Research Methodology and Statistics MSC 502, Monsoon Semester, 2022-2023

Probability Theory

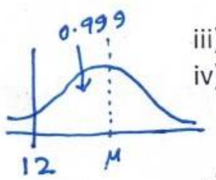
1. Each order placed by the Purchase Department has a 10% chance of being wrong. Find the probability that in a sample of 18 orders collected in a particular month,
   a) Contain exactly two wrong orders? $P(X=2) = {}^{18}C_2 (0.1)^2 (0.9)^{18} = 0.2835$
   b) contain at most four wrong orders? $P(X \leq 4) = P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4)$
   c) contain at least three wrong orders? $P(X \geq 3) = 1 - P(X \leq 2) = 0.9718$
   d) contain wrong orders between three to six? $P(3 \leq X \leq 6) = P(X=3) + P(X=4) + \cdots P(X=6) = 0.265$ $= 0.2662$

   X = # wrong order / month
   $n = 18$, $p = 0.1$
   $X \sim Bin(18, 0.1)$
   $P(X=x) = {}^nC_x\, p^x (1-p)^{n-x}$

2. The number of cracks in a certain section of highway that require repair follow a Poisson distribution with a mean of 2 cracks per mile. What is the probability that
   a) there are no cracks that require repair in 5 miles of highway? X = # cracks in 5 miles, $\lambda = 2 \times 5 = 10$ cracks/5 mi
   $P(X=0) = \frac{\lambda^x e^{-\lambda}}{x!} = \frac{10^0 \cdot e^{-10}}{0!} = 0.0000454$
   b) at least one crack require repair in one-half mile of highway?
   X = # cracks in ½ miles. $\lambda = 1$ crack/½ mile.
   $P(X \geq 1) = 1 - P(X<1) = 1 - \{P(X=0)\} = 0.6321$

3. In the past 20 years, on average only 3% of all cheques written to a company ABC have bounced. This month ABC received 200 cheques. What is the probability that
   $p = 0.03$, $n = 200$ (use Poisson distribution with $\lambda = np = 6$)
   a) exactly 4 of these cheques bounced? $P(X=4) = 0.1339$
   b) at least 4 of the cheques bounced? $P(X \geq 4) = 1 - P(X<4) = 0.8488$
   c) at most 4 of the cheques bounced? $P(X \leq 4) = 0.2851$

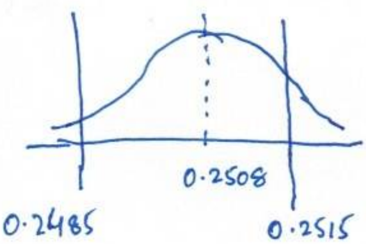
4. The fill-volume of an automated filling machine used for filling cans of carbonated beverage is normally distributed with a mean of 12.4 fluid ounces and a standard deviation of 0.1 fluid ounce.
   $\mu = 12.4$
   $\sigma = 0.1$
   i) What is the probability that a fill-volume is less than 12.26 fluid ounce? $P(X < 12.26) = 0.080$
   ii) If all cans less than 12.1 or greater than 12.6 ounces are scrapped, what proportion of cans is scrapped? $1 - P(12.1 < X < 12.6) = 0.0241$
   iii) Determine the specifications that are symmetric about mean that includes 99% of all cans? $P(-z < Z < z) = 0.99$
   iv) The mean of the filling operation can be easily adjusted easily, but the standard deviation remains at 0.1 ounce. At what value should the mean be set so that 99.9% of all cans exceed 12 ounces?



   $z = 3.08$
   $\mu = 12 + 3.08 \times 0.1 = 12.308$
   v) At what value should the mean be set so that 99.9% of all cans exceed 12 ounces if the standard deviation can be reduced to 0.05 fluid ounce? $\mu = 12 + 3.08 \times 0.05 = 12.0154$

5. The diameter of a shaft in an optical storage drive is normally distributed with mean 0.2508 inch and standard deviation 0.0005 inch. The specifications on the shaft are 0.2500±0.0015 inch. What proportion of shafts conforms to specifications? If the process is centered so that the process mean is equal to the target value of 0.2500, then find the percentage defects?

$\mu = 0.2508$, $\sigma = 0.0005$

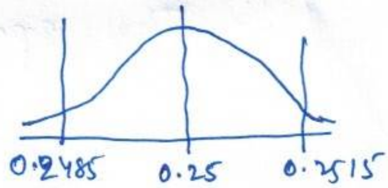


$P(z_1 < Z < z_2) = 0.5 + 0.4192 = 0.9192$

$z_1 = \frac{0.2485 - 0.2508}{0.0005} = -4.6$

$z_2 = \frac{0.2515 - 0.2508}{0.0005} = 1.4$

If the process is centred,

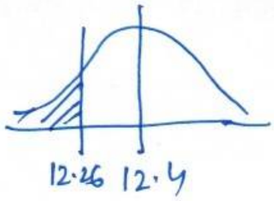


$z_1 = \frac{0.2485 - 0.25}{0.0005} = -3$

$z_2 = \frac{0.2515 - 0.25}{0.0005} = 3$

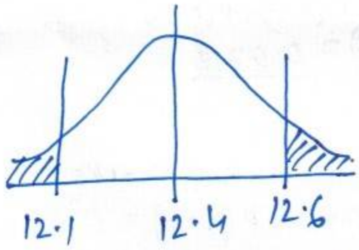
$P(-3 < Z < 3) = 0.4987 \times 2 = 0.9974$

4 (i)



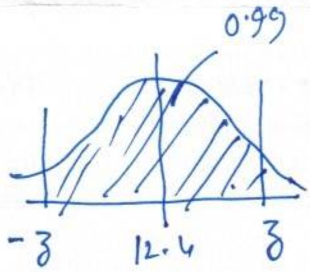
$$P(X < 12.26) = P\left(Z < \frac{12.26 - 12.4}{0.1}\right) = P\left(Z < -1.4\right) = 0.5 - 0.4192$$
$$= 0.0808$$

(ii)



$$P(12.1 < X < 12.6) = P\left(\frac{12.1 - 12.4}{0.1} < Z < \frac{12.6 - 12.4}{0.1}\right)$$
$$= P(-3 < Z < 2) = 0.4987 + 0.4772 = 0.9759$$
$$1 - P(12.1 < X < 12.6) = 1 - 0.9759 = 0.0241$$

(iii)



$$P(-z < Z < z) = 0.99,$$

From Table, $P(0 < Z < 2.575) = 0.495$

$\therefore z = 2.575$

Since, $\pm z = \frac{x - \mu}{\sigma}$ or $\cancel{2.575 = \frac{x - 12.4}{0.1}}$ or $\cancel{x =}$

or $x = \mu \pm z\sigma$ or $x = 12.4 + 2.575 \times 0.1 = 12.6575$

or $x = 12.4 - 2.575 \times 0.1 = 11.7425$

$\therefore$ The specifications will be $(11.7425, 12.6575)$