FUZZY INFERENCE SYSTEM

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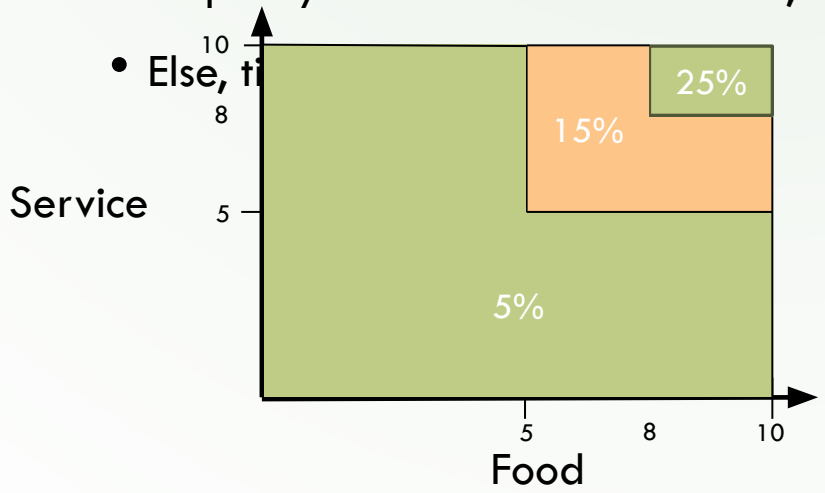
Problems with Traditional (Boolean) Logic

- A two-valued logic system: every statement is either True (1) or False (0).
- A scenario of restaurant:
  - Two factors: Quality of food and quality of service
  - Outcome: Tip
    - If quality >= 8 and service >= 8, tip is 25%
    - If quality >= 5 and service >= 5, tip is 15%
    - Else, ti

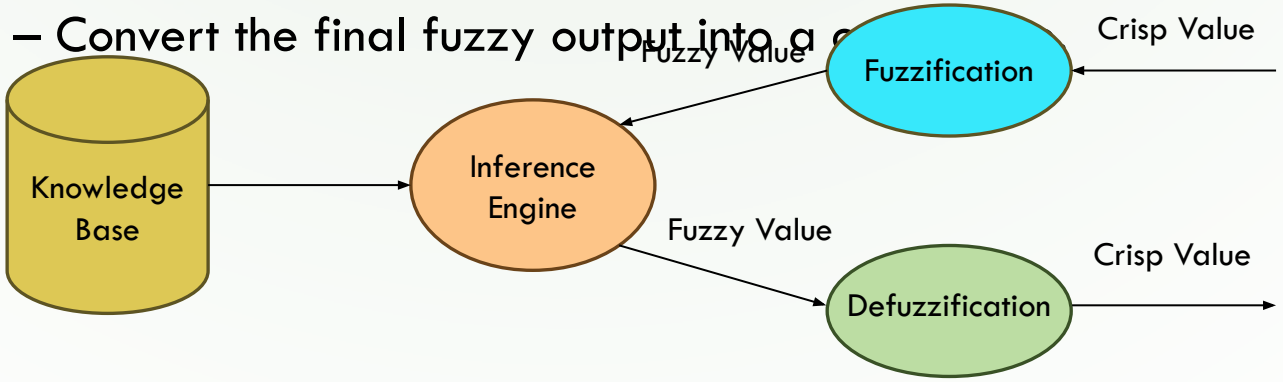


Fuzzy Logic

- Fuzzy logic is a form of many-valued logic, where truth values are expressed in degrees between 0 and 1, instead of just true (1) or false (0).
- A Fuzzy Inference System (FIS) is a way of mapping an input space to an output space using fuzzy logic.
- FIS (also called a Fuzzy Expert System) uses membership functions and fuzzy rules instead of Boolean logic to reason with uncertain data.
- The rules in FIS are fuzzy production rules of the form:
  - If P then Q, where P and Q are fuzzy statements.
  - P is called antecedent and Q is called conclusion.
  - E.x.: If service is poor or food is bad then tip is cheap.

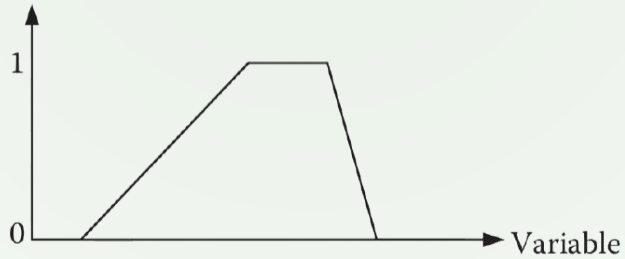
Fuzzy Inference System

- The set of rules in a fuzzy expert system is known as the knowledge base.
- The main functional steps of a fuzzy expert system are:
  - Fuzzification – Convert crisp inputs into fuzzy values using membership functions.
  - Fuzzy Inference – Apply the fuzzy rules and inference method.
  - Aggregation – Combine the outputs of all rules.
  - Defuzzification – Convert the final fuzzy output into a c

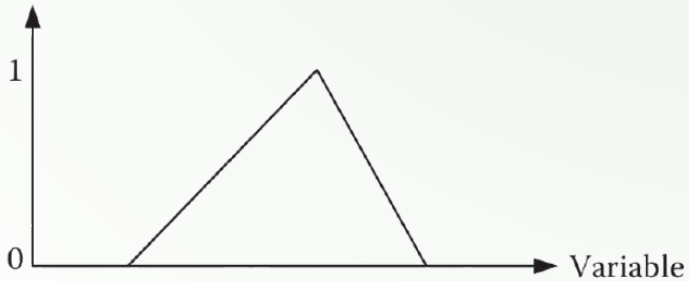


Fuzzy Inference System – Cont'd

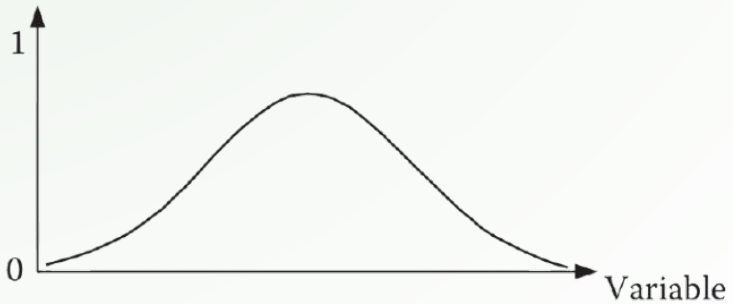
- Some common membership functions:



A trapezoidal membership function



A triangular membership function



A gaussian membership function

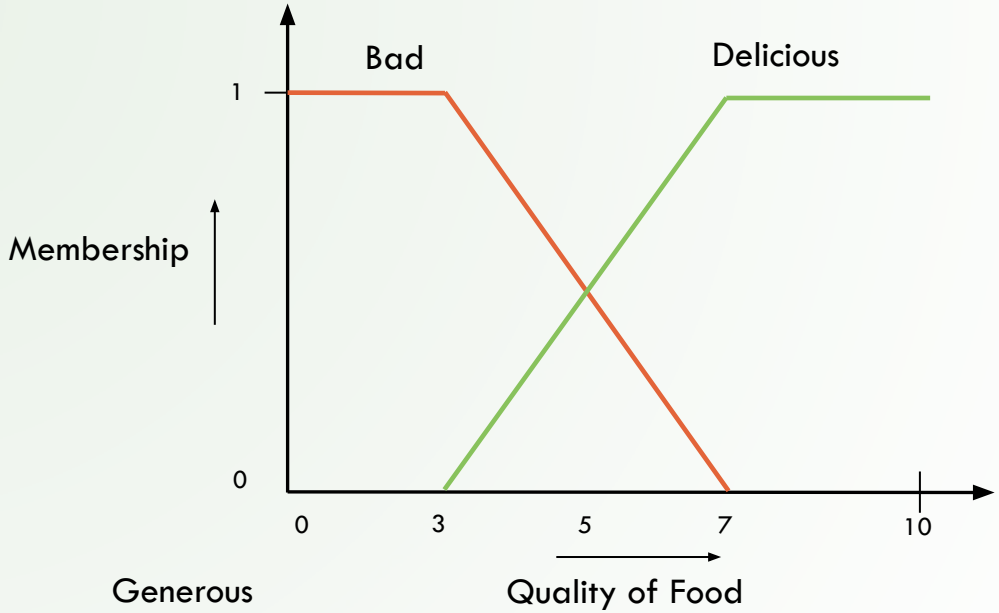
Mamdani Inference System

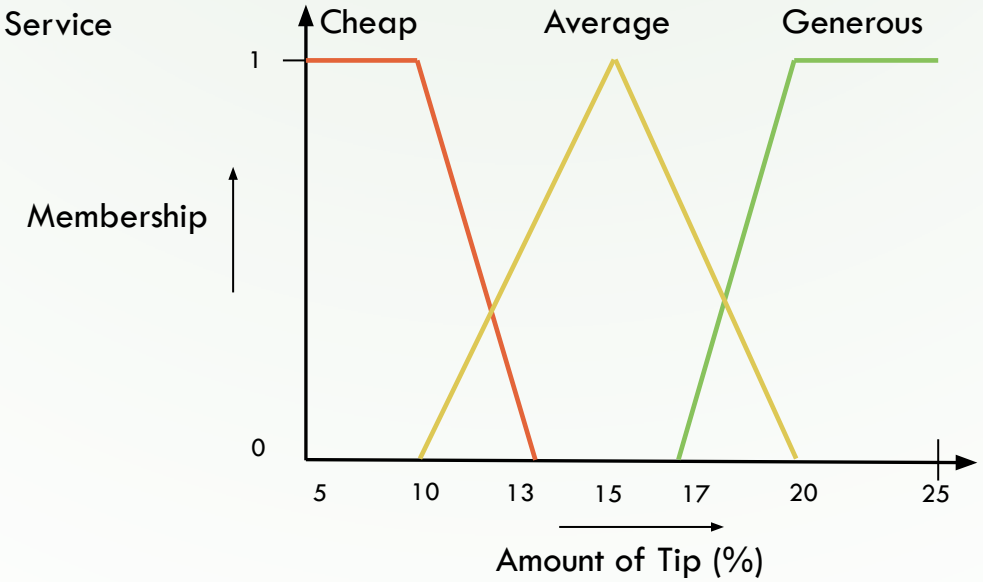
- The most well-known fuzzy inference system is the Mamdani rule-based system.
- Its functional steps are:
  - Fuzzify all input values into fuzzy membership functions.
  - Execute all applicable rules in the rulebase to compute the fuzzy output functions.
  - Defuzzify the fuzzy output functions to get "crisp" output values.
- Example: We will consider the restaurant tipping scenario.
  - Two inputs: Quality of food and service at a restaurant rated at scale from 0-10.
  - One output: Amount of tip to be given (in the range 5%-25% of total bill paid).

Rules for Tipping

- Let us consider the following three rules:
  - If service is poor or food is bad, then tip is cheap .
  - If service is good, then tip is average.
  - If service is excellent or food is delicious, then tip is generous.
- Input variables:
  - Service : represented by poor, good, excellent
  - Food : represented by bad, delicious
- Output Variable:
  - Tip : represented by cheap, average, generous

Membership Functions

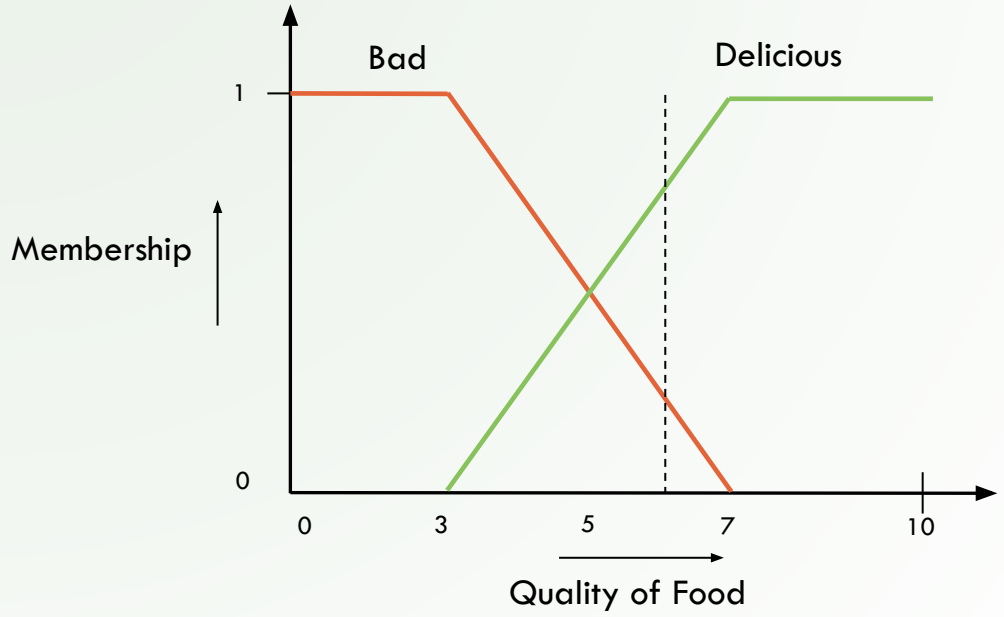




Fuzzification

Let:

Service = 3, Food = 6



$$y = y_1 + \frac{(y_2 - y_1)}{(x_2 - x_1)}(x - x_1)$$

Service(3) = (0.5, 0.33, 0)

Food(6) = (0.25, 0.75)

Inferencing (Executing Rules)

Boolean	Fuzzy
AND(x, y)	MIN(x, y)
OR(x, y)	MAX(x, y)
NOT(x)	1 – x

Service(3) = (0.5, 0.33, 0) Food(6) = (0.25, 0.75)

If service is poor or food is bad, then tip is cheap .

OR(0.5, 0.25) = MAX(0.5, 0.25)

If service is good, then tip is average.

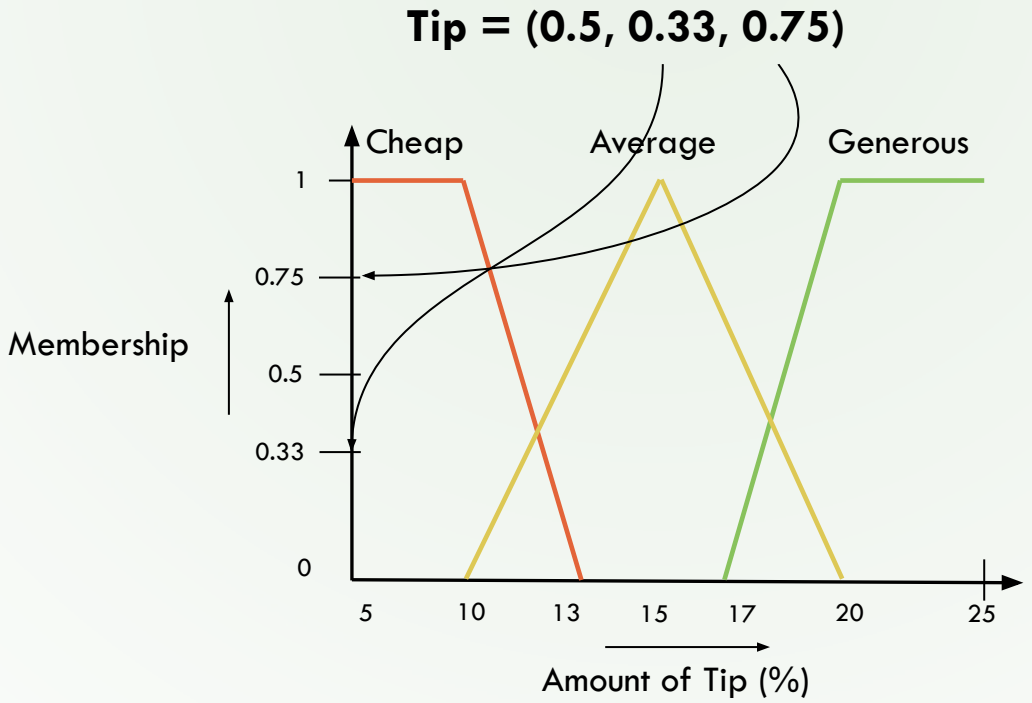
0.33

If service is excellent or food is delicious, then tip is generous.

OR(0, 0.75) = MAX(0, 0.75)

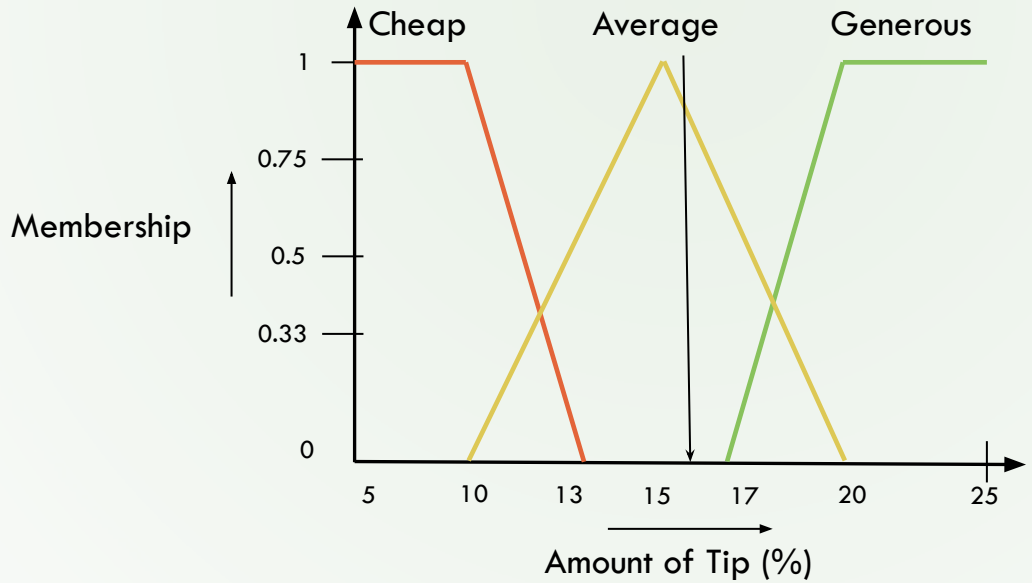
Tip = (0.5, 0.33, 0.75)

Defuzzification



There are several defuzzification methods, here we will use centroid technique.
It finds the point where a vertical line would slice the aggregate set into two equal masses.

Defuzzification – Cont'd

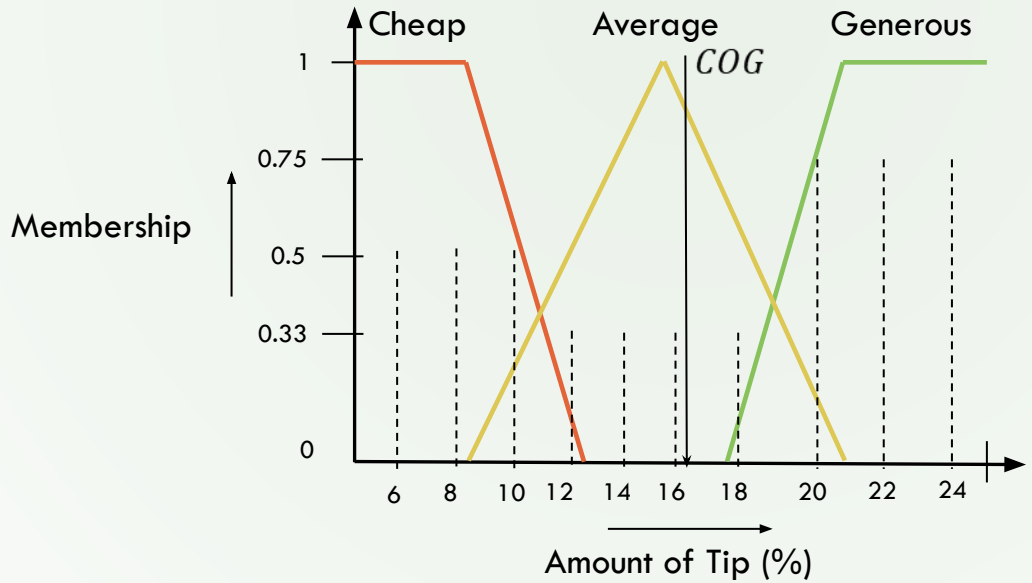


A centroid defuzzification method finds a point representing the center of gravity of the fuzzy set, A, on the interval, ab is given by the equation:

$$\mathrm{COG} = \frac{\sum_{x=a}^{b} \mu_A(x)x}{\sum_{x=a}^{b} \mu_A(x)}$$

$\mu_A(x)$ represents the degree of membership for a value inside the interval.

Defuzzification – Cont'd



$$COG = \frac{(6+8+10)*0.5+(12+14+16+18)*0.33+(20+22+24)*0.75}{0.5+0.5+0.5+0.33+0.33+0.33+0.33+0.75+0.75+0.75}$$

$$= 16.03\%$$

Visualization

Code: https://colab.research.google.com/drive/1gp3VcCxj3t3XyK3c9DCfT1G7B3HWjFLr?usp=sharing

Slide download: https://h1.nu/Fuzzy

THANK YOU