Theory of Computation
Quiz 1

September 7, 2023
Time: 11:50am to 1.00pm
Total marks: 50

Write clear and precise answers.

(1) Let $\Sigma = \{0, 1\}$. Let $L \subset \Sigma^*$ be set of all strings in which the number of ones is a multiple of three and the number of zeros is multiple of two. Design a DFA for L. Draw the state transition diagram with an explanation of how the DFA works. **10 marks**

(2) Let $\Sigma = \{0, 1, 2, 3, 4\}$. Design an ϵ-NFA for the following language using as much nondeterminism as possible. Let $w \in \Sigma^*$ be the set of strings such that the second last symbol of w occurs on odd number of times in w. **10 marks**

(3) For a string $w \in \{0, 1\}^*$, let $ham_2(w)$ be the set of all strings of length $|w|$ which are at Hamming distance 2 from w (i.e. they differ from w in exactly two positions). For example if $w = 000$ then $ham_2(w) = \{110, 011, 101\}$. Let $A \subseteq \{0, 1\}*$ be any regular language. Let

$$B = \bigcup_{w \in A} ham_2(w).$$

Show that B is regular (give a DFA or an NFA or an ϵ-NFA for it). **10 marks**

(4) *Construct a DFA with five states for the language denoted by the regular expression $(0 + 1)^*(00 + 11)$. Can it have a DFA with fewer states? Justify your answer.* **10 marks**

(5) *Consider the finite language*

$$L_2 = \{ww^R \mid w \in \{0, 1\}^k\},$$

where $k > 0$ is a constant and w^R denotes the reverse of the string w. Show that any DFA for L_2 requires at least 2^k states. **10 marks**