Linear Algebra

Matrices
Systems of linear equations
Eigen values & Eigen vectors

Prob: Find the eigen values and eigen vectors of the

matrix $A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$.

Sol:

$|A - \lambda I| = 0$

$$\begin{vmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{vmatrix} - \begin{vmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{vmatrix} = 0$$

$$\begin{vmatrix} 3-\lambda & 1 & 4 \\ 0 & 2-\lambda & 6 \\ 0 & 0 & 5-\lambda \end{vmatrix} = 0$$

$$\begin{vmatrix} 3-\lambda & 1 & 4 \\ 0 & 2-\lambda & 6 \\ 0 & 0 & 5-\lambda \end{vmatrix} = 0$$

$(3-\lambda)\left[(2-\lambda)(5-\lambda) - 0\right] - 1\left[0 - 0\right] + 4\left[0 - 0\right] = 0$

$\boxed{(3-\lambda)(2-\lambda)(5-\lambda) = 0}$ ✓

$\boxed{\lambda = 3, \lambda = 2, \lambda = 5}$ ✓

$\Rightarrow$ eigen Vector for $\lambda = 2$ is

$$(A - \lambda I)\bar{x} = 0$$

$$\begin{bmatrix} 1 & 1 & 4 \\ 0 & 0 & 6 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$x + y + 4z \overset{0}{\cancel{=}} 0$

$\left. \begin{array}{l} 0 + 0 + 6z = 0 \\ 3z = 0 \end{array} \right\} \rightarrow \boxed{z = 0}$

$x + y + 4z \overset{0}{\cancel{=}} 0$

$\left. \begin{array}{l} 0 + 0 + 6z = 0 \\ 3z = 0 \end{array} \right\} \rightarrow \boxed{z = 0}$

$x + y + 4z \overset{0}{\cancel{=}} 0$ ✓

$\left. \begin{array}{l} 0 + 0 + 6z = 0 \\ 3z = 0 \end{array} \right\} \rightarrow \boxed{z = 0}$

let $\boxed{y = K_1} \Rightarrow \boxed{x = -K_1} \Rightarrow \bar{x} = \begin{bmatrix} -K_1 \\ K_1 \\ 0 \end{bmatrix}$

$x + y + 4z \overset{0}{\cancel{=}} 0$

$\left. \begin{array}{l} 0 + 0 + 6z = \\ 3z = 0 \end{array} \right.$

let $\boxed{y = K_1}$ =

$$(A - \lambda I) = \begin{bmatrix} 3-\lambda & 1 & 4 \\ 0 & 2-\lambda & 6 \\ 0 & 0 & 6-\lambda \end{bmatrix}$$

$\rightarrow$ eigen vector $\lambda = 3$ is:

$$(A - 3I)\bar{X} = 0$$

$$\begin{bmatrix} 0 & 1 & 4 \\ 0 & -1 & 6 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$\left. \begin{array}{l} y + 4z = 0 \\ -y + 6z = 0 \end{array} \right\} \rightarrow \boxed{y = 0}$

$2z = 0 \rightarrow \boxed{z = 0}$

let, $\boxed{x = K_2} \Rightarrow \bar{X} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} K_2 \\ 0 \\ 0 \end{bmatrix} = K_2 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ ✓

$y +$

$-y$

let

> eigen vector for $\lambda = 5$ is:

$$\begin{bmatrix} 3-2 & 1 & 9 \\ 0 & 3-2 & 6 \\ 0 & 0 & 5-2 \end{bmatrix}$$

$$(A-5I)\bar{x} = 0$$

$$\begin{bmatrix} -2 & 1 & 9 \\ 0 & -3 & 6 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$-2x + y + 9z = 0 \implies 2x = 2k_3 + 4k_3 \implies \boxed{x = 3k_3}$

$0 = 2y + z + z$

$-3y + 6z = 0 \checkmark \implies 3y = 6k_3$

$\boxed{y = 2k_3} \checkmark$

let $\boxed{z = k_3}$ $\checkmark$

$$\implies \bar{x} = \begin{bmatrix} 3k_3 \\ 2k_3 \\ k_3 \end{bmatrix} = k_3\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} \ ll \checkmark$$

Q] Find the eigen values and eigen vectors of the following matrix.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -2 & 6 \\ 0 & 0 & -3 \end{bmatrix}$$

$$|A - \lambda I| = 0$$

$$\left| \begin{bmatrix} 1 & 2 & 3 \\ 0 & -2 & 6 \\ 0 & 0 & -3 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right| = 0$$

$$\begin{vmatrix} 1-\lambda & 2 & 3 \\ 0 & -2-\lambda & 6 \\ 0 & 0 & -3-\lambda \end{vmatrix} = 0$$

$$(1-\lambda)\left[(-2-\lambda)(-3-\lambda)-0\right] - 2\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} + 3\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$$

$$(1-\lambda)(-2-\lambda)(-3-\lambda) = 0$$

$$(1-\lambda)(2+\lambda)(3+\lambda) = 0$$

$$\boxed{\lambda = 1, -2, -3}$$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -2 & 6 \\ 0 & 0 & -3 \end{bmatrix}$$

$$\boxed{\lambda = 1, -2, -3}$$

$$\Rightarrow \boxed{\lambda = 1} \Rightarrow (A - \lambda I)\bar{x} = 0$$

$$\begin{bmatrix} 1-\lambda & 2 & 3 \\ 0 & -2-\lambda & 6 \\ 0 & 0 & -3-\lambda \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\begin{bmatrix} 0 & 2 & 3 \\ 0 & -3 & 6 \\ 0 & 0 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

1st (

$$0 + 2y + 3z = 0$$

$$0 - 3y + 6z = 0 \longleftarrow \boxed{y=0}$$

$$0 + 0 - 4z = 0 \longrightarrow \boxed{z=0}$$

$$\boxed{\lambda = 1} \Rightarrow \text{The eigen vector is } \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} k_1 \\ 0 \\ 0 \end{bmatrix}$$

$$= k_1 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$\boxed{\lambda = -2}$

$$\begin{bmatrix} 1-\lambda & 2 & 3 \\ 0 & -2-\lambda & 6 \\ 0 & 0 & -3-\lambda \end{bmatrix}$$

$(A - \lambda I)\bar{x} = 0$

$$\begin{bmatrix} -3 & 2 & 3 \\ 0 & 0 & 6 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$3x + 2y + 3z = 0$

$\left. \begin{array}{l} 0 + 0 + 6z = 0 \\ 0 + 0 - z = 0 \end{array} \right\} \rightarrow \boxed{z = 0}$

$3x + 2y = 0$

let $\boxed{y = k_2}$

$3x = -2k_2 \Rightarrow \boxed{x = \dfrac{-2k_2}{3}}$

$\lambda = -2 \Rightarrow \begin{bmatrix} \dfrac{-2k_2}{3} \\ k_2 \\ 0 \end{bmatrix} \Rightarrow k_2 \begin{bmatrix} -2/3 \\ 1 \\ 0 \end{bmatrix}$

$\boxed{\lambda = -3}$

$$\begin{bmatrix} 4 & 2 & 3 \\ 0 & 1 & 6 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$4x + 2y + 3z = 0$

$0 + y + 6z = 0 \Rightarrow \boxed{z = k_3}$

$0 + 0 + 0 = 0.$

$\boxed{y = -6k_3}$

$4x + 2(-6k_3) + 3k_3 = 0$

$4x - 12k_3 + 3k_3 = 0$

Finish it up from here...

$4x = 9k_3 \Rightarrow \boxed{x = \dfrac{9k_3}{4}}$