Decision Modelling Quiz 1 (Date: 12.09.2022, Total Marks: 20, MS 209 offline)

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Q1. Which of the following is correct about a linear programming problem (LPP)? B
(1 Mark)

A. The feasible region for an LP is the set of all points that dissatisfies all the LP's constraints and sign restrictions.
B. The feasible region for an LP is the set of all points that satisfies all the LP's constraints and sign restrictions.
C. The infeasible region for an LP is the set of all points that dissatisfies all the LP's constraints and sign restrictions.
D. The infeasible region for an LP is the set of all points that satisfies all the LP's constraints and sign restrictions.

Q2. What is the optimal solution of the following LP? (3 Marks)

$$\max z = 2x_1 - x_2$$

$$\begin{aligned} \text{s.t.} \quad x_1 - x_2 &\le 1 \\ 2x_1 + x_2 &\ge 6 \\ x_1, x_2 &\ge 0 \end{aligned}$$

Ans: Unbounded

Q3. Which of the statement is true about simplex method for linear programming? C
(1 Mark)

A. It cannot be used for two-variable problems.
B. Inequalities should not be converted into equalities.
C. It is an iterative approach to reach the optimal solution.
D. All of the above

Q4. Write the NBVs of initial basic feasible solution for the following LP: (1 Mark)

$$\max z = 60x_1 + 30x_2 + 20x_3$$

$$\begin{aligned} \text{s.t.} \quad 8x_1 + 6x_2 + x_3 &\le 48 \\ 4x_1 + 2x_2 + 1.5x_3 &\le 20 \\ 2x_1 + 1.5x_2 + 0.5x_3 &\le 8 \\ x_2 &\le 5 \\ x_1, x_2, x_3 &\ge 0 \end{aligned}$$

Ans: NBV $= \{x_1, x_2, x_3\}$

Q5. When do we need to introduce the artificial variables in the Simplex method to solve an LPP. (2 Marks)

Ans: when we do not get the initial basic feasible solⁿ of an LP even after converting into std. form, we introduce artificial variables in those eqⁿs.

Q6. A set of m *supply points* from which a good is shipped. Supply point i can supply at most si units. A set of n *demand points* to which the good is shipped. Demand point j must receive at least dj units of the shipped good. Each unit produced at supply point i and shipped to demand point j incurs a *variable cost* of c_{ij}. Let x_{ij} represents the number of units shipped from supply point i to demand point j then the general formulation of a transportation problem is: (3 Marks)

Ans: general formulation of a transportation problem is :

$$\min \sum_{i=1}^{i=m} \sum_{j=1}^{j=n} c_{ij} x_{ij} \qquad x_{ij} \geq 0$$

$$\text{s.t.} \sum_{j=1}^{j=n} x_{ij} \leq S_i \quad (i=1,2 \ldots m) \quad \text{Supply constrant}$$

$$\sum_{i=1}^{m} x_{ij} \geq d_j \quad (j=1,2 \ldots n) \quad \text{demand Constant.}$$

$\sum S_i = \sum d_j \Rightarrow$ for Balanced TP.

Q7. a. Which method is best to Solve the following LP and why? (2 Marks)

$$\max z = x_1 + x_2$$
$$\text{s.t.} \quad x_1 + x_2 \leq 4$$
$$x_1 - x_2 \geq 5$$
$$x_1, x_2 \geq 0$$

Ans: Graphical method is best because it is two variable-based LP which is easy to draw and find the visual & feasible solution simplex & other methods may become lengthy.

Q7. b. What is the optimal solution of the LP given at Q7 a. (3 Marks)

Ans LP is infeasible

Q8. The penalty of a row in a transportation problem is obtained by C (1 Mark)

A. Deducting the smallest element in the row from all other elements of the row
B. Adding the smallest element in the row to all other elements of the row
C. Deducting the smallest element in the row from the next highest element of the row
D. Deducting the smallest element in the row from the highest element in that row

Q9. What is the requirement of conducting the 'Ratio test' in the simplex method? (2 Marks)

Ans: when we try to enter a variable into the basis, it is required to identify the pivot row; ratio test helps to identify the row to enter that variable into basis.

Q10. For a maximization problem, an optimal solution to an LP is a point in the feasible region with the largest objective function value. Similarly, for a minimization problem, an optimal solution is a point in the feasible region with the smallest objective function value. (1 Mark)

True (T) Or False (F)

Ans: T

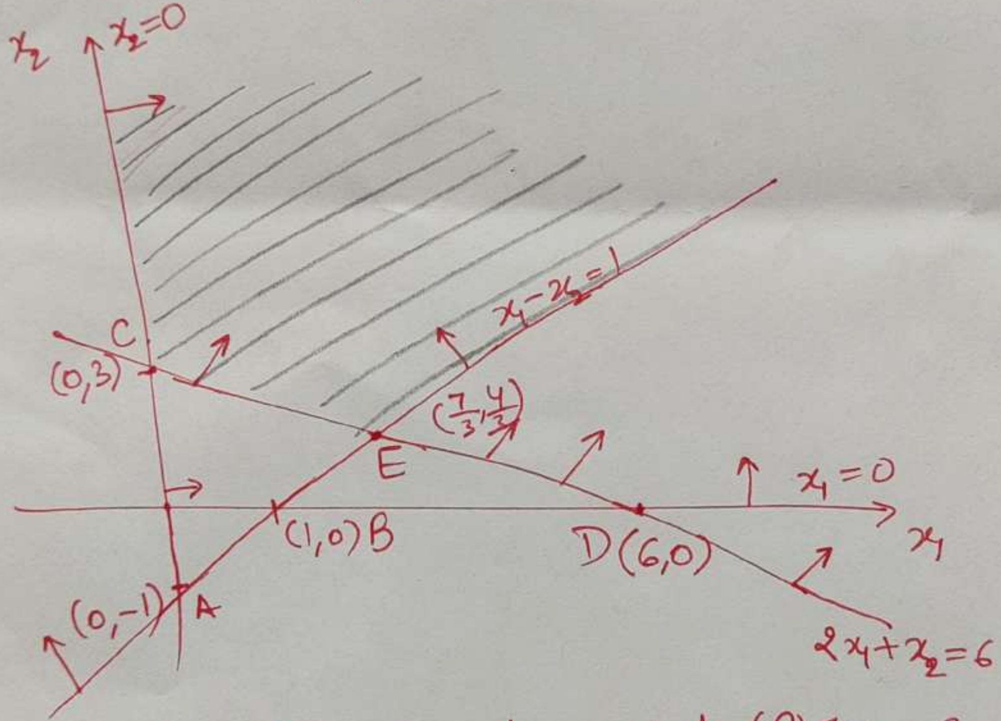
Ans 2. $\max Z = 2x_1 - x_2$

s.t $\quad x_1 - x_2 \le 1$

$2x_1 + x_2 \ge 6$

$x_1, x_2 \ge 0$

$x_1 - x_2 \le 1$	$2x_1 + x_2 = 6$	$x_1 - x_2 = 1$
If $x_1 = 0 \Rightarrow x_2 = -1$ (A)	If $x_1 = 0 \Rightarrow x_2 = 3$ (C)	$2x_1 + x_2 = 6$
If $x_2 = 0 \Rightarrow x_1 = 1$ (B)	If $x_2 = 0 \Rightarrow x_1 = 6$ (D)	$3x_1 = 7$
		$x_1 = \frac{7}{3}, \; x_2 = \frac{4}{3}$ (E)



Objective function value at extreme points (C) $Z = -3$

(E) $Z = \frac{14}{3} - \frac{4}{3} = \frac{10}{3} = 3.3$

Can the Z be improved?
If I say (0,100) yes this is a point within the feasible region but can I get the better Z?

$Z = -100$. It got worst.

So, we can see that for all those points in the unbounded region where $2x_1 > x_2 \Rightarrow Z$ will be having +ve values & increasing if you go towards Southeast.
else it will decrease.

Hence, the LP is unbounded & difficult to predict the single optimal point.

(4) $\max Z = 60x_1 + 30x_2 + 20x_3$

S.t.
$$8x_1 + 6x_2 + x_3 \le 48$$
$$4x_1 + 2x_2 + 1.5x_3 \le 20$$
$$2x_1 + 1.5x_2 + 0.5x_3 \le 8$$
$$x_2 \le 5$$
$$x_1, x_2, x_3 \ge 0$$

Std. form:

$$8x_1 + 6x_2 + x_3 + S_1 = 48$$
$$4x_1 + 2x_2 + 1.5x_3 + S_2 = 20$$
$$2x_1 + 1.5x_2 + 0.5x_3 + S_3 = 8$$
$$x_2 + S_4 = 5$$

$$x_1, x_2, x_3, S_1, S_2, S_3, S_4 \ge 0$$

Initial Basic feasible solution

$$x_1, x_2, x_3 = 0$$
$$S_1 = 48, \quad S_2 = 20, \quad S_3 = 8, \quad S_4 = 5 \Rightarrow Z = 0$$

$NBV = \{x_1, x_2, x_3\}$

$BV = \{S_1, S_2, S_3, S_4\}$

Ans 7 $\max Z = x_1 + x_2$

s.t. $x_1 + x_2 \leq 4$

$x_1 - x_2 \geq 5$

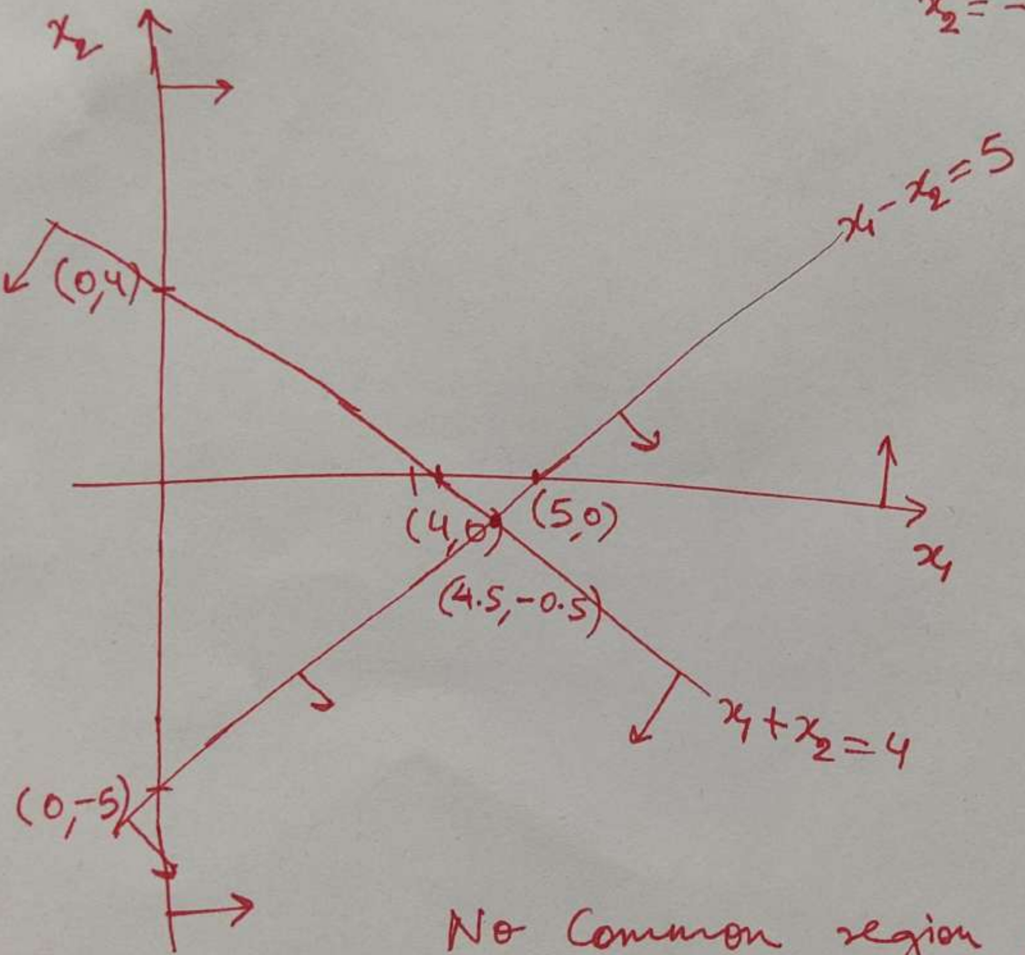
$x_1, x_2 \geq 0$

$x_1 + x_2 = 4$

$x_1 - x_2 = 5$

$\Rightarrow x_1 = 4.5$

$x_2 = -0.5$



No Common region exists

LP is Infeasible