INSIDE BUSINESS 1–3

Joining the Jet Set

Recently, a major airline offered a one-year membership in its Air Club for \$125. Alternatively, one could purchase a three-year membership for \$300. Many managers and executives join air clubs because they offer a quiet place to work or relax while on the road; thus, productivity is enhanced.

Let's assume you wish to join the club for three years. Should you pay the up-front \$300 fee for a three-year membership or pay \$125 per year for three years for total payments of \$375? For simplicity, let's suppose the airline will not change the annual fee of \$125 over the next three years.

On the surface it appears that you save \$75 by paying for three years in advance. But this approach ignores the time value of money. Is paying for all three years in advance profitable when you take the time value of money into account?

The present value of the cost of membership if you pay for three years in advance is \$300, since all of that money is paid today. If you pay annually, you pay \$125 today, \$125 one year from today, and \$125 two years from today. Given an interest rate of 5 percent, the present value of these payments is

$$PV = \$125 + \frac{\$125}{1.05} + \frac{\$125}{(1.05)^2}$$

or

$$\begin{aligned} PV &= 125 + 119.05 + 113.38 \\ &= \$357.43 \end{aligned}$$

Thus, in present value terms, you save \$57.43 if you pay for three years in advance. If you wish to join for three years and expect annual fees to either remain constant or rise over the next three years, it is better to pay in advance. Given the current interest rate, the airline is offering a good deal, but the present value of the savings is \$57.43, not \$75.00.

While the notion of the present value of a firm is very general, the simplified formula presented above is based on the assumption that the growth rate of the firm's profits is constant. In reality, however, the investment and marketing strategies of the firm will affect its growth rate. Moreover, the strategies used by competitors generally will affect the growth rate of the firm. In such instances, there is no substitute for using the general present value formula and understanding the concepts developed in later chapters in this book.

Use Marginal Analysis

Marginal analysis is one of the most important managerial tools—a tool we will use repeatedly throughout this text in alternative contexts. Simply put, *marginal analysis* states that optimal managerial decisions involve comparing the marginal (or incremental) benefits of a decision with the marginal (or incremental) costs. For example, the optimal amount of studying for this course is determined by comparing (1) the improvement in your grade that will result from an additional hour of studying and (2) the additional costs of studying an additional hour. So long as the benefits of studying an additional hour exceed the costs of studying an additional hour, it is profitable to continue to study. However, once an additional hour of studying adds more to costs than it does to benefits, you should stop studying.

More generally, let $B(Q)$ denote the total benefits derived from Q units of some variable that is within the manager's control. This is a very general idea: $B(Q)$ may be the revenue a firm generates from producing Q units of output; it may be the benefits associated with distributing Q units of food to the needy; or, in the context of our previous example, it may represent the benefits derived by studying Q hours for an exam. Let $C(Q)$ represent the total costs of the corresponding level of Q. Depending on the nature of the decision problem, $C(Q)$ may be the total cost to a firm of producing Q units of output, the total cost to a food bank of providing Q units of food to the needy, or the total cost to you of studying Q hours for an exam.

Discrete Decisions

We first consider the situation where the managerial control variable is discrete. In this instance, the manager faces a situation like that summarized in columns 1 through 3 in Table 1–1. Notice that the manager cannot use fractional units of Q; only integer values are possible. This reflects the discrete nature of the problem. In the context of a production decision, Q may be the number of gallons of soft drink produced. The manager must decide how many gallons of soft drink to produce (0, 1, 2, and so on), but cannot choose to produce fractional units (for example, one pint). Column 2 of Table 1–1 provides hypothetical data for total benefits; column 3 gives hypothetical data for total costs.

Suppose the objective of the manager is to maximize the net benefits

$$N(Q) = B(Q) - C(Q),$$

which represent the premium of total benefits over total costs of using Q units of the managerial control variable, Q. The net benefits—$N(Q)$—for our hypothetical

TABLE 1–1 Determining the Optimal Level of a Control Variable: The Discrete Case

(1) Control Variable Q	(2) Total Benefits $B(Q)$	(3) Total Costs $C(Q)$	(4) Net Benefits $N(Q)$	(5) Marginal Benefit $MB(Q)$	(6) Marginal Cost $MC(Q)$	(7) Marginal Net Benefit $MNB(Q)$
Given	Given	Given	(2) − (3)	Δ(2)	Δ(3)	Δ(4) or (5) − (6)
0	0	0	0	—	—	—
1	90	10	80	90	10	80
2	170	30	140	80	20	60
3	240	60	180	70	30	40
4	300	100	200	60	40	20
5	350	150	200	50	50	0
6	390	210	180	40	60	−20
7	420	280	140	30	70	−40
8	440	360	80	20	80	−60
9	450	450	0	10	90	−80
10	450	550	−100	0	100	−100

example are given in column 4 of Table 1–1. Notice that the net benefits in column 4 are maximized when net benefits equal 200, which occurs when 5 units of Q are chosen by the manager.[5]

marginal benefit
The change in total benefits arising from a change in the managerial control variable Q.

To illustrate the importance of marginal analysis in maximizing net benefits, it is useful to define a few terms. *Marginal benefit* refers to the additional benefits that arise by using an additional unit of the managerial control variable. For example, the marginal benefit of the first unit of Q is 90, since the first unit of Q increases total benefits from 0 to 90. The marginal benefit of the second unit of Q is 80, since increasing Q from 1 to 2 increases total benefits from 90 to 170. The marginal benefit of each unit of Q—$MB(Q)$—is presented in column 5 of Table 1–1.

marginal cost
The change in total costs arising from a change in the managerial control variable Q.

Marginal cost, on the other hand, is the additional cost incurred by using an additional unit of the managerial control variable. Marginal costs—$MC(Q)$—are given in column 6 of Table 1–1. For example, the marginal cost of the first unit of Q is 10, since the first unit of Q increases total costs from 0 to 10. Similarly, the marginal cost of the second unit of Q is 20, since increasing Q from 1 to 2 increases total costs by 20 (costs rise from 10 to 30).

Finally, the *marginal net benefits* of Q—$MNB(Q)$—are the change in net benefits that arise from a one-unit change in Q. For example, by increasing Q from 0 to 1, net benefits rise from 0 to 80 in column 4 of Table 1–1, and thus the marginal net benefit of the first unit of Q is 80. By increasing Q from 1 to 2, net benefits increase from 80 to 140, so the marginal net benefit due to the second unit of Q is 60. Column 7 of Table 1–1 presents marginal net benefits for our hypothetical example. Notice that marginal net benefits may also be obtained as the difference between marginal benefits and marginal costs:

$$MNB(Q) = MB(Q) - MC(Q)$$

Inspection of Table 1–1 reveals a remarkable pattern in the columns. Notice that by using 5 units of Q, the manager ensures that net benefits are maximized. At the net-benefit-maximizing level of Q (5 units), the marginal net benefits of Q are zero. Furthermore, at the net-benefit-maximizing level of Q (5 units), marginal benefits equal marginal costs (both are equal to 50 in this example). There is an important reason why $MB = MC$ at the level of Q that maximizes net benefits: So long as marginal benefits exceed marginal costs, an increase in Q adds more to total benefits than it does to total costs. In this instance, it is profitable for the manager to increase the use of the managerial control variable. Expressed differently, when marginal benefits exceed marginal costs, the net benefits of increasing the use of Q are positive; by using more Q, net benefits increase. For example, consider the use

[5]Actually, net benefits are equal to 200 for either 4 or 5 units of Q. This is due to the discrete nature of the data in the table, which restricts Q to be selected in one-unit increments. In the next section, we show that when Q can be selected in arbitrarily small increments (for example, when the firm can produce fractional gallons of soft drink), net benefits are maximized at a single level of Q. At this level of Q, marginal net benefits are equal to zero, which corresponds to 5 units of Q in Table 1–1.

of 1 unit of Q in Table 1–1. By increasing Q to 2 units, total benefits increase by 80 and total costs increase by only 20. Increasing the use of Q from 1 to 2 units is profitable, because it adds more to total benefits than it does to total costs.

Principle

Marginal Principle
To maximize net benefits, the manager should increase the managerial control variable to the point where marginal benefits equal marginal costs. This level of the managerial control variable corresponds to the level at which marginal net benefits are zero; nothing more can be gained by further changes in that variable.

Notice in Table 1–1 that while 5 units of Q maximizes net benefits, it does not maximize total benefits. In fact, total benefits are maximized at 10 units of Q, where marginal benefits are zero. The reason the net-benefit-maximizing level of Q is less than the level of Q that maximizes total benefits is that there are costs associated with achieving more total benefits. The goal of maximizing net benefits takes costs into account, while the goal of maximizing total benefits does not. In the context of a firm, maximizing total benefits is equivalent to maximizing revenues without regard for costs. In the context of studying for an exam, maximizing total benefits requires studying until you maximize your grade, regardless of how much it costs you to study.

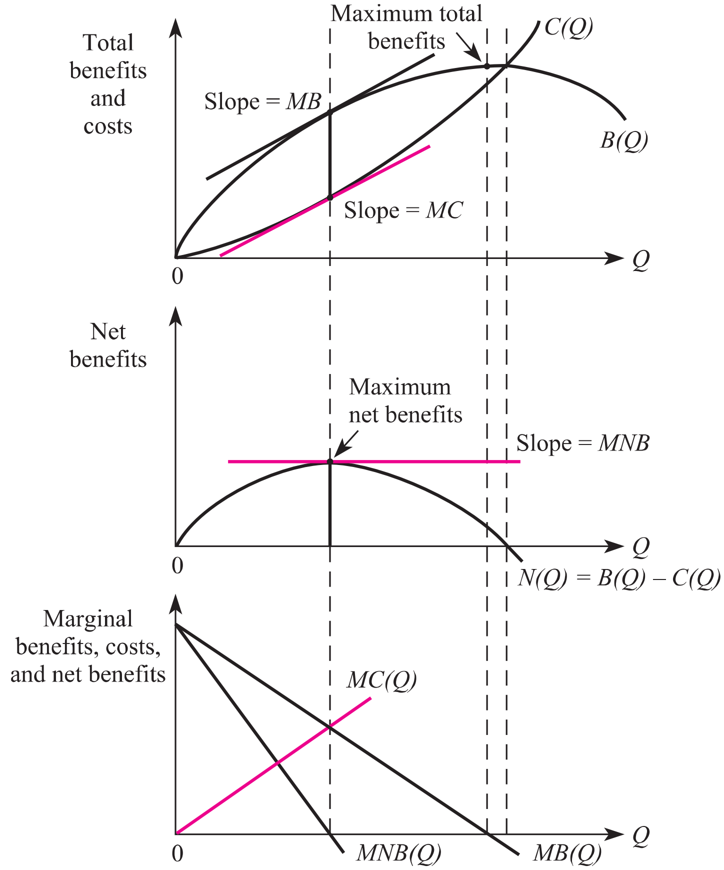
Continuous Decisions

The basic principles for making decisions when the control variable is discrete also apply to the case of a continuous control variable. The basic relationships in Table 1–1 are depicted graphically in Figure 1–2. The top panel of the figure presents the total benefits and total costs of using different levels of Q under the assumption that Q is infinitely divisible (instead of allowing the firm to produce soft drinks only in one-gallon containers as in Table 1–1, it can now produce fractional units). The middle panel presents the net benefits, $B(Q) - C(Q)$, and represents the vertical difference between B and C in the top panel. Notice that net benefits are maximized at the point where the difference between $B(Q)$ and $C(Q)$ is the greatest in the top panel. Furthermore, the slope of $B(Q)$ is $\Delta B/\Delta Q$, or marginal benefit, and the slope of $C(Q)$ is $\Delta C/\Delta Q$, or marginal cost. The slopes of the total benefits curve and the total cost curve are equal when net benefits are maximized. This is just another way of saying that when net benefits are maximized, $MB = MC$.

Principle

Marginal Value Curves Are the Slopes of Total Value Curves
When the control variable is infinitely divisible, the slope of a total value curve at a given point is the marginal value at that point. In particular, the slope of the total benefit curve at a given Q is the marginal benefit of that level of Q. The slope of the total cost curve at a given Q is the marginal cost of that level of Q. The slope of the net benefit curve at a given Q is the marginal net benefit of that level of Q.

FIGURE 1–2 Determining the Optimal Level of a Control Variable: The Continuous Case



A Calculus Alternative

Since the slope of a function is the derivative of that function, the preceding principle means that the derivative of a given function is the marginal value of that function. For example,

$$MB = \frac{dB(Q)}{dQ}$$

$$MC = \frac{dC(Q)}{dQ}$$

$$MNB = \frac{dN(Q)}{dQ}$$

The bottom panel of Figure 1–2 depicts the marginal benefits, marginal costs, and marginal net benefits. At the level of Q where the marginal benefit curve intersects the marginal cost curve, marginal net benefits are zero. That level of Q maximizes net benefits.