The synth_runner Package: Utilities to Automate Synthetic Control Estimation Using synth

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Introduction

- Abadie and Gardeazabal (2003) and Abadie et al. (2010) introduced Synthetic Control Methods (SCM), to identifying treatment effects in case-studies.
- Abadie et al. (2010) released synth for Stata to perform a Synthetic Control estimation.
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Outline

- Synthetic Control Methods
 - Classic SCM
 - Multiple Treatments
- Stata Module
 - Single Treatment Example 1
 - Single Treatment Example 2
 - Multiple Treatments Example 3
 - Installation
- 3 Discussion
 - Future Work
 - Q & A

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- Abadie and Gardeazabal (2003) and Abadie et al. (2010) introduced Synthetic Control Methods (SCM).
- Allows for estimating treatment effects even if treated unit is not on a parallel time trend as the mean of the untreated units.
- Constructs a counterfactual by finding a weighted average of non-treated units that resembles the treated unit in the pre-treatment period.

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DGP for Synthetic Controls

Synthetic Controls can be applied in multiple settings, but most common setting where there are theoretical results is:

• Units $\{1,...,J+1\}$ with 1 being treated and J untread "donors". D_{jt} is the treatment indicator. T_0 pre-treatment periods and T total time periods. Factor Model:

$$Y_{jt} = \alpha_{jt} D_{jt} + Y_{jt}^{N}$$

$$Y_{jt} = \alpha_{jt} D_{jt} + (\theta_{t}' \mathbf{Z}_{j} + \delta_{t} + \lambda_{t}' \mu_{j} + \varepsilon_{jt})$$

 α_{jt} are time-varying treatment effects, Y_{jt}^{AV} is the no-treated counterfactual, θ_t are unknown parameters, \mathbf{Z}_j are observed unaffected covariates, δ_t is an unknown time factor, λ_t are unknown factors, μ_j are unknown factor loadings, and the error ε_{jt} is independent across units and time with zero mean.

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- Let ${\bf W}$ be a weight vector over the other units such that ${\bf W} \geq {\bf 0}$ and $\sum_j w_j = 1$.
- Split pre-/post-treatment Y as $(\mathbf{\hat{Y}} \setminus \mathbf{\hat{Y}})$.
- Let \mathbf{Y}_0 be the $(T \times J)$ matrix of outcomes for donors (similar for $\mathbf{Z}_0, \mathbf{Y}, \mathbf{Y})$.
- Suppose a **W** can be found such that the synthetic control matches the treated unit in pre-treatment:

$$\mathbf{\tilde{Y}}_1 = \mathbf{\tilde{Y}}_0 \mathbf{W}$$
 $\mathbf{Z}_1 = \mathbf{Z}_0 \mathbf{W}$

- ullet Estimate $ec{\mathbf{Y}}_1^N$ as $ec{\mathbf{Y}}_0 \mathbf{W}$ so $\hat{lpha}_1 = ec{\mathbf{Y}}_1 ec{\mathbf{Y}}_0 \mathbf{W}$
- Then $Bias(\hat{\alpha}) \to 0$ as T_0 grows large relative to ε_{it} .

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- Let "predictors" \underline{X} be comprised of Z and some linear combinations of Y.
- Define $\|\mathbf{A}\| = \sqrt{\mathbf{A}cols(\mathbf{A})^{-1}\mathbf{A}}$ and $\|\mathbf{A}\|_{\mathbf{V}} = \sqrt{\mathbf{A}\mathbf{V}^{-1}\mathbf{A}}$. • $\|\mathbf{Y}_1 - \mathbf{Y}_0\mathbf{W}\| = s_1$ is the root mean squared prediction error (RMSPE).
- ullet As matching may only hold approximately, we need a set of predictor weights V that prioritizes which variables to match better.
- Given V, then

$$\mathbf{W}^* = \arg\min_{\mathbf{W}} \|\mathbf{X}_1 - \mathbf{X}_0 \mathbf{W}\|_{\mathbf{V}}$$

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• Factor model $(\lambda'_t \mu_j)$ accommodates many forms:

- Standard panel model with time and units fixed effects from $\mu_j=(1,\gamma_j)$ and $\lambda_t=(\xi_t,1)'$
- Unit-specific (non-parallel) time trends: $\mu_j = (\gamma_j)$ and $\lambda_t = (t)$
- Need $\sum_{t=1}^{T_0} \lambda_t \lambda_t'$ non-singular. Implies:
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- SCM relies on in-place placebo (permutation) tests for inference.
- Re-estimate synthetic controls on all J donors to get $\{\hat{\alpha}_p\}$. Call this the placebo distribution $\hat{\alpha}_1^{PL}$.
- Compare $\hat{\alpha}_1$ to $\hat{\alpha}_1^{PL}$. The two-sided p-value for $\hat{\alpha}_{1t}$ is then

$$p\text{-value} = \Pr\left(|\hat{\alpha}_{1t}^{PL}| \ge |\hat{\alpha}_{1t}|\right)$$
$$= \frac{\sum_{j \ne 1} 1(|\hat{\alpha}_{jt}| \ge |\hat{\alpha}_{1t}|)}{J}$$

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Interpretation of Inference

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- Randomization tests have J+1 in the denominator.
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Accounting for Match Quality

- Post-treatment differences may be large just because of poor match quality.
- Can take quality of match for pre-treatment period into
 - Limit donor pool to cases that matched as well as the treated
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- Inference can then be conducted over four quantities $(\hat{\alpha}_{jt}, \vec{s}_j, \hat{\alpha}_{jt}/\vec{s}_j, \vec{s}_j/\vec{s}_j)$ and the comparison set can also be limited by the choice of m.

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For each $g \in \{1, ..., G\}$:

- Estimate vector $\hat{\alpha}_a$
- Estimate set of vectors $\hat{\alpha}_q^{PL} = \{\hat{\alpha}_i^g\}_{i \in J}$ assuming treatment
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- An averge of picks is $\bar{\alpha}(\tilde{j}) = G^{-1} \sum_{q \in G} \hat{\alpha}^g_{\tilde{j}[q]}$
- Define set of vectors $\bar{\alpha}^{PL} = \{\bar{\alpha}(\tilde{j}) | \tilde{j} \in \tilde{J}\}$

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- Runs placebo tests and outputs p-values and confidence
- Allows for matching on trends in the outcome variable rather
- Handles multiple-treatments
- Automates the process of splitting pre-treatment periods into
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Run estimation similar to ADH10

```
*Data setup the same for all examples
sysuse smoking
tsset state year

tempfile keepfile
*similar syntax to -synth-
synth_runner cigsale beer(1984(1)1988) lnincome(1972(1)
1988) retprice age15to24 cigsale(1988) cigsale(1980)
cigsale(1975), trunit(3) trperiod(1989) keep(`keepfile
')
```

Returned Values

Post-treatment joint significance

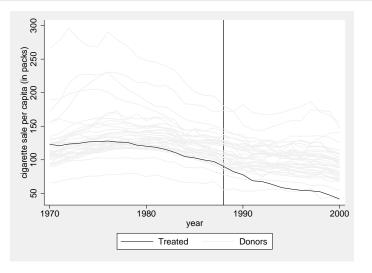
```
e(pval_joint_post) = .1315789473684211
e(pval_joint_post_std) = 0
```

Diagnostics. If too small then SCM may not be appropriate for this unit. If conducting over multiple treatments, good to assess individual match quality and discard some units.

```
e(avg_pre_rmspe_p) = .9210526315789474
```

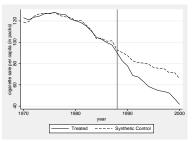
Make Graphs

Outcome Trends

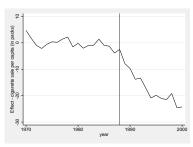


NB: Not a full test of parallel trends assumption as does not account for regression adjustment done in DiD.

Treated and Synthetic Control

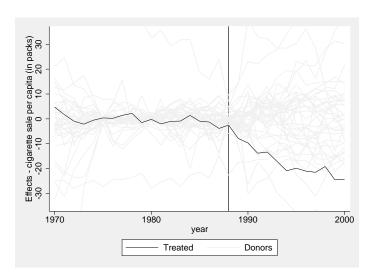


(a) Treated and Control

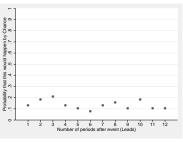


(b) Difference

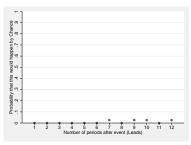
Differences



P-Values



(c) Effect



(d) Studentized Effect

More complicated example

```
gen byte D = (state==3 & year>=1989)
tempfile keepfile2
synth_runner cigsale beer(1984(1)1988) lnincome(1972(1)
    1988) retprice age15to24, trunit(3) trperiod(1989)
    trends training_propr(`=13/18') pre_limit_mult(10)
    keep(`keepfile2')
```

Returned Values

Post-treatment joint significance

```
e(pval_joint_post) = .0263157894736842
e(pval_joint_post_std) = 0
```

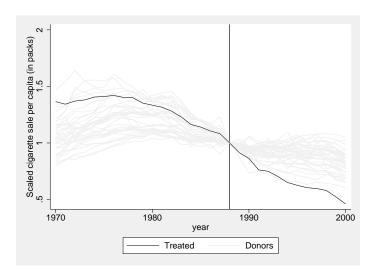
Diagnostics (want large values)

```
e(avg_pre_rmspe_p) = .631578947368421
e(avg_val_rmspe_p) = .8421052631578947
```

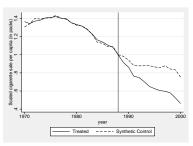
Make Graphs

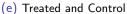
```
merge 1:1 state year using `keepfile2', nogenerate
gen cigsale_scaled_synth = cigsale_scaled - effect_scaled
single_treatment_graphs, depvar(cigsale_scaled) effect_var
    (effect_scaled) trunit(3) trperiod(1989)
effect_graphs , depvar(cigsale_scaled) depvar_synth(
    cigsale_scaled_synth) effect_var(effect_scaled) trunit
        (3) trperiod(1989)
pval_graphs
```

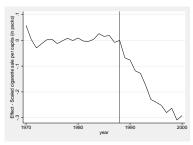
Outcome Trends



Mean Treated and Synthetic Control

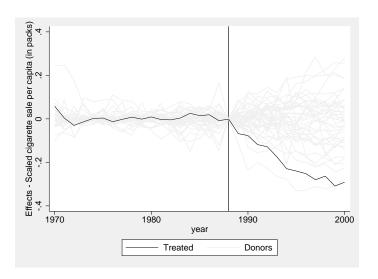




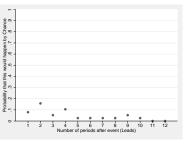


(f) Difference

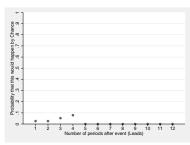
Differences



P-Values



(g) Effect



(h) Studentized Effect

- The number of SC estimates grows linearly $(J \cdot G)$ while the size of the placebo set grows exponentially (J^G)
- Inference can quickly become infeasible
- By default, there is a maximum number (max) of averages computed for inference (1,000,000)
 - ullet If $J^G < max$ then all J^G are used for inference
 - If $J^G>\max$ then \max are drawn at random (with replacement)
 - $(max \text{ is modifyable and can be set to } \infty)$
- Space requirements scale exponentially if confidence intervals as desired, otherwise scales linearly.

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Estimate with Multiple Treatments

Returned Values

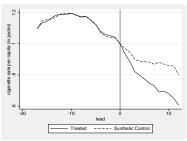
Post-treatment joint significance

```
e(pval_joint_post) = .0423666910153397
e(pval_joint_post_std) = 0
```

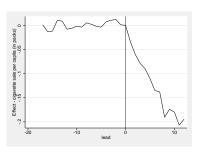
Diagnostics (want large values)

```
e(avg_pre_rmspe_p) = .9298758217677137
e(avg_val_rmspe_p) = .9598246895544192
```

Mean Treated and Synthetic Control

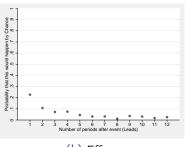


(i) Mean Treated and Control

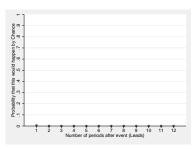


(j) Mean Difference

P-Values



(k) Effect



(I) Studentized Effect

Installation

- Stable Version: http://econweb.umd.edu/~galiani/code.html
- Development: https://github.com/bquistorff/synth_runner
 - Can submit bug reports and patches.

- Cross-validation for picking regressors (Dube and Zipperer, 2015).
- Graph the ratio of post/pre RMSPE (as in ?)
- Leave-one-observation-out robustness:
 - Small number: Graph as in ?. Would also want to show
 - Large number: Standard deviation of the change of the estimate
- Allow per-treatment donor sets e.g. to limit interpolation bias
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 - Small number: Graph as in ?. Would also want to show changing significance levels.
 - Large number: Standard deviation of the change of the estimate $\{(\hat{\alpha} \hat{\alpha}_{-j})\}_j$ (Athey and Imbens, 2015). What proportion lose/gain significance?
- Allow per-treatment donor sets e.g. to limit interpolation bias (non-linear DGP) or spill-overs.
- CV for picking V (mentioned in Abadie et al. 2010 and used in ?) using training period

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Q & A

Appendix Slides

- Alberto Abadie and Javier Gardeazabal. The economic costs of conflict: A case study of the Basque country. *American Economic Review*, 93(1):113–132, March 2003. doi:10.1257/000282803321455188.
- Alberto Abadie, Alexis Diamond, and Jens Hainmueller. Synthetic control methods for comparative case studies: Estimating the effect of California's Tobacco Control Program. *Journal of the American Statistical Association*, 105(490):493–505, June 2010. doi:10.1198/jasa.2009.ap08746.
- Susan Athey and Guido Imbens. A measure of robustness to misspecification. *American Economic Review*, 105(5):476–480, May 2015. doi:10.1257/aer.p20151020.
- Eduardo Cavallo, Sebastian Galiani, Ilan Noy, and Juan Pantano. Catastrophic natural disasters and economic growth. *Review of Economics and Statistics*, 95(5):1549–1561, December 2013. doi:10.1162/rest a 00413.

Arindrajit Dube and Ben Zipperer. Pooling multiple case studies using synthetic controls: An application to minimum wage policies. Technical Report 8944, IZA, March 2015.