Logic Quiz 1

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1. For the following formulas, find a satisfying assignment if one exists. Otherwise, provide a proof that it is not satisfiable.

   (a) $(\neg p_1 \vee p_2 \vee p_3) \wedge ((\neg p_2 \wedge \neg p_3) \vee (p_4 \wedge p_5)) \wedge p_1 \wedge \neg p_5$ (1)

   (b) $(\neg p_1 \vee \neg p_2) \wedge (p_3 \vee (p_4 \wedge p_5)) \wedge p_1 \wedge (p_3 \vee \neg p_2) \wedge (\neg p_3 \vee p_2)$ (1)

2. You are walking in a labyrinth and all of a sudden you find yourself in front of three possible roads: the road on your left is paved with gold, the one in front of you is paved with marble, while the one on your right is made of small stones. Each street is protected by a guardian. You talk to the guardians and this is what they tell you:

   - The guardian of the gold street: "This road will bring you straight to the center. Moreover, if the stones take you to the center, then also the marble takes you to the center."
   - The guardian of the marble street: "Neither the gold nor the stones will take you to the center."
   - The guardian of the stone street: "Follow the gold and you'll reach the center, follow the marble and you will be lost."

   Given that you know that all the guardians are liars, can you choose a road being sure that it will lead you to the center of the labyrinth? If this is the case, which road do you choose?

   Clearly specify the intended meaning associated with every atomic proposition and formula that you use. (2)

3. Suppose α is a formula and X is a finitely satisfiable set of formulas. Prove that at least one of $X \cup \{\alpha\}, X \cup \{\neg\alpha\}$ is finitely satisfiable. (2.5)

4. We know the finite satisfiability lemma, which states that if a set of formulas is not satisfiable, then a finite subset is not satisfiable. This is a semantic concept. Is the corresponding syntactic concept true? I.e., if a set of formulas is inconsistent, does there exist a finite subset that is inconsistent? (2.5)

5. You are given a graph and required to check if it is 3-colourable. Translate the problem into the satisfiability problem of propositional logic. Clearly specify the intended meaning associated with every atomic proposition and formula that you use. (1)