Theory of Computation
Quiz 2

November 2, 2023
Time: 11:50am to 1.00pm
Total marks: 50

Write clear and precise answers.

(1) Recall that a context-free language $L \subseteq \Sigma^*$, for some finite alphabet Σ, is *linear* if it has a context-free grammar $G = (V, \Sigma, P, S)$ with all productions of the form $A \to \alpha B \beta$ or $A \to \alpha$ for variables $A, B \in V$ and terminal strings $\alpha, \beta \in \Sigma^*$.

For a pushdown automaton (PDA) a transition $(p, \gamma) \in \delta(q, a, X)$ is a *pop operation* if $\gamma = \epsilon$. The other transitions are *push operations.*

A *one-turn* pushdown automaton (PDA) is a PDA with the property that, for any input, on any computation path once it does a pop operation it will never use push operations in the rest of the computation.

Show that every linear context-free language can be accepted by a one-turn PDA. **15 marks**

(2) Suppose L is a linear context-free language and R is a regular language. Show that $L \cap R$ is a linear context-free language. **10 marks**

(3) Write the definition of a deterministic PDA. Suppose $L = L(M)$ for a deterministic PDA. Let $L' = \{w \in L \mid \text{no proper prefix of } w \text{ is in } L\}$. Is L' accepted by a deterministic PDA? Justify answer. **10 marks**

(4) Construct a Turing machine M that enumerates $\{0,1\}^*$ in canonical order. Specifically, M will output $x_1\#x_2\#\cdots\#x_i\#x_{i+1}\#\cdots$, where the x_i appear in canonical order on the output tape. It will use as subroutine a Turing machine M' that computes x_{i+1} from x_i. Give the transition function of M' in detail, and a high level description for M. **15 marks**

$\alpha\beta\beta$

$S \to a_1 B_1$

$\epsilon, 0, 1, 10, 11, 1\ldots$

$B_1 \to a_2 B_2$

$00, 01, 10, 11, 000$

$A \to \alpha$

Reversal $\to$

$\#00\$00$

$\alpha\beta$ $\alpha\beta\beta$

a_2 B_2

$\alpha\beta\beta$

$a\,\underbrace{\alpha_1\beta_1}_{\gamma_1}\,\beta$

$S \to$

α β β

α

β

$S \to aV$

q_1

q_0