Quiz (1)

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Q1 for the following formula find a satisfying truth assignment if one exists

(a) $\underbrace{(\neg P_1 \vee P_2 \vee P_3)}_{\xi_1} \wedge ((\underbrace{\neg P_1 \wedge \neg P_3}_{\xi_2}) \vee \underbrace{(P_4 \wedge P_5)}_{\xi_3}) \wedge \underbrace{P_1}_{\xi_4} \wedge \underbrace{(\neg P_5)}_{\xi_5} \Big\} \xi$

(b) $(\underset{F}{\neg P_1} \vee \underset{T}{\neg P_2}) \wedge (\underset{F}{P_3} \vee (P_4 \wedge P_5)) \wedge \underset{T}{P_1} \wedge (\underset{F}{P_3} \vee \underset{T}{\neg P_2}) \wedge (\underset{T}{\neg P_3} \vee \underset{F}{P_2})$

~~Q2 You are walking on a labyrinth~~ later

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(a) No satisfying assignment exists

for $v(\xi)$ to be true $v(\xi_5)$ and $v(\xi_4)$ must be True.

then $v(P_1) = T$

$v(P_5) = F$

But, $v(P_1) = T \Rightarrow v(\neg P_1) = F$

$\Rightarrow v((\neg P_1 \wedge \neg P_3)) = F.$

Similarly, $v(\neg P_5) = T$

$\therefore v(P_5) = F$

$v(P_4 \wedge P_5) = F$

$\therefore$ So, if $v(\xi_4)$ and $v(\xi_5) = $ True

then $v(\xi_2 \vee \xi_3) = $ False

So, no assignment exists.

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(b)

$v(P_1) = T$

$v(P_2) = F$

$v(P_3) = F$

$v(P_4) = T$

$v(P_5) = T$

} Satisfying assignment!

2) You are walking in a labyrinth and suddenly ...

of 3 possible roads.

① left road is paved with gold

② front Road is paved with marble

③ Right Road is paved with small stones

Each street is protected by a guardian.

You talk to the guardians

① LG/Guard : "This road will take you to the center. Moreover, if the stone takes you to the center, then the marble also.

② RG : "follow the gold and you'll reach the center. Follow the marble ; you will be lost"

③ FG : " Non the gold, non the stone will take you to the center"

<u>Now</u>

Gold Road will take to the center $= G$

Marble Road will take to the center $= M$

Stone Road " " " " $= S$

1. Guardian says $(G \wedge (S \Rightarrow M))$

F " " : $(\neg G \wedge \neg S)$

R " " : $(G \wedge \neg M)$

$v(\neg G \wedge \neg S) = F$

$v(G \wedge \neg M) = F$ $\quad$ $v(G \wedge (S \Rightarrow M)) = F$

~~$v(G \vee S) = T$~~ $\quad$ ~~$v(\neg G \vee M) = T$~~

G	S	M	$\neg G \wedge \neg S$	$G \wedge \neg M$	$G \wedge (S \Rightarrow M)$
T	T	T	F	F	T
T	T	F	F	T	F
T	F	T	F	F	T
T	F	F	F	T	T
F	T	T	F	F	F
F	T	F	F	F	F
F	F	T	T	F	F
F	F	F	T	F	F

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when $v(G) = F$, $v(S) = T$, $v(M) = T$ or $v(G) = F$, $v(S) = T$, $v(M) = F$ three of them will be satisfies

Now, as in both cases, going through stone path will take me to the labyrinth

- I should take stone path.

syntactic counterpart will be

If ~~every~~ every finite subset of Δ is consistent, then Δ is consistent

So, let us assume that

Δ is inconsistent then we know

$\Delta \vdash \varphi$ and ~~[illegible]~~ $\Delta \vdash \neg\varphi$ for ~~any~~ some formula φ.

Now, let $\{\alpha_1 \ldots \alpha_n\}$ be the hypothesis in Δ that are used to

derive φ using Modus Ponens.

Then all the same hypothesis can be used $\{\alpha_1 \ldots \alpha\}$ to derive

$\neg\varphi$ using modus ponens.

$\Delta_f = \{\alpha_1 \ldots \alpha_n\}$ and $\Delta_f \vdash \varphi$

$\therefore \Delta_f \subseteq_{fin} \Delta$. $\Delta_f \nvdash \varphi$

therefore Δ_f is finite and Δ_f is inconsistent

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~~Now, $\Delta_f \vdash \varphi$~~

we can always construct $\Delta_f = \{\alpha_1 \ldots \alpha_n\}$ as if $\Delta_{f_1} \vdash \varphi$ and $\Delta_{f_2} \vdash \neg\varphi$, then Δ_f might be $(\Delta_{f_1} \cup \Delta_{f_2})$ which will both $\vdash \varphi$ and $\vdash \neg\varphi$. So, Δ_f is inconsistent

It will be finite

$\bigwedge_{0<i\le k} \left(\bigvee_{0<j\le 3} e_{ijj} \right)$ will be false if every edge has different colour

$\therefore \left(\bigwedge_{0<i\le n} \left(\wedge V_{ik_1} \to \neg V_{ik_2} \right) \right) \wedge \left(\bigwedge_{0<i\le n} \left\{ \bigvee_{0<j\le 3} v_{ij} \right\} \right)$

$\wedge \neg \left(\wedge \left(\bigvee_{0<j\le 3} e_{ijj} \right) \right)$ ~~[illegible]~~ must be satisfiable

if the graph is 3 colourable

Q (5) let the colors be, $C = \{c_1, c_2, c_3\}$

let the vertices be $V = \{v_1, v_2 \ldots v_n\}$

and let the ~~let~~ edges be, $E = \{\{v_i, v_j\} \mid i \neq j,$ $0 < i,j \leq n$ and there exists a edge between v_i and $v_j\}$

has cardinality K

~~$P = \{p \mid p \in (V \cup E)\}$~~

~~Map =~~

$P = \{p \mid p \in (M \cup N)\}$

$M = \{v_{ij} \mid v_i$ has c_j color$\}$

$N = \{e_{ilm} \mid e_{ilm}$ is ith element of E and l, m is color of $v_i, v_j\}$ $\quad 0 < i \leq K$

$$\bigwedge_{0<l\leq n} \Big\{ \bigwedge_{\substack{1<k_1,k_2\leq 3 \\ k_1 \neq k_2}} (v_{ik_1} \Rightarrow \neg v_{ik_2}) \Big\}$$

will be true ~~if only one~~ vertex has only one color for all vertices.

$$\bigwedge_{0<i\leq n} \Big\{ \bigvee_{0<j\leq 3} v_{ij} \Big\}$$

will be true if every vertex has atleast one color.

Therefore

$V_1 = \{$all valuation $v \mid v(X_1) = T\}$

now no element of V_1, v will satisfy α as $X_1 \cup \{\alpha\}$ is unsatisfiable

$v \models X_1 \Rightarrow v \not\models \alpha$

$v \models X_1 \Rightarrow v \models \neg\alpha$

$\therefore$ ~~$(v \models X_1) \subseteq (v \models \neg\alpha)$~~ $\{v \mid v \models X_1\} \subseteq \{v \mid v \models \neg\alpha\}$

Similarly ~~$(v \models X_2) \subseteq (v \models \alpha)$~~ $\{v \mid v \models X_2\} \subseteq \{v \mid v \models \alpha\}$

now. no v exists that will both satisfy α and $\neg\alpha$

$\therefore$ ~~$v(\models \neg\alpha)$~~ $\cap$

$\therefore \{v \mid v \models \neg\alpha\} \cap \{v \mid v \models \alpha\} = \phi$

So. $\{v \mid v \models X_1\} \cap \{v \mid v \models X_2\}$ will be ϕ

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$\therefore$ no such v exists $v(X_1 \cup X_2)$ is true

So, $X_1 \cup X_2$ is unsatisfiable

but $X_1 \cup X_2$ is a subset of X which is finite satisfiable

$\therefore$ ~~X_1 is~~ contradiction.

So, one of $X \cup \{\alpha\}$ and $X \cup \{\neg\alpha\}$ is satisfiable.

9(3) Suppose α is a formula and X is a finitely satisfiable set of formula. then ~~either~~ one of $X \cup \{\alpha\}$, $X \cup \{\neg\alpha\}$ is finitely satisfiable

we will prove using contradiction

Let $X \cup \{\alpha\}$ and $X \cup \{\neg\alpha\}$ both are not finitely satisfiable

$\therefore X_1, X_2 \subseteq_{fin} X$ (X_1, X_2 any two finite subsets of X)

Now both X_1 and X_2 are satisfiable (finite satisfiability lemma)

$\therefore$ ~~[illegible] but [illegible]~~ $X \cup \{\alpha\}$ is not finite satisfiable

$\therefore$ there exists a $X_3 \subseteq_{fin} X \cup \{\alpha\}$ that is not satisfiable

claim:

but X_3 must have ~~[illegible]~~ α in it.

if X_3 does not include α

then $X_3 \subseteq X$, which must be ~~fin~~ satisfiable

~~$X_3 = X_1 \cup \{\alpha\}$~~ So, $X_3 = X_1 \cup \{\alpha\}$

Similarly $X_4 \subseteq_{fin} X \cup \{\neg\alpha\}$

$X_4 = X_2 \cup \{\neg\alpha\}$

now $X_1 \cup \{\alpha\} \subseteq_{fin} X \cup \{\alpha\}$

$X_2 \cup \{\neg\alpha\} \subseteq_{fin} X \cup \{\neg\alpha\}$

and they are finitely insatisfiable