

Question 1

Statistic - is the practice or science of collecting and analysing numerical data in large quantities.
⇒ It is the science of collecting, analyzing, presenting and interpreting data.

Stratified Sampling - is a method of sampling from a population which can be partitioned in subpopulation.
⇒ It is a type of sampling method in which the total population is divided into smaller group or strata to complete the sampling process.

Usefulness of statistic in Engineering

- It helps in improvement of product design & testing product performance
- It helps in determining reliability and maintainability
- It helps in work out safe system of flight control for airports etc

- ✓ It is a critical tool for Robustness analysis
- ✓ It is use for measurement of system error analysis
- It is use for test data analysis.
- It is use for probabilistic risk assessment.

Statistic is a fact or piece of data obtained from a study of a large quantity of numerical data

⑥ 3 red, 4 white, 5 black Total = 3 + 4 + 5 = 12

$$P(3 \text{ balls of same colour}) = P(3 \text{ red}) + P(3 \text{ white}) + P(3 \text{ black})$$

$$P(3 \text{ red}) = \frac{3}{12} \times \frac{3}{12} \times \frac{3}{12} = \frac{1}{64}$$

$$P(3 \text{ white}) = \frac{4}{12} \times \frac{4}{12} \times \frac{4}{12} = \frac{1}{27}$$

$$P(3 \text{ black}) = \frac{5}{12} \times \frac{5}{12} \times \frac{5}{12} = \frac{125}{1728}$$

$$P(3 \text{ balls of same colour}) = \frac{1}{64} + \frac{1}{27} + \frac{125}{1728} = \frac{1}{8}$$

Question 2

②

Observed data			
	Yellow	Green	Total
Round	315	108	423
Wrinkled	101	32	133
Total	416	140	556

$$\text{Chi-square } \chi^2 = \sum \frac{(\text{Observed} - \text{Expected})^2}{\text{Expected}}$$

Calculating Expected

① H_0 (Null hypothesis): There are no relationship b/w the categorical variable.

② H_A (Alternative hypothesis) There are relationship b/w the categorical variable

	Expected Value		
	Yellow	Green	total
Round	$\frac{410 \times 423}{556} = 312$	$\frac{140 \times 423}{556} = 107$	
Wrinkled	$\frac{410 \times 133}{556} = 98$	$\frac{140 \times 133}{556} = 33$	
Total			

$$\chi^2 = \sum \frac{(O - E)^2}{E}$$

$$\chi^2 = \frac{(315 - 312)^2}{312} + \frac{(108 - 107)^2}{107} + \frac{(101 - 98)^2}{98} + \frac{(32 - 33)^2}{33} = \underline{\underline{0.16}}$$

Degree of freedom $df = n - 1$

$$df = (2 - 1)(2 - 1) = \underline{\underline{1}}$$

we accept (H_0)

Since $0.16 < 3.841$, we have statistically significant evidence at $\alpha = 0.05$ to show that H_0 is True

OR

Do not reject (H_0)

There is not enough evidence to suggest that there is no relationship b/w the categorical variables

Question 3

Binomial Distribution

(i)

$$n = 10$$

$$p = \frac{1}{2} = 0.5$$

$$q = 1 - p = 0.5$$

(1) $x = 3$

$$P(3 \text{ heads}) = {}^n C_x \cdot p^x \cdot q^{n-x}$$
$${}^{10} C_3 \times 0.5^3 \times 0.5^{10-3}$$

$$\frac{10!}{(10-3)!3!} \times 0.5^3 \times 0.5^7 = \underline{0.117}$$

or 11.7%

(ii) less than 3 heads

$$P(< 3H) = P(0) + P(1) + P(2)$$

$x = 0$

$$P(0 \text{ Heads}) = {}^{10} C_0 \times 0.5^0 \times 0.5^{10-0} = 0.00098$$

$$P(1 \text{ Heads}) = {}^{10} C_1 \times 0.5^1 \times 0.5^{10-1} = 0.0098$$

$$P(2 \text{ Heads}) = {}^{10} C_2 \times 0.5^2 \times 0.5^{10-2} = 0.044$$

$$P(< 3 \text{ head}) = 0.044 + 0.0098 + 0.00098$$
$$= \underline{0.0548}$$

$$(11) P(> 2 \text{ heads}) = P(3H) + P(4H) + P(5H) + P(6H) + P(7H) + P(8H) + P(9H) + P(10H)$$

$\begin{matrix} 0.117 & 0.205 & 0.246 & 0.205 & 0.117 & 0.044 \\ & \swarrow & \swarrow & \swarrow & \swarrow & \swarrow \\ & 0.0098 & & 0.00098 & & \end{matrix}$

or

$$P(> 2 \text{ heads}) = 1 - P(\leq 3 \text{ heads})$$

$$1 - 0.0548 = \underline{0.9452}$$

⑥ Poisson Distribution

$$P(x; \mu) = \frac{[e^{-\mu}] [\mu^x]}{x!}$$

$$\mu = 4 \text{ days}$$

① $x = 6 \text{ days}$

$$P(6) = \frac{[e^{-4}] [4^6]}{6!}$$

$$e = 2.718$$

$$= \underline{0.1042}$$

② $P(\leq 2) = P(0 \text{ days}) + P(1 \text{ day}) + P(2 \text{ days})$

$$P(0) = \frac{e^{-4} \times 4^0}{0!} = 0.0183$$

$$P(1) = \frac{e^{-4} \times 4^1}{1!} = 0.073$$

$$P(2) = \frac{e^{-4} \times 4^2}{2!} = 0.147$$

$$\rightarrow P(\leq 2) = \underline{0.2383}$$

Question 4

x	y	xy	x ²	y ²
4	8	32	16	64
8	7	56	64	49
2	11	22	4	121
10	5	50	100	25
6	9	54	36	81
$\Sigma x =$ 30	$\Sigma y =$ 40	$\Sigma xy =$ 214	$\Sigma x^2 =$ 220	$\Sigma y^2 =$ 340

$n = 5$

$$y = a + bx$$

$$\Sigma y = an + b \Sigma x$$

$$\Sigma xy = a \Sigma x + b \Sigma x^2$$

$$40 = 5a + 30b$$

$$214 = 30a + 220b$$

$$a = 11.9$$

$$b = -0.65$$

$$y = 11.9 - 0.65x$$

$$x = a + by$$

$$\Sigma x = an + b \Sigma y$$

$$\Sigma xy = a \Sigma y + b \Sigma y^2$$

$$30 = 5a + 40b$$

$$214 = 40a + 340b$$

$$a = 16.4$$

$$b = -1.3$$

$$x = 16.4 - 1.3y$$

⑥ ① $y = ? \quad x = 12$

$$y = 11.9 - 0.65x \quad 11.9 - [0.65 \times 12] = \underline{4.1}$$

⑥ ② $x = ? \quad y = 6$

$$x = 16.4 - 1.3y \Rightarrow 16.4 - [1.3 \times 6] = 8.6$$

⑦ $K = 5 \text{ car} \quad N = 52 \text{ cars} \quad n = 13 \text{ cars}$

$P(3 \text{ of them sold will be whole car})$

Question 5

(a) Sample distribution is a probability distribution of a statistic obtained from a larger number of samples drawn from a specific population.

(b) $N = 3000$ male $\mu = 172.72$ $\sigma = 7.62 \text{ cm}$
 $n = 80$
 $s = ?$ \bar{x}

Normal distr.

$$Z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

(a) $S = \frac{\sigma}{\sqrt{n}} =$

$$\sigma = 7.62 \text{ cm}$$

$$n = 25 \text{ students}$$

$$S = \frac{7.62}{\sqrt{25}} = \underline{1.524 \text{ cm}}$$

$$\bar{x} = \mu = 172.72 \text{ cm}$$

(b) $\bar{x} = \mu = 172.72 \text{ cm}$ $S = \frac{\sigma}{\sqrt{n}} \times \sqrt{\frac{N-n}{N-1}}$

$$\frac{7.62}{\sqrt{25}} \times \sqrt{\frac{3000-25}{300-1}} = \underline{\underline{1.518 \text{ cm}}}$$

Question 6

Interval estimate is the use of sample data to calculate an interval of possible values of an unknown population parameter; this is in contrast to point estimation which gives a single value.

(b) $N = 100$ voters 55% supporters = 55 voters
 $P = 0.55$

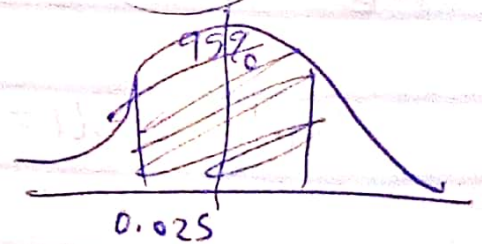
(a) 95% confidence limits $Z = 1.960$

Confidence limit
$$\hat{P} \pm Z \sqrt{\frac{\hat{P}(1-\hat{P})}{n}}$$

$$0.55 \pm 1.960 \times \sqrt{\frac{0.55(1-0.55)}{100}}$$

$$0.55 \pm 0.098$$

95% confident that the proportion is between
 $(0.452, 0.648)$



$$0.025 + 0.95 = \boxed{0.975}$$

Question 7

$$n = 6$$

$$\mu = 15700 \text{ kg} \quad \sigma = 320 \text{ kg}$$

$$\bar{x} = 16210 \text{ kg}$$

(a) 0.05

(b) 0.01

We need check the hypothesis

$$H_0: \mu = 15700 \quad \text{vs} \quad H_a: \mu \geq 15700$$

we use t-test ($n < 30$)

$$t = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}} = \frac{16210 - 15700}{\frac{320}{\sqrt{6}}}$$

$$= \underline{3.904}$$

0.95 Z score for 0.05 = -1.645 — 95% CL

0.99 Z score for 0.01 = -2.325 99% CL

Since $3.904 > 1.63$

we reject H_0 and conclude
new technique can improve the breaking
strength

Q Hypothesis testing is an act in statistics
whereby an analyst tests an assumption
regarding a population parameter

Alternative hypothesis is a ^{position} ~~new theory~~ that
states something is happening, new theory
is preferred instead of an old one.

Type I error is a kind of fault that occurs
during the hypothesis testing process when
a null hypothesis is rejected.