Radboud University
Faculty of Social Sciences
Programme: Bachelor's programme in Artificial Intelligence

Exam: Calculus (SOW-BKI104)
Date:
Time of exam:
Number of pages of which this exam consists: *2*
Number of questions of which this exam consists: *5*

Student number:
Name:

Exam instructions:
- Any exam taken by students who are not eligible to sit that exam will be declared invalid. Registering for the exam is compulsory.
- Mobile phones may not be brought into the exam hall.
- No part of this exam may be reproduced and/or made public by means of writing out, photo, photocopy or other medium.
- Fill in your name, student number and number of the version, where appropriate, on every form you use.
- After the exam, hand in all the forms received to the invigilator.
- Any breach of the above-mentioned rules will be reported to the examination committee as an instance of fraud. As a consequence, your exam could be declared invalid.

Use of aids:
- You can bring a one page formula/notes sheet (front and back). There are no restriction on the nature of this page and it does not need to be handwritten.
- You can use any kind of calculator.
- No other aids are permitted.

Use of the forms handed out:
Answer all the questions on the lined answer sheet.

Use of scrap paper is permitted. Yes.

Exam result
- The exam results will be announced at most 15 working days after the date of the exam.

Access
Date, time and location of access will be announced on Brightspace.

Calculus Exam 4

Luca Ambrogioni

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1 Warm up question (4 points)

Write down the definition of asymptotic relation between two functions $f(x)$ and $g(x)$ for x tending to x_0.

2 Limits and asymptotics (6 points)

2.1 Compute the limit of the following sequences for n tending to infinity: (3 points)

- $S_n = \frac{n^2+3n-1}{8n^2+3n-1/n^3}$
- $S_n = \frac{n^2+4n}{3n^3+n^2}$
- $S_n = \frac{n^6+9n^2+n}{7n^6-100n^4}$

2.2 Compute the limit of the following functions for x tending to 0: (3 points)

- $f(x) = \frac{x^5+3x^3}{3x^5+x^3}$
- $f(x) = \frac{x(e^x-1)}{8x^2+4x^4}$
- $f(x) = \frac{x(e^x-1)}{3x\sin(x)}$

Hint: $e^x - 1 \sim x$, $\sin(x) \sim x$ for $x \to 0$.

3 Derivatives and optimization (10 points)

3.1 Compute the derivative of the following functions: (4 points)

- $f(x) = x^8/8 + x^4/4 + 6x^2 + x$

- $f(x) = 6x^3 e^x$
- $f(x) = x^3 \sin(e^x)$
- $f(x) = \frac{\sin(x)}{(\cos(x))^2}$

3.2 Use the definition of derivative to prove that: (3 points)

$$\frac{de^{x+1}}{dx} = e^{x+1}$$

Hint: $e^x - 1 \sim x$.

3.3 Find the critical points of the following function and determine if they are minima, maxima or saddle points. (3 points)

- $f(x) = x^2 e^x$

4 Integrals (10 points)

4.1 Compute the following integrals: (6 points)

- $$\int_0^3 (3x^3 + 2x^2 + 2x)dx$$
- $$\int_{-2}^1 xe^x dx$$
- $$\int_{-1}^1 (xe^x + x^3)dx$$

4.2 Use the definition of integral to prove that: (4 points)

$$\int_0^b x^2 dx = \frac{1}{3}b^3$$

Hint: you need to use the following formula:

$$\sum_{n=0}^{N-1} n^2 = \frac{(N-1)N(2N-1)}{6}$$

5 Bonus question (5 bonus points)

Explain why a function $f(x)$ cannot be asymptotic to 0 for x tending to 0.