Calculus Exam 4

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1 Warm up question (4 points)

Write down the definition of asymptotic relation between two functions $f(x)$ and $g(x)$ at x tending to x_0.

2 Limits and asymptotics (6 points)

2.1 Compute the limit of the following sequences for n tending to infinity: (3 points)

- $S_n = \frac{n^2+3n-1}{8n^2+3n-1/n^3}$
- $S_n = \frac{n^2+4n}{3n^3+n^2}$
- $S_n = \frac{n^6+9n^2+n}{7n^6-100n^4}$

2.2 Compute the limit of the following functions for x tending to 0: (3 points)

- $f(x) = \frac{(e^x+3)(2x^3+3x)}{3x^5+x^3}$
- $f(x) = \frac{x(e^x-1)}{8x^2+4x^4}$
- $f(x) = \frac{x(e^x-1)}{3x\sin(x)}$

Hint: $e^x - 1 \sim x$, $\sin(x) \sim x$ for $x \to 0$.

3 Derivatives and optimization (10 points)

3.1 Compute the derivative of the following functions: (4 points)

- $f(x) = x^8/8 + x^4/4 + 6x^2 + x$

- $f(x) = 6x^3 e^x$
- $f(x) = x^3 \sin(e^x)$
- $f(x) = \frac{\sin(x)}{(\cos(x))^2}$

3.2 Use the definition of derivative to prove that: (3 points)

$$\frac{de^{x+1}}{dx} = e^{x+1}$$

Hint: $e^x - 1 \sim x$.

3.3 Find the critical points of the following function and determine if they are minima, maxima or saddle points. (3 points)

- $f(x) = x^2 e^x$

4 Integrals (10 points)

4.1 Compute the following integrals: (6 points)

- $$\int_0^3 (3x^3 + 2x^2 + 2x)dx$$
- $$\int_{-2}^1 xe^x dx$$
- $$\int_{-1}^1 (xe^x + x^3)dx$$

4.2 Use the definition of integral to prove that: (4 points)

$$\int_0^b x^2 dx = \frac{1}{2}b^2$$

Hint: you need to use the following formula:

$$\sum_{n=0}^{N-1} n^2 = \frac{(N-1)N(2N-1)}{6}$$

5 Bonus question (5 bonus points)

Explain why a function $f(x)$ cannot be asymptotic to 0 for x tending to 0.