666

## Theory of Computation Midsem Exam

October 4, 2023 Time: 2pm to 4pm Total marks: 100

This is a two hour exam. Write clearly and precisely. In question 8 you can choose between 8(a) and 8(b).

- Let L be any regular language. Consider  $L' = \{w \in L \mid \text{no proper prefix of } w \text{ is in } L\}$ . Prove that L' is regular.
- Let L and L' be regular languages. Prove that their perfect shuffle  $L'' = \{w = a_1b_1a_2b_2\cdots a_kb_k \mid k \geq 0 \text{ where } a_1a_2\cdots a_k \in L \text{ and } b_1b_2\cdots b_k \in L\}$  is regular. 10 marks
- Give a self-contained proof (from first principles) that  $L = \{a^i b^j c^k \mid i, j, k \geq 0 \text{ such that if } i = 1 \text{ then } j = k\}$  is not a regular language.
- Consider the context-free grammar  $E \to E + E \mid E * E \mid (E) \mid x \mid y \mid z$ , with E as start symbol and terminals as  $T = \{(,), x, y, z\}$ . Give a word in  $T^*$  generated by the grammar with two different derivation trees.
- (5) Let  $L = \{a^n b^n \mid n \ge 0\}$ . Prove that its complement  $\overline{L} = \{a, b\}^* \setminus L$  is a context-free language.
- (6) Prove by induction that the context-free grammar consisting of productions  $S \to \epsilon$ ,  $S \to bA$ ,  $S \to aB$ ,  $A \to aS$ ,  $A \to bAA$ ,  $A \to a$ ,  $B \to bS$ ,  $B \to aBB$ ,  $B \to b$ , generates all words with an equal number of a's and b's.
- Is the language over  $\{a, b, c\}$  consisting of all words with an equal number of a's, b's, and c's a context-free language? Justify answer with proof. 10 marks
  - (8) Answer one of the following:
    - (a) Let L and L' be regular languages over alphabet  $\{a,b\}$ . Let  $L''=\{w\in L\mid \text{some }y\in L'\text{ has the same number of }a\text{'s as }w\}$ . Is L'' regular? If yes, give a DFA or NFA for it. Else prove it is not regular.

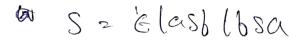
or

(b) Let  $L \subseteq \{a, b\}^*$  be any regular language. Let L' be the context-free language consisting of all words  $w \in \{a, b\}^*$  with an equal number of a's and b's. Is the language L'' =

 $\{w\mid ww'\in L \text{ for some } w'\in L'\}$  regular? If yes, give a DFA or NFA for it. Else prove it is not regular.

10 marks

- (9) A linear context-free grammar over alphabet  $\Sigma$  has only productions of the form  $A \to \alpha B\beta \mid \alpha$  for  $\alpha, \beta \in \Sigma^*$ , where A and B are variables in the grammar. In other words, the RHS of every production has at most one variable.
- (a) Let L be a context-free language with a linear context-free grammar. Show that there is a constant n such that all words  $z \in L$  of length at least n can be written as z = uvwxy where  $|uvxy| \le n$ ,  $|vx| \ge 1$ , and  $uv^iwx^iy \in L$  for all  $i \ge 0$ .
  - (b) Consider the context-free language  $L = \{w \mid w \text{ has an equal number of } a$ 's and b's  $\}$ . Show using part (a) or otherwise that L cannot be generated by a linear context-free grammar.



10+5 marks