Chapter 4 Part 2

Budget Constraint

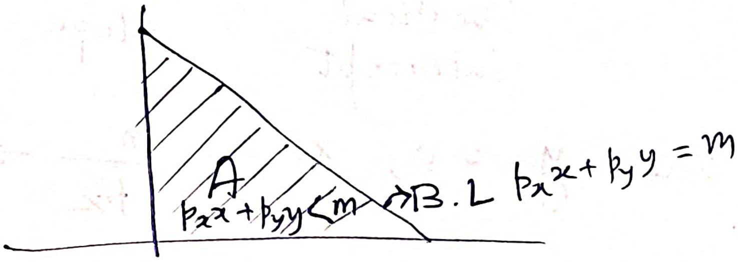
The Budget Constraint describes what the consumer can afford.

Suppose the consumer has a consumption bundle (x, y) and we can observe the prices of x and y as p_x and p_y respectively.

Let the consumer have a certain amount of money, m, to spend on buying (x, y).

Then the budget constraint of the consumer can be written as:

$$\underbrace{p_x x}_{\text{amount consumer spends on } x} + \underbrace{p_y y}_{\text{amount consumer spend on } y} \leq m$$



The consumer's affordable consumption bundles are those that don't cost any more than m. These include all consumption bundles in the area A + all cons. bundles on the budget line. We call this set of affordable cons. bundles at prices (p_x, p_y) and income (m) the BUDGET SET of the consumer.

The budget line is the set of consumption bundles that cost exactly m :

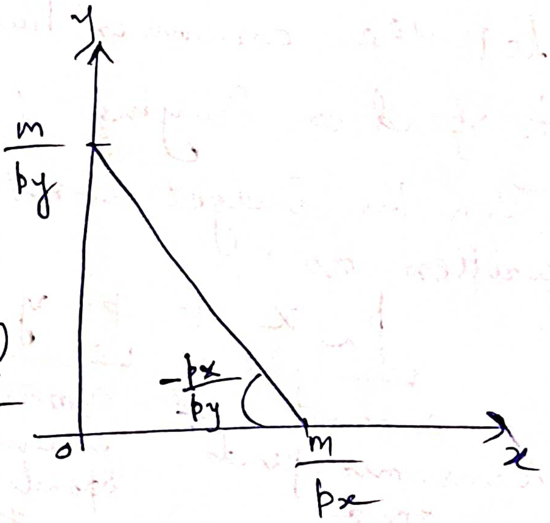
$$p_x x + p_y y = m$$

Assumption

The consumer shall exhaust the income allocated by him/her to a certain cons. bundle.

The budget line can be rearranged as :

$$y = \underbrace{\frac{m}{p_y}}_{\text{vertical intercept}} - \underbrace{\frac{p_x}{p_y}}_{\text{slope}} \cdot x$$



when $y = 0$, $x = \frac{m}{p_x}$ → horizontal intercept

when $x = 0$, $y = \frac{m}{p_y}$ → vertical intercept

The slope of the B.L. measures the rate at which the market is willing to substitute good x for good y.

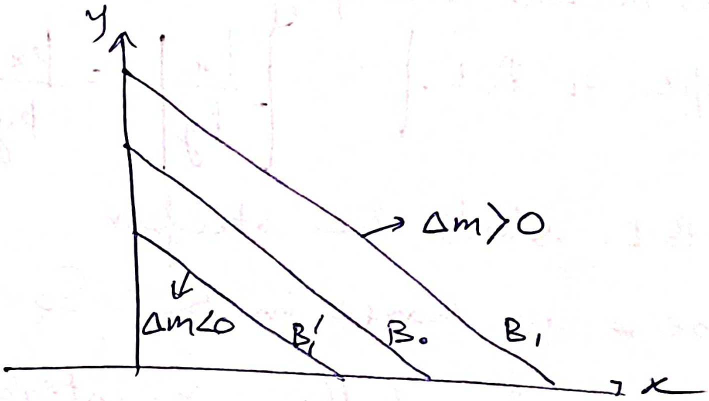
$$\frac{dy}{dx} = -\frac{p_x}{p_y}$$

This means that given the budget constraint, if you want to consume more of x, you must consume less of y.

The absolute value of the slope of the B.L. is the market rate of substitution. It is the opportunity cost of consuming good x in terms of good y as determined by market conditions.

Factors that affect the budget line

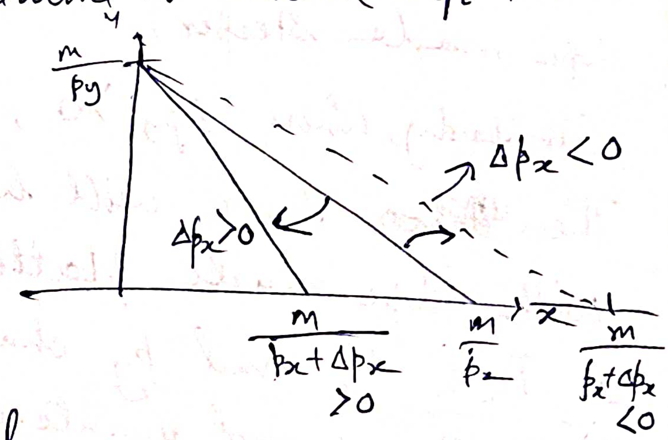
① Income, m



Any change in income by Δm, just changes the intercepts without changing the slope of the initial B.L.

This kind of shift is called parallel outward or inward shift.

② Prices, p_x and p_y



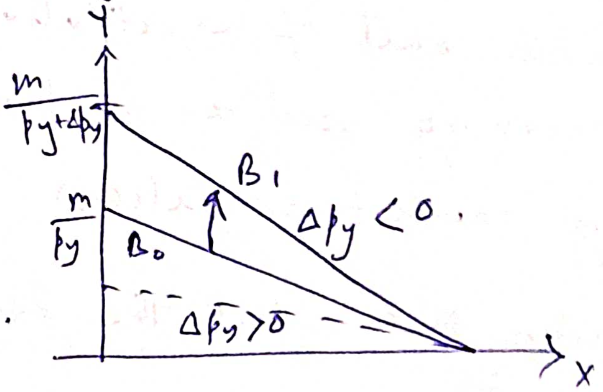
a. If only price of x changes e.g. p_x changes to $p_x + \Delta p_x$ where $\Delta p_x >$ or < 0.

Case study: $p_x + \Delta p_x$ where $\Delta p_x > 0$

This means that price of x has increased. It will reduce x-intercept while y-intercept is unchanged. There shall be inward rotation with slope made steeper.

b. <u>If price of y changes while p_x is unchanged</u>

$p_y \rightarrow p_y + \Delta p_y$



Case

Let $\Delta p_y < 0$

This means price of y has decreased.

$\left|\text{Slope of B.L.}\right| = \left|-\frac{p_x}{p_y}\right| = \frac{p_x}{p_y}$ shall increase.

So B.L. shall be steeper.

There is no change in p_x. So x-intercept is unchanged.

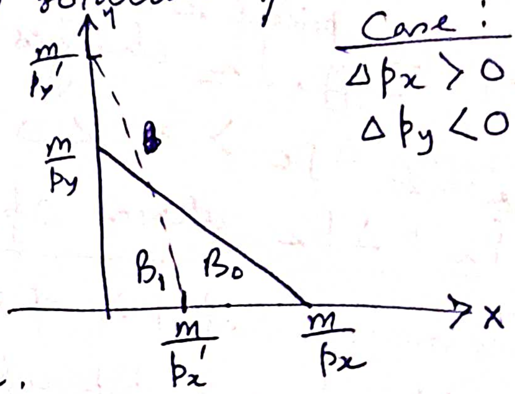
Since p_y has fallen, y-intercept shall increase.

There will be an outward ~~shift~~ rotation of B.L. with slope made steeper.

Similarly, when $\Delta p_y > 0$, price of y rises.

Then there will be an inward rotation of B.L. with slope made flatter.

c. <u>Both p_x and p_y changes.</u>



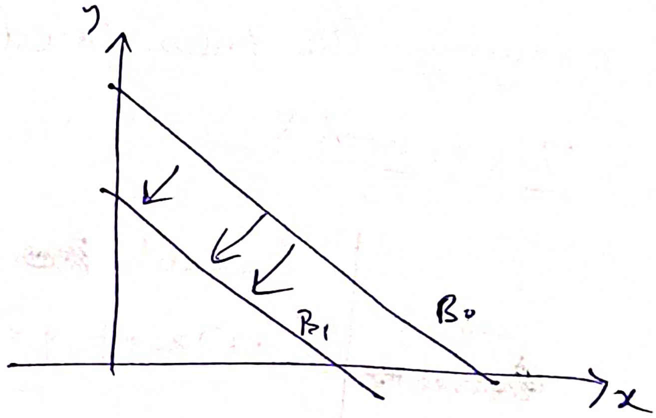
Case:
$\Delta p_x > 0$
$\Delta p_y < 0$

Here both x and y intercepts change.
Depending on the value of Δp_x and Δp_y, we have to see how the slope will change.

$$\frac{p_x + \Delta p_x}{p_y + \Delta p_y}$$

Example :

Suppose both price of x and y are multiplied by the same ratio t, $t>1$
then the new B.L. can be written as :



$$t\,p_x \cdot x + t\,p_y \cdot y = m$$

or $$p_x \cdot x + p_y \cdot y = \frac{m}{t}$$

If $t = 2$,

$$p_x \cdot x + p_y \cdot y = \frac{m}{2}$$

So multiplying both prices by same constant, t, is just like dividing the income by t. so there will be parallel inward shift.

Similarly, when prices are decreased e.g. divided by t, then income is multiplied by t.

If $t = 2$,

$$p_x \cdot x + p_y \cdot y = 2m.$$

Thought experiment

What happens to budget line when both prices and income changes simultaneously ?

Look at the intercepts and slope .

Taxes, Subsidies and Rationing

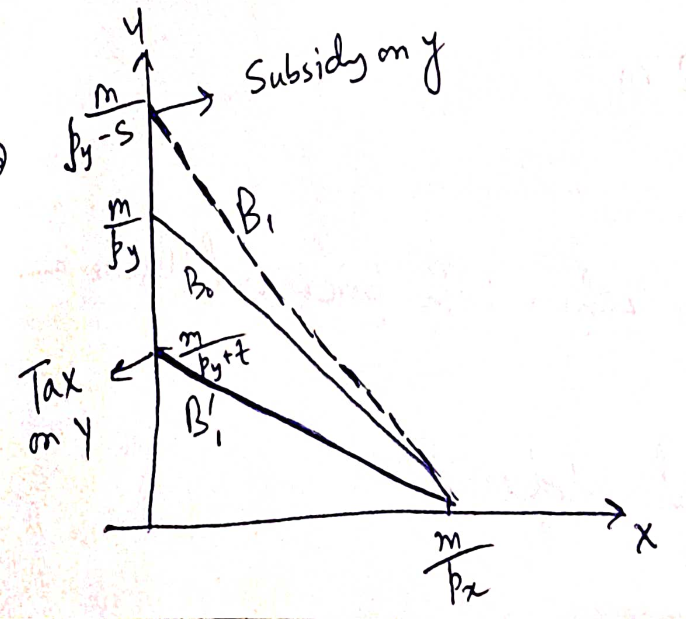
These change the slope and ~~income~~ position of B.L. by changing the prices faced by consumer and his income.

Tax on good X

	Quantity	Value	Lumpsum
(TAX)	$(p_x+t)x + p_y y = m$	$p_x(1+t)x + p_y y = m$	$p_x x + p_y y = m - t$
(SUBSIDY)	$(p_x - s)x + p_y y = m$	$p_x(1-s)x + p_y y = m$	$p_x x + p_y y = m + s$

Tax on good Y

	Quantity	Value	Lumpsum
TAX	$p_x x + (p_y + t)y = m$	$p_x x + p_y(1+t)y = m$	$p_x x + p_y y = m - t$
SUBSIDY	$p_x x + (p_y - s)y = m$	$p_x x + p_y(1-s)y = m$	$p_x x + p_y y = m + s$



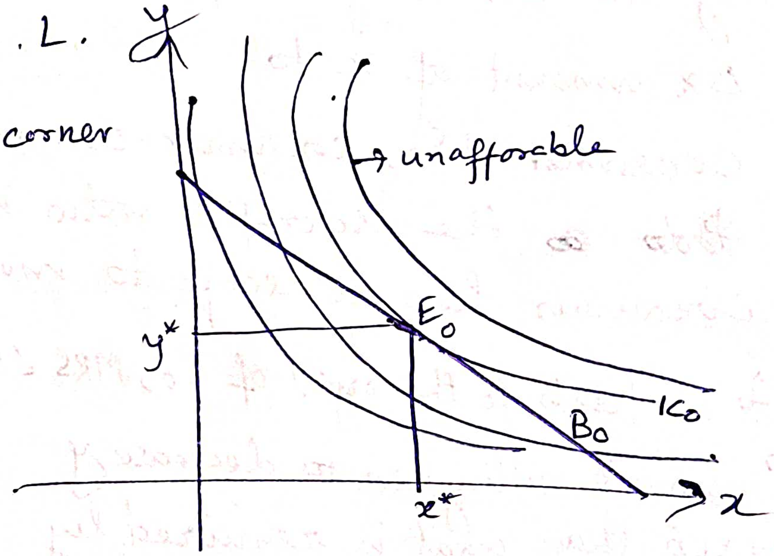
Chapter 4 – Part 3

Optimal Choice.

Let us draw the consumer's budget set and several of the consumer's indifference curves as in Fig 1.

The objective is to find what is the consumer's most preferred bundle given the budget set.

Since 'more is better', the consumer shall always choose a bundle on the B.L.



If we move from right or left corner of the B.L. we shall move from higher to higher indifference curves until we get to IC_0 which is just tangent to B.L. B_0.

The consumer cannot afford to buy any cons. bundle on an IC above IC_0 since his budget is constrained by B.L.

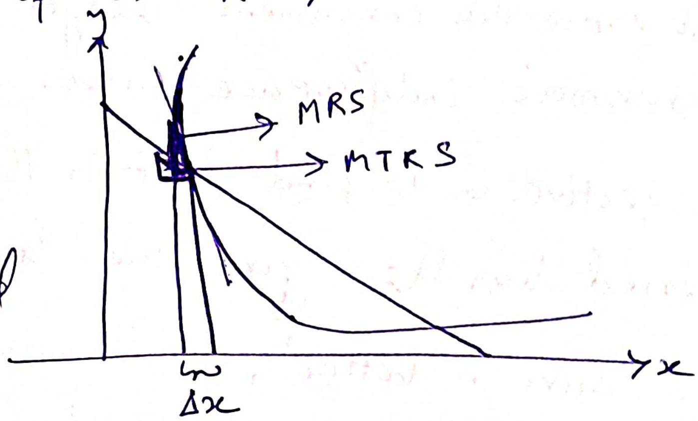
So, consumer's optimal bundle is at E_0 e.g. at the point of tangency b/w IC and B.L. → (x^*, y^*)

This is the best bundle that consumer can afford.

At E_0, MRS = $\underline{\text{MTRS}}$ ↳ Mkt. rate of ~~sb~~ substitution

At all points to the left of E_0, MRS > MTRS

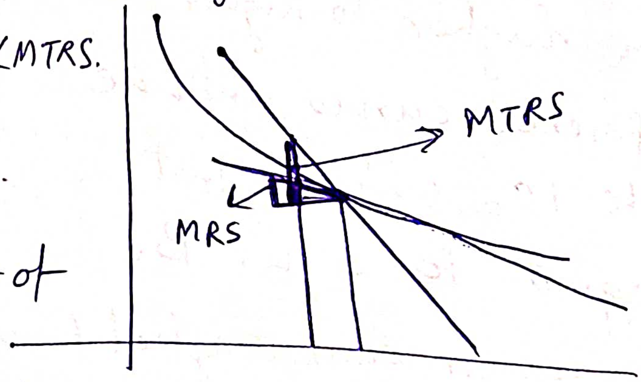
Consumer's willingness to decrease his cons. of y to get additional Δx amount of x is more than what is required by market to give up Δx amount of x to consumer.



So, consumer can move to higher utility until the sacrifice ratio is equal for both. Then consumer has no scope to move to higher utility.

At all points to the right of E_0, MRS < MTRS.

Consumer's willingness to decrease y is less than what is required by market to give him Δx amount of x. So, his utility falls as he goes to the right corner of B.L.



Alternatively, this can be seen as : $\frac{1}{MTRS} < \frac{1}{MRS} \Rightarrow \frac{P_2}{P_1} < \frac{U_2}{U_1} = \frac{dX}{dY}$

Consumer's willingness to decrease X to get an additional unit of Y is more than that what market requires to give up an additional Δy amount to consumer. So consumer moves to higher utility levels until he is at E_0.

The consumer's optimal bundle is characterised by the condition that slope of indifference curve (MRS) will equal the slope of the budget line.

We now derive this utility maximisation problem using actual utility function.

Let the consumer's preferences be represented by a utility fn. $U = u(x, y)$

The consumer faces a budget constraint.

So we have a <u>constrained maximisation problem</u>:

$$\max_{x,y} \; u(x, y) \quad \text{s.t.} \quad p_x x + p_y y = m.$$

We construct an auxilliary fn. <u>Lagrangian</u>.

$$\mathcal{L} = u(x, y) + \lambda [m - p_x x - p_y y]$$

where λ is the Lagrangian multiplier.

Acc. to Lagrangian theorem. optimal choice $[x^*, y^*]$ must satisfy 3 first order conditions [FOCs].

$$\frac{\partial \mathcal{L}}{\partial x} = \frac{\partial u}{\partial x} - \lambda p_x = 0 \quad \text{—— (1)}$$

$$\frac{\partial \mathcal{L}}{\partial y} = \frac{\partial u}{\partial y} - \lambda p_y = 0 \quad \text{—— (2)}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = m - p_x x - p_y y = 0 \quad \text{—— (3)}$$

Equation 1, 2, 3 are three eqns. in 3 unknowns. So the System is solvable.

Dividing 1 by 2, $\dfrac{\frac{\partial u}{\partial x}}{\frac{\partial u}{\partial y}} = \dfrac{\cancel{\lambda} p_x}{\cancel{\lambda} p_y} \Rightarrow \dfrac{u_x}{u_y} = \underbrace{\left(\dfrac{p_x}{p_y}\right)}_{\text{constant}}$ —— 4

Since marginal utility is also a fn. of x and y, we can solve 4 to write y in terms of x

Substituting this value of y in eqn. 3, we solve x^* and y^* in terms of p_x, p_y and m.

These optimal choices are also the demand functions as $x^* = x^*(p_x, p_y, m)$ describes how the demand of x changes as p_x, p_y or m changes.

Similarly $y^* = y^*(p_x, p_y, m)$ describes how the demand of y changes as p_x, p_y, m changes.

<u>Example :</u> C-D Utility fn, $u(x_1, x_2) = x_1^c x_2^d$

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