

## Time Value of Money

- Rs 1 today is worth more than Rs 1 received in the future
- Opportunity cost (OC) of money received in future: forgone interest that could be earned if Rs 1 was received today
- PV of an amount received in the future is the amount that would have to be invested today at the prevailing interest rate to generate the given future value.

**Formula (Present Value).** The present value (PV) of a future value (FV) received n years in the future is

$$PV = \frac{FV}{(1+i)^n} \tag{1-1}$$

where *i* is the rate of interest.

# Present Value Analysis

■ Higher the interest rate, the lower the present value of a future amount

$$PV = \frac{FV_1}{(1+i)^1} + \frac{FV_2}{(1+i)^2} + \frac{FV_3}{(1+i)^3} + \dots + \frac{FV_n}{(1+i)^n}$$

Formula (Present Value of a Stream). When the interest rate is i, the present value of a stream of future payments of  $FV_1, FV_2, \ldots, FV_n$  is

$$PV = \sum_{t=1}^{n} \frac{FV_t}{(1+i)^t}$$

- PV or a ruture payment : difference between the ruture value (FV) and the opportunity cost of waiting (OCW): PV = FV OCW → PV < FV as long as OCW > 0
- Higher the interest rate, lower the PV needed to generate the same FV in future.
- Net present value (NPV) of a project is simply the present value (PVt) of the income stream generated by the project minus the current cost (C0) of the project: NPV = PVt C0.

# Present Value of Indefinitely Lived Assets

$$PV_{Asset} = CF_0 + \frac{CF_1}{(1+i)} + \frac{CF_2}{(1+i)^2} + \frac{CF_3}{(1+i)^3} + \cdots$$

■ Sum1: the value of a perpetual bond that pays the owner Rs 1000 at the end of each year when the interest rate is fixed at 5 percent is given by?

## Value of a firm

■ Value of a firm is the present value of the stream of profits (cash flows) generated by the firm's physical, human, and intangible assets including its current profit

$$PV_{Firm} = \pi_0 + \frac{\pi_1}{(1+i)} + \frac{\pi_2}{(1+i)^2} + \frac{\pi_3}{(1+i)^3} + \cdots$$

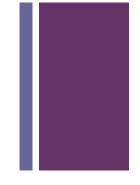
## Marginal (Incremental) Analysis

- Control Variable Examples:
  - Output
  - Price
  - Product Quality
  - Advertising
  - R&D
- Basic Managerial Question: How much of the control variable should be used?
- Firm solves this by finding out the level of control variable that maximizes net benefits. This optimal Y is denoted as Y\*
- Net benefits are maximized when, the marginal change in total benefits (MB) equals the marginal change in total costs (MC).
- So we have to study when we increase the control variable by a small amount, how the benefits and costs increase. In discrete case, we consider increasing the control variable Y by 1 unit. In continuous variable case, we consider increasing the control variable Y by an infinitesimally small amount denoted by  $\Delta Y$  or dY



# **Net Benefits**

- Net Benefits = Total Benefits (B) Total Costs (C)
- Benefits vary with Y | B is a function of Y B(Y)
- Costs vary with Y | C is a function of Y C(Y)
- Objective of manager: Maximize net benefits by choosing optimum Y
- N(Y) = B(Y) C(Y)
- Suppose we increase Y by  $\Delta Y$  unit, then B increases by  $\Delta B$  and C increases by  $\Delta C$
- When  $\triangle B > \triangle C$ , it pays to increase Y, since NB will increase
- When  $\triangle B < \triangle C$ , it pays to decrease Y, since NB will now decrease if Y is increased
- So max is at  $\triangle B = \triangle C$  or MB = MC



# Marginal Benefit (MB)

■ Change in total benefits arising from a change in the control variable, Q:

$$MB = \frac{\Delta B}{\Delta Q}$$

■ This is the Slope (calculus derivative) of the total benefit curve.

# Marginal Cost (MC)

■ Change in total costs arising from a change in the control variable, Q:

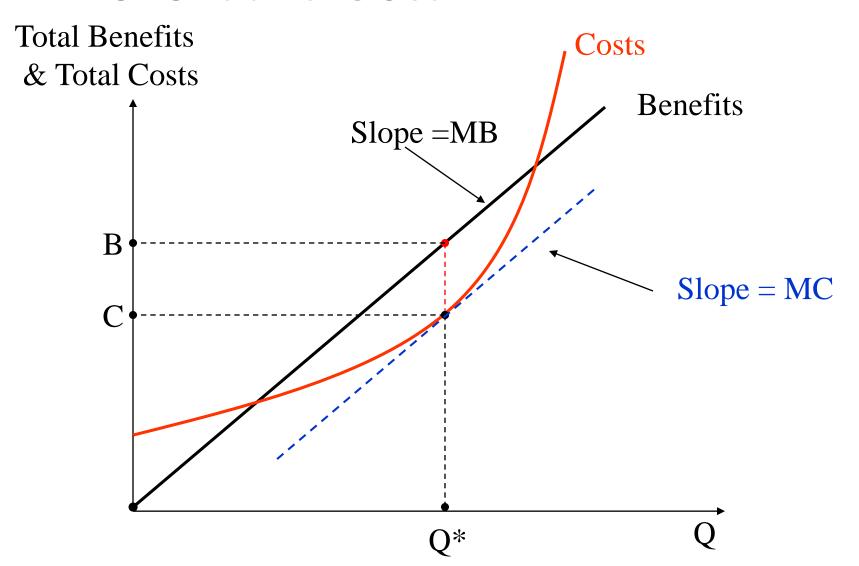
$$MC = \frac{\Delta C}{\Delta Q}$$

■ Slope (calculus derivative) of the total cost curve

## Marginal Principle

- To maximize net benefits, the managerial control variable should be increased up to the point where MB = MC.
- *MB* > *MC* means the last unit of the control variable increased benefits more than it increased costs.
- *MB* < *MC* means the last unit of the control variable increased costs more than it increased benefits.
- Sum

# The Geometry of Optimization: Total Benefit and Cost





# The Geometry of Optimization: Net Benefits

