Assignment 2
AI1110: Probability and Random Variables
Indian Institute of Technology Hyderabad

Arjit Jain
AI22BTECH11002

Chapter 16 , Exercise 16.4

Question 9 :

If 4-digit numbers greater than 5,000 are randomly formed from the digits 0,1,3,5 and 7, what is the probability of forming a number divisible by 5 when :
 1) the digits are repeated?
 2) the repetition of digits are not allowed?

Solution:

 Since 4-digit numbers greater than 5,000 are formed , The thousands place is either 7 or 5.

 1) Digits are repeated

Description	Parameter	Value
Let X_0, X_1, X_2, X_3 be 4 random variables representing Thousandths	X_0	$\{5, 7\}$
, Hundredths , Tenths and Ones place respectively and 'A' be the	X_1	$\{0, 1, 3, 5, 7\}$
Event representing all the 4-digits numbers greater than 5,000	X_2	$\{0, 1, 3, 5, 7\}$
divisible by 5 when digits are repeated.	X_3	$\{0, 1, 3, 5, 7\}$

TABLE 1

 Given probabilities :

$$\Pr(X_0 = 5, X_1 = 0, X_2 = 0, X_3 = 0) = \left(\frac{1}{2} \times \frac{1}{5} \times \frac{1}{5} \times \frac{1}{5} \right) \tag{1}$$

$$\Pr(X_0 = 5, X_3 = 0) = \left(\frac{1}{2} \times 1 \times 1 \times \frac{1}{5} - \Pr(X_0 = 5, X_1 = 0, X_2 = 0, X_3 = 0) \right) \tag{2}$$

$$\Pr(X_0 = 5, X_3 = 0) = \left(\frac{1}{2} \times 1 \times 1 \times \frac{1}{5} - \frac{1}{2} \times \frac{1}{5} \times \frac{1}{5} \times \frac{1}{5} \right) \tag{3}$$

$$\Pr(X_0 = 5, X_3 = 5) = \left(\frac{1}{2} \times 1 \times 1 \times \frac{1}{5} \right) \tag{4}$$

$$\Pr(X_0 = 7, X_3 = 0) = \left(\frac{1}{2} \times \frac{1}{5} \right) \tag{5}$$

$$\Pr(X_0 = 7, X_3 = 5) = \left(\frac{1}{2} \times \frac{1}{5} \right) \tag{6}$$

Desired probability :

$$\Pr(A) = \Pr(X_0 = 5, X_3 = 0) + \Pr(X_0 = 5, X_3 = 5) + \Pr(X_0 = 7, X_3 = 0) + \Pr(X_0 = 7, X_3 = 5) \tag{7}$$

$$= \frac{24}{250} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} \tag{8}$$

$$= \frac{99}{250} \tag{9}$$

2) Digits are not repeated

$$X_0 \implies 2 \text{ choices}$$
$$X_3 \implies 4 \text{ choices}$$
$$X_1 \implies 3 \text{ choices}$$
$$X_2 \implies 2 \text{ choices}$$

'B' = Event representing all the 4-digits numbers greater than 5,000 divisible by 5 when digits are not repeated.
Given probabilities :

$$\Pr(X_0 = 5, X_3 = 0) = \left(\frac{1}{2} \times 1 \times 1 \times \frac{1}{4} \right) \tag{10}$$

$$\Pr(X_0 = 7, X_3 = 0) = \left(\frac{1}{2} \times 1 \times 1 \times \frac{1}{4} \right) \tag{11}$$

$$\Pr(X_0 = 7, X_3 = 5) = \left(\frac{1}{2} \times 1 \times 1 \times \frac{1}{4} \right) \tag{12}$$

Desired probability :

$$\Pr(B) = \Pr(X_0 = 5, X_3 = 0) + \Pr(X_0 = 7, X_3 = 0) + \Pr(X_0 = 7, X_3 = 5) \tag{13}$$

$$= \frac{1}{8} + \frac{1}{8} + \frac{1}{8} \tag{14}$$

$$= \frac{3}{8} \tag{15}$$