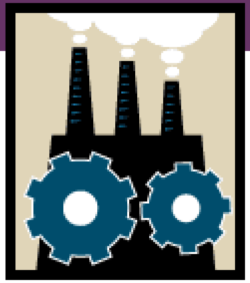
& Business Strategy

Chapter 5

The Production Process and Costs



McGraw-Hill/Irwin
Michael R. Baye, Managerial Economics and Business Strategy

Overview

I. Production Analysis

- **Total Product, Marginal Product, Average Product**
- **Isoquants**
- **Isocosts**
- **Cost Minimization**

II. Cost Analysis

- Total Cost, Variable Cost, Fixed Costs
- Cubic Cost Function
- Cost Relations

Production Analysis

- Technology provides the feasible means of converting raw inputs, such as steel, labor, and machinery, into an output such as an automobile.
- Production function is an engineering relation that defines the maximum amount of output that can be produced with a given set of inputs. $Q = F(K,L)$
  - Q is quantity of output produced.
  - K is capital input.
  - L is labor input.
  - F is a functional form relating the inputs to output.

Short-Run vs. Long-Run Decisions

- Fixed vs. Variable Inputs
- Fixed factors are the inputs the manager cannot adjust in the short run.
- Variable factors are the inputs a manager can adjust to alter production
- Short run is defined as the time frame in which there are fixed factors of production. Level of capital is fixed in the short run.
- $Q = F \quad (\quad K, \quad L)$
- Long run is defined as the horizon over which the manager can adjust all factors of production e.g. all factors of production are variable.
  $Q = F(\bar{K},L)$

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TABLE 5–1 The Production Function

(1) K^* Fixed Input (Capital) [Given]	(2) L Variable Input (Labor) [Given]	(3) ΔL Change in Labor [Δ(2)]	(4) Q Output [Given]	(5) $\frac{\Delta Q}{\Delta L} = MP_L$ Marginal Product of Labor [Δ(4)/Δ(2)]	(6) $\frac{Q}{L} = AP_L$ Average Product of Labor [(4)/(2)]
2	0	—	0	—	—
2	1	1	76	76	76
2	2	1	248	172	124
2	3	1	492	244	164
2	4	1	784	292	196
2	5	1	1,100	316	220
2	6	1	1,416	316	236
2	7	1	1,708	292	244
2	8	1	1,952	244	244
2	9	1	2,124	172	236
2	10	1	2,200	76	220
2	11	1	2,156	−44	196

Short run production function

- Since one input (capital, K) is fixed, short-run production function is essentially only a function of labor

- $Y=f(L)$

- Example: $Y=f(L) = L^2 - 10L$

- $Y\big|_{L = 20} = 400 – 200 = 200$ units

Production Function Algebraic Forms

- Linear production function: inputs are perfect substitutes.

$$Q = F(K, L) = aK + bL$$

- Leontief production function: inputs are used in fixed proportions.

$$Q = F(K, L) = \min\{bK, cL\}$$

- Cobb-Douglas production function: inputs have a degree of substitutability.

$$Q = F(K, L) = K^a L^b$$

Productivity Measures: Total Product

- Total Product (TP): maximum output produced with given amounts of inputs.
- Example: Cobb-Douglas Production Function:

  $Q = F(K,L) = K^{.5} L^{.5}$

  - K is fixed at 16 units.
  - Short run Cobb-Douglass production function:

    $Q = (16)^{.5} L^{.5} = 4 L^{.5}$
  - Total Product when 100 units of labor are used?

    $Q = 4 (100)^{.5} = 4(10) = 40$ units

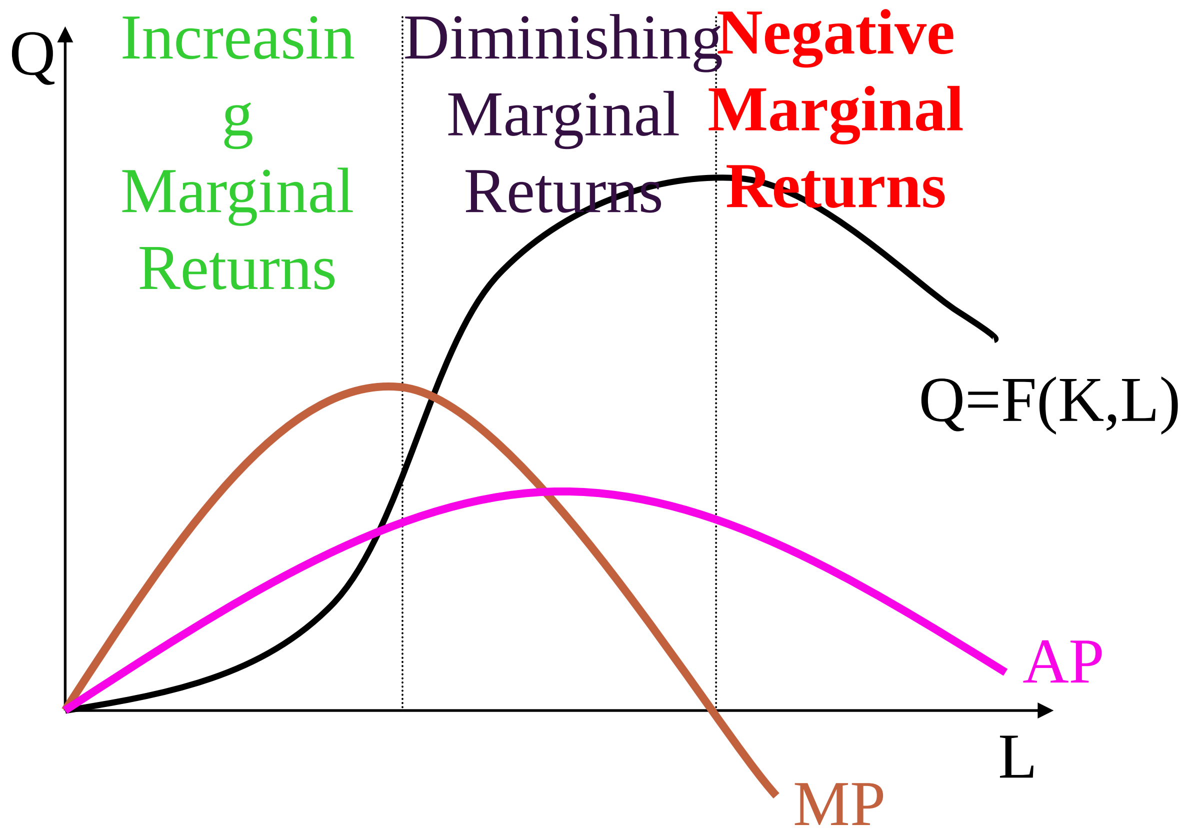
Productivity Measures: Average Product of an Input

- Average Product of an Input: measure of output produced per unit of input.
  - Average Product of Labor: $AP_L = Q/L$.
    - Measures the output of an "average" worker.
    - Example: $Q = F(K,L) = K^{.5} L^{.5}$
      - If the inputs are K = 16 and L = 16, then the average product of labor is $AP_L = [(16)^{0.5}(16)^{0.5}]/16 = 1$.
  - Average Product of Capital: $AP_K = Q/K$.
    - Measures the output of an "average" unit of capital.
    - Example: $Q = F(K,L) = K^{.5} L^{.5}$
      - If the inputs are K = 16 and L = 16, then the average product of capital is $AP_K = [(16)^{0.5}(16)^{0.5}]/16 = 1$.

+ Productivity Measures: Marginal Product of an Input

- Marginal Product on an Input: change in total output attributable to the last unit of an input.
  - Marginal Product of Labor: $MP_L = \Delta Q/\Delta L$
    - Measures the output produced by the last worker.
    - Slope of the short-run production function (with respect to labor).
  - Marginal Product of Capital: $MP_K = \Delta Q/\Delta K$
    - Measures the output produced by the last unit of capital.
    - When capital is allowed to vary in the short run, MP_K is the slope of the production function (with respect to capital).

Increasing, Diminishing and Negative Marginal Returns



Guiding the Production Process

- Producing on the production function
  - Aligning incentives to induce maximum worker effort.
- Employing the right level of inputs
  - When labor or capital vary in the short run, to maximize profit a manager will hire
    - labor until the value of marginal product of labor equals the wage: $VMP_L = w$, where $VMP_L = P \times MP_L$.
    - capital until the value of marginal product of capital equals the rental rate: $VMP_K = r$, where $VMP_K = P \times MP_K$.

Isoquant

- Illustrates the long-run combinations of inputs (K, L) that yield the producer the same level of output.
- The shape of an isoquant reflects the ease with which a producer can substitute among inputs while maintaining the same level of output.

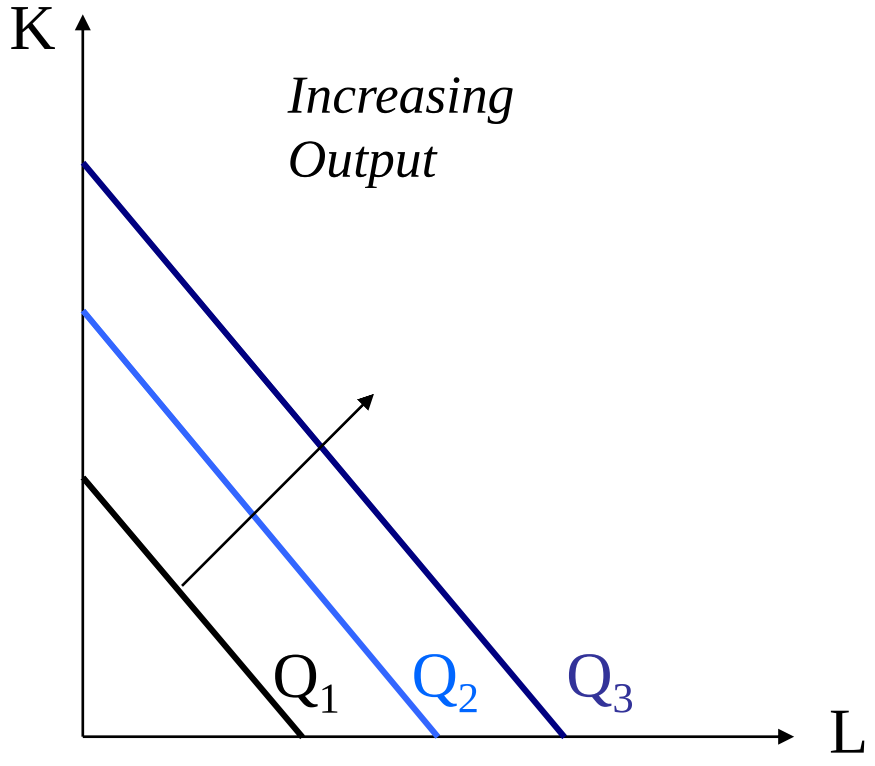
Marginal Rate of Technical Substitution (MRTS)

- The rate at which two inputs are substituted while maintaining the same output level.

$$MRTS_{KL} = \frac{MP_L}{MP_K}$$

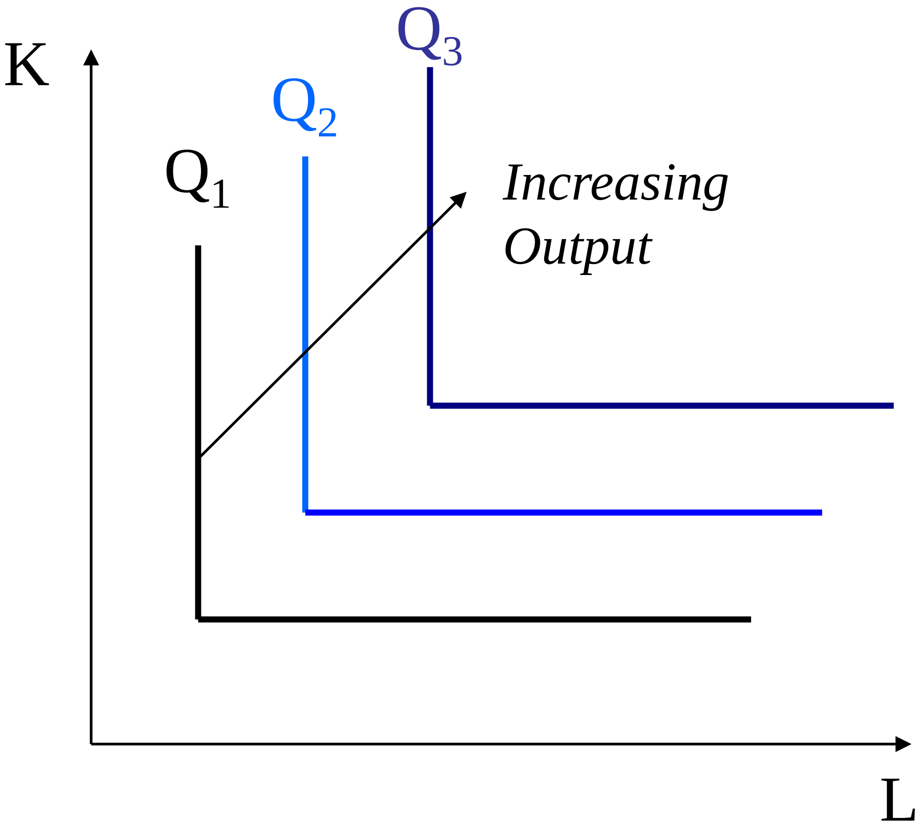
Linear Isoquants

- Capital and labor are perfect substitutes
  - $Q = aK + bL$
  - $MRTS_{KL} = b/a$
  - Linear isoquants imply that inputs are substituted at a constant rate, independent of the input levels employed.



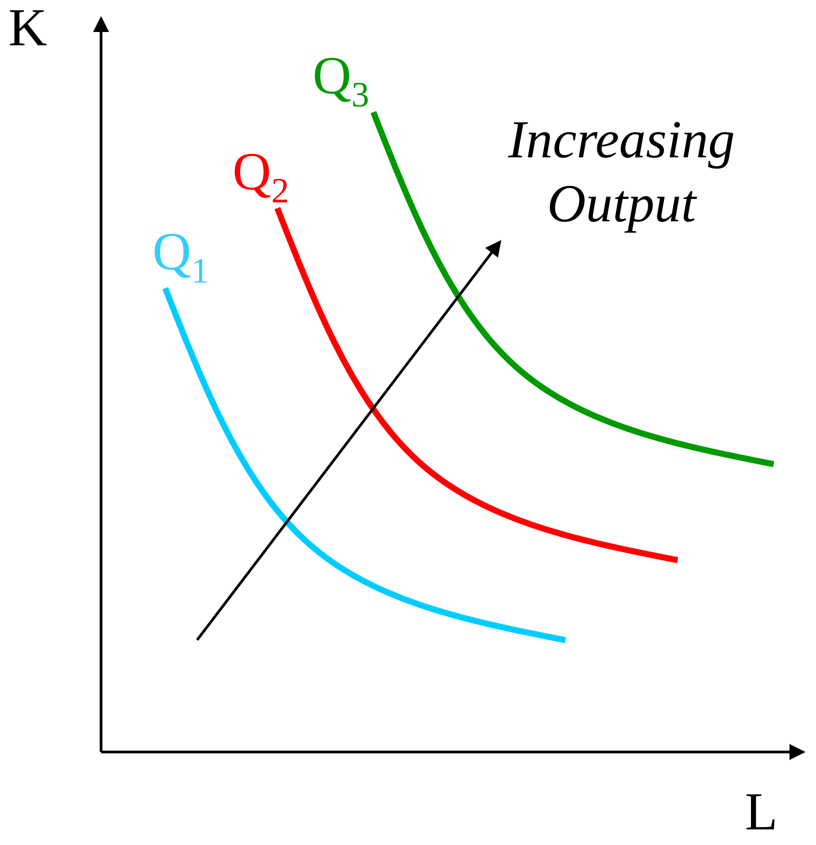
Leontief Isoquants

- Capital and labor are perfect complements.
- Capital and labor are used in fixed-proportions.
- $Q = \min\{bK, cL\}$
- Since capital and labor are consumed in fixed proportions there is no input substitution along isoquants (hence, no $MRTS_{KL}$).



Cobb-Douglas Isoquants

- Inputs are not perfectly substitutable.
- Diminishing marginal rate of technical substitution.
  - As less of one input is used in the production process, increasingly more of the other input must be employed to produce the same output level.
- $Q = K^a L^b$
- $MRTS_{KL} = MP_L/MP_K$



Isocost

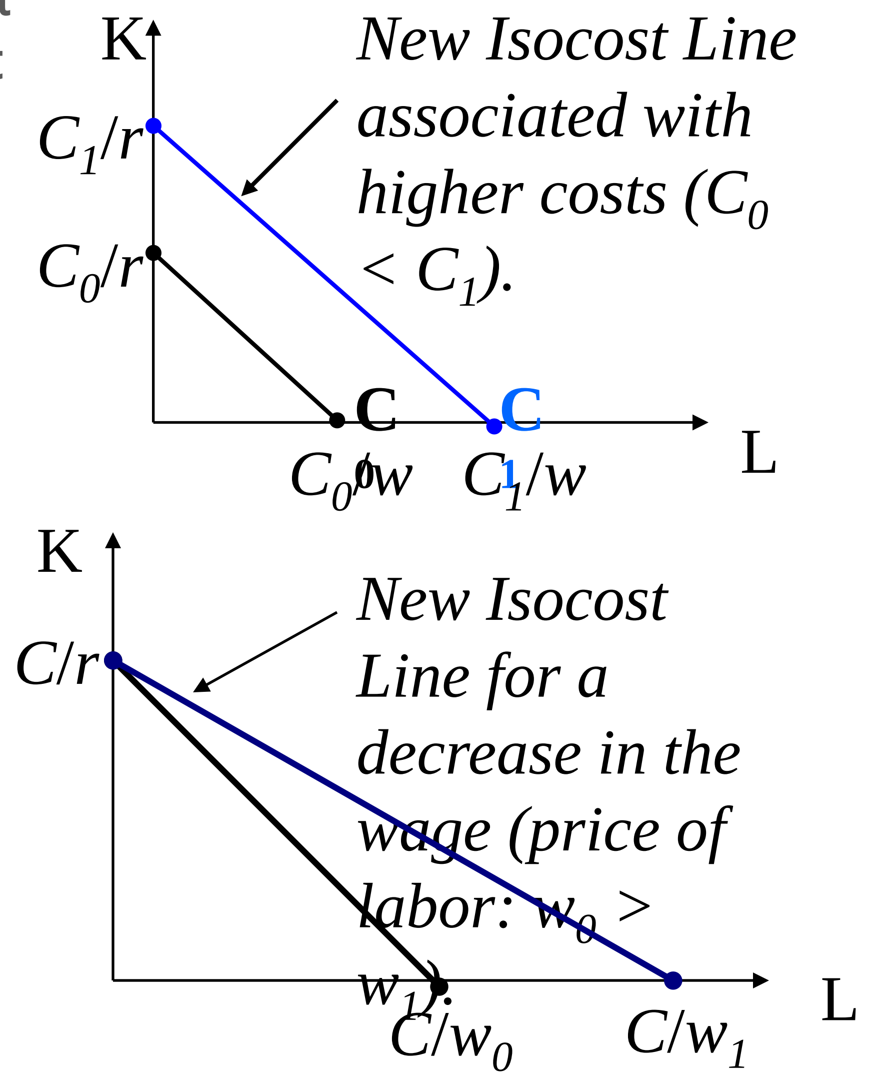
- The combinations of inputs that produce a given level of output at the same cost:

$$wL + rK = C$$

- Rearranging,

$$K = (1/r)C - (w/r)L$$

- For given input prices, isocosts farther from the origin are associated with higher costs.
- Changes in input prices change the slope of the isocost line.



Cost Minimization

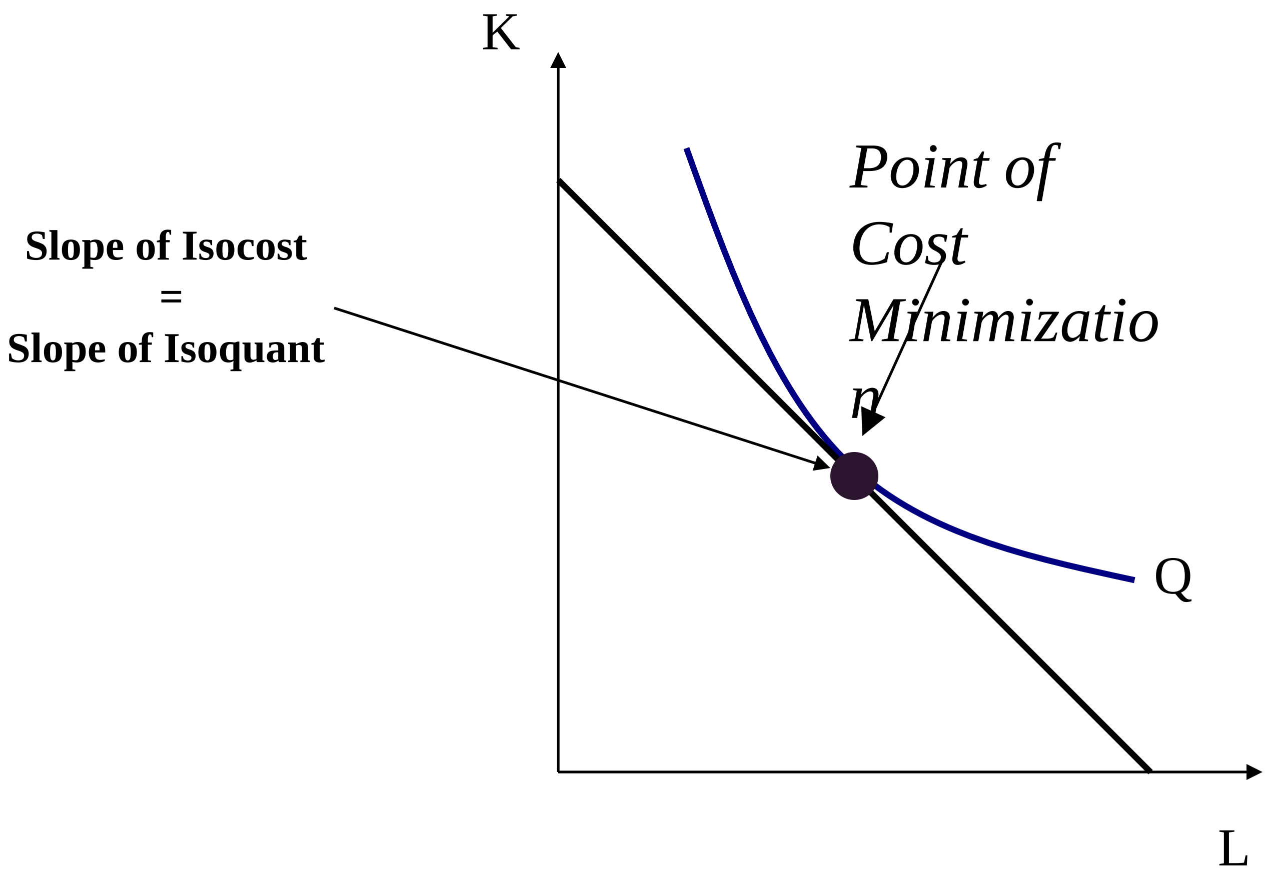
- Marginal product per dollar spent should be equal for all inputs:

$$\frac{MP_L}{w} = \frac{MP_K}{r} \Leftrightarrow \frac{MP_L}{MP_K} = \frac{w}{r}$$

- But, this is just

$$MRTS_{KL} = \frac{w}{r}$$

Cost Minimization



Optimal Input Substitution

- A firm initially produces Q_0 by employing the combination of inputs represented by point A at a cost of C_0.
- Suppose w_0 falls to w_1.
  - The isocost curve rotates counterclockwise; which represents the same cost level prior to the wage change.
  - To produce the same level of output, Q_0, the firm will produce on a lower isocost line (C_1) at a point B.
  - The slope of the new isocost line represents the lower wage relative to the rental rate of capital.