Assignment Problem

The assignment problem in which n workers are assigned to n jobs can be represented as an LP model in the following manner: Let c_{ij} be the cost of assigning worker i to job j, and define

$$x_{ij} = \begin{cases} 1, & \text{if worker } i \text{ is assigned to job } j \\ 0, & \text{otherwise} \end{cases}$$

Then the LP model is given as

$$\text{Minimize } z = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij}$$

subject to

$$\sum_{j=1}^{n} x_{ij} = 1, i = 1, 2, \ldots, n$$

$$\sum_{i=1}^{n} x_{ij} = 1, j = 1, 2, \ldots, n$$

$$x_{ij} = 0 \text{ or } 1$$

Transportation Problem

1 A set of m *supply points* from which a good is shipped. Supply point i can supply at most s_i units.

2 A set of n *demand points* to which the good is shipped. Demand point j must receive at least d_j units of the shipped good.

3 Each unit produced at supply point i and shipped to demand point j incurs a *variable cost* of c_{ij}.

Let

$$x_{ij} = \text{number of units shipped from supply point } i \text{ to demand point } j$$

then the general formulation of a transportation problem is

$$\min \sum_{i=1}^{i=m} \sum_{j=1}^{j=n} c_{ij} x_{ij}$$

$$\text{s.t.} \quad \sum_{j=1}^{j=n} x_{ij} \le s_i \quad (i = 1, 2, \ldots, m) \qquad \text{(Supply constraints)}$$

$$\sum_{i=1}^{i=m} x_{ij} \ge d_j \quad (j = 1, 2, \ldots, n) \qquad \text{(Demand constraints)}$$

$$x_{ij} \ge 0 \quad (i = 1, 2, \ldots, m;\ j = 1, 2, \ldots, n)$$

Hungarian Algorithm

Step 1. Determine *pi*, the minimum cost element of row *i* in the original cost matrix, and subtract it from all the elements of row *i*, *i* = 1, 2, 3.
Step 2. For the matrix created in step 1, determine *qj*, the minimum cost element of column *j*, and subtract it from all the elements of column *j*, *j* = 1, 2, 3.
Step 3. From the matrix in step 2, attempt to find a *feasible* assignment among all the resulting zero entries. Assign the Zeros Row wise, then assign the zeros column wise
3a. If all the jobs are assigned, it is optimal.
3b. Else, additional calculations are needed

Step 3b. If no feasible zero-element assignments can be found,

(i) Draw the *minimum* number of horizontal and vertical lines in the last reduced matrix to cover *all* the zero entries.

-Cover all the zeros by drawing a minimum number of straight lines as follows:

(a) Mark the rows that do not have assignment.

(b) Mark the columns (not already marked) that have zeros in marked rows.

(c) Mark the rows (not already marked) that have assignments in marked columns.

(d) Repeat (b) and (c) until no more marking is required.

(e) Draw lines through all unmarked rows and marked columns. If the number of these lines is equal to the order of the matrix then it is an optimum solution otherwise go to Step 4.

Step 4:

(i) Select the *smallest uncovered* entry, subtract it from every uncovered entry, and then add it to every entry at the intersection of two lines. The entries having single line will be unaffected.

(ii) If no feasible assignment can be found among the resulting zero entries, repeat step 3a.

Unmarked rows represent the assigned rows and the marked columns represent the assigned columns.

Each column and each row can contain just one assigned zero.

Problem1:

15	11	14	21	14
17	25	11	15	17
17	17	19	17	18
22	14	17	20	18
16	12	20	23	14

Problem 2

9	22	58	11	19
43	78	72	50	63
41	28	91	37	45
74	42	27	49	39
36	11	57	22	25