Time Value of Money

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(1) Costs of a project are borne today, while benefits from the project or income stream or profits are received in future over several periods 1, 2, . . . , n.

(2) If we consider future income or cash flows at face value, then we are ignoring the fact that Rs 1 received or owned today is worth more than Rs 1 received in future because the opportunity cost (OC) of receiving the Rs 1 in future is the interest income you forego.

(3) Every amount received in future must be discounted for the foregone interest income i.e. calculate its Present Value (PV)

(4) Then, all the discounted future benefits/income/cashflow should be added and compared to the cost incurred today in the project / investment / production etc.

Discounted future benefit is what is present value (PV).

Present value (PV) of an amount received n years into the future is the amount that would have to be invested today at the prevailing interest rate (i) to generate the given future value (FV)

Formula 1 : $$PV_n = \frac{FV_n}{(1+i)^n}$$

For year 1 : $PV(1+i) = FV \implies PV = FV - \underbrace{PV.i}_{\substack{OCW \\ \text{Opportunity cost} \\ \text{of waiting}}}$ —— (i)

Following eqn. (i), higher the int rate i, the lower the value of a future income/cash flow in present time, since OCW is higher at higher interest rate.

Formula 2 : $$PV_{Total} = \sum_{t=1}^{n} \frac{FV_t}{(1+i)^t}$$

When there are a series of future payments $FV_1, FV_2 \ldots \ldots FV_n$ over n years, the total present value PV_{total} is the sum of all the PVs corresponding to the future payments.

$$PV_{total} = PV_1 + PV_2 + \cdot \cdot \cdot \cdot + PV_n$$

$$= \frac{FV_1}{(1+i)} + \frac{FV_2}{(1+i)^2} + \cdot \cdot \cdot \cdot + \frac{FV_n}{(1+i)^n}$$

$$= \sum_{t=1}^{n} \frac{FV_t}{(1+i)^t}$$

Future value (FV) can be written as future profits ($\pi_0, \pi_1, \ldots$) or future cash flows ($CF_1, CF_2 \ldots$)

Net Present Value (NPV)

It is the present value of the income stream generated by the project minus the current cost (C_0) of the project (we assume no costs are borne in future periods)

0	1	2	. . . .
$-C_0$	FV_1 $\downarrow$ PV_1	FV_2 $\downarrow$ PV_2	FV_n $\downarrow$ PV_n

It is an important concept to find the profitability of any decision

Formula 3:
$$NPV = \sum_{t=1}^{n} \frac{FV_t}{(1+i)^t} - C_0$$

If first term exceeds C_0, then $NPV > 0$, which means project is profitable.

If first term is less than C_0 then $NPV < 0$, means project should be rejected.

Application 1

The same concept can be utilised to compare between different projects.

Project A

Initial Cost : C_{0A}

PV_{TA} : Total present value of all future income streams generated from the project

$$: \frac{FV_{1A}}{(1+i)} + \frac{FV_{2A}}{(1+i)^2} + \cdots \frac{FV_{n}}{(1+}$$

Project B

Initial Cost : C_{0B}

$$PV_{TB} : \frac{FV_{1B}}{(1+i)} + \frac{FV_{2B}}{(1+i)^2} + \cdots + \frac{FV_{n}}{(1+}$$

Project C

Initial Cost : C_{0C}

$$PV_{TC} : \frac{FV_{1C}}{(1+i)} + \frac{FV_{2C}}{(1+i)^2} + \cdots + \frac{FV_{n}}{(1+}$$

Step 1 : Calculate the NPV of each project using Formula (3).

Step 2 : Select the project with highest NPV
For example, if $NPV_B > NPV_C > NPV_A$, then we can say Project B is most profitable.

Application 2

Application 2

Now, we use the same concept to find the value of an asset that generates cash flows (CF) that continue indefinitely.

0	1	2	. . . . . . n
CF_0	CF_1	CF_2	. . . . CF_n

One such financial asset is the "perpetuity" bond that generates identical future cash flows indefinitely (∞) ($CF_1 = CF_2 = \ldots CF_n = CF$) and $CF_0 = 0$ because current period no coupon payment is there.

$$PV_{perpetuity} = \frac{CF_1}{(1+i)} + \frac{CF_2}{(1+i)^2} + \ldots \infty$$

$$= \frac{CF}{(1+i)}\left[1 + \frac{1}{(1+i)} + \ldots \infty\right]$$

Using infinite GP series,

Formula 4 $$PV_{perpetuity} = \frac{CF}{(1+i)}\left[\frac{1}{1-\frac{1}{1+i}}\right] = \frac{CF}{\cancel{(1+i)}} \cdot \frac{\cancel{(1+i)}}{i} = \frac{CF}{i}$$

~~Formula 4~~ Application 3 : Calculating PV of a firm

PV_{firm} can be calculated using the same concept.

If a firm has a current profit of π_0 and if we can form an idea that every year the firm's profits grow at "g" rate of growth, then the stream of profits can be written as:

0	1	2	3	. . n . .	∞
π_0	π_1	π_2	π_3	π_n	π_∞

Same can be written as :

If Π_0 grows at a constant rate of 'g' then each year it will increase by a factor of $(1+g)$

	0	1	2	3	
FVs $\rightarrow$	Π_0	$\Pi_0 + \Pi_0 . g = \Pi_0(1+g)$	$\Pi_0(1+g) + g.\Pi_0(1+g)$ $\Pi_0(1+g)^2$	$\Pi_0(1+g)^3$	
PVs $\rightarrow$	Π_0	$\dfrac{\Pi_0(1+g)}{1+i}$	$\dfrac{\Pi_0(1+g)^2}{(1+i)^2}$	. . . . .	

Yr 1: $\Pi_0(1+g)$

Yr 2:
$\Pi_0(1+g) + g\Pi_0(1+g)$
$= \Pi_0(1+g)(1+g)$
$= \Pi_0(1+g)^2$

Yr 3:
$\Pi_0(1+g)^2 + g[\Pi_0(1+g)^2]$
$= \Pi_0(1+g)^3$

$$\therefore \; PV_{firm} = \Pi_0 + \Pi_0 \frac{(1+g)}{(1+i)} + \Pi_0 \frac{(1+g)^2}{(1+i)^2} + \ldots \infty$$

Using infinite GP series formula

$$a + a.r + a.r^2 + a.r^3 + \ldots \infty$$

where r is a multiplicative factor and $|r| < 1$

$|r| < 1 \rightarrow$ series converges

$$= a\left[1 + r + r^2 + \ldots r^n + \ldots \infty\right]$$

$$= \frac{a}{1-r}$$

$$PV_{firm} = \Pi_0 \left[1 + \overset{r}{\left(\frac{1+g}{1+i}\right)} + \left(\frac{1+g}{1+i}\right)^2 + \ldots \infty\right]$$

$$= \Pi_0 \left[\frac{1}{1 - \left(\frac{1+g}{1+i}\right)}\right] = \Pi_0 \left[\frac{1}{\frac{\cancel{1}+i-\cancel{1}-g}{1+i}}\right] = \Pi_0 . \frac{(1+i)}{(i-g)}$$

Formula 5

$$PV_{firm} = \Pi_0 \frac{(1+i)}{(i-g)}$$

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$$= \pi_0 \left[\frac{1}{1 - \left(\frac{1+g}{1+i}\right)}\right] = \pi_0 \left[\frac{1}{\frac{\cancel{1}+i-\cancel{1}-g}{1+i}}\right] = \pi_0 \cdot \frac{(1+i)}{(i-g)}$$

Formula 5

$$PV_{firm} = \pi_0 \frac{(1+i)}{(i-g)}$$

where π_0 is the profit in current period, g is the growth rate, i is the interest rate

Application 4

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Application 4

Suppose you know that every year a project will generate a given cash flow ~~or futu~~ and every year the project will require some investment in resources. In this case, the future value of the project will be, in each year $(CF_n - Cost_n)$ ie. substract costs borne in that year from the cash flow of that year and you arrive at that years future value.

Step 1 : Find the actual future value by substracting the cost borne in that year

Step 2 : For each future value, calculate the present value using

$$PV_n = \frac{FV_n}{(1+i)^n}$$

Step 3 : Add the calculated PVs to arrive at the NPV of the firm.

If the firm / project has some initial cost C_0 please substract it without any discounting.

Example

Each year a project generates a profit of 100,000 Rs, while to keep the project running each year you have to invest 35,000 Rs. The project runs for only 3 years. Int rate is 4 %

Step 1

0	1	2	3	. . . . . .
	FV_1 = 100000 − 35000	FV_2 = 100,000 − 35,000	FV_3 = 100,000 − 35,000	. . . . . .
	= 65,000	= 65,000	= 65,000	

Calculate the actual future value by substracting cost / investment in project from profit

Step 2 Calculate each year's PV

1	2	3
$PV_1 = \frac{65{,}000}{(1+0.04)}$	$PV_2 = \frac{65{,}000}{(1+0.04)^2}$	$PV_3 = \frac{65{,}000}{(1+0.04)^3}$

Step 3 Calculate the project's PV by adding yearly PVs.

$$PV_{project} = PV_1 + PV_2 + PV_3$$

$$= 65{,}000 \left[\frac{1}{1.04} + \frac{1}{1.04^2} + \frac{1}{1.04^3} \right]$$

~~= 65,000 [0.96150 + 0.9245 + ...]~~

Please calculate