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1 (a) Decision problems: are those problem, given in instance of the problem answer will be 'yes' or 'No'

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(b) The Class P: are those problem, one instance of the problem can be solved in polynomial time (or $O(n^k)$, where n is the size of the input and K is some fixed constant.

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What does "can be solved" mean? E.g. is it Ok if the solving happens via a separate algorithm for, say, each input size?

(c) The class NP: are those problem which can be non-deterministically solved in Polytime. or.

What does this mean?

Given an instance of the problem and a certificate to the problem, the certificate can be verified in polynomial time

By whom? What does verifying mean?

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(d) $A \leq_p B$ means a general ~~an~~ instance of problem A can be ~~poly~~ redu[ced]

what does this mean?

to a special ~~an~~ instance of problem B in polynomial time

Special in what way?

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e) The Class NP-hard: are those problem which are as hard as any

other problems in NP.

or,

for any problem in NP (say A),

$A \leq_p B$ ~~where B is also in NP, a~~

then B is NP-hard. (Any NP problem can be reduced to an NP-hard problem in poly-time)

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The Class NP-complete: are those problem which

1) are in NP.

2) are also in NP-hard

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means?

(a) ① Clique is NP: Given a set of vertices we can ~~verify~~ verify if the set has size $\geq K$ and for every two nodes in the subset, if there exists an edge between them. in polynomial time

So, Clique is NP.

② Clique is NP-hard: will show Vertex cover $\leq_p$ clique.

~~Given a vertex co~~

Input for vertex cover, $G=(V,E), K$

$G'=(V,\bar{E})$

let G' be a graph with such that

for any two pairs of vertices u, v in (G) if there exists an edge between u, v. then G' will have no edge between them, else, they have an edge between them.

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~~Now if G'~~

Also, n be the number of vertices.

So, Clique $(G', n-K)$ will be the instance of Clique problem

Why? Vertex cover (G,K) will only have solution iff clique $(G', n-K)$ have a solution

and reduction takes polynomial time

So, $VC \leq_p$ Clique $\Rightarrow$ Clique is NP-hard

Hence proved

(b) Vertex cover is NP :-

at most

Given a set of vertices, we can check if size ~~...~~ is (3) (c

more than K and, ~~...~~ if every edge has atleast one endpoint

in the given subset. It can be done in poly time

VC is NP-hard :- will show clique $\leq_p$ VC

Let G be a graph st $G = V, E$

G' be the the graph $G' = (V, \bar{E})$

(Similar ~~reduct~~ reduction to G')

So, if (G, K) is an instance of Clique problem

we can reduce the G to G' in polytime

Why is this reduction correct?

and ask Vertex Cover $(G', n-K)$

So, clique $\leq_p$ VC $\Rightarrow$ VC is NP-hard.

$\therefore$ Clique is NP-complete

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(3) (a)

Assuming set of vertices has an order (1 .. n)

G is the graph
P is the partition

Global array →

number of vertices

Store [k][n] = all empty

length of simple path

will store the list of paths.

Simple Path (K, G, P)

if (K==1):

for v in G.V

denotes the set of vertices

~~[illegible]~~

Simple Path [1] [v] = [[v]]

else,

for v in G.V:

for e in G.E:

i.e. e = (v,u) or (u,v)

if v in e;
u be the another point in e
set of paths = Simple Path [K-1] [u]

for path in set of Paths:

if v not in path

then (simple path [K][v] . append

$\underbrace{([v] + \text{path})}$)

adding v to the ~~set of~~ list of vertices in path and then append it

~~for (i, j) in SimplePath~~

for j in G.V
if SimplePath [k][j] has kSRed

array.
then return 'Yes'.

(b)