

Assignment 2

AI1110: Probability and Random Variables

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Chapter 16 , Exercise 16.4

Question 9 :

If 4-digit numbers greater than 5,000 are randomly formed from the digits 0,1,3,5 and 7, what is the probability of forming a number divisible by 5 when :

- 1) the digits are repeated?
- 2) the repetition of digits are not allowed?

Solution:

Since 4-digit numbers greater than 5,000 are formed , The thousands place is either 7 or 5.

- 1) Digits are repeated

Description	Parameter	Value
Let W , X , Y , Z be 4 random variables representing Thousands , Hundredths , Tenths and Units place respectively	W	{5, 7}
and 'A' be the Event representing all the 4-digits numbers greater than 5,000 divisible by 5 when digits are repeated.	X	{0, 1, 3, 5, 7}
	Y	{0, 1, 3, 5, 7}
	Z	{0, 1, 3, 5, 7}

TABLE I

$$\Pr(W = 5, X = 0, Y = 0, Z = 0) = \left(\frac{1}{2} \times \frac{1}{5} \times \frac{1}{5} \times \frac{1}{5}\right) \quad (1)$$

$$\Pr(W = 5, Z = 0) = \left(\frac{1}{2} \times 1 \times 1 \times \frac{1}{5} - \Pr(W = 5, X = 0, Y = 0, Z = 0)\right) \quad (2)$$

$$\Pr(W = 5, Z = 0) = \left(\frac{1}{2} \times 1 \times 1 \times \frac{1}{5} - \frac{1}{2} \times \frac{1}{5} \times \frac{1}{5} \times \frac{1}{5}\right) \quad (3)$$

$$\Pr(W = 5, Z = 5) = \left(\frac{1}{2} \times 1 \times 1 \times \frac{1}{5}\right) \quad (4)$$

$$\Pr(W = 7, Z = 0) = \left(\frac{1}{2} \times \frac{1}{5}\right) \quad (5)$$

$$\Pr(W = 7, Z = 5) = \left(\frac{1}{2} \times \frac{1}{5}\right) \quad (6)$$

$$(7)$$

$$\Pr(A) = \Pr(W = 5, Z = 0) + \Pr(W = 5, Z = 5) + \Pr(W = 7, Z = 0) + \Pr(W = 7, Z = 5) \quad (8)$$

$$= \frac{24}{250} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} \quad (9)$$

$$= \frac{99}{250} \quad (10)$$

2) Digits are not repeated

W \Rightarrow 2 choices

Z \Rightarrow 4 choices

X \Rightarrow 3 choices

Y \Rightarrow 2 choices

'B' = Event representing all the 4-digits numbers greater than 5,000 divisible by 5 when digits are not repeated.

$$\Pr(W = 5, Z = 0) = \left(\frac{1}{2} \times 1 \times 1 \times \frac{1}{4}\right) \quad (11)$$

$$\Pr(W = 7, Z = 0) = \left(\frac{1}{2} \times 1 \times 1 \times \frac{1}{4}\right) \quad (12)$$

$$\Pr(W = 7, Z = 5) = \left(\frac{1}{2} \times 1 \times 1 \times \frac{1}{4}\right) \quad (13)$$

$$\Pr(B) = \Pr(W = 5, Z = 0) + \Pr(W = 7, Z = 0) + \Pr(W = 7, Z = 5) \quad (14)$$

$$= \frac{1}{8} + \frac{1}{8} + \frac{1}{8} \quad (15)$$

$$= \frac{3}{8} \quad (16)$$