Calculus Exam 3

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October 2019

1 Warm up question (4 points)

Write down the definition of with respect to a differential:

$$\int_a^b f(x)dg(x) =?$$

2 Limits and asymptotics (6 points)

2.1 Compute the limit of the following sequences for n tending to infinity: (3 points)

- $S_n = \frac{-n^2+3n}{3n^2+5n-1/n^2}$
- $S_n = \frac{3n^3+1}{4n^3+n^2+n}$
- $S_n = \frac{n^2+4n+1}{7n^2-100n+1000}$

2.2 Compute the limit of the following functions for x tending to 0: (3 points)

- $f(x) = \frac{(\sin(x)+1)(2x^6+3x^2-6x)}{7x^6}$
- $f(x) = \frac{x\sin(x)}{3x^2}$
- $f(x) = \frac{xe^{x^2}-x}{3x^3+x^2}$

Hint: $e^x - 1 \sim x$, $e^{x^2} - 1 \sim x^2$, $\sin(x) \sim x$ for $x \to 0$.

3 Derivatives and optimization (10 points)

3.1 Compute the derivative of the following functions: (4 points)

- $f(x) = -2x^5 + 3x^4 - 3x^2 + 1$
- $f(x) = 2x^2e^x + 3$
- $f(x) = x^3 \cos(x^2 + 3x)$
- $f(x) = \frac{\sin(x)^2}{\cos(x)}$

3.2 Use the definition of derivative to prove that: (3 points)

$$\frac{dx^3}{dx} = 3x^2$$

3.3 Find the critical points of the following function and determine if they are minima, maxima or saddle points. (3 points)

- $f(x) = xe^x + 1$

4 Integrals (10 points)

4.1 Compute the following integrals: (6 points)

- $$\int_0^4 (9x^4 + 2x^2 + x)dx$$
- $$\int_0^1 xe^x dx$$
- $$\int_{-1}^1 (x^2e^x + x^2 + 1)dx$$

4.2 Use the definition of integral to prove that: (4 points)

$$\int_0^b x^2 dx = \frac{1}{3}b^3$$

Hint: you need to use the following formula:

$$\sum_{n=0}^{N-1} n^2 = \frac{(N-1)N(2N-1)}{6}$$

5 Bonus question (5 bonus points)

Prove that the limit of $S_n = 1/n^3$ is 0 using the definition of limit.