the budget line ... decreases (in absolute value), ... the budget line increases less than price 1, the budget line ...

2.5 The Numeraire

The budget line is defined by two prices and one income, but one of these variables is redundant. We could peg one of the prices, or the income, to some fixed value, and adjust the other variables so as to describe exactly the same budget set. Thus the budget line

$$p_1 x_1 + p_2 x_2 = m$$

is exactly the same budget line as

$$\frac{p_1}{p_2} x_1 + x_2 = \frac{m}{p_2}$$

or

$$\frac{p_1}{m} x_1 + \frac{p_2}{m} x_2 = 1,$$

since the first budget line results from dividing everything by p_2, and the second budget line results from dividing everything by m. In the first case, we have pegged $p_2 = 1$, and in the second case, we have pegged $m = 1$. Pegging the price of one of the goods or income to 1 and adjusting the other price and income appropriately doesn't change the budget set at all.

When we set one of the prices to 1, as we did above, we often refer to that price as the **numeraire** price. The numeraire price is the price relative to which we are measuring the other price and income. It will occasionally be convenient to think of one of the goods as being a numeraire good, since there will then be one less price to worry about.

2.6 Taxes, Subsidies, and Rationing

Economic policy often uses tools that affect a consumer's budget constraint, such as taxes. For example, if the government imposes a **quantity tax**, this means that the consumer has to pay a certain amount to the government for each unit of the good he purchases. In the U.S., for example, we pay about 15 cents a gallon as a federal gasoline tax.

How does a quantity tax affect the budget line of a consumer? From the viewpoint of the consumer the tax is just like a higher price. Thus a quantity tax of t dollars per unit of good 1 simply changes the price of good 1 from p_1 to $p_1 + t$. As we've seen above, this implies that the budget line must get steeper.

Another kind of tax is a **value** tax. As the name implies this is a tax on the value—the price—of a good, rather than the quantity purchased of a good. A value tax is usually expressed in percentage terms. Most states in the U.S. have sales taxes. If the sales tax is 6 percent, then a good that is priced at $1 will actually sell for $1.06. (Value taxes are also known as **ad valorem** taxes.) P (1+0.06)

If good 1 has a price of p_1 but is subject to a sales tax at rate τ, then the actual price facing the consumer is $(1 + \tau)p_1$.[2] The consumer has to pay p_1 to the supplier and τp_1 to the government for each unit of the good so the total cost of the good to the consumer is $(1 + \tau)p_1$.

A **subsidy** is the opposite of a tax. In the case of a **quantity subsidy**, the government *gives* an amount to the consumer that depends on the amount of the good purchased. If, for example, the consumption of milk were subsidized, the government would pay some amount of money to each consumer of milk depending on the amount that consumer purchased. If the subsidy is s dollars per unit of consumption of good 1, then from the viewpoint of the consumer, the price of good 1 would be $p_1 - s$. This would therefore make the budget line flatter.

$(1-s) = 1$
$-s = \frac{1}{2}$
$s = 1 - \frac{1}{2}$
$= \frac{1}{2}$
$= .5$
$= 50\%$

Similarly an ad valorem subsidy is a subsidy based on the price of the good being subsidized. If the government gives you back $1 for every $2 you donate to charity, then your donations to charity are being subsidized at a rate of 50 percent. In general, if the price of good 1 is p_1 and good 1 is subject to an ad valorem subsidy at rate σ, then the actual price of good 1 facing the consumer is $(1 - \sigma)p_1$.[3]

You can see that taxes and subsidies affect prices in exactly the same way except for the algebraic sign: a tax increases the price to the consumer, and a subsidy decreases it.

Another kind of tax or subsidy that the government might use is a **lump-sum** tax or subsidy. In the case of a tax, this means that the government takes away some fixed amount of money, regardless of the individual's behavior. Thus a lump-sum tax means that the budget line of a consumer will shift inward because his money income has been reduced. Similarly, a lump-sum subsidy means that the budget line will shift outward. Quantity taxes and value taxes tilt the budget line one way or the other depending

[2] The Greek letter τ, tau, rhymes with "wow."

[3] The Greek letter σ is pronounced "sig-ma."

on which good is being taxed, but a lump-sum tax shifts the budget line inward.

Governments also sometimes impose *rationing* constraints. This means that the level of consumption of some good is fixed to be no larger than some amount. For example, during World War II the U.S. government rationed certain foods like butter and meat.

Suppose, for example, that good 1 were rationed so that no more than $\overline{x}_1$ could be consumed by a given consumer. Then the budget set of the consumer would look like that depicted in Figure 2.4: it would be the old budget set with a piece lopped off. The lopped-off piece consists of all the consumption bundles that are affordable but have $x_1 > \overline{x}_1$.

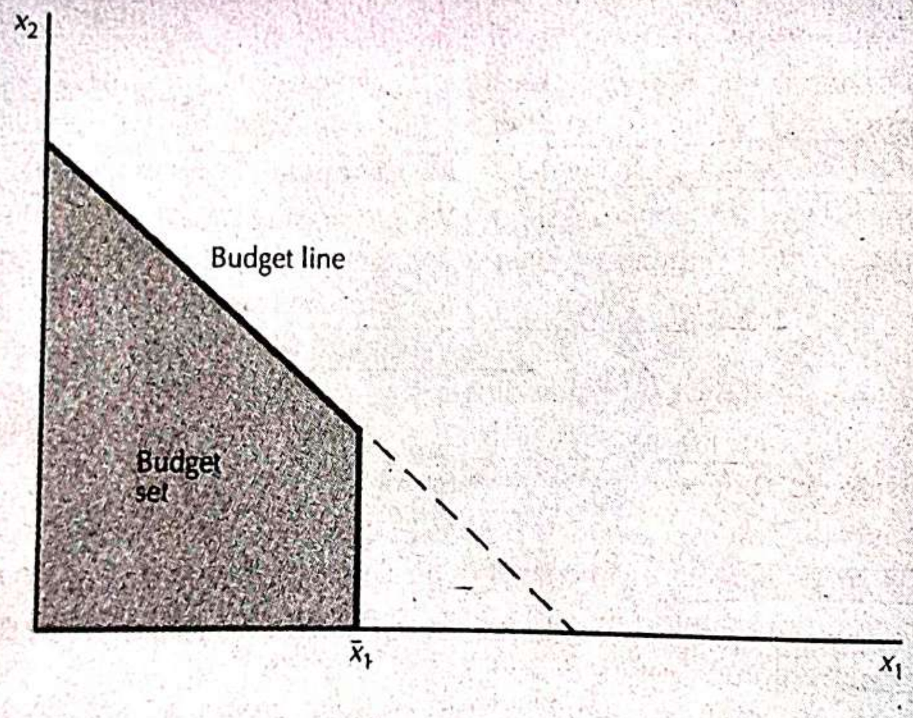
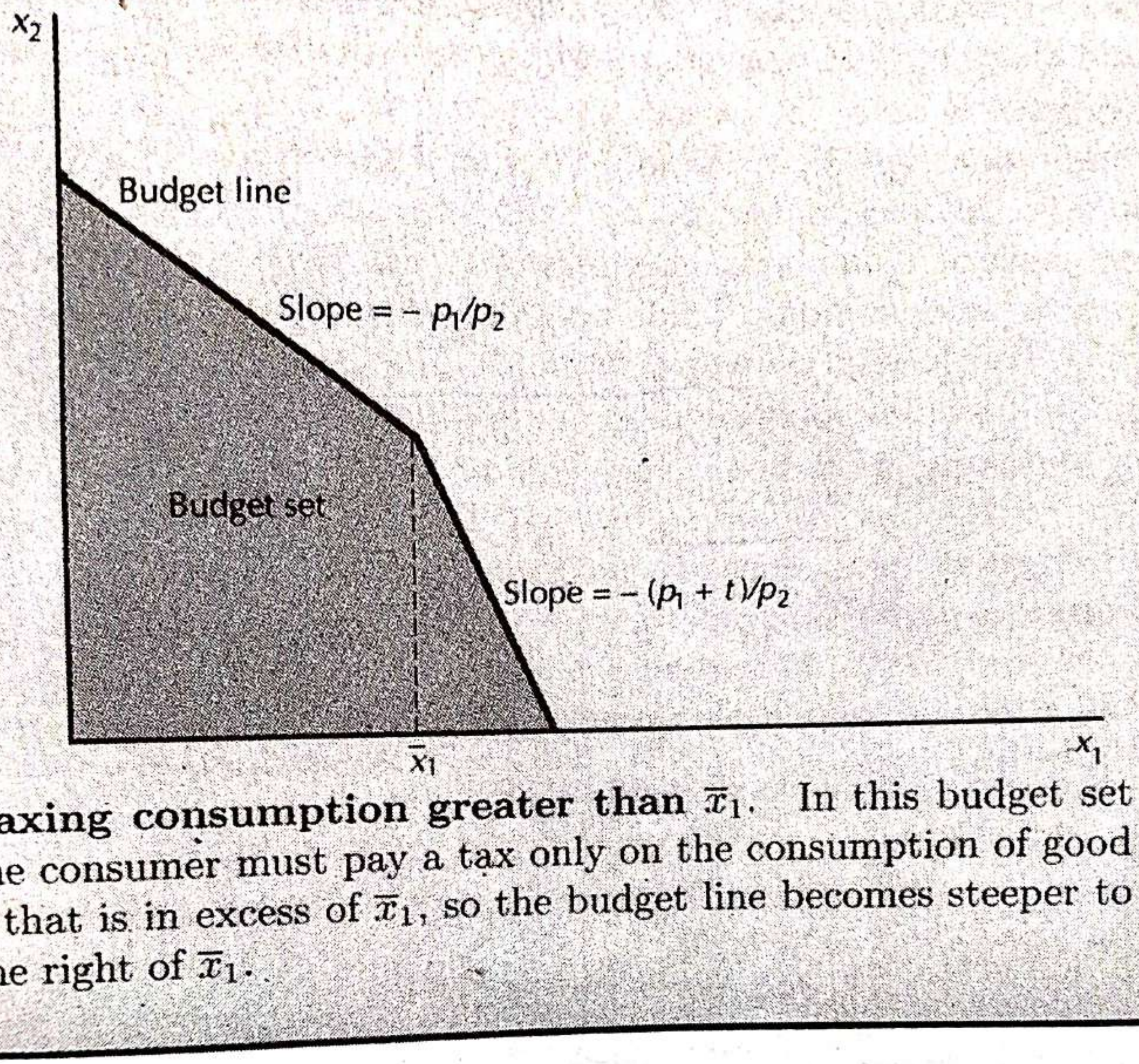


Figure 2.4

Budget set with rationing. If good 1 is rationed, the section of the budget set beyond the rationed quantity will be lopped off.

Sometimes taxes, subsidies, and rationing are combined. For example, we could consider a situation where a consumer could consume good 1 at a price of p_1 up to some level $\overline{x}_1$, and then had to pay a tax t on all consumption in excess of $\overline{x}_1$. The budget set for this consumer is depicted in Figure 2.5. Here the budget line has a slope of $-p_1/p_2$ to the left of $\overline{x}_1$, and a slope of $-(p_1 + t)/p_2$ to the right of $\overline{x}_1$.



Taxing consumption greater than $\overline{x}_1$. In this budget set the consumer must pay a tax only on the consumption of good 1 that is in excess of $\overline{x}_1$, so the budget line becomes steeper to the right of $\overline{x}_1$.

sumer would prefer to specialize, at least to some degree, and to consume only one of the goods. However, the normal case is where the consumer would want to trade some of one good for the other and end up consuming some of each, rather than specializing in consuming only one of the two goods.

In fact, if we look at my preferences for *monthly* consumption of ice cream and olives, rather than at my immediate consumption, they would tend to look much more like Figure 3.10A than Figure 3.10C. Each month I would prefer having some ice cream and some olives—albeit at different times—to specializing in consuming either one for the entire month.

Finally, one extension of the assumption of convexity is the assumption of **strict convexity**. This means that the weighted average of two indifferent bundles is *strictly* preferred to the two extreme bundles. Convex preferences may have flat spots, while *strictly* convex preferences must have indifferences curves that are "rounded." The preferences for two goods that are perfect substitutes are convex, but not strictly convex.

3.6 The Marginal Rate of Substitution

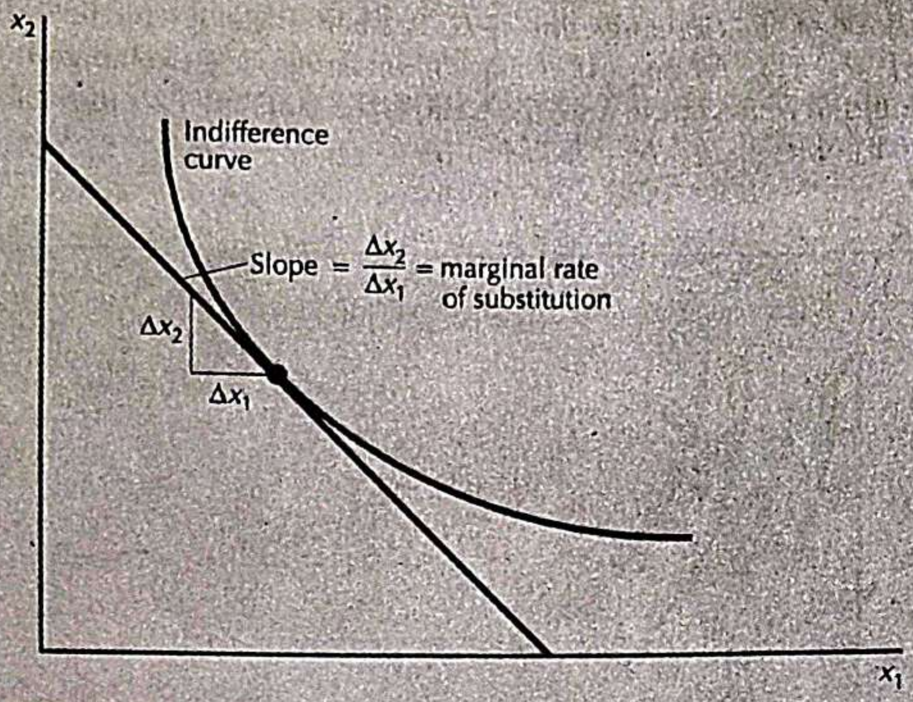
We will often find it useful to refer to the slope of an indifference curve at a particular point. This idea is so useful that it even has a name: the slope of an indifference curve is known as the **marginal rate of substitution (MRS)**. The name comes from the fact that the MRS measures the rate at which the consumer is just willing to substitute one good for the other.

Suppose that we take a little of good 1, Δx_1, away from the consumer. Then we give him Δx_2, an amount that is just sufficient to put him back on his indifference curve, so that he is just as well off after this substitution of x_2 for x_1 as he was before. We think of the ratio $\Delta x_2/\Delta x_1$ as being the *rate* at which the consumer is willing to substitute good 2 for good 1.

Now think of Δx_1 as being a very small change—a marginal change. Then the rate $\Delta x_2/\Delta x_1$ measures the *marginal* rate of substitution of good 2 for good 1. As Δx_1 gets smaller, $\Delta x_2/\Delta x_1$ approaches the slope of the indifference curve, as can be seen in Figure 3.11.

When we write the ratio $\Delta x_2/\Delta x_1$, we will always think of both the numerator and the denominator as being small numbers—as describing *marginal* changes from the original consumption bundle. Thus the ratio defining the MRS will always describe the slope of the indifference curve: the rate at which the consumer is just willing to substitute a little more consumption of good 2 for a little less consumption of good 1.

One slightly confusing thing about the MRS is that it is typically a *negative* number. We've already seen that monotonic preferences imply that indifference curves must have a negative slope. Since the MRS is the numerical measure of the slope of an indifference curve, it will naturally be a negative number.



The marginal rate of substitution (MRS). The marginal rate of substitution measures the slope of the indifference curve.

Figure 3.11

The marginal rate of substitution measures an interesting aspect of the consumer's behavior. Suppose that the consumer has well-behaved preferences, that is, preferences that are monotonic and convex, and that he is currently consuming some bundle (x_1, x_2). We now will offer him a trade: he can exchange good 1 for 2, or good 2 for 1, in any amount at a "rate of exchange" of E.

That is, if the consumer gives up Δx_1 units of good 1, he can get $E\Delta x_1$ units of good 2 in exchange. Or, conversely, if he gives up Δx_2 units of good 2, he can get $\Delta x_2/E$ units of good 1. Geometrically, we are offering the consumer an opportunity to move to any point along a line with slope $-E$ that passes through (x_1, x_2), as depicted in Figure 3.12. Moving up and to the left from (x_1, x_2) involves exchanging good 1 for good 2, and moving down and to the right involves exchanging good 2 for good 1. In either movement, the exchange rate is E. Since exchange always involves giving up one good in exchange for another, the exchange *rate* E corresponds to a *slope* of $-E$.

We can now ask what would the rate of exchange have to be in order for the consumer to want to stay put at (x_1, x_2)? To answer this question, we simply note that any time the exchange line *crosses* the indifference curve, there will be some points on that line that are preferred to (x_1, x_2)—that lie above the indifference curve. Thus, if there is to be no movement from

(x_1, x_2), the exchange line must be tangent to the indifference curve. That is, the slope of the exchange line, $-E$, must be the slope of the indifference curve at (x_1, x_2). At any other rate of exchange, the exchange line would cut the indifference curve and thus allow the consumer to move to a more preferred point.

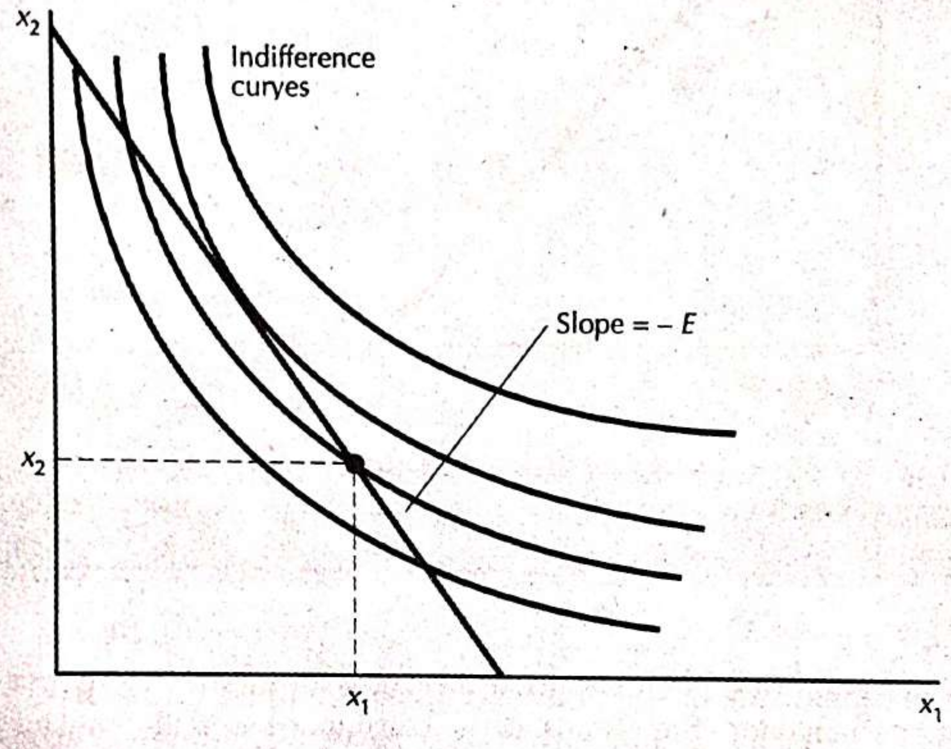
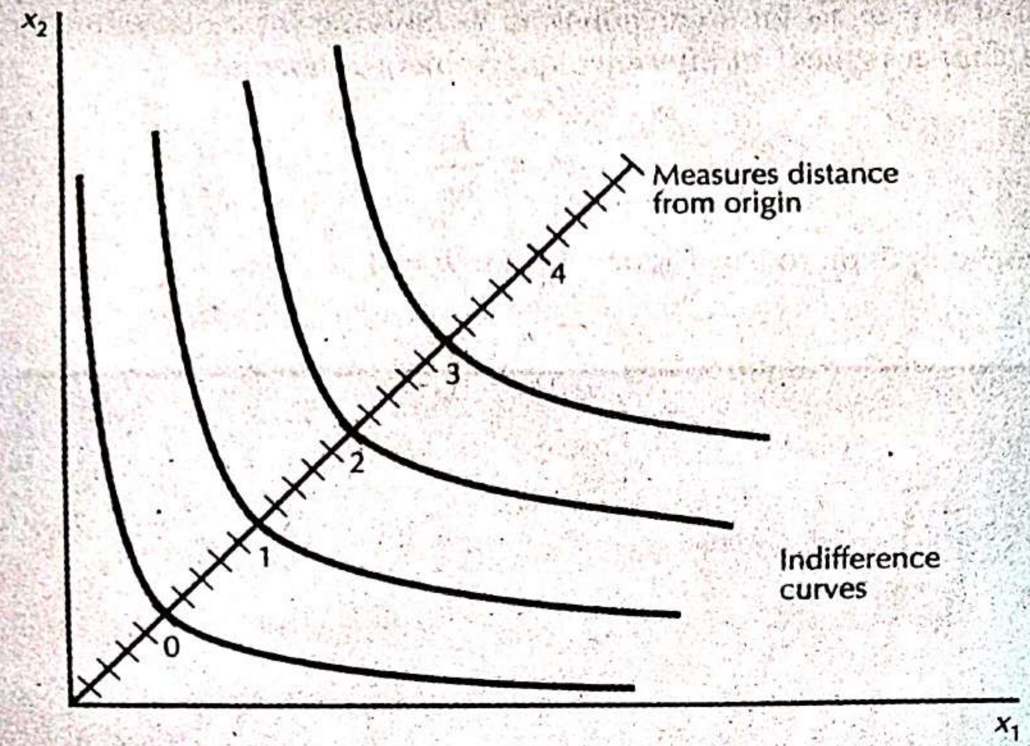


Figure 3.12 **Trading at an exchange rate.** Here we are allowing the consumer to trade the goods at an exchange rate E, which implies that the consumer can move along a line with slope $-E$.

Thus the slope of the indifference curve, the marginal rate of substitution, measures the rate at which the consumer is just on the margin of trading or not trading. At any rate of exchange other than the MRS, the consumer would want to trade one good for the other. But if the rate of exchange equals the MRS, the consumer wants to stay put.

3.7 Other Interpretations of the MRS

We have said that the MRS measures the rate at which the consumer is just on the margin of being willing to substitute good 1 for good 2. We could also say that the consumer is just on the margin of being willing to "pay" some of good 1 in order to buy some more of good 2. So sometimes



Constructing a utility function from indifference curves. Draw a diagonal line and label each indifference curve with how far it is from the origin measured along the line.

Figure 4.2

This gives us one way to find a labeling of indifference curves, at least as long as preferences are monotonic. This won't always be the most natural way in any given case, but at least it shows that the idea of an ordinal utility function is pretty general: nearly any kind of "reasonable" preferences can be represented by a utility function.

4.3 Some Examples of Utility Functions

In Chapter 3 we described some examples of preferences and the indifference curves that represented them. We can also represent these preferences by utility functions. If you are given a utility function, $u(x_1, x_2)$, it is relatively easy to draw the indifference curves: you just plot all the points (x_1, x_2) such that $u(x_1, x_2)$ equals a constant. In mathematics, the set of all (x_1, x_2) such that $u(x_1, x_2)$ equals a constant is called a **level set**. For each different value of the constant, you get a different indifference curve.

EXAMPLE: Indifference Curves from Utility

Suppose that the utility function is given by: $u(x_1, x_2) = x_1 x_2$. What do the indifference curves look like?

We know that a typical indifference curve is just the set of all x_1 and x_2 such that $k = x_1x_2$ for some constant k. Solving for x_2 as a function of x_1, we see that a typical indifference curve has the formula:

$$x_2 = \frac{k}{x_1}.$$

This curve is depicted in Figure 4.3 for $k = 1, 2, 3 \cdots$.

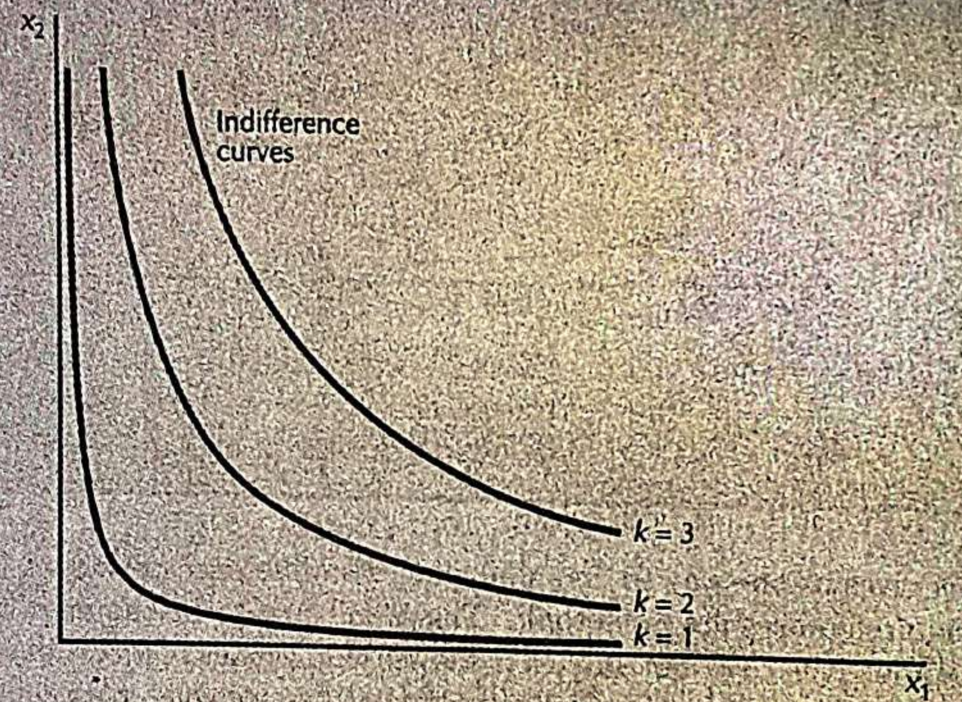


Figure 4.3 **Indifference curves.** The indifference curves $k = x_1x_2$ for different values of k.

Let's consider another example. Suppose that we were given a utility function $v(x_1, x_2) = x_1^2x_2^2$. What do its indifference curves look like? By the standard rules of algebra we know that:

$$v(x_1, x_2) = x_1^2x_2^2 = (x_1x_2)^2 = u(x_1, x_2)^2.$$

Thus the utility function $v(x_1, x_2)$ is just the square of the utility function $u(x_1, x_2)$. Since $u(x_1, x_2)$ cannot be negative, it follows that $v(x_1, x_2)$ is a monotonic transformation of the previous utility function, $u(x_1, x_2)$. This means that the utility function $v(x_1, x_2) = x_1^2x_2^2$ has to have exactly the same shaped indifference curves as those depicted in Figure 4.3. The labeling of the indifference curves will be different—the labels that were $1, 2, 3, \cdots$ will now be $1, 4, 9, \cdots$—but the set of bundles that has $v(x_1, x_2) =$

9 is exactly the same as the set of bundles that has $u(x_1, x_2) = 3$. Thus $v(x_1, x_2)$ describes exactly the same preferences as $u(x_1, x_2)$ since it *orders* all of the bundles in the same way.

Going the other direction—finding a utility function that represents some indifference curves—is somewhat more difficult. There are two ways to proceed. The first way is mathematical. Given the indifference curves, we want to find a function that is constant along each indifference curve and that assigns higher values to higher indifference curves.

The second way is a bit more intuitive. Given a description of the preferences, we try to think about what the consumer is trying to maximize—what combination of the goods describes the choice behavior of the consumer. This may seem a little vague at the moment, but it will be more meaningful after we discuss a few examples.

Perfect Substitutes

Remember the red pencil and blue pencil example? All that mattered to the consumer was the total number of pencils. Thus it is natural to measure utility by the total number of pencils. Therefore we provisionally pick the utility function $u(x_1, x_2) = x_1 + x_2$. Does this work? Just ask two things: is this utility function constant along the indifference curves? Does it assign a higher label to more-preferred bundles? The answer to both questions is yes, so we have a utility function.

Of course, this isn't the only utility function that we could use. We could also use the *square* of the number of pencils. Thus the utility function $v(x_1, x_2) = (x_1 + x_2)^2 = x_1^2 + 2x_1x_2 + x_2^2$ will also represent the perfect-substitutes preferences, as would any other monotonic transformation of $u(x_1, x_2)$.

What if the consumer is willing to substitute good 1 for good 2 at a rate that is different from one-to-one? Suppose, for example, that the consumer would require *two* units of good 2 to compensate him for giving up one unit of good 1. This means that good 1 is *twice* as valuable to the consumer as good 2. The utility function therefore takes the form $u(x_1, x_2) = 2x_1 + x_2$. Note that this utility yields indifference curves with a slope of -2.

In general, preferences for perfect substitutes can be represented by a utility function of the form

$$u(x_1, x_2) = ax_1 + bx_2.$$

Here a and b are some positive numbers that measure the "value" of goods 1 and 2 to the consumer. Note that the slope of a typical indifference curve is given by $-a/b$.

Perfect Complements

This is the left shoe–right shoe case. In these preferences the consumer only cares about the number of *pairs* of shoes he has, so it is natural to choose the number of pairs of shoes as the utility function. The number of complete pairs of shoes that you have is the *minimum* of the number of right shoes you have, x_1, and the number of left shoes you have, x_2. Thus the utility function for perfect complements takes the form $u(x_1, x_2) = \min\{x_1, x_2\}$.

To verify that this utility function actually works, pick a bundle of goods such as $(10, 10)$. If we add one more unit of good 1 we get $(11, 10)$, which should leave us on the same indifference curve. Does it? Yes, since $\min\{10, 10\} = \min\{11, 10\} = 10$.

So $u(x_1, x_2) = \min\{x_1, x_2\}$ is a possible utility function to describe perfect complements. As usual, any monotonic transformation would be suitable as well.

What about the case where the consumer wants to consume the goods in some proportion other than one-to-one? For example, what about the consumer who always uses 2 teaspoons of sugar with each cup of tea? If x_1 is the number of cups of tea available and x_2 is the number of teaspoons of sugar available, then the number of correctly sweetened cups of tea will be $\min\{x_1, \frac{1}{2}x_2\}$.

This is a little tricky so we should stop to think about it. If the number of cups of tea is greater than half the number of teaspoons of sugar, then we know that we won't be able to put 2 teaspoons of sugar in each cup. In this case, we will only end up with $\frac{1}{2}x_2$ correctly sweetened cups of tea. (Substitute some numbers in for x_1 and x_2 to convince yourself.)

Of course, any monotonic transformation of this utility function will describe the same preferences. For example, we might want to multiply by 2 to get rid of the fraction. This gives us the utility function $u(x_1, x_2) = \min\{2x_1, x_2\}$.

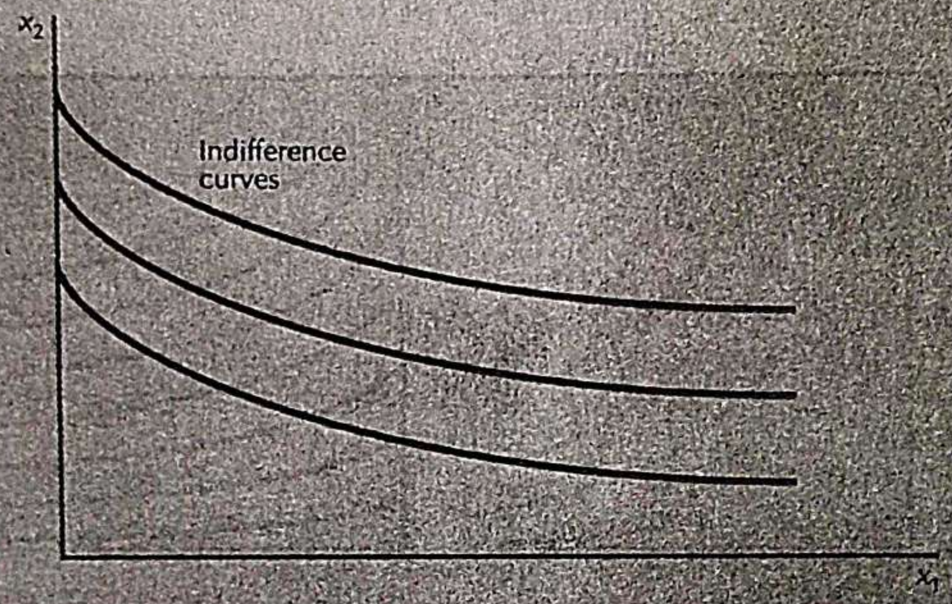
In general, a utility function that describes perfect-complement preferences is given by

$$u(x_1, x_2) = \min\{ax_1, bx_2\},$$

where a and b are positive numbers that indicate the proportions in which the goods are consumed.

Quasilinear Preferences

Here's a shape of indifference curves that we haven't seen before. Suppose that a consumer has indifference curves that are vertical translates of one another, as in Figure 4.4. This means that all of the indifference curves are just vertically "shifted" versions of one indifference curve. It follows that the equation for an indifference curve takes the form $x_2 = k - v(x_1)$, where k is a different constant for each indifference curve. This equation says that the height of each indifference curve is some function of x_1, $-v(x_1)$, plus a constant k. Higher values of k give higher indifference curves. (The minus sign is only a convention; we'll see why it is convenient below.)



Quasilinear preferences. Each indifference curve is a vertically shifted version of a single indifference curve.

Figure 4.4

The natural way to label indifference curves here is with k—roughly speaking, the height of the indifference curve along the vertical axis. Solving for k and setting it equal to utility, we have

$$u(x_1, x_2) = k = v(x_1) + x_2.$$

In this case the utility function is linear in good 2, but (possibly) nonlinear in good 1; hence the name **quasilinear utility**, meaning "partly linear" utility. Specific examples of quasilinear utility would be $u(x_1, x_2) = \sqrt{x_1} + x_2$, or $u(x_1, x_2) = \ln x_1 + x_2$. Quasilinear utility functions are not particularly realistic, but they are very easy to work with, as we'll see in several examples later on in the book.

Cobb-Douglas Preferences

Another commonly used utility function is the **Cobb-Douglas** utility function

$$u(x_1, x_2) = x_1^c x_2^d,$$

you give up one unit of good 1, you can buy p_1/p_2 units of good 2. If the consumer is at a consumption bundle where he or she is willing to stay put, it must be one where the MRS is equal to this rate of exchange:

$$\text{MRS} = -\frac{p_1}{p_2}.$$

Another way to think about this is to imagine what would happen if the MRS were different from the price ratio. Suppose, for example, that the MRS is $\Delta x_2/\Delta x_1 = -1/2$ and the price ratio is $1/1$. Then this means the consumer is just willing to give up 2 units of good 1 in order to get 1 unit of good 2—but the market is willing to exchange them on a one-to-one basis. Thus the consumer would certainly be willing to give up some of good 1 in order to purchase a little more of good 2. Whenever the MRS is different from the price ratio, the consumer cannot be at his or her optimal choice.

5.2 Consumer Demand

The optimal choice of goods 1 and 2 at some set of prices and income is called the consumer's **demanded bundle**. In general when prices and income change, the consumer's optimal choice will change. The **demand function** is the function that relates the optimal choice—the quantities demanded—to the different values of prices and incomes.

We will write the demand functions as depending on both prices and income: $x_1(p_1, p_2, m)$ and $x_2(p_1, p_2, m)$. For each different set of prices and income, there will be a different combination of goods that is the optimal choice of the consumer. Different preferences will lead to different demand functions; we'll see some examples shortly. Our major goal in the next few chapters is to study the behavior of these demand functions—how the optimal choices change as prices and income change.

5.3 Some Examples

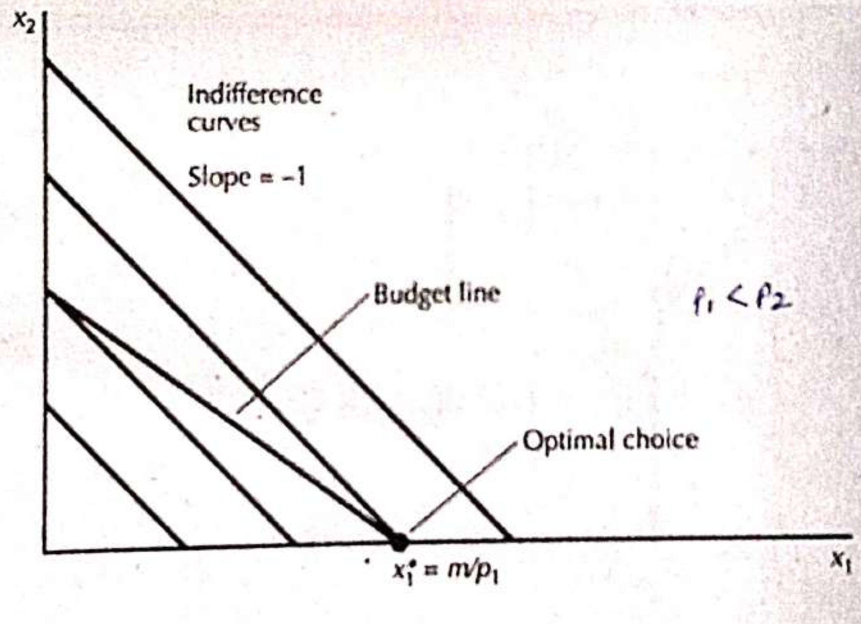
Let us apply the model of consumer choice we have developed to the examples of preferences described in Chapter 3. The basic procedure will be the same for each example: plot the indifference curves and budget line and find the point where the highest indifference curve touches the budget line.

Perfect Substitutes

The case of perfect substitutes is illustrated in Figure 5.5. We have three possible cases. If $p_2 > p_1$, then the slope of the budget line is flatter than the slope of the indifference curves. In this case, the optimal bundle is where the consumer spends all of his or her money on good 1. If $p_1 > p_2$, then the consumer purchases only good 2. Finally, if $p_1 = p_2$, there is a whole range of optimal choices—any amount of goods 1 and 2 that satisfies the budget constraint is optimal in this case. Thus the demand function for good 1 will be

$$x_1 = \begin{cases} m/p_1 & \text{when } p_1 < p_2; \\ \text{any number between 0 and } m/p_1 & \text{when } p_1 = p_2; \\ 0 & \text{when } p_1 > p_2. \end{cases}$$

Are these results consistent with common sense? All they say is that if two goods are perfect substitutes, then a consumer will purchase the cheaper one. If both goods have the same price, then the consumer doesn't care which one he or she purchases.



Optimal choice with perfect substitutes. If the goods are perfect substitutes, the optimal choice will usually be on the boundary.

Figure 5.5

Perfect Complements

The case of perfect complements is illustrated in Figure 5.6. Note that the optimal choice must always lie on the diagonal, where the consumer is purchasing equal amounts of both goods, no matter what the prices are.

In terms of our example, this says that people with two feet buy shoes in pairs.[2]

Let us solve for the optimal choice algebraically. We know that this consumer is purchasing the same amount of good 1 and good 2, no matter what the prices. Let this amount be denoted by x. Then we have to satisfy the budget constraint

$$p_1 x + p_2 x = m.$$

Solving for x gives us the optimal choices of goods 1 and 2:

$$x_1 = x_2 = x = \frac{m}{p_1 + p_2}.$$

The demand function for the optimal choice here is quite intuitive. Since the two goods are always consumed together, it is just as if the consumer were spending all of her money on a single good that had a price of $p_1 + p_2$.

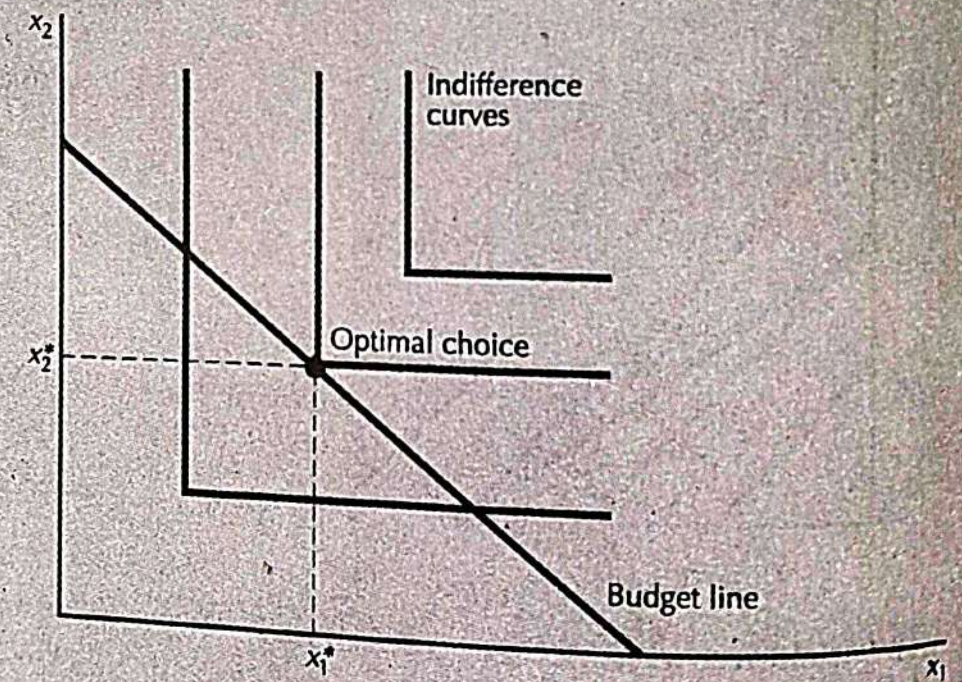


Figure 5.6

Optimal choice with perfect complements. If the goods are perfect complements, the quantities demanded will always lie on the diagonal since the optimal choice occurs where x_1 equals x_2.

[2] Don't worry, we'll get some more exciting results later on.

5.6 Choosing Taxes

Even the small bit of consumer theory we have discussed so far can be used to derive interesting and important conclusions. Here is a nice example describing a choice between two types of taxes. We saw that a **quantity tax** is a tax on the amount consumed of a good, like a gasoline tax of 15 cents per gallon. An **income tax** is just a tax on income. If the government wants to raise a certain amount of revenue, is it better to raise it via a quantity tax or an income tax? Let's apply what we've learned to answer this question.

First we analyze the imposition of a quantity tax. Suppose that the original budget constraint is

$$p_1x_1 + p_2x_2 = m.$$

What is the budget constraint if we tax the consumption of good 1 at a rate of t? The answer is simple. From the viewpoint of the consumer it is just as if the price of good 1 has increased by an amount t. Thus the new budget constraint is

$$(p_1 + t)x_1 + p_2x_2 = m. \tag{5.1}$$

Therefore a quantity tax on a good increases the price perceived by the consumer. Figure 5.9 gives an example of how that price change might affect demand. At this stage, we don't know for certain whether this tax will increase or decrease the consumption of good 1, although the presumption is that it will decrease it. Whichever is the case, we do know that the optimal choice, (x_1^*, x_2^*), must satisfy the budget constraint

$$(p_1 + t)x_1^* + p_2x_2^* = m. \tag{5.2}$$

The revenue raised by this tax is $R^* = tx_1^*$.

Let's now consider an income tax that raises the same amount of revenue. The form of this budget constraint would be

$$p_1x_1 + p_2x_2 = m - R^*$$

or, substituting for R^*,

$$p_1x_1 + p_2x_2 = m - tx_1^*.$$

Where does this budget line go in Figure 5.9?

It is easy to see that it has the same slope as the original budget line, $-p_1/p_2$, but the problem is to determine its location. As it turns out, the budget line with the income tax must pass through the point (x_1^*, x_2^*). The way to check this is to plug (x_1^*, x_2^*) into the income-tax budget constraint and see if it is satisfied.

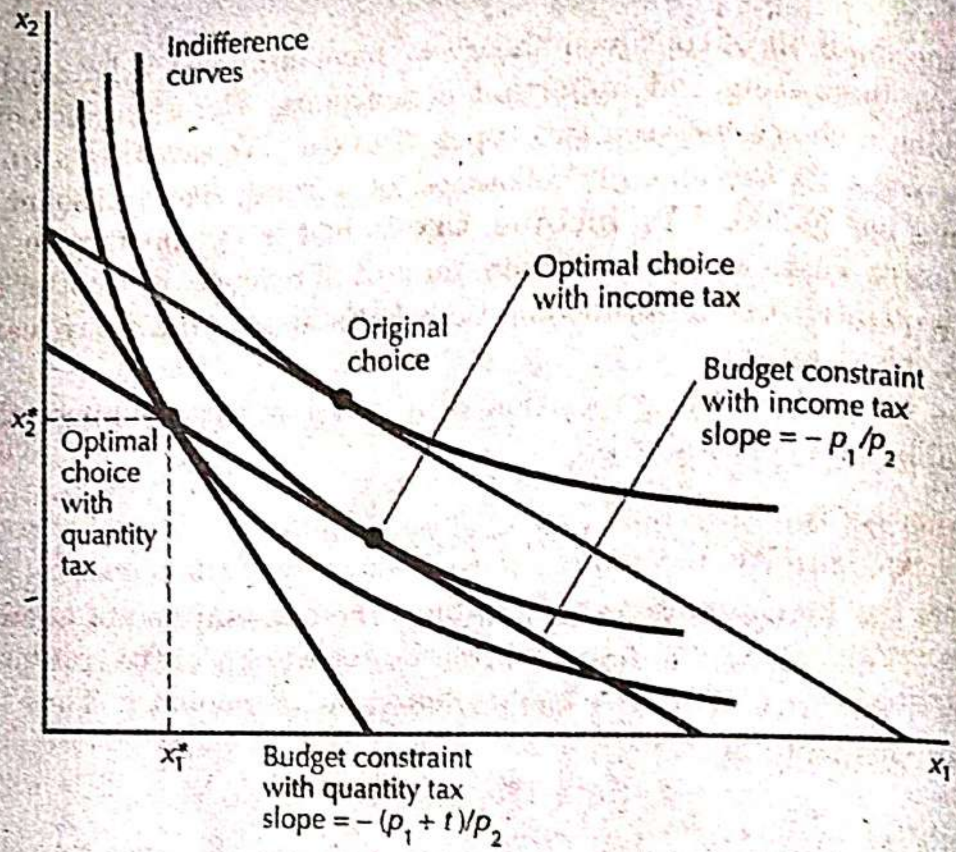


Figure 5.9

Income tax versus a quantity tax. Here we consider a quantity tax that raises revenue R^* and an income tax that raises the same revenue. The consumer will be better off under the income tax, since he can choose a point on a higher indifference curve.

Is it true that

$$p_1x_1^* + p_2x_2^* = m - tx_1^*?$$

Yes it is, since this is just a rearrangement of equation (5.2), which we know to be true.

This establishes that (x_1^*, x_2^*) lies on the income tax budget line: it is an *affordable* choice for the consumer. But is it an optimal choice? It is easy to see that the answer is no. At (x_1^*, x_2^*) the MRS is $-(p_1+t)/p_2$. But the income tax allows us to trade at a rate of exchange of $-p_1/p_2$. Thus the budget line cuts the indifference curve at (x_1^*, x_2^*), which implies that there will be some point on the budget line that will be preferred to (x_1^*, x_2^*).

Therefore the income tax is definitely superior to the quantity tax in the sense that you can raise the same amount of revenue from a consumer and still leave him or her better off under the income tax than under the quantity tax.

This is a nice result, and worth remembering, but it is also worthwhile understanding its limitations. First, it only applies to one consumer. The argument shows that for any given consumer there is an income tax that will raise as much money from that consumer as a quantity tax and leave him or her better off. But the amount of that income tax will typically differ from person to person. So a *uniform* income tax for all consumers is not necessarily better than a *uniform* quantity tax for all consumers. (Think about a case where some consumer doesn't consume any of good 1—this person would certainly prefer the quantity tax to a uniform income tax.)

Second, we have assumed that when we impose the tax on income the consumer's income doesn't change. We have assumed that the income tax is basically a lump sum tax—one that just changes the amount of money a consumer has to spend but doesn't affect any choices he has to make. This is an unlikely assumption. If income is earned by the consumer, we might expect that taxing it will discourage earning income, so that after-tax income might fall by even more than the amount taken by the tax.

Third, we have totally left out the supply response to the tax. We've shown how demand responds to the tax change, but supply will respond too, and a complete analysis would take those changes into account as well.

Summary

1. The optimal choice of the consumer is that bundle in the consumer's budget set that lies on the highest indifference curve.

2. Typically the optimal bundle will be characterized by the condition that the slope of the indifference curve (the MRS) will equal the slope of the budget line.

3. If we observe several consumption choices it may be possible to estimate a utility function that would generate that sort of choice behavior. Such a utility function can be used to predict future choices and to estimate the utility to consumers of new economic policies.

4. If everyone faces the same prices for the two goods, then everyone will have the same marginal rate of substitution, and will thus be willing to trade off the two goods in the same way.

REVIEW QUESTIONS