Assignment 2
AI1110: Probability and Random Variables
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Chapter 16 , Exercise 16.4

Question 9 :

If 4-digit numbers greater than 5,000 are randomly formed from the digits 0,1,3,5 and 7, what is the probability of forming a number divisible by 5 when :

 1) the digits are repeated?

 2) the repetition of digits are not allowed?

Solution:

 Since 4-digit numbers greater than 5,000 are formed , The thousands place is either 7 or 5.

 1) Digits are repeated

Description	Parameter	Value
Let X_{Th}, X_H, X_T, X_O be 4 random variables representing Thousandths , Hundredths , Tenths and Ones place respectively and 'A' be the Event representing all the 4-digits numbers greater than 5,000 divisible by 5 when digits are repeated.	X_{Th} X_H X_T X_O	$\{5, 7\}$ $\{0, 1, 3, 5, 7\}$ $\{0, 1, 3, 5, 7\}$ $\{0, 1, 3, 5, 7\}$

TABLE 1

 Given probabilities :

$$\Pr(X_{Th} = 5, X_H = 0, X_T = 0, X_O = 0) = \left(\frac{1}{2} \times \frac{1}{5} \times \frac{1}{5} \times \frac{1}{5}\right) \tag{1}$$

$$\Pr(X_{Th} = 5, X_O = 0) = \left(\frac{1}{2} \times 1 \times 1 \times \frac{1}{5} - \Pr(X_{Th} = 5, X_H = 0, X_T = 0, X_O = 0)\right) \tag{2}$$

$$\Pr(X_{Th} = 5, X_O = 0) = \left(\frac{1}{2} \times 1 \times 1 \times \frac{1}{5} - \frac{1}{2} \times \frac{1}{5} \times \frac{1}{5} \times \frac{1}{5}\right) \tag{3}$$

$$\Pr(X_{Th} = 5, X_O = 5) = \left(\frac{1}{2} \times 1 \times 1 \times \frac{1}{5}\right) \tag{4}$$

$$\Pr(X_{Th} = 7, X_O = 0) = \left(\frac{1}{2} \times \frac{1}{5}\right) \tag{5}$$

$$\Pr(X_{Th} = 7, X_O = 5) = \left(\frac{1}{2} \times \frac{1}{5}\right) \tag{6}$$

Desired probability :

$$\Pr(A) = \Pr(X_{Th} = 5, X_O = 0) + \Pr(X_{Th} = 5, X_O = 5) + \Pr(X_{Th} = 7, X_O = 0) + \Pr(X_{Th} = 7, X_O = \tag{7}$$

$$= \frac{24}{250} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} \tag{8}$$

$$= \frac{99}{250} \tag{9}$$

2) Digits are not repeated

$$X_{Th} \implies 2 \text{ choices}$$
$$X_O \implies 4 \text{ choices}$$
$$X_H \implies 3 \text{ choices}$$
$$X_T \implies 2 \text{ choices}$$

'B' = Event representing all the 4-digits numbers greater than 5,000 divisible by 5 when digits are not repeated.
Given probabilities :

$$\Pr(X_{Th} = 5, X_O = 0) = \left(\frac{1}{2} \times 1 \times 1 \times \frac{1}{4}\right) \tag{10}$$

$$\Pr(X_{Th} = 7, X_O = 0) = \left(\frac{1}{2} \times 1 \times 1 \times \frac{1}{4}\right) \tag{11}$$

$$\Pr(X_{Th} = 7, X_O = 5) = \left(\frac{1}{2} \times 1 \times 1 \times \frac{1}{4}\right) \tag{12}$$

Desired probability :

$$\Pr\left(B\right) = \Pr\left(X_{Th} = 5, X_O = 0\right) + \Pr\left(X_{Th} = 7, X_O = 0\right) + \Pr\left(X_{Th} = 7, X_O = 5\right) \quad (13)$$

$$= \frac{1}{8} + \frac{1}{8} + \frac{1}{8} \quad (14)$$

$$= \frac{3}{8} \quad (15)$$