WP Observation Simulation

J. G. P. Vermeulen

August 2, 2021

1 Vis

2 IR

Space-based thermal infrared studies of asteroids (Mainzer et al 2015 in Asteroids IV)

From energy balance:

$$L_e = L_a \tag{1}$$

Emitted radiation is absorbed radiation. Then, from albedo:

$$A_p S = L_r + L_e \tag{2}$$

And

$$\frac{L_r}{L_e} = \frac{A}{1-A} \tag{3}$$

With:

- A_p: Projected area
- S: Solar flux
- L_r: Reflected radiation
- L_e: Emitted radiation

Then there's a mess for the thermal model:

$$L_e = \epsilon\eta\sigma R^2 \int_{-\pi} \pi \int_{-\pi/2} \pi/2 T^4(\theta, \phi)\cos(\phi) d\phi d\theta \tag{4}$$

with longitude θ and latitude ϕ measured from the subsolar point. Then, with angular distance from the subsolar point ζ:

$$T(\theta, \phi) = \begin{cases} T_{ss}\cos^{1/4}\zeta & \text{if } \zeta < \pi/2 \\ 0 & \text{if } \zeta \leq \pi/2 \end{cases} \tag{5}$$

With temperature at the subsolar point:

$$T_{ss} = \left(\frac{S(1-A)}{\eta\epsilon\sigma}\right)^{1/4} \tag{6}$$

Here, we have more parameters:

- η: beaming parameter = 0.756
- ϵ: emissivity at wavelength
- σ: Stefan-Boltzmann constant
- R Object radius

From The COBE Diffuse Infrared Background Experiment Search for the Cosmic Infrared Background. II. Model of the Interplanetary Dust Cloud (Kelsall et al 1997):
We can approximate the infrared zodiacal brightness through this model:

$$Z_\lambda(p,t) = \Sigma_c \int n_c(X,Y,Z)[A_{c,\lambda}F_\lambda^{\odot}\Phi_\lambda(\Theta) + (1-A_{c,\lambda})E_{c,\lambda}B_\lambda(T)K_\lambda(T)]ds \quad (7)$$

The line of sight integral for each pixel p at time t of scattered infrared light and thermal emissions, summed over the components c. With:

- $n_c(X,Y,Z)$: the three-dimensional density for each of the components.
- $A_{c,\lambda}$: Albedo for component c at wavelength λ.
- $F_\lambda^{\odot}$: Solar flux.
- $\Phi_\lambda(\Theta)$: Phase function at scattering angle Θ
- $E_{c,\lambda}$: An important parameter, an emmissivity modification factor measuring deviation from blackbody.
- $B_\lambda(T)$: Blackbody radiance function
- $T(R)$: Temperature
- $K_\lambda(T)$: Color correction factor, can be ignored.

First off, albedo is assumed zero at thermal infrared wavelengths, and the color correction factor is unneccessary for the level of required detail. Therefore, the formula reduces to:

$$Z_\lambda(p,T) = \Sigma_c \int n_c(X,Y,Z)E_{c,\lambda}B_\lambda(T)ds \quad (8)$$

Blackbody radiance is given by Planck's law:

$$B_\lambda = \frac{2hc^2/\lambda^5}{e^{\frac{hc}{\lambda k_B T}} - 1} \quad (9)$$

With:

- Planck constant $h = 6.626 \cdot 10^{-34}$ J $\cdot$ Hz^{-1}
- Boltzmann constant $k_B = 1.380 \cdot 10^{-23}$ J $\cdot$ K^{-1}
- Lightspeed $c = 299792458$ m/s

We can model the temperature according to grey body thermal equilibrium:

$$T(R) = T_0 R^{-\delta} \tag{10}$$

With $\delta = 0.467$.

Then, we have the following coordinate transformations for a spacecraft in the elliptical plane:

$$X = R_{S/C} \cos \lambda_{S/C} + s \cos \beta \cos \lambda \tag{11}$$
$$Y = R_{S/C} \sin \lambda_{S/C} + s \cos \beta \sin \lambda \tag{12}$$
$$Z = s \sin \beta \tag{13}$$
$$R = \sqrt{X^2 + Y^2 + Z^2} \tag{14}$$

With $R_{S/C}$ the heliocentric radius of the spacecraft and $\lambda_{S/C}$ the heliocentric longitude of the spacecraft. Then with longitude λ and latitude β. Integration is suggested to be performed from 0 to 5.2 AU from the Sun (the orbit of Jupiter). We can find an approriate end value of s as follows (WolframAlpha, exact solution):

$$\begin{aligned} s = \frac{1}{5}(-\frac{1}{10}\sqrt{(}(50R_{S/C} \cos \lambda_{S/C} \cos \beta \cos \lambda + \\ 50R_{S/C} \sin \lambda_{S/C} \cos \beta \sin \lambda)^2 - 100(25R_{S/C}^2 - 676)) \\ -5R_{S/C} \cos \lambda_{S/C} \cos \beta \cos \lambda - 5R_{S/C} \sin \lambda_{S/C} \cos \beta \sin \lambda) \end{aligned} \tag{15}$$

Lastly, we have the densities of the smooth dust cloud, the bands, and the circumsolar ring. The Earth-trailing blob is neglected.

Smooth cloud:
Firstly, the cloud is offset from the Sun:

$$X' = X - X_0 \tag{16}$$
$$Y' = Y - Y_0 \tag{17}$$
$$Z' = Z - Z_0 \tag{18}$$
$$R_c = \sqrt{X'^2 + Y'^2 + Z'^2} \tag{19}$$

Furthermore, the cloud is tilted:

$$Z_c = X' \sin \Omega \sin i - Y' \cos \Omega \sin i + Z' \cos i \tag{20}$$

Then, the cloud is modelled as a modified fan:

$$n_c(X, Y, Z) = n_0 R_c^{-\alpha} e^{-\beta g \gamma} \tag{21}$$

With:

$$\zeta \equiv |Z_c / R_c| \tag{22}$$

$$g = \begin{cases} \zeta^2/2\mu & \forall\ \zeta < \mu \\ \zeta - \mu/2 & \forall\ \zeta \leq \mu \end{cases} \tag{23}$$

With $n_0, \alpha, \beta, \gamma, \mu$ as free parameters.

Dust bands:
The density for the rings is given by:

$$\begin{aligned} n_{Bi}(X,Y,Z) = \frac{3n_{3B1}}{R} \exp\left(-\left(\frac{\zeta_{Bi}}{\delta_{\zeta Bi}}\right)^6\right) \left(v_{Bi} + \left(\frac{\zeta_{Bi}}{\delta_{\zeta Bi}}\right)^{p_{Bi}}\right) \\ \cdot \left(1 - \exp\left(-\left(\frac{R}{\delta_{RBi}}\right)^{20}\right)\right) \end{aligned} \tag{24}$$