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(4.5)

(2) let X_1 has sentences, $\phi_1, \phi_2, \ldots, \phi_n$

X_2 has sentences, $\psi_1, \psi_2, \ldots, \psi_m$

what if X_1 is infinite?

now, $\exists$ ~~no model~~ M s.t.

(0.5)

$$M \models (\phi_1 \wedge \cdots \wedge \phi_n) \wedge (\psi_1, \ldots \psi_m)$$

$\Downarrow$

this is not satisfiable (or not consistent).

Obs 1: All the models satisfying $(\phi_1 \wedge \cdots \wedge \phi_n)$ will not satisfy atleast one of $\psi_1 \ldots \psi_n$

similarly the converse is true.

let, δ be such a sentence

$$\therefore \delta = (\phi_1 \wedge \phi_2 \wedge \phi_3 \cdots \phi_n) \wedge (\neg\psi_1 \vee \neg\psi_2 \vee \neg\psi_3 \cdots \vee \neg\psi_m)$$

claim: all models of X_1 satisfy δ.

Proof: all models of X_1 satisfy $\phi_1 \wedge \phi_2 \cdots \phi_n$ and they also satisfy atleast one of the negations of ψ_i (else that $M \models (\phi_1 \wedge \cdots \wedge \phi_n) \wedge (\psi_1 \wedge \cdots \wedge \psi_m)$)

$$\neg\delta = (\psi_1 \wedge \psi_2 \wedge \cdots \wedge \psi_m) \vee (\neg\phi_1 \vee \neg\phi_2 \vee \neg\phi_3 \cdots)$$

claim All models of X_2 are also models of $\neg\delta$.

Proof: all models of X_2 satisfy $(\psi_1 \wedge \psi_2 \wedge \psi_3 \cdots \wedge \psi_n)$

So they will also satisfy $\neg\delta$.

$L = (>,)$

FO formula: $\phi = \forall x \exists y (>(y,x) \wedge \neg(y \equiv x) \wedge \neg >(x,y))$

② $\phi = (\forall x \forall y \forall z (\neg R(x,x) \wedge (R(x,y) \wedge R(y,z) \rightarrow R(x,z))$

$\wedge \neg (\exists x \forall y \neg R(x,y))$

Rephrasing ϕ we get,

$$\phi = (\overbrace{\forall x \forall y \forall z\, R(x,y) \wedge R(y,z) \rightarrow R(x,z)}^{\text{transitive}}) \wedge$$

$$\overbrace{\forall x \neg R(x,x)}^{\text{irreflexive}} \wedge \forall x \exists y R(x,y))$$

Such a language will be. $L = (R)$

$\boxed{\text{for every } x \text{ we will find an } y \text{ st } R(x,y)}$

formula says, R is a relation that is binary, irreflexive and transitive

Claim: Model satisfying this will have infinite elements

Proof:- Observation 1: every element x has an element y st $R(x,y)$

If we try to construct an universe satisfying the formula

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say, we have already constructed / added k elements

$x_1, x_2 \cdots \quad x_k$ in this order

and $R(x_1, x_2) \quad R(x_2, x_3) \quad R(x_1, x_3)$

$\forall_i \forall_j \, (i < j) \Rightarrow R(i, j)$

Now, for x_k we will find a x_{k+1}, now.

Subclaim 2 : x_{k+1} is not already added

say x_{k+1} is already in in the set say x_l

$\therefore R(x_l, x_{l+1})$ and $R(x_{l+1}, x_{l+2}) \cdots$

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$\cdots \qquad R(x_{k-1}, x_k)$ and $R(x_k,$

So, $R(x_l, x_{l+1}) \wedge R(x_{l+1}, x_{l+2}) \cdot \; R(x_{k-1}, x_k) \Rightarrow$

$R(x_l, x_k)$

$R(x_l, x_k) \wedge$

and $R(x_k, x_l) = R(x_k, x_k)$

But. $\neg R(x_k, x_k)$

Contradiction.

So, x_{k+1} should be the new element.

So, universe should be infinite

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3> Let ϕ be the formula.

let's take a point on such an universe that satisfies ϕ.

$R_1(x) = T/F$

$R_2(x) = T/F$

$\vdots$

$R_n(x) = T/F$

Then ϕ will have all possible combination of $R_1 \ldots R_2 \quad R_n$.

If some universe satisfy that there must be 2^n elements in the universe as each point satisfies a subset of all the Relations.

$\phi = \cancel{\exists x_1 \exists x_2 \ldots \exists x_n \left(\bigwedge_{i=1}^{n} \neg R_i(x_i) \wedge \bigwedge_{i \neq j} \neg (x_i \doteq x_j) \right)}$

~~for every i, j, k, l $R_i(x_k) \Rightarrow R_j(x_l) \wedge R_j($~~

for every element, x_k if Relation R_i satisfies it then we will find another Relation that disagrees on other two elements.)

$\forall_{i,j,k,l,m=1}^{n} \left(R_i(x_k) \Rightarrow R_j(x_l) \wedge \neg R_j(x_m) \wedge x_j \neq x_m \wedge x_j \neq x_l \right.$

and initialy we need to have two elements with one unary symbol

$(\forall x \, \exists y \;\; R_i(x) \Rightarrow \neg R_i(y))$

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So, $\phi = \underbrace{\forall x \exists y \, (R_i(x) \Rightarrow \neg R_i(y))}_{\text{Base case}} \wedge$

$$\exists x_k \, x_l \, x_m \left(\bigwedge_{i=1}^{n} \left(R_i(x_k) \Rightarrow R_j(x_l) \wedge \neg R_j(x_m)\right) \wedge \neg(x_i = x_j) \wedge \neg(x_j \doteq x_m) \wedge \neg(x_j \neq x_m) \right)$$

So, for each two elements and a Relation we will get two different ~~relation~~ elements and a relation, satisfying them or not

1 But this will only ensure n elements. We need 2^m

another might be

$$\phi' = \exists x_1 \ldots \exists x_n \left(\bigwedge_{i=1}^{n} R_i(x_i) \wedge \neg(x_i = x_j) \right)$$

Another approach

	R_1	R_2	$\cdots$	R_n
x_1	T	F		
$\vdots$	F	T		
x_{2^n}				
$\vdots$				
x_∞				

All possible 2^n combination

$\therefore$ ~~there exists~~

one of the x should satisfy a set of $\{R\}$ and satisfy the negation of the remaining elements

		R_1	R_2		R_3			R_{n-1}	R_n
say	x	T	T	F	T	F	$\cdots$	T	F

$\therefore \left(R_1(x) \wedge R_2(x) \wedge R_3(x) \wedge R_{n-1}(x)\right) \wedge \left(\neg R_4(x) \wedge \neg R_5(x) \cdots \wedge \neg R_n(x)\right.$

P.T.O

$\phi =$

$$\exists x \left(\bigwedge_{i \in K} R_i(x) \wedge \bigwedge_{j \notin K} \neg R_j(x) \right)$$

where K ranges over all possible subset of $\{1, \ldots N\}$

But this will result in an exponentially sized formula