Force
CVE 204

- Force an agent which produces or tends to produce, destroys, or tends to destroy motion.

Question...
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STRENGTH OF MATERIALS I 2 UNITS

SYNOPSIS

- FORCE EQUILIBRIUM – Free body diagrams
- Concept of Stress Strain, Tensile test
- Young's modulus and other strength factors.
- Axially loaded bars, Composite bars
  Temperature Stress and simple indeterminate problems.
- Hoop stress in Cylinders and Rings
- Bending moment Shear force and axial force diagrams for simple cases,
- Simple torsion and Application.

INTRODUCTION

When an external force acts on a body, the body tends to undergo deformation.
Due to cohesion between the molecules, the body resists deformation.
The resistance by which material of the body opposes the deformation is known as Strength of Material.

1 Within a certain limit, (elastic range/stage) the resistance offered by the material is proportional to the deformation brought out on the material by the external force.

2 Also within elastic limit, the resistance is equal to the external force (or applied load)

(3) But beyond the elastic limit, the resistance offered by the material is less than the applied load. In such a case, the deformation continues, until failure takes place.

Within elastic stage, the resisting force equals applied load.

i.e. Within Elastic Stage
Resistance force of the material
=
External force (Applied load) on the material.
Action and Reaction equal and opposite.

Strength of Materials.
It is the branch of Mechanics that deals with the behaviour of solid matter under external actions.
Now, what are the external Actions?

- External force (Applied load.)
- Temperature Change &
- Displacement.

STRENGTH OF MATERIALS STRESS & STRAIN.
We have already established that
The strength of a material is its ability to withstand an applied load (external force) without failure.
A load applied to a mechanical member will induce internal forces within the member.
The internal forces are called Stresses within the member.

The stresses acting on the material cause deformation of the material in various manner

Deformation of the material is called Strain.

STRESS:

The force of resistance per unit area, offered by a body against deformation is known as Stress.

External force acting on the body = load or Force.

The load is applied on the body while the Stress is induced in the material of the body.

A loaded member remains in equilibrium when the resistance offered by the member against deformation and the applied load are equal. (3)

Mathematically

Stress written as $\sigma = \frac{P}{A}$

σ = Stress (also called Intensity of Stress)
P = External force or load; and
A = Cross sectional area.

Units of Stress

The unit of stress depends upon the unit of (load (or force) and unit of area.
In metre, Kilogram, and/or second (MKS) units,

Force is expressed in Kgf.
Area in metre square m^2 or Cm^2

Hence stress = Kgf/cm^2
Kgf/m^2.

In the S.I units,

Force is expressed in Newtons (N)

Area in m^2.

Hence

Stress = N/m^2

$$1\ N/m^2 = 1\ N/(100\ cm)^2$$

$$= 1\ N/10^4\ cm^2$$

$$= 10^{-4}\ N/cm^2$$

$$= 10^{-6}\ N/mm^2$$

i.e. $1\ N/mm^2 = 10^6\ N/m^2$

And $1\ N/m^2 = 1$ Pascal $= 1$ Pa

Large quantities

Kilo = 10^3 ... K

Mega = 10^6 ... M

Giga = 10^9 ... G

Terra = 10^{12} ... T

ie $1\ KN = 1000\ N$

$1\ MN = 10^6\ N$

As in

~~1MB~~ 1KB = 1000 Bytes

1 MB = 10^6 Bytes

1 GB = 10^9 Bytes

1 TB = 10^{12} Bytes

Symbol

$1\ MPa = 1$ mega pascal which means.
10^6 pascal
or $= 10^6\ N/m^2$.

Small quantities:

Milli	$= 10^{-3}$	----	m
Micro	$= 10^{-6}$	----	μ (G. Small letter Mu)
Nano	$= 10^{-9}$	----	η (Greek Small letter Eta)
Pico	$= 10^{-12}$	------	p.

Note:

(1) Newton is a force acting on a mass of 1 kg and produces an acceleration of $1\ m/s^2$
i.e

$$1\ N = 1\ kg \times 1\ m/s^2$$

(2) S.I unit of Stress $= N/m^2$ or N/mm^2

(3) Stress $1\ N/mm^2 = 10^6\ N/m^2 = MN/m^2$
Thus one N/mm^2 = one MN/m^2.

(4) 1 Pascal is written as 1 Pa and is equal to $1\ N/m^2$.

STRAIN

When a body is subjected to some external force, there is some change of dimension of the body.

The ratio of change of dimension of the body. The ratio of change of dimension of the body to the original dimension is known as Strain

Strain may be:

Tensile Strain

Compressive Strain

Volumetric Strain

Sheer Strain

If there is some increase in length of a body due to external force (Applied load), then the ratio of increase of length to the original length is called Tensile Strain

But

If there is some decrease in length . . . . Then the ratio of decrease of the length of the body . . . . . Compressive Strain

The ratio of change of volume of the body to the original volume is called Volumetric S-

The strain produced by sheer stress is known as sheer strain

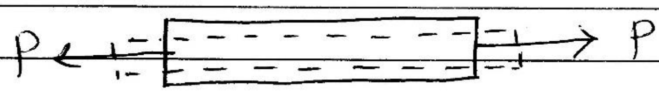
TYPES OF STRESSES.

- Normal stress or
- Shear Stress.

Normal stress is the stress which acts in a direction perpendicular to the area. It is represented by σ (Sigma). The normal stress is further divided into tensile stress and compressive stress.

Tensile Stress

Stress induced in a body, when subjected to two equal and opposite pulls

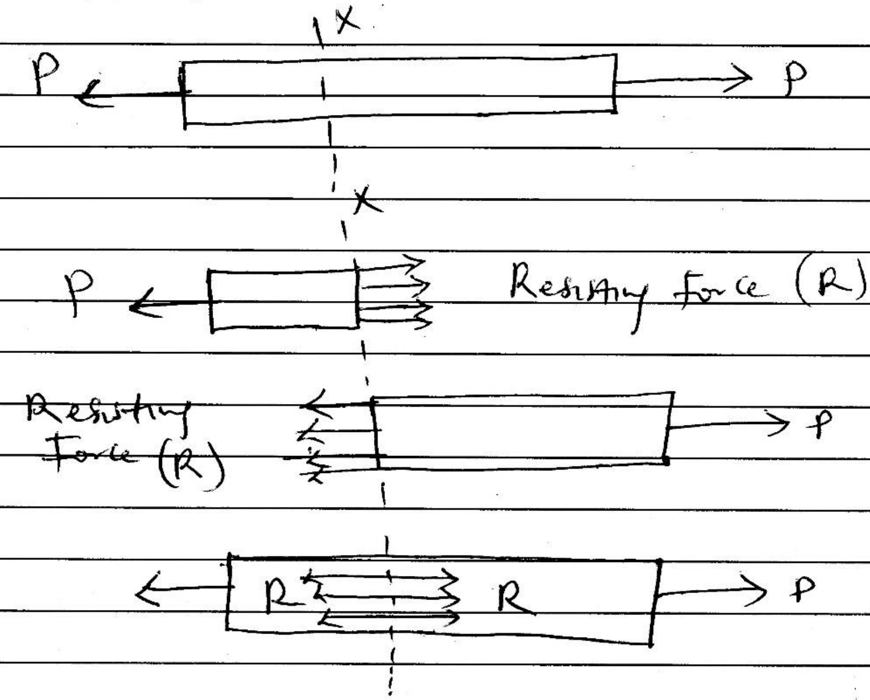


As a result of which there is an increase in length, is known as tensile stress.

The ratio of increase in length to the original length is known as tensile strain.

The tensile stress acts normal to the area and it pulls on the area.

Let P = Pull (or force) acting on the body
A = Cross-sectional area of the body
L = Original length of the body
dL = Increase in length ~~of the~~ due to pull 'P' acting on the body
σ = Stress induced in the body and
e = Strain (i.e tensile strain).



Consider a section x-x which divides the bar into 2 parts. The part left to section x-x will be in equilibrium if

$$P = \text{Resisting force } (R)$$

Similarly the part right to x-x will be in equilibrium if

$$P = R$$

This resisting force per unit area is known as stress or intensity of stress

$$\therefore \text{Tensile stress} = \sigma = \frac{\text{Resisting force } R}{\text{Cross-sectional Area}}$$

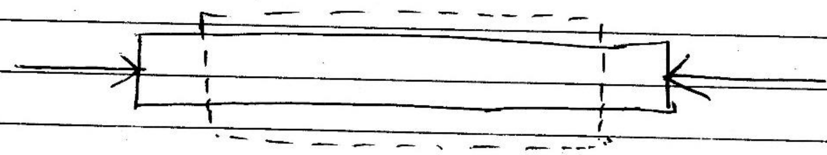
$$= \frac{\text{Tensile load } (P)}{A}$$

$$\sigma = \frac{P}{A}$$

[redacted] Tensile Strain $e = \frac{\text{Increase in length}}{\text{Original length}} = \frac{dL}{L}$

COMPRESSIVE STRESS

Stress induced in a body when subjected to 2 equal and opposite pushes

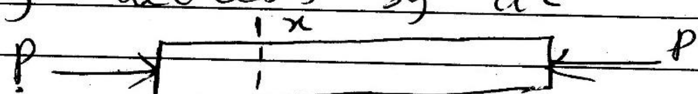


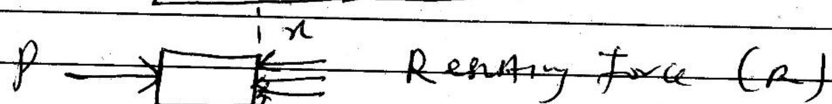
as a result of which there is a decrease in length of the body, is known as Compressive Stress.

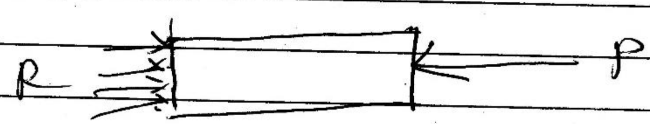
The ratio of decrease in length to the original length is known as Compressive ~~stress~~ Strain

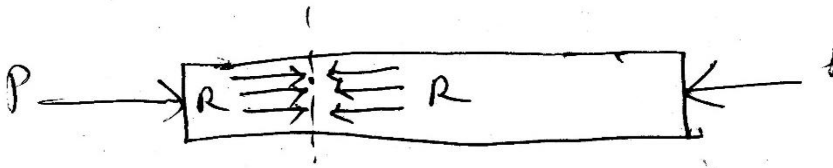
~~And the ratio~~ The Compressive Stress acts normal to the area and it pushes on the area.

Let an axial push P acting on a body with Cross-Sectional area A. Due to external push P, let the original length L of the body decreases by dL







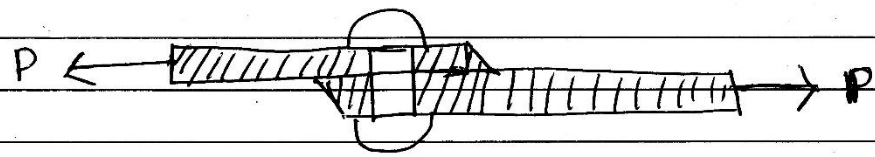


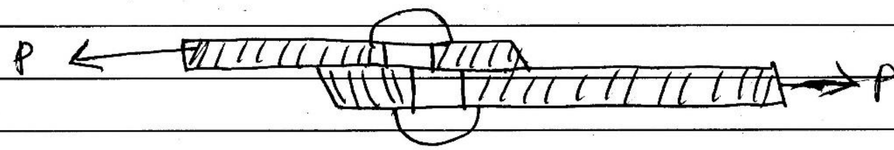
Compressive stress $\sigma = \frac{R}{A} = \frac{\text{Push } (P)}{\text{Area } (A)} = \frac{P}{A}$

Compressive Strain

$$e = \frac{\text{Decrease in length}}{\text{Original length}} = \frac{dL}{L}$$

SHEAR STRESS





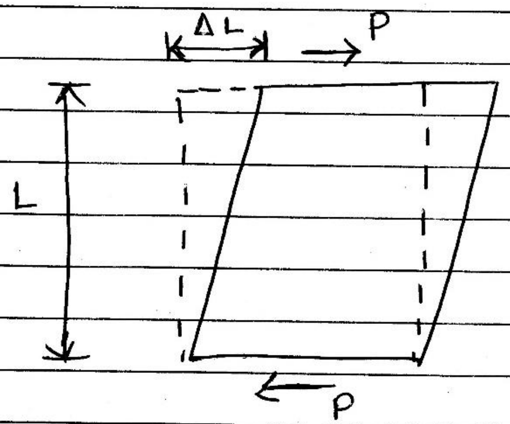
The stress induced in a body, when subjected to two equal and opposite forces which are acting tangentially across the resisting section as shown above as a result of which the body tends to shear off across the section, is known as shear stress. The corresponding strain is known as shear strain.

The shear stress is the stress which acts tangetial to the area.

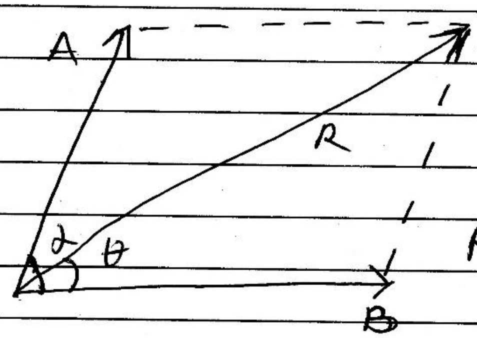
It is represented by τ

Mathematically

Shear Stress $\tau = \dfrac{R}{A} = \dfrac{\text{Shear Resistance}}{\text{Shear Area}}$



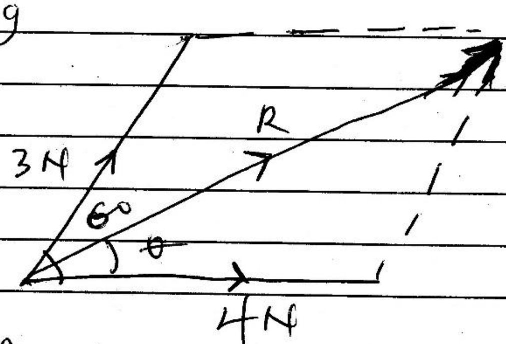
Parallelogram Law of Forces.



$R^2 = A^2 + B^2 + 2AB\cos\alpha$

$R = \sqrt{A^2 + B^2 + 2AB\cos\alpha}$

e.g



$R^2 = 3^2 + 4^2 + 2 \times 3 \times 4 \cos 60$

$R = 6.08\,N$ magnitude

Direction of R is given by θ

$$\tan\theta = \frac{A\sin 60}{B + A\cos 60}$$

ELASTICITY AND ELASTIC LIMIT

When an external force acts on a body, the body tends to undergo some deformation. If the external force is removed and the body comes back to its original shape and size (which means the deformation disappears completely), the body is know as Elastic body.

This property of which certain materials return back to their original position after the removal of external force is called elasticity.

There is a limit value of force up to and within which, the deformation entirely disappears on the removal of force.

$\frac{100}{100} + \frac{30}{100} = 1.3$ $\quad$ $\overline{10}$ $\quad$ 0.3

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(13)

PRINCIPLE OF SUPERPOSITION.

When a number of loads are acting on a body, the resulting strain, according to principle of superposition, will be algebraic sum of strains caused by individual loads.

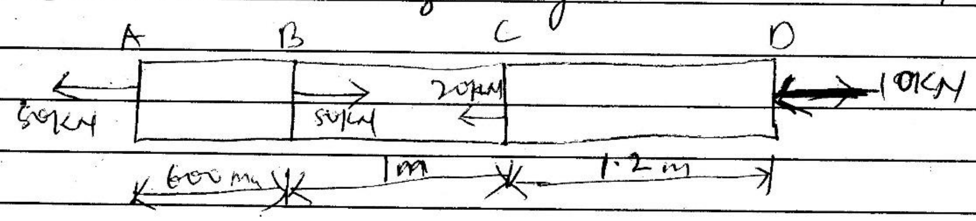
While using this principle for an elastic body which is subjected to a number of direct forces (tensile or compressive) at different sections along the length of the body, first

- the free body diagram of individual section is drawn.

- Then the deformation of the each section is obtained.

- Lastly, The total deformation of the body will be then equal to the algebraic sum of deformations of the individual sections.

Ex 2 A brass ~~bar~~, having cross-sectional area of $1000\ mm^2$, is subjected to axial forces as shown. Find the total elongation of the bar. $E = 1.05 \times 10^5\ N/mm^2$

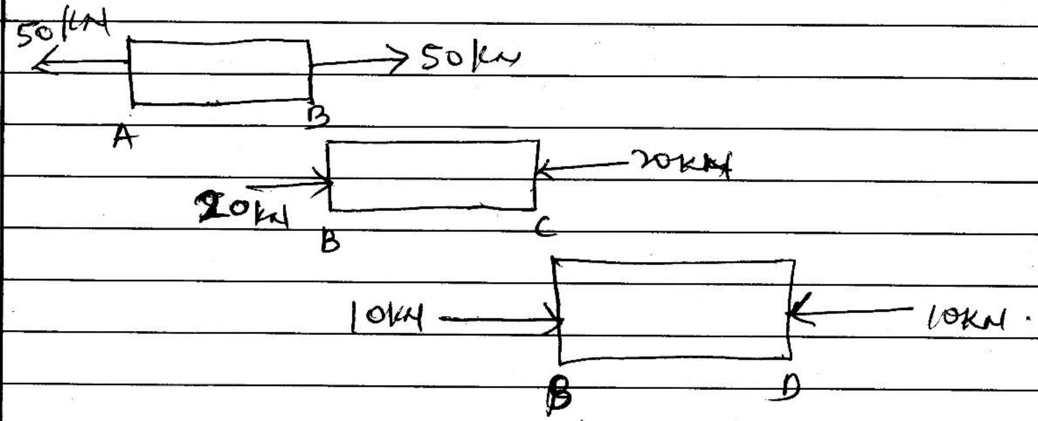


$A = 1000\ mm^2$ (the same $\times$ sectional area).

$E = 1.05 \times 10^5\ N/mm^2$

Think about equilibrium first ----------- Balance first

(1) Draw the free body diagram.



Part AB :- is subjected to tensile force of 50KN.

Increase in length of AB — Elongation

$L_{AB} = 600\,mm$

$P_1 = 50\,KN$

$= 50000\,N$

$$\delta L_{AB} = \frac{P_1 L_{AB}}{AE}$$

$$= \frac{50 \times 1000 \times 600}{1000 \times 1.05 \times 10^5}$$

$$= 0.2857$$

Part BC :- Subjected to compressive force of 20KN

decrease in length of BC ((deformation))

deformation

$P_2 = 20KN$

$= 20,000\,N$

$$\delta L_{BC} = \frac{P_2 L_{BC}}{AE}$$

$L_{BC} = 1m$

$= 1000mm$

$A = 1000\,mm$

$E = 1.05 \times 10^5\ N/mm^2$

Free body diagram (FBD)

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(15)

$\delta L_{BC} = \frac{20 \times 1000 \times 1000}{1000 \times 1.05 \times 10^5}$

$= 0.1904$. (–ve sign) decrease

~~Part BD : Subjected to tensile~~

Part BD : Subjected to compressive → ← force 10 kN

decrease in length of BD (deformation).

$\delta_{BD} = \frac{P_3 L_{BD}}{AE}$

$P_3 = 10 kN$
$= 10,000 N$

$L_{BD} = (1 + 1.2) = 2.2 m$
$= 2200 mm$.

$= \frac{10000 \times 2200}{1000 \times 1.05 \times 10^5} = 0.2095$ (–ve sign) decrease.

∴ Total Elongation of the bar = algebraic sum of elongations (or deformations) as the case may be. of the individuals sections.

$\delta L = \delta_{AB} + \delta_{BC} + \delta_{BD}$

$= 0.2857 - 0.1904 - 0.2095$ (shortenings)

$= -0.1142$ mm. (decrease)

Negative sign shows, that there will be decrease in length of the bar.

Shortening.

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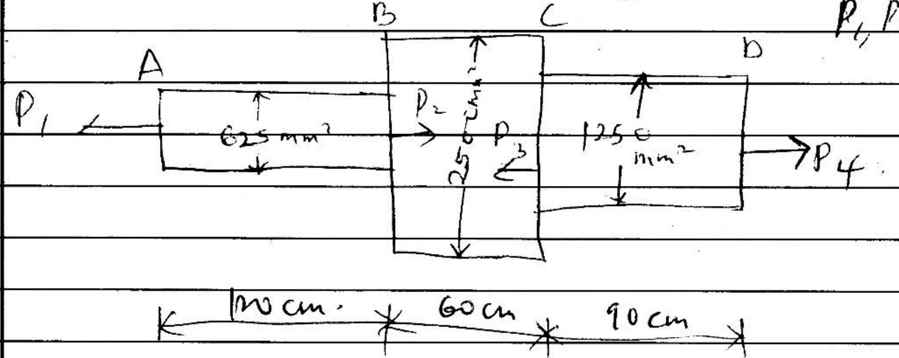
Ex 2

A member ABCD is subjected to point loads P_1, P_2, P_3 & P_4 as shown



Calculate the force P_2 necessary for equilibrium

If $P_1 = 45$ kN, $P_3 = 450$ kN, and $P_4 = 130$ kN.

Determine the total elongation of the member, assuming the modulus of elasticity to be 2.1×10^5 N/mm²

Soln

Given Part AB $A_1 = 625$ mm²
$L_1 = 120$ cm $= 1200$ mm

BC $= A_2 = 2500$ mm²
$L_2 = 60$ cm $= 600$ mm

CD $A_3 = 1250$ mm²
$L_3 = 90$ cm $= 900$ mm²

$E = 2.1 \times 10^5$ N/mm²

(i) Value of P_2 necessary for equilibrium

Resolving the forces on the rod along its axis

i.e. $P_1 - P_2 + P_3 - P_4 = 0$

or $P_1 + P_3 = P_2 + P_4$

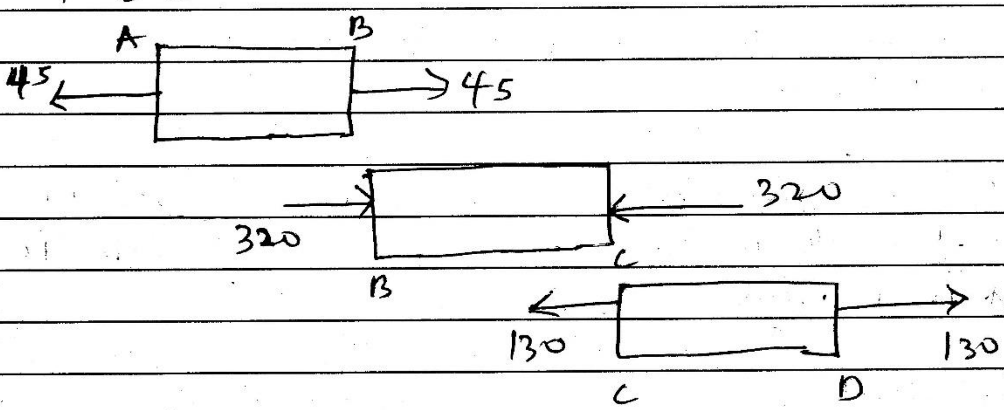
Substituting load given i.e. P_1, P_3 & P_4.

~~$P_2 = 45+$~~

$45 + 450 = P_2 + 130$

$P_2 = 495 - 130 = 365$ kN.

FBD



Part AB → Increase in length tensile ← →

$\frac{PL}{A_1 E} = \frac{45 \times 1000 \times 1200}{625 \times 2.1 \times 10^5}$

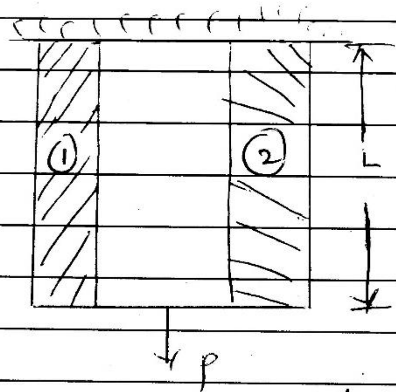
Decrease (−ve sign) $= 0.4114$ mm

$BC = \frac{320{,}000 \times 600}{2500 \times 2.1 \times 10^5} = 0.3657$ mm

Increase
$CD = \frac{130{,}000 \times 900}{1250 \times 2.1 \times 10^5} = 0.4457$ mm

Total change in length $= 0.4114 - 0.3657 - 0.4457 = 0.4914$ mm

ANALYSIS OF BARS OF COMPOSITE SECTIONS.



A bar, made up of two or more bars of equal lengths but of different materials rigidly fixed with each other and behaving as one unit for extension or compression when subjected to an axial tensile or compressive force, is called a composite bar.

For composite bar the following 2 points are important:-

(1) The extension or compression in each bar is equal. Hence deformation per unit length i.e strain in each bar is equal.

(2) The total external load on the composite bar is equal to the sum of the loads carried by each different material.

(19)

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let P = Total load on the Composite bar
L = Length of Composite bar and also length of bars of different materials
A_1 = Area of Cross-Section of bar 1
A_2 = Area of Cross-Section of bar 2
E_1 = Young modulus of bar 1
E_2 = Young modulus of bar 2
P_1 = Load shared by bar 1
P_2 = load shared by bar 2
σ_1 = Stress induced by bar 1
σ_2 Stress induced by bar 2

Now total load on the Composite bar is equal to the sum of the load carried by the two bars.

i.e $\quad P = P_1 + P_2 \quad$ —— (1)

$$\text{Stress in bar 1} = \frac{\text{load carried by bar 1}}{\text{Area of x-Section of bar 1}}$$

$$\sigma_1 = \frac{P_1}{A_1} \quad \text{or} \quad P_1 = \sigma_1 A_1 \quad \text{— (2)}$$

Similarly

$$\sigma_2 = \frac{P_2}{A_2} \quad \therefore \quad P_2 = \sigma_2 A_2 \quad \text{— (3)}$$

Substituting

$$P = \sigma_1 A_1 + \sigma_2 A_2 \quad \text{—— (4)}$$

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Since the ends of the two bars are rigidly Connected, each bar will Change in length by the same amount.

Also the length of each bar is ~~same~~ Same and hence the ratio of change in length to the original length (Strain) will be same for each bar.

But Strain in bar 1 = $\frac{\text{Stress in bar 1}}{\text{Young's modulus of bar 1}}$

$$= \frac{\sigma_1}{E_1}$$

Similarly in bar 2

$$= \frac{\sigma_2}{E_2}$$

But Strain in bar 1 = Strain in bar 2.

i.e. $\frac{\sigma_1}{E_1} = \frac{\sigma_2}{E_2}$ —— (5)

From Equation (4) & (5), the stresses σ_1 and σ_2 can be determined.

By Substituting the values of σ_1 and σ_2 in equations (ii) and (iii), the load carried by different materials may be computed.

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The ratio of $\frac{E_1}{E_2}$ is called the moduler ratio.

of the first material to the second.

Ex

A reinforced concrete circular column section of 50,000 mm^2 cross sectional area carries 6 reinforcing bars whose total area is 500 mm^2. Find the safe load the column can carry. If the concrete is not to be stressed more than 3-5 MPa. Take moduler ratio for steel and concrete as 18

Soln

Given : Area of column = 50,000 mm^2

No of reinforcing bars = 6

Total area of steel bars A_s = 500 mm^2

Minimum stress in concrete σ_c = 3·5 mPa = 3·5 N/mm^2

Moduler ratio $\frac{E_s}{E_c}$ = 18

Area of concrete is given as A_c

Area of concrete + Area of steel = Area of column.

i.e

$A_c + A_s$ = Area of column.

A_c = 50,000 − 500

= 49 500 mm^2.

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Stress in Steel :

$$\frac{\sigma_1}{E_1} = \frac{\sigma_2}{E_2}$$

i.e $$\frac{\sigma_s}{E_s} = \frac{\sigma_c}{E_c}$$

$$\sigma_s = \frac{E_s}{E_c} \sigma_c$$

$$= 18 \times 3.5$$

$$\sigma_s = 63 \text{ N/mm}^2$$

(2) Safe load the Column Can Carry :

$$P = P_c + P_s$$

$$P = \sigma_1 A_1 + \sigma_2 A_2$$

$$P = \sigma_c A_c + \sigma_s A_s$$

$$= (3.5 \times 49500) + (63 \times 500)$$

$$= 204750 \text{ N}$$

$$= 204.75 \text{ kN}$$

Questions

Question...

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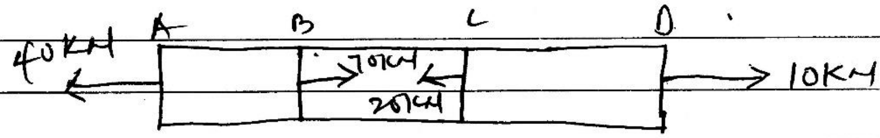
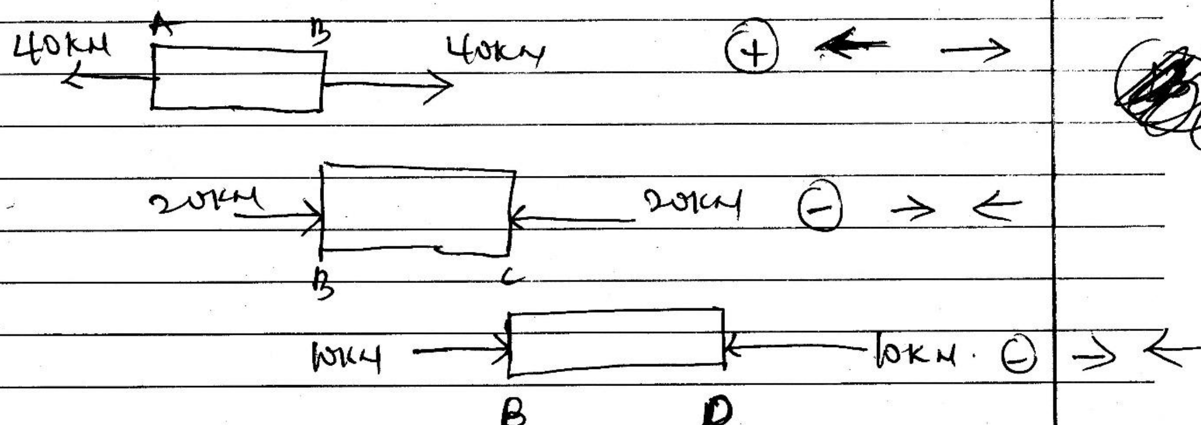
A brass bar having cross-section of 900 mm m^2 is subjected to an axial forces as shown below in which AB = 0·6m BC = 0·8m and CD = 1·0m. Find the total elongation of the bar. Take $E = 1.0 \times 10^5\ N/mm^2$.



Solution

(1) Draw the free body diagram.



Part AB – tensile force 40KN = 40,000 N

L_{AB} length of AB = 0·6m = 600mm.

$$\delta L_{AB} = \frac{P L_{AB}}{AE} = \frac{40,000 \times \cancel{600}}{\cancel{900} \times 1 \times 10^5}$$

$$= 0.2667\,mm \quad (+)$$

Part BC = Compressive force 20KN = 20,000N

L_{BC} = 0·8m = 800mm.

$$\delta L_{BC} = \frac{P L_{BC}}{AE} = \frac{20,000 \times \cancel{800}}{\cancel{900} \times 1 \times 10^5}$$

$$= 0.1778\,mm \quad (-)$$

Part BD Compressive force 10kN = 10,000 N.

$L_{BD} = 1.8m = 1800\ mm$.

$$\delta L_{BD} = \frac{P L_{BD}}{AE} = \frac{10,000 \times \cancel{1800}^{2}}{\cancel{900} \times 1\times10^5}$$

$= 0.2mm$ (−)

Total elongation

$= 0.2667 - 0.1778 - 0.2$

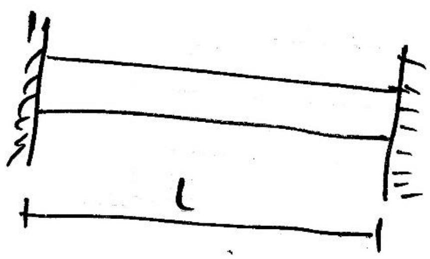
$= -0.1111mm$. Shortening of the bar

or decrease in length of the bar

Temperature Stresses (25)

Materials expand or contract with rise or fall in temperatures. However, if this expansion or contraction is wholly or partially resisted, stresses set up in the body.

Consider a bar restrained at both ends undergoing variations in temperature.



let L = original length of the bar

t = Increase in temperature and

α = Coefficient of linear expansion

The increase in length due to increase in temperature is given as

$$\delta L = L \cdot \alpha \cdot t .$$

If the ends of the bar are fixed to rigid supports, so that its expansion is prevented, then compressive strain induced in the bar

$$\varepsilon = \frac{\delta L}{L} = \frac{L \cdot \alpha \cdot t}{L} = \alpha \cdot t .$$

Stress $\sigma = \varepsilon E = \alpha \cdot t \cdot E$

$$\sigma = \alpha t E$$

(26)

Ex

A rod is 2m long at a temperature of 10°C. Find the expansion of the rod when the temperature is raised to 80°C. If this expansion is prevented, then find the stress in the material of the rod. $E = 100$ GPa and $\alpha = 0.000012$ per degree Celsius

Soln

Given length $l = 2\ m$

$2 \times 10^3\ mm$

$t = 80 - 10 = 70\ °C$

$\alpha = 0.000012\ /°C$

Expansion of the rod

$\delta l = L \cdot \alpha \cdot t$

$= 2 \times 10^3 \times 0.000012 \times 70$

$= 1.68\ mm$.

Stress in the material of the rod

$\sigma = \alpha t E$

$E = 100\ GPa$

$= 100 \times 10^3\ N/mm^2$

$\sigma = 0.000012 \times 70 \times 100 \times 10^3$

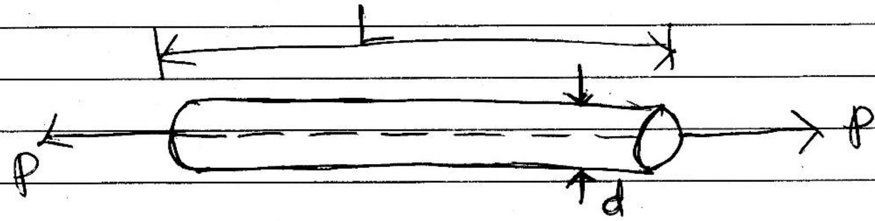
$= 84\ N/mm^2$

$= 84\ MPa$.

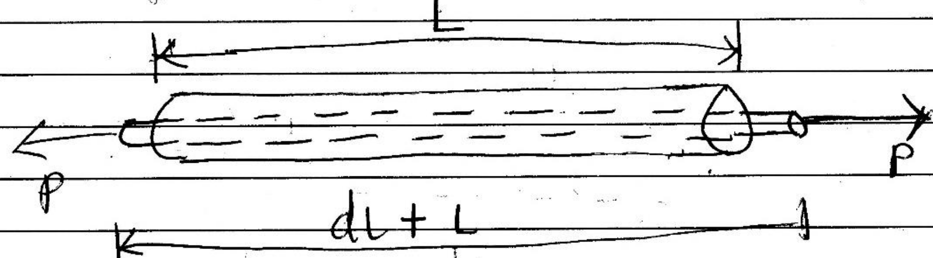
Poisson Ratio

We have already discussed that ~~whatever~~ whenever some external force acts on a body, it undergoes deformation.

Consider a circular bar, subjected to a tensile force as shown below.



L = length of the bar
d = diameter of the bar
dL = Increase in length of the bar as a result of the tensile force.



When there's tensile force (Pull) on a material, the length increases but the cross-sectional area of the material reduces (decrease in diameter).

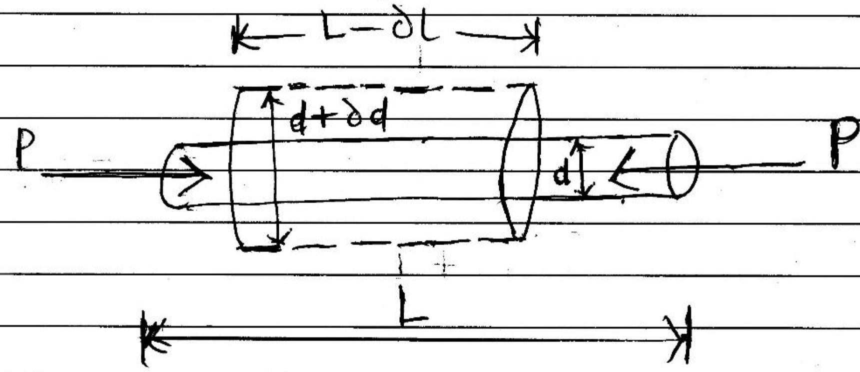
The deformation of the bar per unit length in the direction of the force is known as

i.e $\dfrac{\delta L}{L}$ is known as

primary / Linear / longitudinal strain

If we actually study the deformation of the bar, we will find that the bar has extended through length δL, which will be followed by the decrease of diameter d to $(d - \delta d)$

Similarly if the bar is subjected to a compressive force.



the length of the bar will decrease by δL which will be followed by the increase of diameter d to $(d + \delta d)$

It is thus obvious that every direct stress is always accompanied by a strain in its own direction and an opposite kind of strain in every direction at right angles to it. Such strain is known as secondary ~~and~~ or lateral strain.

The ration

$$\frac{\text{Lateral strain}}{\text{Linear (longitudinal) strain}} = \text{Constant}$$

i.e The ratio lateral strain to the longitudinal (linear) strain is known as poisson ratio (μ)

$$\frac{\text{Secondary strain}}{\text{primary strain}} = \frac{\text{Lateral strain}}{\text{Linear strain}} = \mu$$

i.e Lateral strain = Poisson ratio $\times$ Linear strain

i.e. Lateral strain = $\mu \varepsilon$

ε = linear / longitudinal strain

Example

A steel bar 2m long 40mm wide and 20mm thick is subjected to an axial pull of 160KN in the direction of its length. Find the changes in length, width and thickness of the bar.

Take $E = 200$ GPa and

Poisson's ratio $= 0.3$

Solution

Length $L = 2m = 2 \times 10^3$ mm

width $b = 40$ mm.

thickness $t = 20$mm.

Axial Pull $P = 160$ KN $= 160 \times 10^3$ N

Modulus of Elasticity $E = 200$ GPa

$$200 \text{ GPa} = 200 \times 10^9 \text{ Pa}$$

$$10^6 \text{ Pa} = 1 \text{ N/mm}^2$$

$$200 \times 10^9 \text{ Pa} = \left(\frac{200 \times 10^9}{10^6} \right) \text{N/mm}^2$$

$$= 200 \times 10^3 \text{ N/mm}^2$$

(i) Change in length δL

$$\delta L = \frac{PL}{AE}$$

$$= \frac{(160\times10^3)(2\times10^3)}{(40\times20)(200\times10^3)}$$

$$\delta L = 2\,mm.$$

~~Change in width.~~

Linear strain $\varepsilon = \frac{dL}{L}$

$$\varepsilon = \frac{2}{2\times10^3}$$

$$\varepsilon = 0.001$$

Lateral strain $= \mu\varepsilon$

$$= 0.3\times0.001$$

$$= 0.0003$$

(2) Change in width.

$$\delta b = b \times \text{Lateral Strain}$$
$$= 40 \times 0.0003$$
$$= 0.012 \text{ mm}.$$

(3) Change in thickness

$$\delta t = t \times \text{lateral strain}$$
$$= 20 \times 0.0003$$
$$= 0.006 \text{ mm}$$

Ex 2

A metal bar 50mm × 50mm in Section is subjected to an axial Compressive load of 500kN. If the contraction of 200mm gauge length was found to be 0.5mm and the increase in thickness 0.04 mm. Find the Young Modulus and poisson's ratio for the bar material.

Soln

Given
width $b = 50$mm
thickness $t = 50$mm
Axial Compressive load $P = 500$ kN
500×10^3 N

length $L = 200\,mm$
Change in length $\delta L = 0.5\,mm$
Change in thickness $= 0.04\,mm$.

Value of E (young modulus of the bar material)

$$\delta L = \frac{PL}{AE}$$

i.e $$0.5 = \frac{(500 \times 10^3)(200)}{(50 \times 50) \times E}$$

$$E = 80 \times 10^3\ N/mm^2$$

(2) Value of poissons ratio μ

Linear strain $\varepsilon = \frac{\delta L}{L} = \frac{0.5}{200} =$
$= 0.0025$

lateral strain $= \mu \times 0.0025$.

but $0.04 = t \times$ lateral strain

i.e $0.04 =$ Change in thickness.

i.e $\frac{0.04}{50} = \mu \times 0.0025$

$\mu = 0.32$.

Answers

(1)

- Show that deformation δl of a body due to force acting on it is given as

$$\delta L = \frac{PL}{AE}$$

Solution

We know that Stress $\sigma = \frac{P}{A}$ — (2)

And Strain $\varepsilon = \frac{\sigma}{E}$ — (2)

i.e $\varepsilon = \frac{P}{AE}$ — (1)

And deformation (change in length) δL is from Strain $\varepsilon = \frac{\delta L}{L}$ $\frac{\text{Change in length}}{\text{Original length}}$ — (2)

i.e $\delta L = \varepsilon . L$ — (1)

$\varepsilon = P/AE$

$\therefore \delta L = \frac{PL}{AE}$ — (2)

(2)

Solution

Given length $l = 2m = 2 \times 10^3 mm$ (1/2)

Outside diameter $D = 50mm$

Inside diameter $d = 30mm$

load $P = 25KN = 25 \times 10^3 N$ — (1)

Modulus of Elasticity $E = 100GPa$

$= 100 \times 10^3 N/mm^2$ (1)

Cross-sectional area of the hollow Cylinder A,

$$A = \frac{\pi}{4}(D^2 - d^2)$$

$$= \frac{\pi}{4}(50^2 - 30^2)$$

$$= \underline{\underline{1257 mm^2}}$$ (2 1/2)

Stress in the Cylinder σ

$$\sigma = \frac{P}{A} = \frac{25 \times 10^3}{1257}$$

$$= 19.9\ N/mm^2 \text{ or } 19.9\ MPa \quad (2\tfrac{1}{2})$$

Deformation of the Cylinder δL

$$\delta l = \frac{P\,L}{A\,E} = \frac{(25\times10^3)\times(2\times10^3)}{1257 \times (100\times10^3)}$$

$$= 0.397 \approxeq \underline{0.4mm} \quad (2\tfrac{1}{2})$$

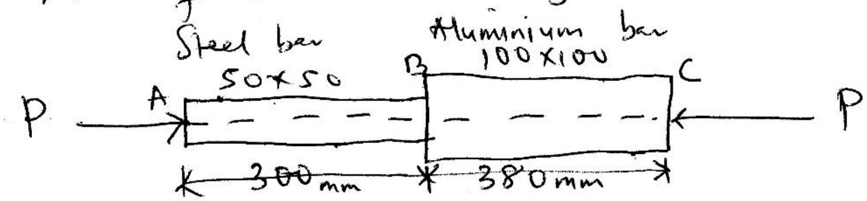
(3) Solution : Given ;

Decrease in length (δL) = 0.25mm (1/2)

Modulus of Elasticity for Steel E_S = 210 GPA = $= 210 \times 10^3\ N/mm^2$ (1/2)

Modulus of Elasticity for Aluminium E_A = 70 GPA = $= 70 \times 10^3\ N/mm^2$. (1/2)

Area of Steel Section A_S = $50 \times 50 = 2500\,mm^2$ (1/2)



Area of Aluminium Section $A_A = 100 \times 100$
$= 10000\ mm^2$ (1/2)

Length of Steel Section l_S = 300mm (1/2).

length of Aluminium Section l_A = 380mm. (1/2)

let P = Magnitude of force in KN (1/2)

We Know that decrease in the length of the member δl

i.e $\delta l = \frac{P l_s}{A_s E_s} + \frac{P l_A}{A_A E_A}$

i.e $\delta l = P\left[\frac{l_s}{A_s E_s} + \frac{l_A}{A_A E_A}\right]$ ✓ (2)

i.e $0.25 = P\left[\frac{300}{2500 \times (210 \times 10^3)} + \frac{380}{10000 \times (70 \times 10^3)}\right]$

$0.25 = \frac{780 P}{700 \times 10^6}$ ✓ (1)

$P = \frac{0.25 \times (700 \times 10^6)}{780}$ ✓ (1)

$P = 224.4 \times 10^3 \, N$ ✓ (1)

$P = 224.4 \, KN$ ✓ (1)

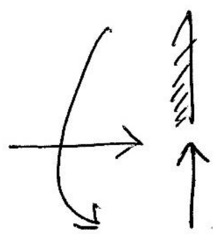
(4) (a)

The Shearing force at any section of a beam is the algebraic sum of all the lateral components of the forces acting on either side of the section. (2)

The bending moment about any section of a beam is the algebraic sum of all the moments about that section of all forces acting on either side of the section.

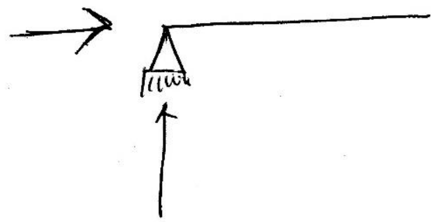
4 (b)

(i) Fixed / Rigid Support (2)



This Support supplys three reactive forces; horizontal vertical; and fixing moments.

(ii) Pinned or Hinged Support (2)



This Supports Supplys two reactions; horizontal and vertical forces.

(iii) Roller or Rocker Support.

This Support supplys one reaction; vertical forces.

Shear Force and Bending Moment Diagram of Beams

1.0 General Background

1.1 Beam – structural member loaded perpendicular to its longitudinal axis

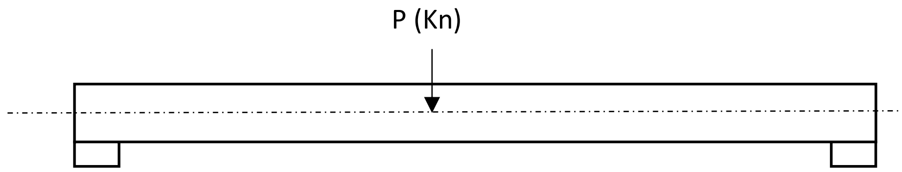
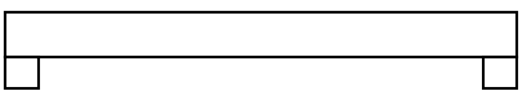
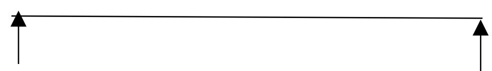


Figure 1

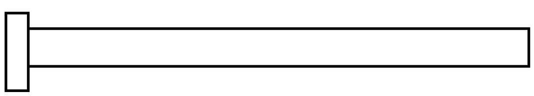
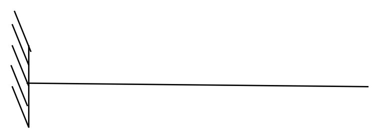
Beams transmit loads by the development of BM and SF at different sections. Beams can be classified in many ways

i. Classification based on Support system

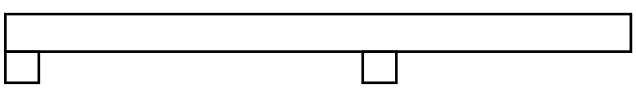
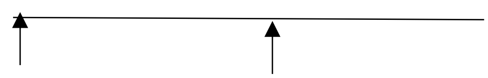
- Simply supported

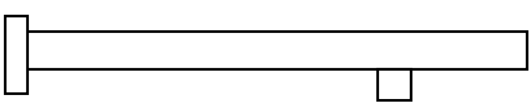
- Cantilever

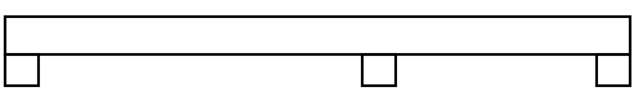
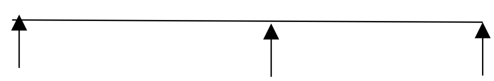
- Simply supported with overhangs

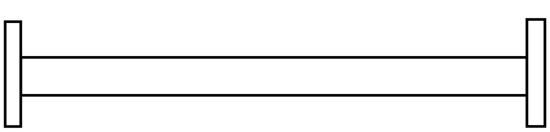
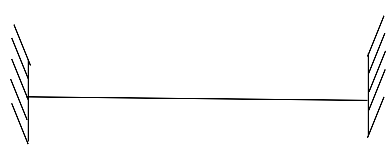
- Propped cantilever




- Continuous

- Fixed ends

1.2 **Types of Support in Beams Structure**

The supports in beams (framed structure) supply the necessary reactive forces to maintain the structure in equilibrium as a result of applied load. There are many types of supports from which a system of supports can result. Some of these supports are:

a) Fixed End /Bulk-In/EnCastre

Figure 1.2.1: Fixed Support

Three supports are capable of supplying three reactive forces: horizontal, vertical, and a fixing moment. This type of support is fixed so that it cannot move or rotate under the action of superimposes load.

b) Pin Support

Figure 1.2.2: Pinned Support

This is assumed to be free to rotate under the applied load, but cannot move either vertically or horizontally. This type of support can produce two support reactions. These are: horizontal and vertical forces.

c) The Roller Bearing

This can only supply one reaction. This is the case when a beam simply rests on a support. The bean can rotate. It can move horizontally. But it is restrained in the vertical direction only. When we sit down on a chair, this is the type of support that develop

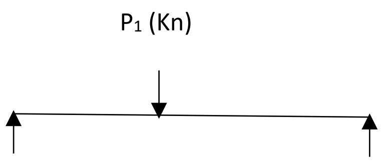
1.2.3: Roller Bearing Support

1.3 Types of Loads on Beams Structure

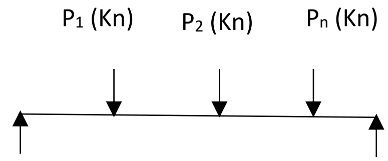
Many forms of loads are dealt with in structural analysis. They include:

1. Static Load

   i) Point Static. Also known as concentrated load



One (1) Point Load

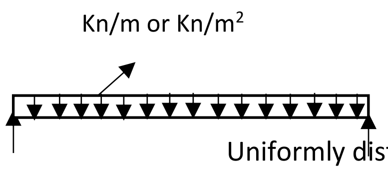


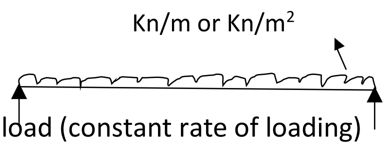
Many Point Loads

   ii) Distributed Load (udl)

   This is when a load is applied in such a way that it spread over the entire or part of the length.

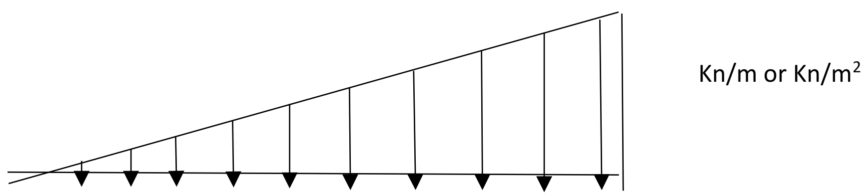
   a. When the loading rate is uniform, it is called **uniformly distributed load**.



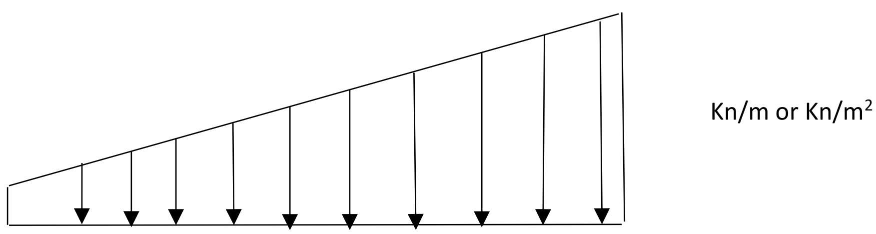


Uniformly distributed load (constant rate of loading)

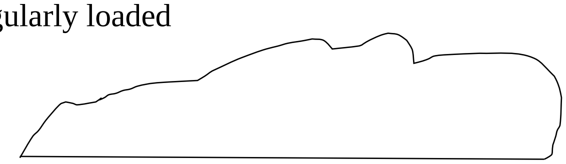
   b. Triangular Load (varying rate of loading). For example, soil pressure on wall, etc.



   c. Trapezoidal loaded (Varying rate of loading). For example, soil pressure + permanent load

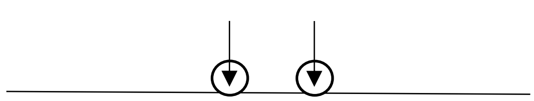


   d. Irregularly loaded



2. Dynamic Loads

These are moving loads. It usually occurs in highway/bridge analysis and design.



1.4 SHEAR FORCE AND BENDING MOMENTS

The forces acting in a structural beam member may include some or all the following:

a. Forces normal to elemental axis e.g. shear force.

b. Forces parallel to elemental axis e.g. Axial or normal forces

c. Bending Actions. For example, bending moment and twisting moment

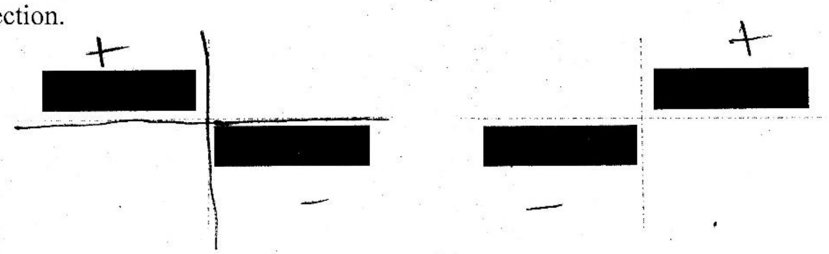
Beans subjected to Bending Actions

In the beam type of structure, analysis involves the determination of the magnitude of the Shear force (SF) and the Bending moment (BM) at all points of the beam.

The algebraic sum of the vertical forces at any section of a beam to the right or left of the beam is known as shear force.

The Shearing Force (SF)

The shearing Force at any section of a beam is defined as the unbalanced vertical force to the right or left of the section.



a) Positive S.F b) Negative SF

The Shearing force at any section is the algebraic sum of all the lateral components of the forces acting on either side of the section.

Shearing force is +ve when the resultant to the left is upward, and –ve when resultant to the right is downward.

The shear force varies along the length of the beam; and this variation is represented by a diagram. The **shearing force diagram** is the one which shows the variation of shearing force along the length of the beam

The algebraic sum of the moments of all forces acting to the right or left of the section is known as bending moment.

The Bending Moment (BM)

The bending moment about any section is the algebraic sum of all the moments about that section of all the forces acting on either side of the section

The Bending moment diagram is the one which shows the variation of bending moment along the length of the beam

Bending moment can be of two types:

a. Sagging moment → +ve

b.) Hogging moment → -ve

Also, bending moment at a section is described in terms of clockwise and anticlockwise. It is taking to be POSITIVE when clockwise and NEGATIVE when anticlockwise

a) Positive Moment

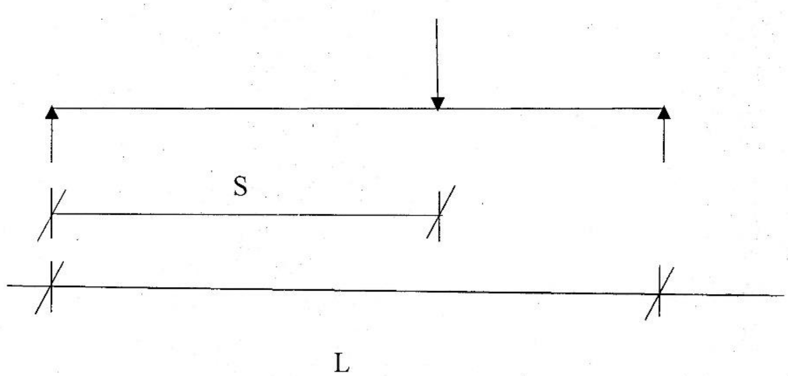
b) Negative Moment

Note:

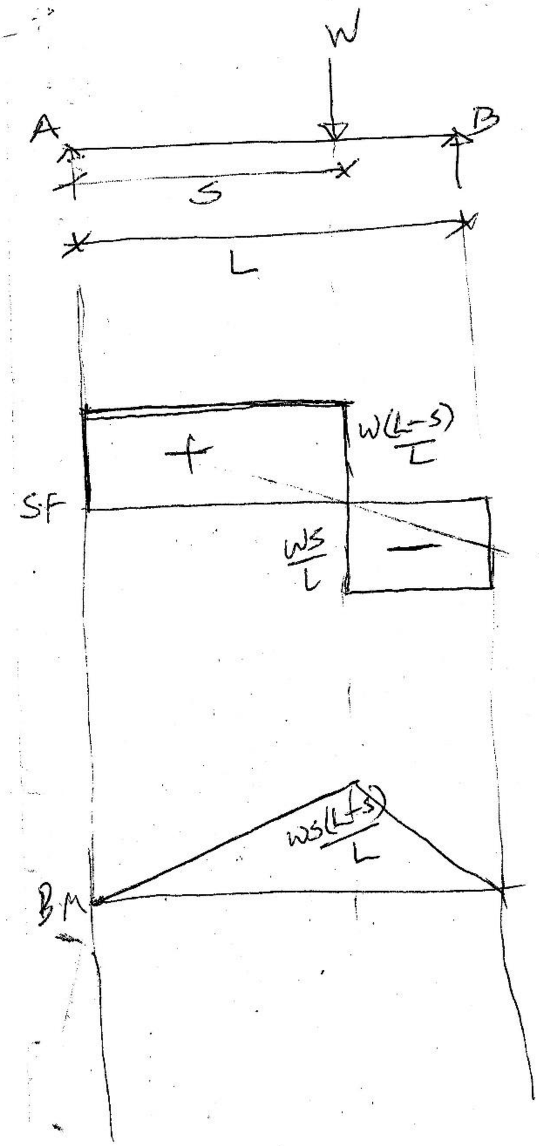
1. The beam is assumed to be weightless when calculating SF and BM in a section
2. When drawing the SF and BM diagrams, all positive values are plotted above the base line and the negatives are plotted below the base line.

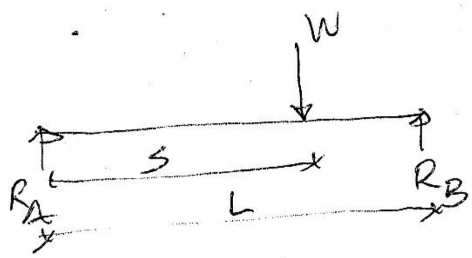
Example 1

Draw the shear force and bending moment diagram for a simply supported beam with a point load as shown



Example 1 Solution





Find the Rxns A & B

$\Sigma F_y = 0$

$R_A + R_B = W$ —— (1)

$\Sigma M_A = 0 \quad -R_B L + W s = 0$

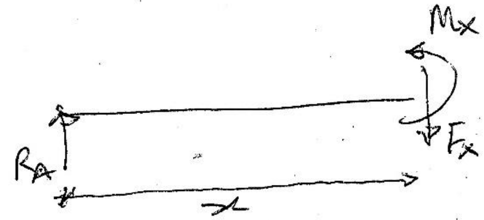
(1) $R_B L = W s$

$R_B = \frac{W s}{L}$ —— (2)

from eqn 1

$$R_A = W - R_B$$
$$= W - \frac{W s}{L}$$
$$= W\left(1 - \frac{s}{L}\right) \quad \text{—— 3a}$$
$$= \frac{W(L-s)}{L} \quad \text{—— 3b}$$

Section 1 $0 \le x \le s$



SF

$\Sigma F_y = 0$

$R_A - F_x = 0$

$$F_x = R_A = \frac{W(L-s)}{L}$$

This is the general eqn of SF for the section. Constant. Not fn

at $x = 0$; $F_0 = \frac{W(L-s)}{L}$

at $x = s$ $\quad F_s = \frac{W(L-s)}{L}$

BM

$\Sigma M_s = 0$

$R_A x - M_x = 0$

$M_x = R_A x$; $= \frac{W(L-s)x}{L}$

is BM eqn. It is a [illegible] expression in x

[illegible] $x = 0$

$$M_{x=0} = \frac{W(L-S)}{L}(0)$$

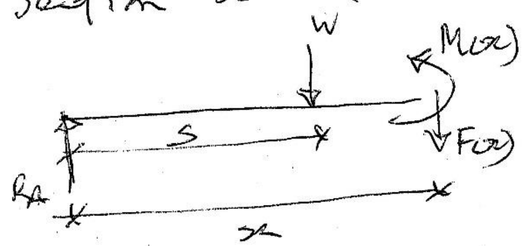
$$= 0$$

at $x = S$

$$M_{x=S} = \frac{W(L-S)S}{L}$$

$$= \frac{WS(L-S)}{L}$$

Section 2 $S \leq x \leq L$



S.F

$$\Sigma F_y = 0$$

$$R_A - W - F_x = 0$$

$$F_x = \cancel{W - R_A}\ R_A - W$$

$$= \frac{W(L-S)}{L} - W$$

$$=$$

$$F_x = -\frac{WS}{L}$$

this is expression. It is [illegible]

at $x = 0$

$$F_x = -\frac{WS}{L}$$

(2)

BM

$$R_A x - W(x-S) - M_{xy} = 0$$

$$M_x = R_A x - W(x-S)$$

$$= \frac{W(L-S)}{L}x - W(x-S)$$

$$= \cancel{Wx} - \frac{WSx}{L} - \cancel{Wx} + WS$$

$$= -\frac{WSx}{L} + WS$$

This is the BM expression

at $x = S$

$$M_{(x=S)} = -\frac{WSS}{L} + WS$$

$$= WS\left(1 - \frac{S}{L}\right)$$

$$= WS\frac{(L-S)}{L}$$

at $x = L$

$$M_{(x=L)} = -\frac{WSL}{L} + WS$$

$$= -WS + WS$$

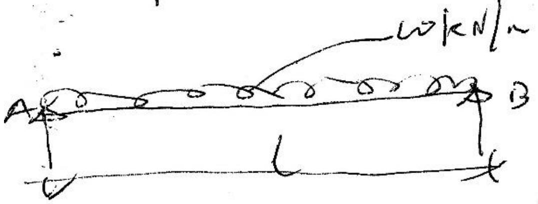
$$= 0\ \text{kNm}$$

Conclusion

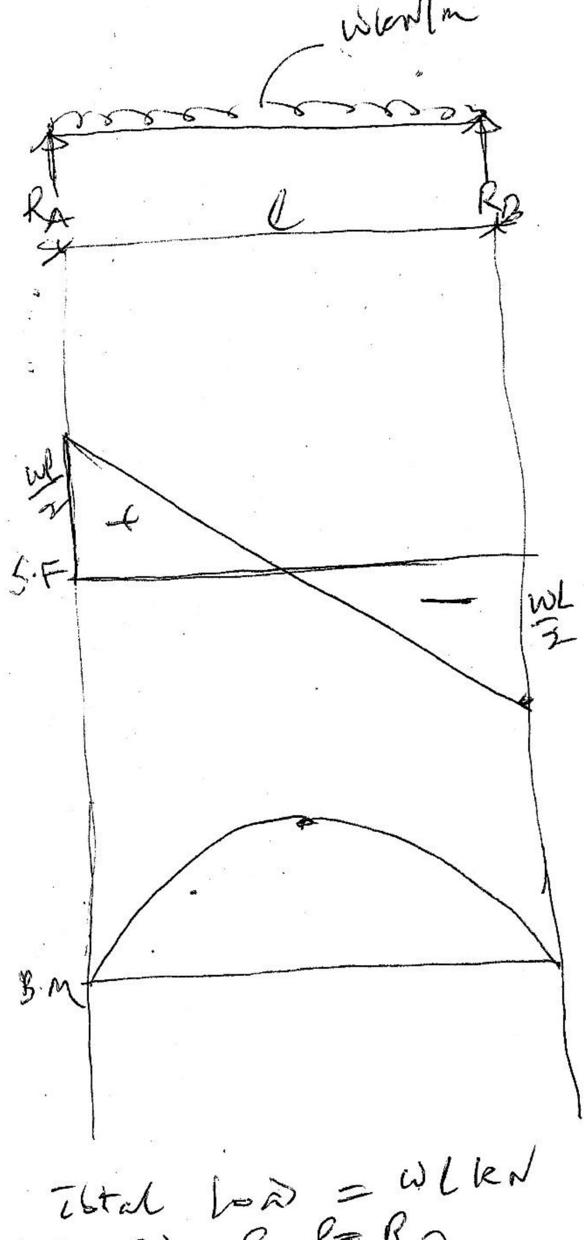
For simply supported beam & point load

P.T.O

Example 2 (3)



Solution



Total load = wl kN

To find R_A & R_B

$\Sigma F_y = 0$

$R_A + R_B = wl$ —— (1)

$\Sigma M_A = 0$

$-R_B l + wl \cdot \frac{l}{2} = 0$

$R_B l = \frac{wl^2}{2}$

$R_B = \frac{wl}{2}$ kN —— (2)

from eqn 1

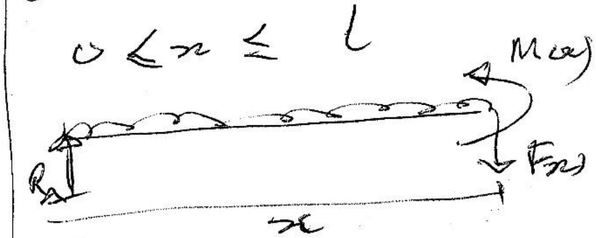
$R_A = wl - R_B$

$= wl - \frac{wl}{2}$

$= \frac{wl}{2}$ —— (3)

Just one section is needed

$0 \leq x \leq l$



S.F

$\Sigma F_y = 0$

$R_A - wx - F_x = 0$

$F_x = R_A - wx$

$= \frac{wl}{2} - wx$ —— (4)

this is a general eqn

at $x = 0$

$F_{(x=0)} = \frac{wl}{2} - 0$

$= \frac{wl}{2}$ kN

at $x = l$

$F_{(x=l)} = \frac{wl}{2} - wl$

B.M.

$\Sigma M_D = 0$

$$R_A x - Wx \cdot \frac{x}{2} - M_x = 0$$

$$M_{xx} = R_A x - \frac{Wx^2}{2}$$

$$= \frac{wlx}{2} - \frac{wx^2}{2} \longleftarrow (2)$$

This is the B.m. expression & it is quadratic

at $x = 0$

$$M_{(x=0)} = \frac{wl(0)}{2} - \frac{w(0)^2}{2}$$

$$= 0 \text{ kN.m}$$

at $x = L$

$$M_{(x=L)} = \frac{wlL}{2} - \frac{wL^2}{2}$$

$$= 0 \text{ kN.m}$$

For a simply supported beam carrying a udl, the relationship between the SF & BM is that at the point of zero shear, the BM is maximum

$$F = \frac{dM}{dx} = 0$$

$$= R_A - Wx$$

$$= 0$$

$$x = \frac{R_A}{W} = \frac{\frac{wl}{2}}{W} = \frac{L}{2} \text{ m}$$

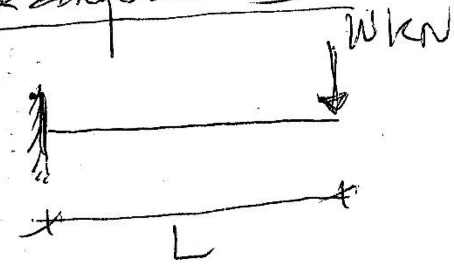
Thus the B.M at $x = \frac{L}{2}$

$$M_{(x=\frac{L}{2})} = \frac{wL}{2}\frac{L}{2} - \frac{w\left(\frac{L}{2}\right)^2}{2}$$

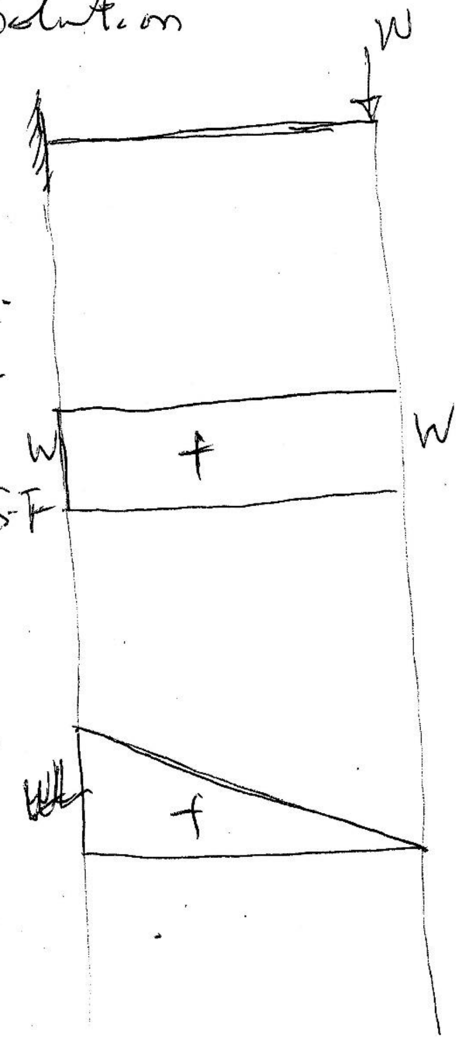
$$= \frac{wL^2}{4} - \frac{wL^2}{8}$$

$$= \frac{wL^2}{8} \text{ kN-m}$$

Example 3 ⑥



Solution



Only one section will be cut referenced at RHS.

$0 < x < l$

SF

$\Sigma F_y = 0$

$F - W = 0$

$F_{(x)} = W$ —— (1)

That is, independent of the beam span. Constant through all the length

at $x = 0$

$F_{(x=0)} = W$ KN

at $x = l$

$F_{(x=l)} = W$ KN

B.M

$\Sigma M_x = 0$

$Wx - M_x = 0$

$M_{(x)} = Wx$ —— (2)

A fcn of x.

at $x = 0$

$M_{(x=0)} = W \times 0$

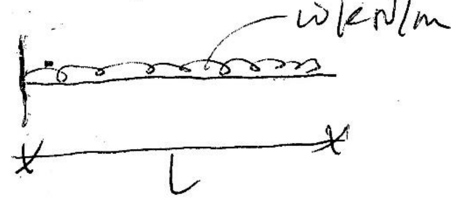
$= 0$ KN.m

at $x = l$

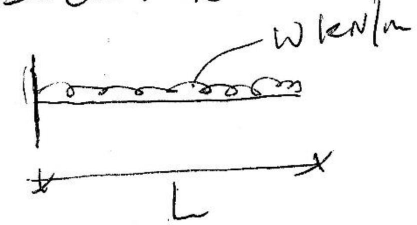
$M_{(x=l)} = WL$

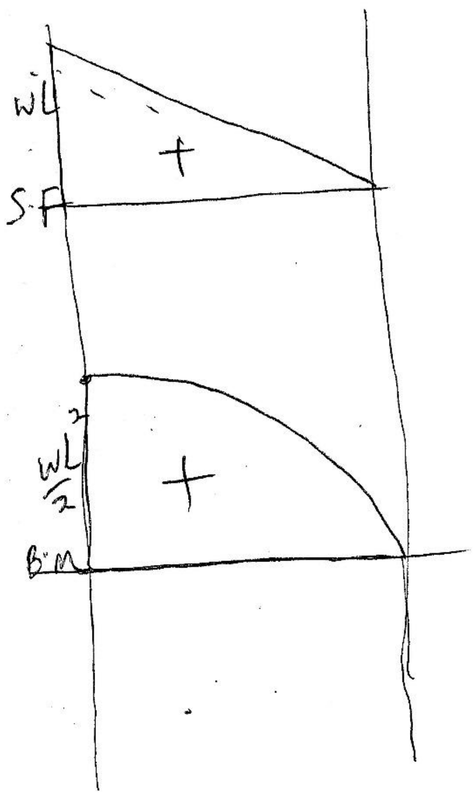
$= Wl$ (KN.m)

Example 4

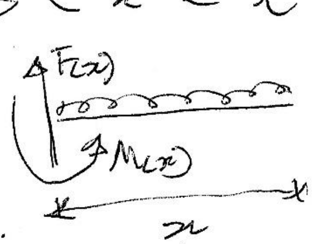


Solution





$0 < x < l$



S.F

$\Sigma F_y \Rightarrow$

$F(x) - wx = 0$

$F(x) = wx$ —— (1)

That is a linear function of x

at $x = 0$

$F_{(x=0)} = w \times 0 = 0\,kN$

at $x = l$

$F_{(x=l)} = wl$

$= wl\,kN$

B.M

$\Sigma M_{(x)} = 0$

$-M_{(x)} + wx \cdot \frac{x}{2} \Rightarrow$

$M_{(x)} = \frac{wx^2}{2}$ —— (2)

A quadratic expression

at $x = 0$

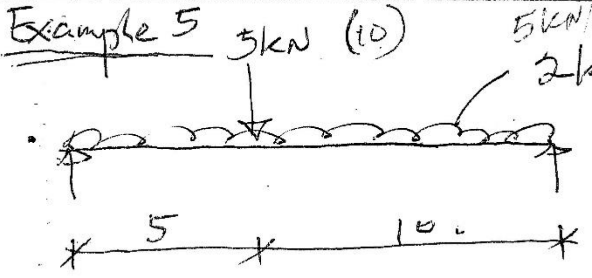
$M_{(x=0)} = 0\,kN.m$

at $x = l$

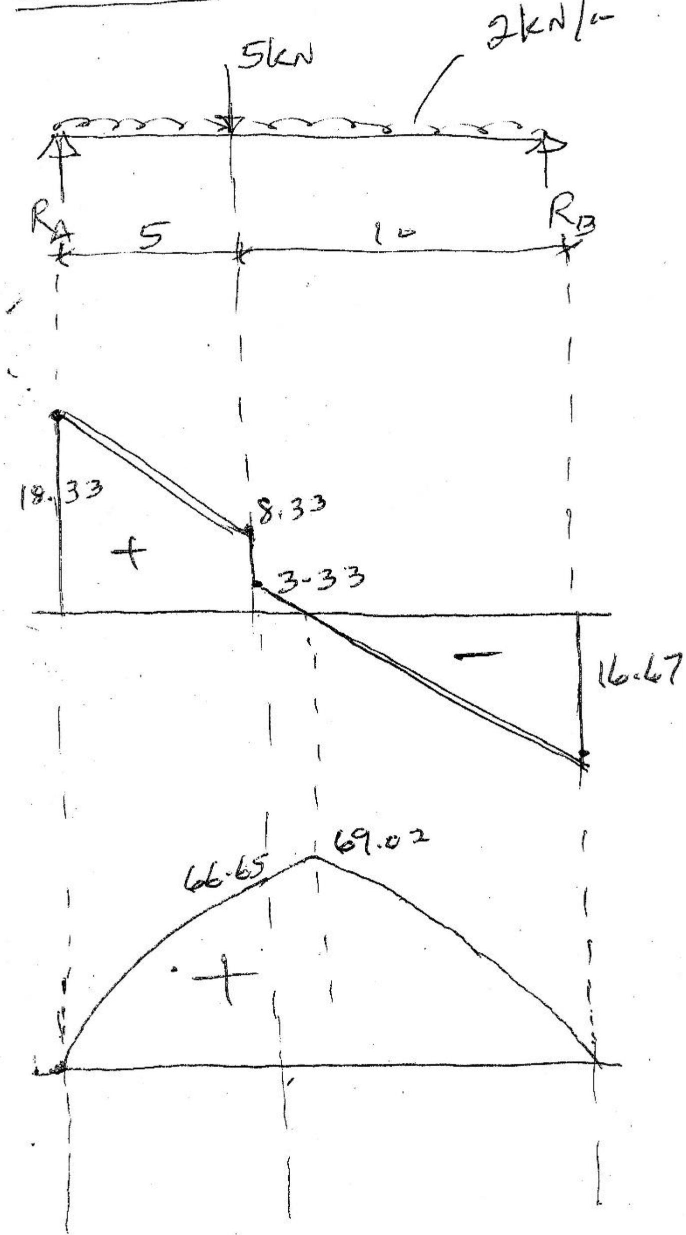
$M_{(x=l)} = \frac{wl^2}{2}$

$= \frac{wl^2}{2}\,kNm$

Example 5 (10)



Solution



$R_A + R_B = 5 + 2 \times 15$
$= 5 + 30$
$= 35\,kN$ — (1)

Moment about A

$5 \times 5 + 2 \times 15 \cdot \frac{15}{2} - 15R_B = 0$

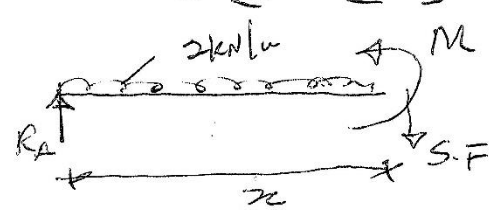
$15R_B = 25 + 225 = 250$

$\underline{R_B = 16.67\,kN}$

from eqn 1

$R_A = 35 - 16.67$
$\underline{= 18.33\,kN}$

$0 \leq x \leq 5$



SF

$\Sigma F_y = 0$

$R_A - 2x - F(x) = 0$

$F(x) = R_A - 2x$
$= 18.33 - 2x$ (2)

at $x = 0$
$F(x=0) = 18.33\,kN$

at $x = 5$
$F(x=5) = 18.33 - 2 \times 5$
$= 8.33\,kN$

BM

$R_A x - 2x \cdot \frac{x}{2} - M(x) = 0$

$M(x) = R_A x - x^2$

$M_{(x)} = 18.33x - x^2$

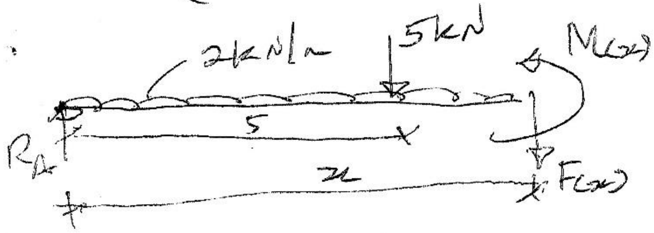
at $x = 0$

$M_{(x=0)} = 0 \text{ kN.m}$

at $x = 5$

$M_{(x=5)} = 18.33 \times 5 - (5)^2$
$= 91.65 - 25$
$= 66.65 \text{ kN.m}$

$5 \leq x \leq 15$



S.F

$\Sigma F_y = 0$

$R_A - 2x - 5 - F_{(x)} = 0$

$F_{(x)} = R_A - 2x - 5$
$= 18.33 - 2x - 5$
$= 13.33 - 2x$ (*)

at $x = 5$

$F_{(x=5)} = 13.33 - 2 \times 5$
$= 13.33 - 10$
$= 3.33 \text{ kN}$

at $x = 15$

$F_{(x=15)} = 13.33 - 2 \times 15$
$= -16.67 \text{ kN}$

(9) At the point where S.F crosses the axis,

$F_{(x)} = 0$

That is

$0 = 13.33 - 2x$

$\underline{x = 6.67 \text{m from A}}$

BM

$\Sigma M_o = 0$

$R_A x - 2 \cdot x \cdot \frac{x}{2} - 5(x-5) - M_{(x)}$

$M_{(x)} = R_A x - x^2 - 5x + 25$
$= 18.33x - x^2 - 5x + 25$
$= -x^2 + 13.33x + 25$

at $x = 5$

$M_{(x=5)} = -(25) + 66.65 + 25$
$= 66.65 \text{ kN.m}$

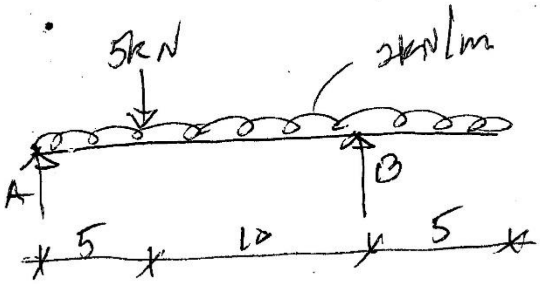
at $x = 15$

$M_{(x=15)} = -225 + 199.95 + 25$
$= 0.05 \text{ kN.m}$
$\simeq 0 \text{ kN.m}$

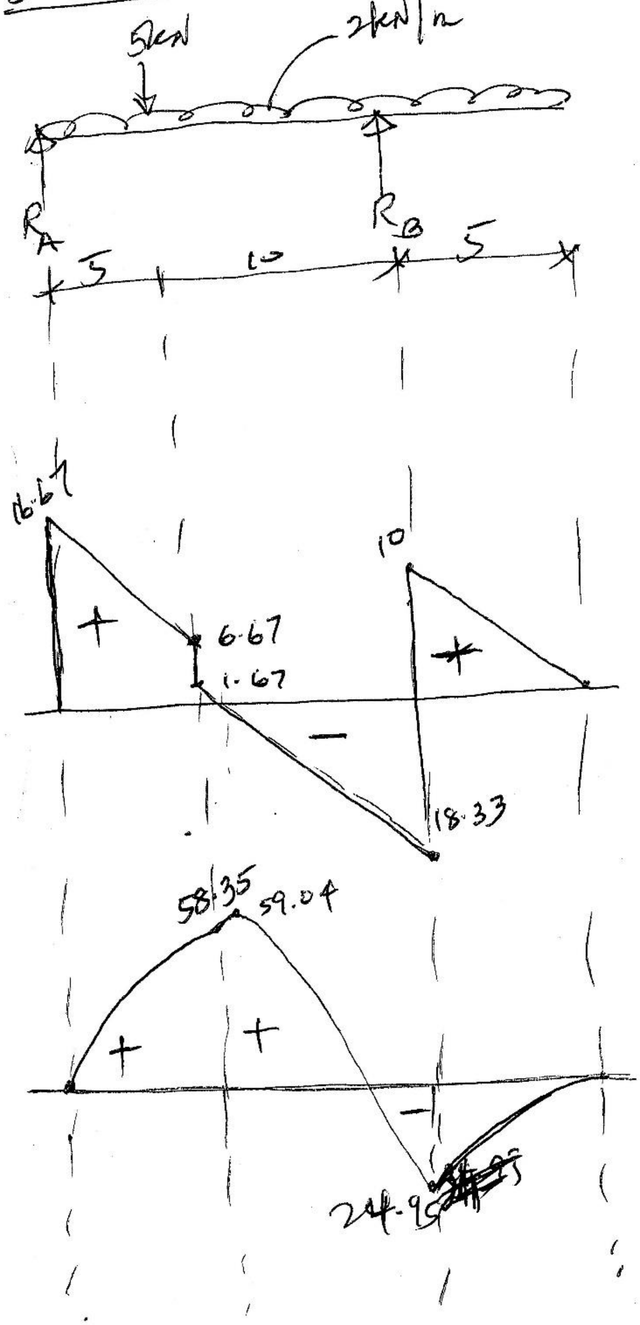
The max Moment at
$x = 6.67 \text{m}$

$M_{max} = -(44.89) + 88.91 + 25$
$= 69.02 \text{ kN.m}$

Example 6 (10)



Solution



Find the reactions R_A & R_B

$$R_A + R_B - 2\times 20 - 5 = 0$$

$$R_A + R_B = 45 \quad \text{---} (1)$$

M_A

$$-15R_B + 5\times 5 + 2\times 20 \times \frac{20}{2} = 0$$

$$-15R_B + 25 + 400 = 0$$

$$15R_B = 425$$

$$R_B = 28.33\,KN$$

from eqn 1

$$R_A = 45 - R_B$$

$$= 45 - 28.33$$

$$= 16.67\,KN$$

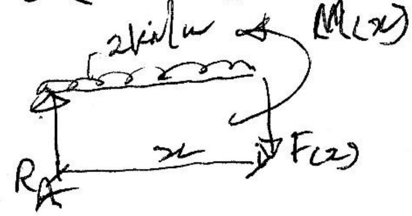
Sections

(1) $0 < x < 5$

(2) $5 < x < 15$

(3) $15 < x < 20$

(1) $0 < x < 5$



S.F.

$$R_A - 2x - F(x) = 0$$

$$F(x) = R_A - 2x$$

$$\boxed{= 16.67 - 2x} \quad (2)$$

at $x = 0$ m

$$F(x) = \underline{16.67\,KN}$$

$x = 5$ m

$$F(5) = 16.67 - 2\times 5 = 6.67$$

B.M

$R_A x - 2\cdot x\cdot \frac{x}{2} - M_{(x)} = 0$

$M_{(x)} = R_A x - x^2$

$\boxed{= 16.67x - x^2}$ (3)

at $x = 0$

$M_{(x)} = 0$

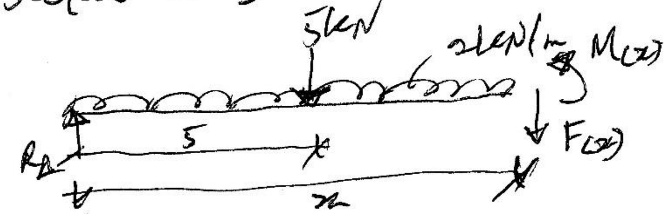
at $x = 5$ m

$M_{(x)} = 16.67\times 5 - (5)^2$

$= 83.35 - 25$

$= 58.35$ kN.m

section 2 $5 < x < 15$



S.F.

$R_A - 2x - 5 - F_{(x)} = 0$

$F_{(x)} = R_A - 2x - 5$

$= 16.67 - 2x - 5$

$\boxed{= 11.67 - 2x}$ — (4)

at $x = 5$ m

$F_{(x)} = 11.67 - 10$

$= 1.67$ kN

$x = 15$ m

$F_{(x)} = 11.67 - 2\times 15$

$= 11.67 - 30$

$= -18.33$ kN

At point where S.F $= 0$

eqn 4 $= 0$

That is

$11.67 - 2x = 0$

$2x = 11.67$

$\boxed{x = 5.84\text{ m}}$ (from the left)

B.M

$R_A x - 5(x-5) - 2\cdot x\cdot\frac{x}{2} - M_x = 0$

$M_{(x)} = R_A x - 5x + 25 - x^2$

$= 16.67x - 5x + 25 - x^2$

$= 11.67x + 25 - x^2$

$\boxed{= -x^2 + 11.67x + 25}$ — (5)

at $x = 5$ m

$M_{(x)} = \overset{-(5)^2}{-25} + 58.35 + 25$

$= 58.35$ kN.m

at $x = 15$ m

$M_{(x)} = -(15)^2 + 175.05 + 25$

$= -225 + 175.05 + 25$

$= -24.95$ kN.m

Max Moment at $x = 5.84$

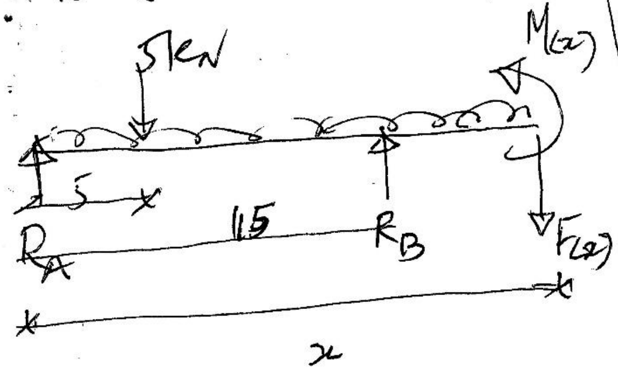
$M_{max} = -(5.84)^2 + 11.67\times 5.84 + 25$

$= -34.11 + 68.15 + 25$

$\boxed{= 59.04\text{ kN.m}}$

Section 3 (12)

$15 < x < 20$



S.F..

$R_A - 5 - 2x + R_B - F(x) = 0$

$F(x) = R_A + R_B - 5 - 2x$

$= 16.67 + 28.33 - 5 - 2\cancel{x}$

$= 45 - 5 \cancel{-40} - 2x$

$\boxed{= 40 - 2x}$ (6)

at $x = 15\,m$

$F(x) = 40 - 30$

$= 10\,kN$

at $x = 20\,m$

$F(x) = 40 - 40$

$= 0\,kN$

BM

$R_A x - 5(x-5) - 2 \cdot x \cdot \frac{x}{2} + R_B(x-15) - M(x) = 0$

$R_A x - 5x + 25 - x^2 + R_B x - 15R_B - M(x) = 0$

$x(R_A - 5 + R_B) + 25 - 15R_B - x^2 = M(x)$

$40x + 25 - 15 \times 28.33 - x^2 = M(x)$

$40x - 399.95 - x^2 = M(x)$

$M(x) = -x^2 + 40x - 399.95$

at $x = 15\,m$

$M(x) = -225 + 600 - 399.95$

$= 24.95\,kNm$

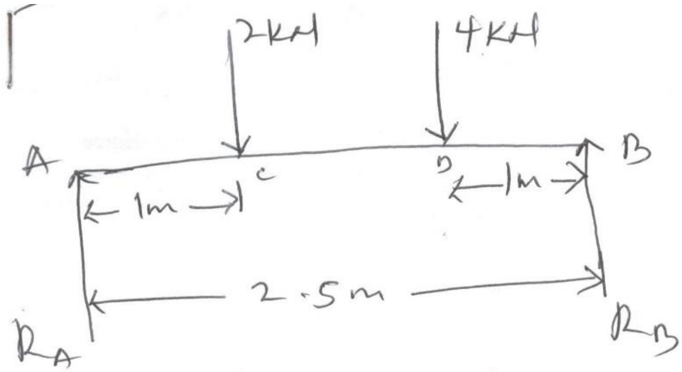
at $x = 20\,m$

$M(x) = -(20)^2 + 40 \times 20 - 399.95$

$= -400 + 800 - 399.95$

$= 0\,kN \cdot m$

Symmetric loadin



$\Sigma M_B = 0$

$2.5R_A - 2(1.5) - 4(1) = 0$

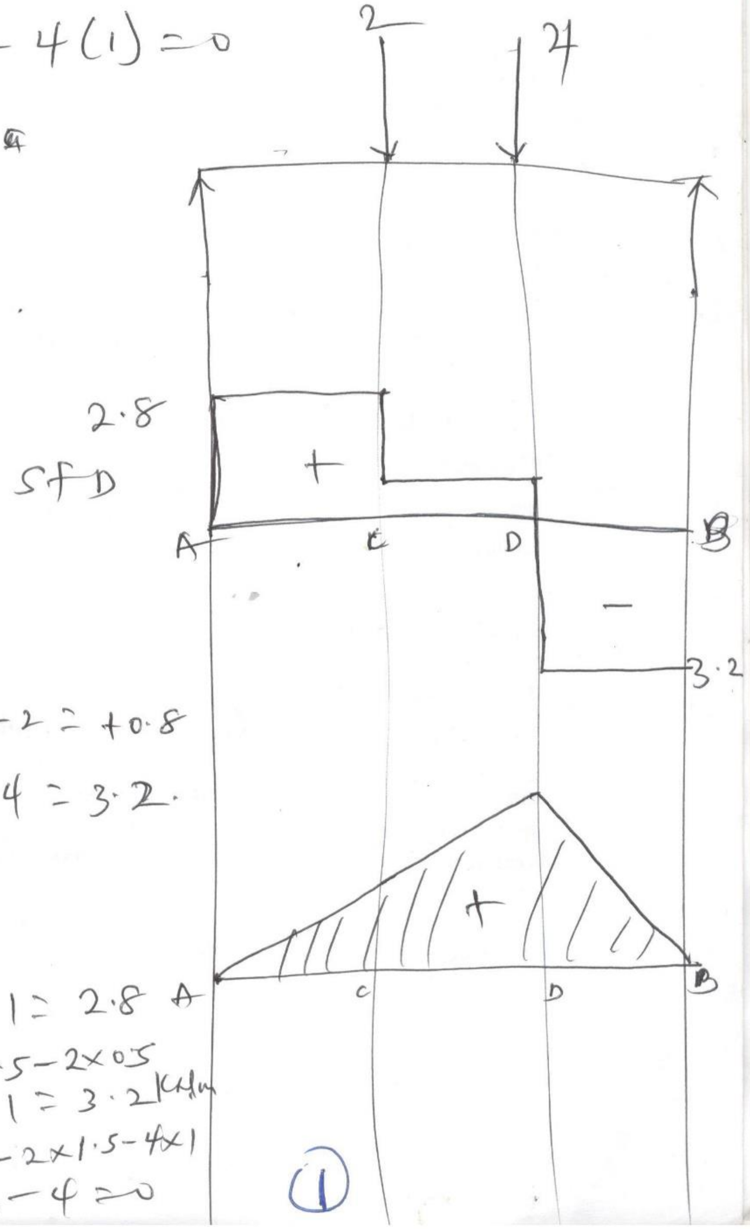
$2.5R_A = 3 + 4$

$2.5R_A = 7$

$R_A = 2.8$ KN.

$R_B = 6 - 2.8$ KN

$= 3.2$ KN.



SFD

At A : SF = +2.8

At C : SF = +2.8 − 2 = +0.8

At D : SF = +0.8 − 4 = 3.2.

BMD

At A : BM = 0

At C : BM = $R_A \times 1 = 2.8$

At D = BM = $R_A \times 1.5 - 2 \times 0.5$

$= 4.2 - 1 = 3.2$ KNm

At B = BM = $R_A \times 2.5 - 2 \times 1.5 - 4 \times 1$

$= 7 - 3 - 4 = 0$

1