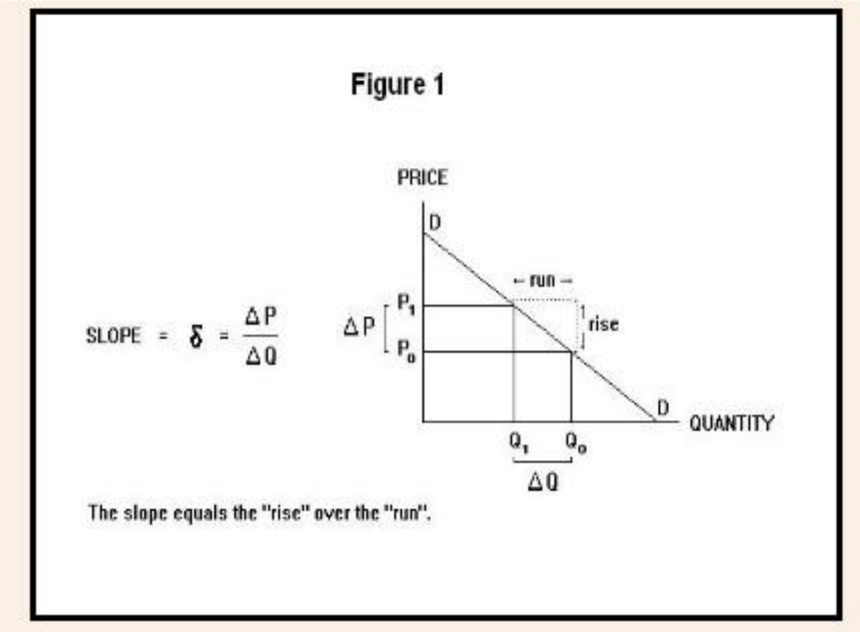
Elasticity, Total Revenue and Marginal Revenue

1. Let us start with the inverse demand function, P = function (Q) = P(Q) = a - bQ.

The slope of the demand curve is shown in Figure 1.

Slope of the demand curve, $dP/dQ = \delta < 0$

Along a linear demand curve, the slope δ is constant at every point of the demand curve.



2. Elasticity: How responsive is Q to a change in variable "P" ***in absolute terms***

By measuring the responsiveness of quantity to changes in price using the concept of elasticity, we avoid this dependence on units of measurement.

The elasticity of demand is defined as the relative change (or percentage change) in quantity divided by the relative (or percentage) change in price.

Let us use the greek symbol ε (epsilon) to denote the elasticity of demand. Then we can write

$$\varepsilon = \left|\frac{(dQ/Q)}{(dP/P)}\right| \quad \text{(please note mod means absolute value)}$$

or, $\varepsilon = -\left(\frac{dQ}{dP}\frac{P}{Q}\right)$ [Since slope of the demand curve = δ = dP /dQ <0]

----------------------------------(1)

So, the elasticity is the reciprocal of the slope multiplied by the ratio of price over quantity.

It turns out that even though the slope is constant, the elasticity will not be constant as we move along the curve because $\frac{P}{Q}$ is changing.

As should be clear from Equation 1, given a constant slope, the elasticity will decline as P / Q declines as we move down to the right along the straight-line demand curve.

At the vertical axis where Q is zero the elasticity is infinite and at the quantity axis where P is zero the elasticity is zero. All this is illustrated in Figure 2 where the elasticity of demand is measured relative to the price-quantity ratio P/Q.

So we have three distinct regions: Elastic, Unitary Elastic and Inelastic on the demand curve.

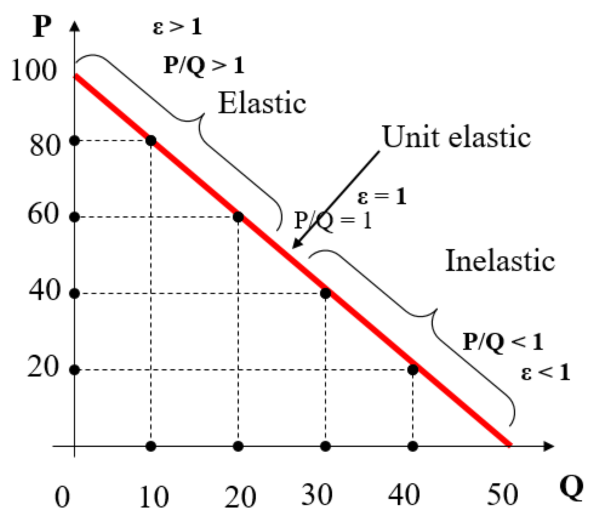


Figure 2

3. Relationship between Total Revenue, Marginal Revenue and Elasticity:

The total revenue to the seller of a commodity, is obtained by multiplying the price by the quantity.

TR = P(Q) Q ------------------------------(2)

It appears in Figure 3 as the area of a rectangle whose bottom left corner is the origin and top right corner is a point on the demand curve. The top left and bottom right corners equal price and quantity respectively. The shaded rectangle in Figure 4, for example, gives the total revenue at point c on the demand curve---the product of the price P0 and the quantity Q0. The total revenue at point c is the rectangle

P_0 c Q_0 0.

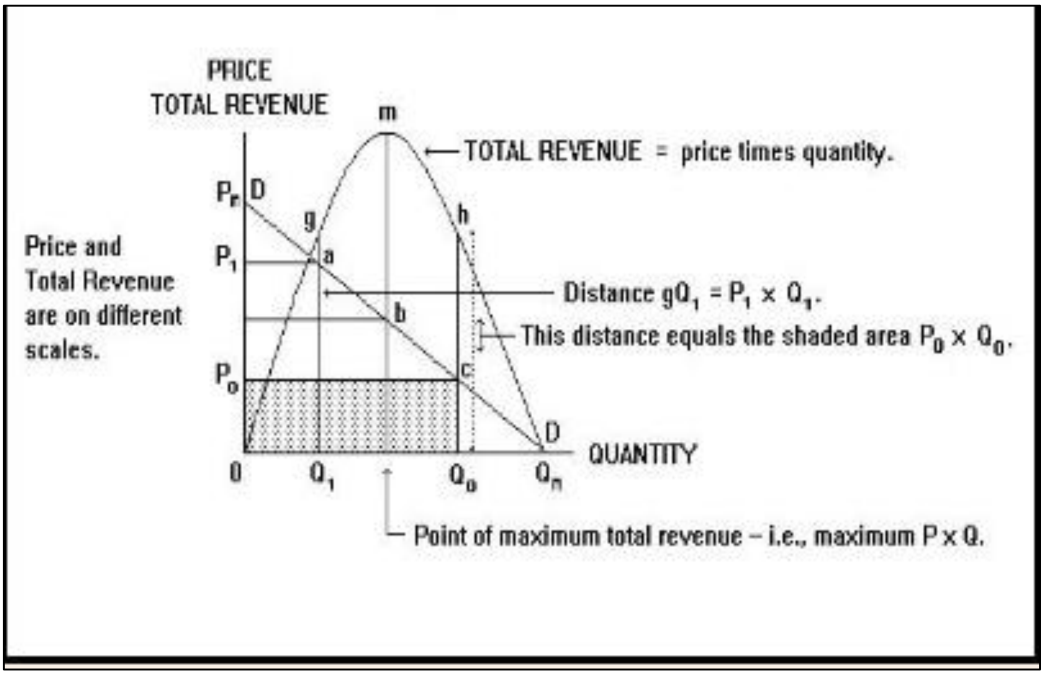


Figure 3

The total revenue varies as we move along the demand curve.

The total revenue at zero quantity and price Pm is zero. As we move down along the demand curve, the total revenue increases (this is the elastic part of the demand curve), reaching its maximum at the point b (which is unitary elastic part of the demand curve) and then declines (this is the inelastic part of the demand curve), reaching zero again at price zero and quantity Qm. So, initially the slope of the TR curve, which is nothing but the Marginal Revenue (MR), is positive and crosses the zero line when TR is maximum and becomes negative beyond that point.

The derivation of this relationship is as follows:

TR = P(Q) Q

MR = d(TR)/dQ = P(Q) + (dP(Q)/dQ). Q

$$= P(Q)\left[1+ \frac{dP(Q)}{dQ}\frac{Q}{P}\right]$$

$$= P(Q)\left[1- \left(-\frac{dP(Q)}{dQ}\frac{Q}{P}\right)\right]$$

$$= P(Q)\left[1 - \left(\frac{1}{\boldsymbol{\varepsilon}}\right)\right] \quad \{\text{since } \boldsymbol{\varepsilon} = (-dQ/dP)(P/Q)\}$$

$$= P(Q)\left(\frac{\boldsymbol{\varepsilon} - 1}{\boldsymbol{\varepsilon}}\right) \quad \text{------------------------------(3)}$$

From above equation (3),

When $\varepsilon > 1$, MR > 0 (TR rising) ➔ Elastic portion of demand curve

When $\varepsilon = 1$, MR = 0 (TR maximum) ➔ Unitary Elastic portion of demand curve

When $\varepsilon < 1$, MR < 0 (TR falling) ➔ Inelastic portion of demand curve