23	18	20	127	21.17
							383	15.96

- This is an example of a completely randomized single-factor experiment with four levels of the factor.
- The levels of the factor are called **treatments,** and each treatment has six observations or **replicates.**
- The role of **randomization** in this experiment is extremely important. By randomizing the order of the 24 runs, the effect of any nuisance variable that may influence the observed tensile strength is approximately balanced out.
- For example, suppose that there is a warm-up effect on the tensile testing machine; that is, the longer the machine is on, the greater the observed tensile strength. If all 24 runs are made in order of increasing hardwood concentration (that is, all six 5% concentration specimens are tested first, followed by all six 10% concentration specimens, etc.), then any observed differences in tensile strength could also be due to the warm-up effect.

Figure presents box plots of tensile strength at the four hardwood concentration levels.



ANOVA: ONE-WAY

Typical Data for a Single-Factor Experiment

Treatment	Observations				Totals	Averages
1	y_{11}	y_{12}	...	y_{1n}	$y_{1\cdot}$	$\bar{y}_{1\cdot}$
2	y_{21}	y_{22}	...	y_{2n}	$y_{2\cdot}$	$\bar{y}_{2\cdot}$
.	.	.	...	.	.	.
.	.	.	...	.	.	.
.	.	.	...	.	.	.
a	y_{a1}	y_{a2}	...	y_{an}	$y_{a\cdot}$	$y_{a\cdot}$
					$y_{\cdot\cdot}$	$\bar{y}_{\cdot\cdot}$

The Analysis of Variance for a Single-Factor Experiment

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0
Treatments	$SS_{\text{Treatments}}$	$a-1$	$MS_{\text{Treatments}}$	$\dfrac{MS_{\text{Treatments}}}{MS_E}$
Error	SS_E	$a(n-1)$	MS_E	
Total	SS_T	$an-1$		

$$SS_T = \sum_{i=1}^{a}\sum_{j=1}^{n} y_{ij}^2 - \frac{y_{\cdot\cdot}^2}{N}$$

$$SS_{\text{Treatments}} = \sum_{i=1}^{a} \frac{y_{i\cdot}^2}{n} - \frac{y_{\cdot\cdot}^2}{N}$$

$$SS_E = SS_T - SS_{\text{Treatments}}$$

reject H_0 if $F_0 > F_{\alpha,a-1,a(n-1)}$

Tensile Strength of Paper (psi)

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5	7	8	15	11	9	10	60	10.00
10	12	17	13	18	19	15	94	15.67
15	14	18	19	17	16	18	102	17.00
20	19	25	22	23	18	20	127	21.17
							383	15.96

$$SS_T = \sum_{i=1}^{4}\sum_{j=1}^{6} y_{ij}^2 - \frac{y_{..}^2}{N}$$

$$= (7)^2 + (8)^2 + \cdots + (20)^2 - \frac{(383)^2}{24} = 512.96$$

$$SS_{\text{Treatments}} = \sum_{i=1}^{4} \frac{y_{i.}^2}{n} - \frac{y_{..}^2}{N}$$

$$= \frac{(60)^2 + (94)^2 + (102)^2 + (127)^2}{6} - \frac{(383)^2}{24} = 382.79$$

$$SS_E = SS_T - SS_{\text{Treatments}}$$

$$= 512.96 - 382.79 = 130.17$$

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Analysis of Variance
Source     DF        SS        MS        F
Factor      3    382.79    127.60    19.61
Error      20    130.17      6.51
Total      23    512.96
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$F_{0.01,3,20} = 4.94$, we reject H_0