

CHAPTER 1

1. If $f(x) = x^2$ and $g(x) = x^2 + 1$

i) $g(g(0))$

$$g(0) = 0^2 + 1 = 1$$

$$g(1) = 1^2 + 1 = 2$$

ii) $f(g(2))$

$$g(2) = 2^2 + 1 = 5$$

$$f(5) = 5^2 = 25$$

2. $f(x) = 2^x$

i) $f(x^2)f(1)$

$$f(x^2) = 2^{x^2}$$

$$f(1) = 2^1$$

$$f(x^2)f(1) = 2^{x^2} \times 2^1$$

$$= 2^{x^2+1}$$

ii) $\frac{f(x+3)}{f(-x)}$

$$f(x+3) = 2^{x+3}$$

$$f(-x) = 2^{-x}$$

$$\frac{f(x+3)}{f(-x)} = \frac{2^{x+3}}{2^{-x}}$$

$$= 2^{x+3 - (-x)}$$

$$= 2^{x+3+x}$$

$$= 2^{2x+3}$$

iii) $f(\sqrt{x}) = 2^{\sqrt{x}}$

iv) $f(x^2+1) = 2^{x^2+1}$

3. $f(x) = 10^x$

$$f(x) + f(2+x) = 10^x + 10^{2+x}$$

$$= 10^x + 10^2 \cdot 10^x$$

$$= 10^x(1 + 10^2)$$

$$= 101 \cdot 10^x$$

ii) $f(x)f(2+x)$

$$= 10^x \cdot 10^{2+x}$$

$$= 10^{x+2+x}$$

$$= 10^{2x+2}$$

$$= 10^{2(x+1)}$$

iii) $\frac{f(x)}{f(2+x)}$

$$= \frac{10^x}{10^{2+x}}$$

$$= 10^{x-2-x}$$

$$= 10^{-2}$$

iv) $f(f(2+x))$

$$f(2+x) = 10^{2+x}$$

$$f(10^{2+x}) = 10^{10^{2+x}}$$

v) $f(\sin^2 x) \cdot f(\cos^2 x)$

$$= 10^{\sin^2 x} + 10^{\cos^2 x}$$

$$= 10^{\sin^2 x + \cos^2 x}$$

$$= 10$$

vi) THE QUESTION IS FAULTY

$$4) \log_a 216 = 3$$

$$216 = a^3$$

$$a^3 = 216$$

$$a^3 = 6^3$$

$$a = 6$$

$$\text{ii) } \log_a 625 = 4$$

$$625 = a^4$$

$$a^4 = 625$$

$$a^4 = 5^4$$

$$a = 5$$

$$\text{iii) } \log_a 1/4a = -2$$

$$\frac{1}{4a} = a^{-2}$$

$$\frac{1}{4a} = \frac{1}{a^2}$$

$$a^2 = 4a$$

$$a^2 - 4a = 0$$

$$a(a - 4) = 0$$

$$a = 0 \text{ or } a = 4$$

$$\text{iv) } \log_{14} x = 1/4$$

$$x = 14^{1/4}$$

$$x = 1.93$$

$$5) 4^x = 7$$

Take \ln of both sides

$$\ln 4^x = \ln 7$$

$$x \ln 4 = \ln 7$$

$$x = \frac{\ln 7}{\ln 4} = 1.404$$

$$\text{b) } 3^x = 6^{x+3}$$

Take \ln of both sides

$$\ln 3^x = \ln 6^{x+3}$$

$$x \ln 3 = (x + 3) \ln 6$$

$$x \ln 3 = x \ln 6 + 3 \ln 6$$

$$x \ln 3 - x \ln 6 = 3 \ln 6$$

$$x(\ln 3 - \ln 6) = 3 \ln 6$$

$$x = \frac{3 \ln 6}{\ln 3 - \ln 6} = -7.755$$

$$\text{c) } 5^{x+1} = 9$$

Take \ln of both sides

$$\ln 5^{x+1} = \ln 9$$

$$(x + 1) \ln 5 = \ln 9$$

$$x \ln 5 + \ln 5 = \ln 9$$

$$x \ln 5 = \ln 9 - \ln 5$$

$$x = \frac{\ln 9 - \ln 5}{\ln 5} = 0.3652$$

$$\text{d) } 2^{x-1} = 5^{2x+1}$$

Take \ln of both sides

$$\ln 2^{x-1} = \ln 5^{2x+1}$$

$$(x - 1) \ln 2 = (2x + 1) \ln 5$$

$$x \ln 2 - \ln 2 = 2x \ln 5 + \ln 5$$

$$x \ln 2 - 2x \ln 5 = \ln 2 + \ln 5$$

$$x(\ln 2 - 2 \ln 5) = \ln 2 + \ln 5$$

$$x = \frac{\ln 2 + \ln 5}{\ln 2 - 2 \ln 5} = -0.912$$

$$\text{e) } 8^{x+2} = 3^{2x-1}$$

Take \ln of both sides

$$\ln 8^{x+2} = \ln 3^{2x-1}$$

$$(x + 2) \ln 8 = (2x - 1) \ln 3$$

$$x \ln 8 + 2 \ln 8 = 2x \ln 3 - \ln 3$$

$$x \ln 8 - 2x \ln 3 = -\ln 3 - 2 \ln 8$$

$$x(\ln 8 - 2 \ln 3) = -\ln 3 - 2 \ln 8$$

$$x = \frac{-\ln 3 - 2\ln 8}{\ln 8 - 2\ln 3} = 44.637$$

$$f) 7^x = 4^{2x-1}$$

Take \ln of both sides

$$\ln 7^x = \ln 4^{2x-1}$$

$$x \ln 7 = (2x - 1) \ln 4$$

$$x \ln 7 = 2x \ln 4 - \ln 4$$

$$x \ln 7 - 2x \ln 4 = -\ln 4$$

$$x(\ln 7 - 2\ln 4) = -\ln 4$$

$$x = \frac{-\ln 4}{\ln 7 - 2\ln 4} = 1.677$$

X	Floor	ceiling
5.23	5	6
0.2	0	1
-7.1	-8	-7
3	3	3
-0.4	-1	0
1.9	1	2

CHAPTER 2

$$1.) x + 7 = 0$$

$$x = -7$$

Point of discontinuity occurs when x is -7

$$b) x - 1 = 0$$

$$x = 1$$

Point of discontinuity occurs when x is 1

$$c) 9 + x = 0$$

$$x = -9$$

Point of discontinuity occurs when x is -9

$$2.a) \lim_{x \rightarrow 2} (x^3 + 2x - 6)$$

$$= (2)^3 + 2(2) - 6$$

$$= 8 + 4 - 6 = 6$$

$$b. \lim_{x \rightarrow \infty} \frac{4x^3 - x^2 + x - 2}{x^3 + 3x^2 - 3x + 1}$$

$$\lim_{x \rightarrow \infty} \frac{\frac{4x^3}{x^3} - \frac{x^2}{x^3} + \frac{x}{x^3} - \frac{2}{x^3}}{\frac{x^3}{x^3} + \frac{3x^2}{x^3} - \frac{3x}{x^3} + \frac{1}{x^3}}$$

$$\lim_{x \rightarrow \infty} \frac{4 - \frac{1}{x} + \frac{1}{x^2} - \frac{2}{x^3}}{1 + \frac{3}{x} - \frac{3}{x^2} + \frac{1}{x^3}}$$

$$\Rightarrow \frac{4 - \frac{1}{\infty} + \frac{1}{\infty^2} - \frac{2}{\infty^3}}{1 + \frac{3}{\infty} - \frac{3}{\infty^2} + \frac{1}{\infty^3}}$$

$$\Rightarrow 4$$

3. Yes

4i) $f(x)$ is continuous for all values of x except at $x = -3$

ii) $f(x)$ is continuous for all values of x

$$5. f(x) = \frac{x-1}{x^2+x-2}$$

$$\text{i) } \lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} \frac{x-1}{x^2+x-2}$$

$$\Rightarrow \frac{0-1}{0^2+0-2} = 1/2$$

$$\text{ii) i) } \lim_{x \rightarrow \infty} f(x) = \lim_{x \rightarrow \infty} \frac{x-1}{x^2+x-2}$$

$$= \lim_{x \rightarrow \infty} \frac{\frac{x}{x^2} - \frac{1}{x^2}}{\frac{x^2}{x^2} + \frac{x}{x^2} - \frac{2}{x^2}}$$

$$= \lim_{x \rightarrow \infty} \frac{\frac{1}{x} - \frac{1}{x^2}}{1 + \frac{1}{x} - \frac{2}{x^2}}$$

$$\Rightarrow \frac{\frac{1}{\infty} - \frac{1}{\infty^2}}{1 + \frac{1}{\infty} - \frac{2}{\infty^2}}$$

$$\Rightarrow 0$$

$$6. \lim_{x \rightarrow 0} \frac{f(x)-f(a)}{x-a}; \lim_{t \rightarrow 0} \frac{f(a+t)-f(a)}{t}$$

$$\text{A (i) } f(x) = x^2, a = 3$$

$$\lim_{x \rightarrow 0} \frac{x^2 - a^2}{x - a}$$

$$\lim_{x \rightarrow 0} \frac{(x-a)(x+a)}{x-a}$$

$$\lim_{x \rightarrow 0} x + a$$

$$\Rightarrow 0 + 3 \Rightarrow 3$$

$$\text{ii) } \lim_{t \rightarrow 0} \frac{(a+t)^2 - (a)^2}{t}$$

$$\lim_{t \rightarrow 0} \frac{a^2 + 2at + t^2 - a^2}{t}$$

$$\lim_{t \rightarrow 0} \frac{t(2a+t)}{t}$$

$$\lim_{t \rightarrow 0} (2a+t)$$

$$\Rightarrow (2(3) + 0)$$

$$\Rightarrow 6$$

$$\text{B (i) } f(x) = x^2 + 1, a = 2$$

$$\lim_{x \rightarrow 0} \frac{(x^2 + 1) - (a^2 + 1)}{x - a}$$

$$\lim_{x \rightarrow 0} \frac{(x^2 + 1 - a^2 - 1)}{x - a}$$

$$\lim_{x \rightarrow 0} \frac{x^2 - a^2}{x - a}$$

$$\lim_{x \rightarrow 0} \frac{(x-a)(x+a)}{x-a}$$

$$\lim_{x \rightarrow 0} x + a$$

$$\Rightarrow 0 + 2 \Rightarrow 2$$

$$\text{ii) } \lim_{t \rightarrow 0} \frac{(a+t)^2 + 1 - (a^2 + 1)}{t}$$

$$\lim_{t \rightarrow 0} \frac{a^2 + 2at + t^2 + 1 - a^2 - 1}{t}$$

$$\lim_{t \rightarrow 0} \frac{t(2a+t)}{t}$$

$$\lim_{t \rightarrow 0} (2a+t)$$

$$\Rightarrow (2(2) + 0) = 4$$

$$\text{Ci) } f(x) = 3x^2 - x, a = 0$$

$$\lim_{x \rightarrow 0} \frac{3x^2 - x - (3a^2 - a)}{x - a}$$

$$\lim_{x \rightarrow 0} \frac{3x^2 - x - 3a^2 + a}{x - a}$$

$$\lim_{x \rightarrow 0} \frac{3x^2 - 3a^2 - (x - a)}{x - a}$$

$$\lim_{x \rightarrow 0} \frac{3(x^2 - a^2) - (x - a)}{x - a}$$

$$\lim_{x \rightarrow 0} \frac{3(x+a)(x-a) - (x-a)}{x-a}$$

$$\lim_{x \rightarrow 0} \frac{(x-a)(3(x+a)-1)}{x-a}$$

$$\lim_{x \rightarrow 0} 3(x+a)-1$$

$$\Rightarrow 3(0+0)-1$$

$$\Rightarrow -1$$

$$\text{ii) } \lim_{t \rightarrow 0} \frac{3(a+t)^2 - (a+t) - (3a^2 - a)}{t}$$

$$\lim_{t \rightarrow 0} \frac{3a^2 + 6at + 3t^2 - a - t - 3a^2 + a}{t}$$

$$\lim_{t \rightarrow 0} \frac{t(6a + 3t - 1)}{t}$$

$$\Rightarrow -1$$

$$7. \lim_{x \rightarrow a} \frac{x^4 - a^4}{x - a}$$

$$\lim_{x \rightarrow a} \frac{(x^2 - a^2)(x^2 + a^2)}{x - a}$$

$$\lim_{x \rightarrow a} \frac{(x-a)(x+a)(x^2 + a^2)}{x - a}$$

$$\lim_{x \rightarrow a} (x+a)(x^2 + a^2)$$

$$\Rightarrow (a+a)(a^2 + a^2)$$

$$\Rightarrow 2a \times 2a^2 \Rightarrow 4a^3$$

$$\lim_{h \rightarrow 0} \frac{4(x+h)^3 - 4x^3}{h}$$

$$\lim_{h \rightarrow 0} \frac{4(x^3 + 3x^2h + 3xh^2 + h^3) - 4x^3}{h}$$

$$\lim_{h \rightarrow 0} \frac{4x^3 + 12x^2h + 12xh^2 + 4h^3 - 4x^3}{h}$$

$$\lim_{h \rightarrow 0} \frac{h(12x^2 + 12xh + 4h^2)}{h}$$

$$\lim_{h \rightarrow 0} (12x^2 + 12xh + 4h^2)$$

$$\Rightarrow 12x^2 + 12x(0) + 4(0)^2$$

$$\Rightarrow 12x^2$$

$$\lim_{x \rightarrow 0} \frac{1 - 2^{2x}}{1 + 2^x}$$

$$\lim_{x \rightarrow 0} \frac{(1 - 2^x)(1 + 2^x)}{1 + 2^x}$$

$$\lim_{x \rightarrow 0} (1 - 2^x)$$

$$\Rightarrow 1 - 2^0$$

$$\Rightarrow 0$$

$$\text{iv) } \lim_{x \rightarrow 0} \frac{x^2 - x}{x}$$

$$\lim_{x \rightarrow 0} \frac{x(x-1)}{x}$$

$$\lim_{x \rightarrow 0} (x-1)$$

$$\Rightarrow 0 - 1 = -1$$

$$\text{v) } \lim_{x \rightarrow 10} (1 - \log_{10} x)$$

$$\Rightarrow 1 - \log_{10} 10$$

$$\Rightarrow 1 - 1 = 0$$

CHAPTER 3

$$1. f(x) = \frac{1}{x^2}$$

$$f(x + \Delta x) = \frac{1}{(x + \Delta x)^2}$$

$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \rightarrow 0} \frac{\frac{1}{(x+\Delta x)^2} - \frac{1}{x^2}}{\Delta x}$$

$$= \lim_{\Delta x \rightarrow 0} \frac{\frac{1}{x^2 + 2x\Delta x + \Delta x^2} - \frac{1}{x^2}}{\Delta x}$$

$$\begin{aligned}
&= \lim_{\Delta x \rightarrow 0} \frac{x^2 - x^2 - 2x\Delta x - \Delta x^2}{(x^2)(\Delta x)(x^2 + 2x\Delta x + \Delta x^2)} \\
&= \lim_{\Delta x \rightarrow 0} \frac{\Delta x(-2x - \Delta x)}{x^2 \Delta x (x^2 + 2x\Delta x + \Delta x^2)} \\
&\Rightarrow \frac{-2x-0}{x^2(x^2+0)} \\
&\Rightarrow \frac{-2x}{x^4} \\
f'(x) &= \frac{-2}{x^3}
\end{aligned}$$

2. $f(x) = x^2 + 1$

$$f(x + \Delta x) = (x + \Delta x)^2 + 1$$

$$\begin{aligned}
f'(x) &= \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\
&= \lim_{\Delta x \rightarrow 0} \frac{(x + \Delta x)^2 + 1 - (x^2 + 1)}{\Delta x} \\
&= \lim_{\Delta x \rightarrow 0} \frac{x^2 + 2x\Delta x + \Delta x^2 + 1 - x^2 - 1}{\Delta x} \\
&= \lim_{\Delta x \rightarrow 0} \frac{\Delta x(2x + \Delta x)}{\Delta x} \\
&\Rightarrow 2x + 0 \Rightarrow 2x
\end{aligned}$$

3. $f(x) = \sin 2x$

$$f(x + \Delta x) = \sin 2(x + \Delta x)$$

$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{\sin(2x + 2\Delta x) - \sin 2x}{\Delta x}$$

recall

$$\sin(C + D) = \sin C \cos D + \cos C \sin D$$

$$-\sin(C - D) = \sin C \cos D - \cos C \sin D$$

$$\begin{aligned}
&\Rightarrow \sin(C + D) - \sin(C - D) \\
&= 2\cos C \sin D
\end{aligned}$$

$$\text{let } A = C + D; B = C - D$$

$$\frac{A + B}{2} = C; \frac{A - B}{2} = D$$

$$\Rightarrow \sin A - \sin B = 2\cos \frac{A + B}{2} \sin \frac{A - B}{2}$$

$$\text{where } A = 2x + 2\Delta x; B = 2x$$

$$\begin{aligned}
f'(x) &= \lim_{\Delta x \rightarrow 0} \frac{2\cos \frac{A+B}{2} \sin \frac{A-B}{2}}{\Delta x} \\
&= \lim_{\Delta x \rightarrow 0} \frac{2\cos \left(\frac{2x+2\Delta x+2x}{2} \right) \sin \left(\frac{2x+2\Delta x-2x}{2} \right)}{\Delta x} \\
&= \lim_{\Delta x \rightarrow 0} \frac{2\cos(2x + \Delta x) \sin \Delta x}{\Delta x}
\end{aligned}$$

$$= \lim_{\Delta x \rightarrow 0} 2\cos(2x + \Delta x) \frac{\sin \Delta x}{\Delta x}$$

As limit tends to 0

$$\begin{aligned}
&= 2\cos(2x + 0) \times 1 \left(\text{where } 1 = \lim_{\Delta x \rightarrow 0} \frac{\sin \Delta x}{\Delta x} \right) \\
&= 2\cos 2x
\end{aligned}$$

4. $f(x) = \cos 2x$

$$f(x + \Delta x) = \cos 2(x + \Delta x)$$

$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{\cos(2x + 2\Delta x) - \cos 2x}{\Delta x}$$

$$\cos(C + D) = \cos C \cos D - \sin C \sin D$$

$$-\cos(C - D) = \cos C \cos D + \sin C \sin D$$

$$\begin{aligned}
&\Rightarrow \cos(C + D) - \cos(C - D) = \\
&-2\sin C \sin D
\end{aligned}$$

$$\text{let } A = C + D; B = C - D$$

$$\frac{A + B}{2} = C; \frac{A - B}{2} = D$$

$$\cos A - \cos B = -2\sin \frac{A + B}{2} \sin \frac{A - B}{2}$$

$$\text{where } A = 2x + 2\Delta x; B = 2x$$

$$\begin{aligned}
 f'(x) &= \lim_{\Delta x \rightarrow 0} \frac{-2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}}{\Delta x} \\
 &= \lim_{\Delta x \rightarrow 0} \frac{-2 \sin \left(\frac{2x+2\Delta x+2x}{2} \right) \sin \left(\frac{2x+2\Delta x-2x}{2} \right)}{\Delta x} \\
 &= -2 \lim_{\Delta x \rightarrow 0} \frac{\sin(2x + \Delta x) \sin \Delta x}{\Delta x} \\
 &= -2 \lim_{\Delta x \rightarrow 0} \sin(2x + \Delta x) \frac{\sin \Delta x}{\Delta x}
 \end{aligned}$$

As limit tends to 0

$$\begin{aligned}
 &= -2 \sin(2x + 0) \times 1 \left(\text{where } 1 = \lim_{\Delta x \rightarrow 0} \frac{\sin \Delta x}{\Delta x} \right) \\
 &= -2 \sin 2x
 \end{aligned}$$

$$5. f(x) = x^2 + 3x + 1$$

$$f(x + \Delta x) = (x + \Delta x)^2 + 3(x + \Delta x) + 1$$

$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{(x + \Delta x)^2 + 3(x + \Delta x) + 1 - (x^2 + 3x + 1)}{\Delta x}$$

$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{x^2 + 2x\Delta x + \Delta x^2 + 3x + 3\Delta x + 1 - x^2 - 3x - 1}{\Delta x}$$

$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{\Delta x(2x + \Delta x + 3)}{\Delta x}$$

$$f'(x) = \lim_{\Delta x \rightarrow 0} (2x + \Delta x + 3)$$

$$= (2x + 0 + 3)$$

$$= 2x + 3$$

$$6. f(x) = \frac{x}{x+1}$$

$$f(x + \Delta x) = \frac{x + \Delta x}{x + \Delta x + 1}$$

$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{\frac{x + \Delta x}{x + \Delta x + 1} - \frac{x}{x + 1}}{\Delta x}$$

$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{\frac{(x + \Delta x)(x + 1) - x(x + \Delta x + 1)}{(x + \Delta x + 1)(x + 1)}}{\Delta x}$$

$$= \lim_{\Delta x \rightarrow 0} \frac{x^2 + x + x\Delta x + \Delta x - x^2 - x\Delta x - x}{\Delta x(x + \Delta x + 1)(x + 1)}$$

$$= \lim_{\Delta x \rightarrow 0} \frac{\Delta x}{\Delta x(x + \Delta x + 1)(x + 1)}$$

$$= \lim_{\Delta x \rightarrow 0} \frac{1}{(x + \Delta x + 1)(x + 1)}$$

$$\Rightarrow \frac{1}{(x + 0 + 1)(x + 1)}$$

$$\Rightarrow \frac{1}{(x + 1)^2}$$

$$7. f(x) = \tan 2x$$

$$f(x + \Delta x) = \tan 2(x + \Delta x)$$

$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{\tan(2x + 2\Delta x) - \tan 2x}{\Delta x}$$

$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{\frac{\sin(2x + 2\Delta x)}{\cos(2x + 2\Delta x)} - \frac{\sin 2x}{\cos 2x}}{\Delta x}$$

$$\begin{aligned}
 f'(x) &= \lim_{\Delta x \rightarrow 0} \frac{\frac{\sin(2x + 2\Delta x)\cos 2x - \cos(2x + 2\Delta x)\sin 2x}{\cos(2x + 2\Delta x)\cos 2x}}{\Delta x} \\
 &= \lim_{\Delta x \rightarrow 0} \frac{\sin(2x + 2\Delta x)\cos 2x - \cos(2x + 2\Delta x)\sin 2x}{\Delta x \cos(2x + 2\Delta x)\cos 2x}
 \end{aligned}$$

$$\begin{aligned}
 &\text{recall, } \sin(A - B) \\
 &= \sin A \cos B - \cos A \sin B
 \end{aligned}$$

$$\text{let } A = 2x + 2\Delta x; B = 2x$$

$$\sin(2x + 2\Delta x - 2x) \Rightarrow \sin 2\Delta x$$

$$\Rightarrow \sin(2x + 2\Delta x)\cos 2x - \cos(2x + 2\Delta x)\sin 2x$$

$$\therefore f'(x) = \lim_{\Delta x \rightarrow 0} \frac{\sin 2\Delta x}{\Delta x \cos(2x + 2\Delta x)\cos 2x}$$

$$\text{recall, } \sin 2\Delta x = 2\sin\Delta x \cos\Delta x$$

$$\Rightarrow \lim_{\Delta x \rightarrow 0} \frac{2\sin\Delta x \cos\Delta x}{\Delta x \cos(2x + 2\Delta x) \cos 2x}$$

$$\text{recall, } \lim_{\Delta x \rightarrow 0} \frac{\sin\Delta x}{\Delta x} = 1$$

$$\Rightarrow 2 \times 1 \times \frac{\cos 0}{\cos(2x + 2(0)) \cos 2x}$$

$$= 2 \times \frac{1}{\cos^2 2x}$$

$$= 2 \sec^2 2x$$

$$8. f(x) = x(x^2 + 2x)$$

$$f(x + \Delta x) = (x + \Delta x)((x + \Delta x)^2 + 2(x + \Delta x))$$

$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{(x + \Delta x)((x + \Delta x)^2 + 2(x + \Delta x)) - x(x^2 + 2x)}{\Delta x}$$

$$= \lim_{\Delta x \rightarrow 0} \frac{(x + \Delta x)(x^2 + 2x\Delta x + \Delta x^2 + 2x + 2\Delta x) - x^3 - 2x^2}{\Delta x}$$

$$= \lim_{\Delta x \rightarrow 0} \frac{x^3 + 2x^2\Delta x + x\Delta x^2 + 2x^2 + 2x\Delta x + x^2\Delta x + 2x\Delta x^2 + \Delta x^3 + 2x\Delta x + 2\Delta x^2 - x^3 - 2x^2}{\Delta x}$$

$$= \lim_{\Delta x \rightarrow 0} \frac{\Delta x(2x^2 + x\Delta x + 2x + x^2 + 2x\Delta x + \Delta x^2 + 2x + 2\Delta x)}{\Delta x}$$

$$\Rightarrow 3x^2 + 3x(0) + 4x + (0)^2 + 2(0)$$

$$\Rightarrow 3x^2 + 4x$$

Exercise 2

$$1. y = x^3 - 9x^2 + 24x$$

$$y' = 3x^2 - 18x + 24$$

$$3x^2 - 18x + 24 = 0$$

$$x^2 - 6x + 8 = 0$$

$$x^2 - 4x - 2x + 8 = 0$$

$$x(x - 4) - 2(x - 4) = 0$$

$$(x - 4)(x - 2) = 0$$

$$x = 4 \text{ or } x = 2$$

for $x = 4$ choose the interval $[3, 5]$

$$f(4) = 4^3 - 9(4)^2 + 24(4)$$

$$= 64 - 144 + 96$$

$$= 16$$

$$f(3) = 3^3 - 9(3)^2 + 24(3)$$

$$= 27 - 81 + 72$$

$$= 18$$

$$f(5) = 5^3 - 9(5)^2 + 24(5)$$

$$= 125 - 225 + 120$$

$$= 20$$

since $f(4) < f(3), f(4) < f(5);$

a local minimum occurs at $f(4)$

for $x = 2$ choose the interval $[1, 3]$

$$f(2) = 2^3 - 9(2)^2 + 24(2)$$

$$= 8 - 36 + 48$$

$$= 20$$

$$f(1) = 1^3 - 9(1)^2 + 24(1)$$

$$= 1 - 9 + 24$$

$$= 16$$

$$f(3) = 3^3 - 9(3)^2 + 24(3)$$

$$= 27 - 81 + 72$$

$$= 18$$

since $f(2) > f(1), f(2) > f(3);$

a local maximum occurs at $f(2)$

$$\text{b) } y = x^4 - 2x^2 + 3$$

$$y' = 4x^3 - 4x$$

$$4x^3 - 4x = 0$$

$$4x(x^2 - 1) = 0$$

$$4x = 0 \text{ or } x^2 - 1 = 0$$

$$x = 0 \text{ or } x = \pm 1$$

for $x = 0$ choose the interval $[-1, 1]$

$$f(0) = 0^4 - 2(0)^2 + 3$$

$$= 3$$

$$f(-1) = (-1)^4 - 2(-1)^2 + 3$$

$$= 1 - 2 + 3$$

$$= 2$$

$$f(1) = (1)^4 - 2(1)^2 + 3$$

$$= 1 - 2 + 3$$

$$= 2$$

since $f(0) > f(-1)$ & $f(0) > f(1)$;

a local maximum occurs at $x = 0$

for $x = -1$ choose the interval $[0, -2]$

$$f(-1) = (-1)^4 - 2(-1)^2 + 3$$

$$= 1 - 2 + 3$$

$$= 2$$

$$f(0) = 0^4 - 2(0)^2 + 3$$

$$= 3$$

$$f(-2) = (-2)^4 - 2(-2)^2 + 3$$

$$= 16 - 8 + 3$$

$$= 11$$

since $f(-1) < f(0)$, $f(-1) < f(-2)$;

a local minimum occurs at $x = -1$

for $x = 1$ choose the interval $[0, 2]$

$$f(1) = (1)^4 - 2(1)^2 + 3$$

$$= 1 - 2 + 3$$

$$= 2$$

$$f(0) = 0^4 - 2(0)^2 + 3$$

$$= 3$$

$$f(2) = (2)^4 - 2(2)^2 + 3$$

$$= 16 - 8 + 3$$

$$= 11$$

since $f(1) < f(0)$; $f(1) < f(2)$

a local minimum occurs at $x = 1$

Exercise 3

$$1. y = \frac{x^2}{x+1}$$

given the interval $[1, 2]$

$$f(1) = \frac{1^2}{1+1} = \frac{1}{2}$$

$$f(2) = \frac{2^2}{2+1} = \frac{4}{3}$$

$$\text{recall, } \frac{f(b) - f(a)}{b - a} = f'(c)$$

$$\frac{\frac{4}{3} - \frac{1}{2}}{2 - 1} = \frac{\frac{8-3}{6}}{1}$$

$$= \frac{5}{6} = f'(c)$$

$$f'(x) = \frac{(x+1)(2x) - x^2(1)}{(x+1)^2}$$

$$= \frac{(2x^2 + 2x) - x^2}{(x+1)^2}$$

$$= \frac{x^2 + 2x}{(x+1)^2}$$

$$f'(c) = \frac{c^2 + 2c}{(c+1)^2} = \frac{5}{6}$$

$$6(c^2 + 2c) = 5(c^2 + 2c + 1)$$

$$6c^2 + 12c = 5c^2 + 10c + 5$$

$$c^2 + 2c - 5 = 0$$

$$c = \frac{-2 \pm \sqrt{4 + 20}}{2}$$

$$= \frac{-2 \pm \sqrt{24}}{2}$$

$$= \frac{-2 \pm 2\sqrt{6}}{2}$$

$$= -1 + \sqrt{6} \text{ or } -1 - \sqrt{6}$$

since $-1 + \sqrt{6}$ is within the interval ;

the value of c that would satisfy y is

$$-1 + \sqrt{6}$$

$$2. x^2 - x^3 \quad -2 \leq x \leq 1$$

the equation satisfies the condition for MVT

$$f(-2) = (-2)^2 - (-2)^3 = 12$$

$$f(1) = 1^2 - 1^3 = 0$$

$$f'(c) = \frac{0 - 12}{1 - -2}$$

$$= \frac{-12}{3}$$

$$= -4$$

$$f'(x) = 2x - 3x^2$$

$$f'(c) = 2c - 3c^2 = -4$$

$$3c^2 - 2c - 4 = 0$$

$$c = \frac{- -2 \pm \sqrt{(-2)^2 - 4(3 \times -4)}}{2(3)}$$

$$c = \frac{2 \pm \sqrt{52}}{6}$$

$$= \frac{1 \pm \sqrt{13}}{3}$$

since the value of c within the given boundary is

$$\frac{1 - \sqrt{13}}{3};$$

that is the value of c that will satisfy $f(x)$

$$3. g(x) = 3x^3 - 9x^2 - 3x \quad -2 \leq x \leq 3$$

the equation satisfies the conditions for MVT

$$g(-2) = 3(-2)^3 - 9(-2)^2 - 3(-2)$$

$$= -24 - 36 + 6$$

$$= -54$$

$$g(3) = 3(3)^3 - 9(3)^2 - 3(3)$$

$$= 81 - 81 - 9$$

$$= -9$$

$$\frac{-9 - -54}{3 - -2} = 9 = g'(c)$$

$$g'(x) = 9x^2 - 18x - 3$$

$$g'(c) = 9c^2 - 18c - 3$$

$$9c^2 - 18c - 3 = 9$$

$$9c^2 - 18c - 12 = 0$$

solving quadratically;

$$c = 2.53 \text{ and } -0.53$$

The both values of c gotten are

selected because it falls within the given interval

Exercise 4

$$1. f(x) = x^3 - 6x^2 + 12x - 8$$

$$f'(x) = 3x^2 - 12x + 12$$

$$f'(x) > 0$$

$$3x^2 - 12x + 12 > 0$$

$$x^2 - 4x + 4 > 0$$

$$x^2 - 2x - 2x + 4 > 0$$

$$\Rightarrow x > 2$$

$\therefore f$ is increasing in $(2, \infty)$

$$f'(x) < 0$$

$$\Rightarrow x < 2$$

$\therefore f$ is decreasing in $(-\infty, 2)$

$$2. f(x) = 3x^3 - 6x^2$$

$$f'(x) = 9x^2 - 12x$$

$$f'(x) > 0$$

$$9x^2 - 12x > 0$$

$$3x^2 - 4x > 0$$

$$x(3x - 4) > 0$$

$$\Rightarrow x > 0 \text{ and } x > \frac{4}{3} \text{ or } x < 0 \text{ and } x < \frac{4}{3}$$

$\therefore f$ is increasing in $[-\infty, 0] \& [\frac{4}{3}, \infty]$

$$f'(x) < 0$$

$$\Rightarrow x > 0 \text{ and } x < \frac{4}{3} \text{ or } x < 0 \text{ and } x > \frac{4}{3}$$

$\therefore f$ is decreasing in $[0, \frac{4}{3}]$

$$3. f(x) = x^2 - 6x$$

$$f'(x) = 2x - 6$$

$$f'(x) > 0$$

$$2x - 6 > 0$$

$$\Rightarrow x > 3$$

$\therefore f$ is increasing in $[3, \infty]$

$$f'(x) < 0$$

$$\Rightarrow x < 3$$

$\therefore f$ is decreasing in $[-\infty, 3]$

$$4. f(x) = x^4 - 8x^3 + 10x^2 + 40$$

$$f'(x) = 4x^3 - 24x^2 + 20x$$

$$f'(x) > 0$$

$$x(x^2 - 6x + 5) > 0$$

$$x(x^2 - 5x - x + 5) > 0$$

$$x(x(x - 5) - 1(x - 5)) > 0$$

$$x(x - 1)(x - 5) > 0$$

$$\Rightarrow x > 0; x > 1; x > 5 \text{ or}$$

$$x < 0; x < 1; x < 5$$

$\therefore f$ is increasing in $[5, \infty]$ and $[0, -1]$

$$f'(x) < 0$$

$$\Rightarrow x > 0; x < 1 \text{ or } x < 0; x > 1$$

$$\Rightarrow x > 1; x < 5 \text{ or } x < 1; x > 5$$

$\therefore f$ is decreasing in $[1, 5]$ and $[-\infty, 0]$

CHAPTER 4

EXERCISE 1

$$1. y = e^{5x}(3x + 1)$$

using product rule

$$u = e^{5x}; v = 3x + 1$$

$$\frac{du}{dx} = 5e^{5x}$$

$$\frac{dv}{dx} = 3$$

$$\text{recall, } \frac{dy}{dx} = v \frac{du}{dx} + u \frac{dv}{dx}$$

$$\frac{dy}{dx} = (3x + 1)5e^{5x} + e^{5x}(3)$$

$$= e^{5x}(15x + 5 + 3)$$

$$\frac{dy}{dx} = e^{5x}(15x + 8)$$

$$2. y = x \cos 2x$$

$$u = x; v = \cos 2x$$

$$w = 2x; v = \cos w$$

$$\frac{dv}{dx} = \frac{dv}{dw} \times \frac{dw}{dx}$$

$$= -\sin w \times 2$$

$$= -2\sin w$$

$$= -2\sin 2x$$

$$\frac{du}{dx} = 1$$

$$\frac{dy}{dx} = v \frac{du}{dx} + u \frac{dv}{dx}$$

$$= \cos 2x(1) + x(-2\sin 2x)$$

$$= \cos 2x - 2x\sin 2x$$

$$3. y = x^3 \sin 5x^2$$

$$u = x^3; v = \sin 5x^2$$

$$w = 5x^2; v = \sin w$$

$$\frac{dv}{dx} = \frac{dv}{dw} \times \frac{dw}{dx}$$

$$= \cos w \times 10x$$

$$= 10x \cos 5x^2$$

$$\frac{du}{dx} = 3x^2$$

$$\frac{dy}{dx} = v \frac{du}{dx} + u \frac{dv}{dx}$$

$$= (\sin 5x^2)(3x^2) + x^3(10x \cos 5x^2)$$

$$= 3x^2 \sin 5x^2 + 10x^4 \cos 5x^2$$

$$4. y = x^2 \cos^2 x$$

$$u = x^2; \frac{du}{dx} = 2x$$

$$v = \cos^2 x$$

$$w = \cos x; v = w^2$$

$$\frac{dv}{dx} = \frac{dv}{dw} \times \frac{dw}{dx}$$

$$= 2w \times -\sin x$$

$$= 2(\cos x)(-\sin x)$$

$$= -\sin 2x$$

$$\frac{dy}{dx} = v \frac{du}{dx} + u \frac{dv}{dx}$$

$$\frac{dy}{dx} = \cos^2 x(2x) + (x^2)(-\sin 2x)$$

$$\frac{dy}{dx} = 2x \cos^2 x - x^2 \sin 2x$$

$$\frac{dy}{dx} = x(2 \cos^2 x - x \sin 2x)$$

QUOTIENT RULE

$$1. y = \frac{\sin x}{\cos 9x}$$

$$u = \sin x; v = \cos 9x$$

$$\frac{du}{dx} = \cos x; \frac{dv}{dx} = -9\sin 9x$$

$$\begin{aligned} \frac{dy}{dx} &= \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2} \\ &= \frac{(\cos 9x)(\cos x) - (\sin x)(-9\sin 9x)}{(\cos 9x)^2} \\ &= \frac{\cos 9x \cos x + 9\sin x \sin 9x}{(\cos 9x)^2} \end{aligned}$$

$$2. y = \frac{\sin 2x}{2x+5}$$

$$u = \sin 2x; v = 2x + 5$$

$$\frac{du}{dx} = 2\cos 2x; \frac{dv}{dx} = 2$$

$$\begin{aligned} \frac{dy}{dx} &= \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2} \\ &= \frac{(2x+5)(2\cos 2x) - (\sin 2x)(2)}{(2x+5)^2} \\ &= \frac{2((2x+5)\cos 2x - \sin 2x)}{(2x+5)^2} \end{aligned}$$

$$3. y = \frac{(3x+1)\cos 2x}{e^{2x}}$$

$$v = e^{2x}; \frac{dv}{dx} = 2e^{2x}$$

$$u = (3x+1)\cos 2x$$

$$w = 3x+1; z = \cos 2x$$

$$u = wz$$

$$\frac{dw}{dx} = 3; \frac{dz}{dx} = -2\sin 2x$$

$$\frac{du}{dx} = w \frac{dz}{dx} + z \frac{dw}{dx}$$

$$= (3x+1)(-2\sin 2x) + (\cos 2x)3$$

$$= 3\cos 2x - 2(\sin 2x)(3x+1)$$

$$\begin{aligned} \frac{dy}{dx} &= \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2} \\ &= \frac{(e^{2x})(3\cos 2x - 2(\sin 2x)(3x+1)) - (3x+1)(\cos 2x)(2e^{2x})}{(e^{2x})^2} \\ &= \frac{(3\cos 2x - 2(\sin 2x)(3x+1)) - (3x+1)(2\cos 2x)}{e^{2x}} \end{aligned}$$

$$4. y = \frac{x\sin x}{1+\cos x}$$

$$u = x\sin x; v = 1 + \cos x$$

$$\frac{dv}{dx} = -\sin x$$

$$u = x\sin x$$

$$w = x; \frac{dw}{dx} = 1$$

$$z = \sin x; \frac{dz}{dx} = \cos x$$

$$\frac{du}{dx} = x\cos x + \sin x$$

$$\begin{aligned} \frac{dy}{dx} &= \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2} \\ &= \frac{(1+\cos x)(x\cos x + \sin x) - (x\sin x)(-\sin x)}{(1+\cos x)^2} \\ &= \frac{(1+\cos x)(x\cos x + \sin x) + x\sin^2 x}{(1+\cos x)^2} \end{aligned}$$

$$5. y = \frac{e^{4x}\sin x}{x\cos 2x}$$

$$u = e^{4x}\sin x$$

$$a = e^{4x}; b = \sin x$$

$$\frac{da}{dx} = 4e^{4x}; \frac{db}{dx} = \cos x$$

$$\frac{du}{dx} = e^{4x}(\cos x) + \sin x(4e^{4x})$$

$$= e^{4x}(\cos x + 4\sin x)$$

$$v = x \cos 2x$$

$$c = x; d = \cos 2x$$

$$\frac{dc}{dx} = 1; \frac{dd}{dx} = -2\sin 2x$$

$$\frac{dv}{dx} = x(-2\sin 2x) + \cos 2x$$

$$= \cos 2x - 2x\sin 2x$$

$$\frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

$$\begin{aligned} & (x \cos 2x)(e^{4x}(\cos x + 4\sin x)) - \\ & = \frac{e^{4x} \sin x (\cos 2x - 2x \sin 2x)}{(x \cos 2x)^2} \end{aligned}$$

$$6. y = \frac{x^4}{(x+1)^2}$$

$$u = x^4; \frac{du}{dx} = 4x^3$$

$$v = (x+1)^2$$

$$w = x+1; v = w^2$$

$$\frac{dv}{dx} = \frac{dv}{dw} \times \frac{dw}{dx}$$

$$= 2w \times 1$$

$$= 2(x+1)$$

$$\frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

$$= \frac{(x+1)^2(4x^3) - (x^4)(2)(x+1)}{(x+1)^4}$$

$$= \frac{2x^4 + 4x^3}{(x+1)^3}$$

EXERCISE 3

$$1. y = \frac{e^{4x}}{x^3 \cosh 3x}$$

take \ln of both sides

$$\ln y = \ln e^{4x} - \ln x^3 - \ln \cosh 3x$$

differentiating wrt x

$$\frac{1}{y} \cdot \frac{dy}{dx} =$$

$$\frac{1}{e^{4x}} \cdot 4e^{4x} - \frac{1}{x^3} \cdot 3x^2 - \frac{1}{\cosh 3x} \cdot 3 \sinh 3x$$

$$\frac{dy}{dx} = \frac{e^{4x}}{x^3 \cosh 3x} \left[4 - \frac{3}{x} - 3 \tanh 3x \right]$$

$$2. y = \frac{(3x+1)\cos 2x}{e^{2x}}$$

take \ln of both sides

$$\ln y = \ln (3x+1) + \ln \cos 2x - \ln e^{2x}$$

differentiating wrt x

$$\frac{1}{y} \cdot \frac{dy}{dx} =$$

$$\frac{1}{3x+1} \cdot 3 + \frac{1}{\cos 2x} \cdot (-2\sin 2x) - \frac{1}{e^{2x}} \cdot 2e^{2x}$$

$$\frac{dy}{dx} = \frac{(3x+1)\cos 2x}{e^{2x}} \left[\frac{3}{3x+1} - 2\tan 2x - 2 \right]$$

$$3. y = x^5 \sin 2x \cos 4x$$

take \ln of both sides

$$\ln y = \ln x^5 + \ln \sin 2x + \ln \cos 4x$$

differentiating wrt x

$$\frac{1}{y} \cdot \frac{dy}{dx} =$$

$$\frac{1}{x^5} \cdot 5x^4 + \frac{1}{\sin 2x} \cdot 2\cos 2x + \frac{1}{\cos 4x} \cdot -4\sin 4x$$

$$\frac{dy}{dx}$$

$$= x^5 \sin 2x \cos 4x \left[\frac{5}{x} + 2\cot 2x - 4\tan 4x \right]$$

$$4. y = \frac{(x^3-1)\sin 5x}{x^6}$$

take \ln of both sides

$$\ln y = \ln (x^3 - 1) + \ln \sin 5x - \ln x^6$$

differentiating wrt x

$$\frac{1}{y} \cdot \frac{dy}{dx} =$$

$$\frac{1}{x^3-1} \cdot 3x^2 + \frac{1}{\sin 5x} \cdot 5\cos 5x - \frac{1}{x^6} \cdot 6x^5$$

$$\frac{dy}{dx} = \frac{(x^3-1)\sin 5x}{x^6} \left[\frac{3x^2}{x^3-1} + 5\cot 5x - \frac{6}{x} \right]$$

$$5. y = \frac{\sin 2x \cos 3x}{\cos 4x}$$

take \ln of both sides

$$\ln y = \ln \sin 2x + \ln \cos 3x - \ln \cos 4x$$

differentiating wrt x

$$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{2\cos 2x}{\sin 2x} - \frac{3\sin 3x}{\cos 3x} + \frac{4\sin 4x}{\cos 4x}$$

$$\frac{dy}{dx}$$

$$= \frac{\sin 2x \cos 3x}{\cos 4x} [2\cot 2x - 3\tan 3x + 4\tan 4x]$$

EXERCISE 4.

$$1. y = \cos(7x + 2)$$

$$\text{let } u = 7x + 2; y = \cos u$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$= -\sin u \times 7$$

$$= -7\sin(7x + 2)$$

$$2. y = (4x - 5)^6$$

$$\text{let } u = 4x - 5; y = u^6$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$= 6u^5 \times 4$$

$$= 24u^5$$

$$= 24(4x - 5)^6$$

$$3. y = e^{3-x}$$

$$\text{let } u = 3 - x; y = e^u$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$= e^u \times -1$$

$$= -e^u$$

$$= -e^{(3-x)}$$

$$4. y = \sin 2x$$

$$\text{let } u = 2x; y = \sin u$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$= \cos u \times 2$$

$$= 2\cos u$$

$$= 2\cos 2x$$

$$5. y = \cos(x^2)$$

$$\text{let } u = x^2; y = \cos u$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$= -\sin u \times 2x$$

$$= -2x\sin(x^2)$$

$$6. y = \ln(3 - 4 \cos x)$$

$$\text{let } u = 3 - 4 \cos x; y = \ln u$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$= \frac{1}{u} \times 4 \sin x$$

$$= \frac{4 \sin x}{3 - 4 \cos x}$$

$$7. y = e^{\sin 2x}$$

$$\text{let } v = 2x; u = \sin v; y = e^u$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dv} \times \frac{dv}{dx}$$

$$= e^u \times \cos v \times 2$$

$$= 2e^u \cos v$$

$$= 2e^{\sin 2x} \cos 2x$$

$$8. \text{ check N0. 4}$$

$$9. y = \cos^3(3x)$$

$$\text{let } u = 3x; v = \cos u; y = v^3$$

$$\frac{dy}{dx} = \frac{dy}{dv} \times \frac{dv}{du} \times \frac{du}{dx}$$

$$= 3v^2 \times -\sin u \times 3$$

$$= -9 \cos^2 u \sin u$$

$$= -9 \cos^2 3x \sin 3x$$

$$10. y = \ln \cos 3x$$

$$\text{let } u = \cos 3x; y = \ln u$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$= \frac{1}{u} \times -3 \sin 3x$$

$$= \frac{-3 \sin 3x}{\cos 3x}$$

$$= -3 \tan 3x$$

Exercise 5

$$1. y = 1 + 2x^5$$

$$y' = 10x^4$$

$$y'' = 40x^3$$

$$2. y = (3x^2 - 4)^4$$

$$\text{let } u = 3x^2 - 4; y = u^4$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$= 4u^3 \times 6x$$

$$= 24x(3x^2 - 4)^3$$

$$\text{let } v = (3x^2 - 4)^3$$

$$w = x$$

$$z = 3x^2 - 4$$

$$v = z^3$$

$$\frac{dv}{dx} = \frac{dv}{dz} \times \frac{dz}{dx}$$

$$= 3z^2 \times 6x$$

$$= 18x(3x^2 - 4)^2$$

$$y'' = 24 \left(v \frac{dw}{dx} + w \frac{dv}{dx} \right)$$

$$= 24((3x^2 - 4)^3(1) + x(18x)(3x^2 - 4)^2)$$

$$= 24(3x^2 - 4)^2(21x^2 - 4)$$

$$3. y = x^4 - 8x^2 + 1$$

$$y' = 4x^3 - 16x$$

$$y'' = 12x^2 - 16$$

$$4. y = x^3 + 7$$

$$y' = 3x^2$$

$$y'' = 6x$$

$$5. y = 2x^3 + 3x^2 - 12x + 20$$

$$y' = 6x^2 + 6x - 12$$

$$y'' = 12x + 6$$

EXERCISE 6

$$1. x^3 + y^3 - 3xy = 8$$

$$3x^2 + 3y^2y' - 3[xy' + y] = 0$$

$$3x^2 + 3y^2y' - 3xy' - 3y = 0$$

$$y'[3y^2 - 3x] = 3y - 3x^2$$

$$y' = \frac{3y - 3x^2}{3y^2 - 3x} = \frac{y - x^2}{y^2 - x}$$

$$2. x^2 + y^3 - 4x + 4y = 26$$

$$2x + 3y^2y' - 4 + 4y' = 0$$

$$2 + 6yy'' = 0$$

$$y'' = -\frac{2}{6y}$$

$$= -\frac{1}{3y}$$

$$3. x^3 + y^3 + 4xy^2 = 5$$

$$3x^2 + 3y^2y' + 4[x \cdot 2yy' + y^2] = 0$$

$$3y^2y' + 8xyy' = -4y^2 - 3x^2$$

$$y'[3y^2 + 8xy] = -4y^2 - 3x^2$$

$$y' = \frac{-4y^2 - 3x^2}{3y^2 + 8xy}$$

$$4. x^2 + y^2 - 5xy^3 + 9 = 0$$

$$2x + 2yy' - 5[x \cdot 3y^2y' + y^3] = 0$$

$$y'[2y - 15xy^2] = 5y^3 - 2x$$

$$y' = \frac{5y^3 - 2x}{2y - 15xy^2}$$

CHAPTER 5

Exercise 1

$$1. y = x^2 - x$$

$$Area = \int_0^1 y dx$$

$$= \int_0^1 (x^2 - x) dx$$

$$= \left[\frac{x^3}{3} - \frac{x^2}{2} \right]_0^1$$

$$= \left[\frac{1^3}{3} - \frac{1^2}{2} \right] - \left[\frac{0^3}{3} - \frac{0^2}{2} \right]$$

$$= \frac{1}{3} - \frac{1}{2}$$

$$= \frac{2-3}{6}$$

$$= -\frac{1}{6}$$

$$2. x^2 - 9 = 4 - 7x^2$$

$$x^2 + 7x^2 = 4 + 9$$

$$8x^2 = 13$$

$$x^2 = \frac{13}{8}$$

$$x = \pm \sqrt{\frac{13}{8}}$$

$$A = \int_{-\sqrt{\frac{13}{8}}}^{\sqrt{\frac{13}{8}}} (x^2 - 9) - (4 - 7x^2) dx$$

$$A = \int_{-\sqrt{\frac{13}{8}}}^{\sqrt{\frac{13}{8}}} 8x^2 - 13 dx$$

$$= \left[\frac{8x^3}{3} - 13x \right]_{-\sqrt{\frac{13}{8}}}^{\sqrt{\frac{13}{8}}}$$

$$= \left[\frac{8\left(\sqrt{\frac{13}{8}}\right)^3}{3} - 13\left(\sqrt{\frac{13}{8}}\right) \right] - \left[\frac{8\left(-\sqrt{\frac{13}{8}}\right)^3}{3} - 13\left(-\sqrt{\frac{13}{8}}\right) \right]$$

$$= 5.524 - 16.572 + 5.524 - 16.572$$

$$= -22.096$$

3. The question is faulty

$$4. y = 4 - 3x - x^2$$

$$y = -2x - 2$$

$$4 - 3x - x^2 = -2x - 2$$

$$-x^2 - 3x + 2x + 4 + 2 = 0$$

$$-x^2 - x + 6 = 0$$

$$x^2 + x - 6 = 0$$

$$x^2 + 3x - 2x - 6 = 0$$

$$x(x + 3) - 2(x + 3) = 0$$

$$(x + 3)(x - 2) = 0$$

$$x - 2 = 0 \text{ or } x + 3 = 0$$

$$x = 2 \text{ or } x = -3$$

$$A = \int_{-3}^2 (4 - 3x - x^2) - (-2x - 2) dx$$

$$= \int_{-3}^2 (4 - 3x - x^2 + 2x + 2) dx$$

$$= \int_{-3}^2 (-x^2 - x + 6) dx$$

$$= \left[-\frac{x^3}{3} - \frac{x^2}{2} + 6x \right]_{-3}^2$$

$$= \left[\frac{-(2)^3}{3} - \frac{(2)^2}{2} + 6(2) \right]$$

$$- \left[-\frac{(-3)^3}{3} - \frac{(-3)^2}{2} + 6(-3) \right]$$

$$= \frac{22}{3} - -\frac{27}{2}$$

$$= \frac{125}{6}$$

$$= 20\frac{5}{6}$$

EXERCISE 2

X	2.1	2.4	2.7	3.0	3.3	3.6
Y	3.2	2.7	2.9	3.5	4.1	5.2

$$\int_{2.1}^{3.6} y dx$$

$$\Delta x = \frac{b-a}{n} = \frac{3.6-2.1}{5}$$

$$= 0.3$$

$$\int_a^b f(x)dx \approx \frac{\Delta x}{2} [f(x_0) + f(x_5) + 2(f(x_1) + f(x_2) + f(x_3) + f(x_4))] \\ \approx \frac{0.3}{2} [3.2 + 5.2 + 2(2.7 + 2.9 + 3.5 + 4.1)]$$

$$\approx 5.22$$

EXERCISE 3

$$1. \int_0^4 x^2 dx$$

$$\Delta x = \frac{b-a}{n} = \frac{4-0}{6} = \frac{2}{3}$$

S/ N	x	x^2	F+ L	E	R
1	0	0	0		
2	2/3	4/9		4/9	
3	4/3	16/9			16/9
4	2	4		4	
5	8/3	64/9			64/9
6	10/3	100/9		100/9	
7	4	16	16		
Σ	\Rightarrow	\Rightarrow	16	140/9	80/9

$$A \approx \frac{2}{3} \times \frac{1}{3} [16 + 4\left(\frac{140}{9}\right) + 2\left(\frac{80}{9}\right)]$$

$$A \approx 21\frac{1}{3}$$

$$2. \int_0^2 e^{x^2} dx$$

$$\Delta x = \frac{b-a}{n} = \frac{2-0}{6} = \frac{1}{3}$$

S/ N	x	x^2	e^{x^2}	F+ L	E	R
1	0	0	1	1		
2	1/3	1/9	1.118		1.118	
3	2/3	4/9	1.56			1.56
4	1	1	2.718		2.718	
5	4/3	16/9	5.92			5.92
6	5/3	25/9	16.083		16.083	
7	2	4	54.6	54.6		

Σ	\Rightarrow	\Rightarrow	\Rightarrow	55. 6	19.919	7.48
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$$A \approx \frac{1}{9} [55.6 + 4(19.919) + 2(7.48)]$$

$$A \approx 16.69$$

$$3. \int_0^2 e^{2x} dx$$

$$\Delta x = \frac{b-a}{n} = \frac{2-0}{6} = \frac{1}{3}$$

$$A = \frac{\Delta x}{3} [(F + L) + 4E + 2R]$$

S/ N	x	$2x$	e^{2x}	F+L	E	R
1	0	0	1	1		
2	1/3	2/3	1.948		1.948	
3	2/3	4/3	3.794			3.794
4	1	2	7.389		7.389	
5	4/3	8/3	14.392			14.392
6	5/3	10/3	28.032		28.032	
7	2	4	54.598	54.598		
Σ	\Rightarrow	\Rightarrow	\Rightarrow	55.598	37.369	18.186

$$A \approx \frac{1}{9} [55.598 + 4(37.369) + 2(18.186)]$$

$$A \approx 26.827$$

$$4. \sqrt{1-x^4} dx$$

$$\Delta x = \frac{b-a}{n} = \frac{1-0}{6} = \frac{1}{6}$$

$$A = \frac{\Delta x}{3} [(F + L) + 4E + 2R]$$

S/N	x	x^4	$1-x^4$	$\sqrt{1-x^4}$
1	0	0	1	1
2	1/6	0.0008	0.9992	0.9996

3	1/3	0.0123	0.9877	0.9938
4	1/2	0.0625	0.9375	0.9682
5	2/3	0.1975	0.8025	0.8958
6	5/6	0.4823	0.5177	0.7195
7	1	1	0	0
Σ	\Rightarrow	\Rightarrow	\Rightarrow	\Rightarrow

Table cont'd.....

F+L	E	R
1		
	0.9996	
		0.9938
	0.9682	
		0.8958
	0.7195	
0		
1	2.6873	1.8896

$$A \approx \frac{1/6}{3} [1 + 4(2.6873) + 2(1.8896)]$$

$$A \approx \frac{1}{18} [15.5284]$$

$$A \approx 0.8627$$

1ST EDITION OF CONCISE MTH102 WORKBOOK SOLUTION

APPRECIATION

MY SINCERE APPRECIATION GOES TO ALL
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DEPARTMENT, FUOYE

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REFERENCE

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