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Subject :- DSP

DSP Assignment - 2

Q1. —

=> Let $N = 4$, $\omega_0 = 50000\pi$ as $f_0 = 25KHz$

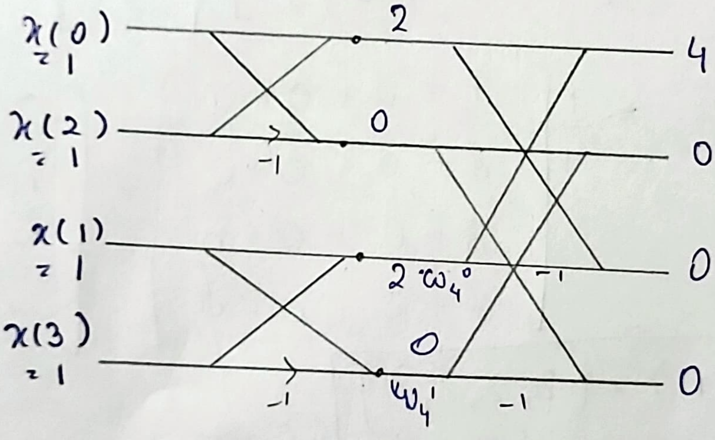
$x(n) = \{1, 1, 1, 1\}$

and $h(n) = \{1, 0, 1, 0\}$

$W_4^0 = 1$

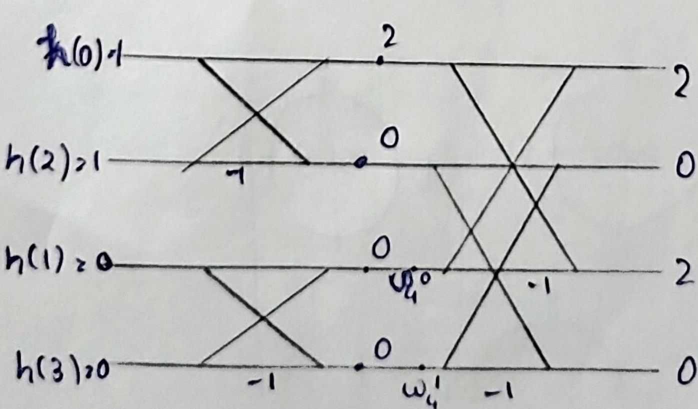
$W_4^1 = 1$

for $X(K)$:-



$$\boxed{X(K) = \{4, 0, 0, 0\}}$$

for $H(K)$



$$\boxed{H(K) = \{\underset{\uparrow}{2}, 0, 2, 0\}}$$

b) Product of H(K) and X(K)

$$Y(K) = H(K) \times X(K)$$

$$\boxed{Y(K) = \{8, 0, 0, 0\}}$$

c) y(n) → inverse DFT of Y(k)

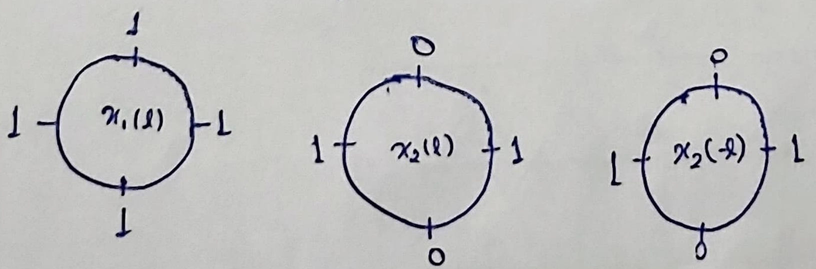
$$\begin{bmatrix} y_1(0) \\ y_1(1) \\ y_1(2) \\ y_1(3) \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix} \begin{bmatrix} 8 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$= \frac{1}{4} \begin{bmatrix} 8+0+0+0 \\ 8+0+0+0 \\ 8+0+0+0 \\ 8+0+0+0 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 2 \\ 2 \end{bmatrix}$$

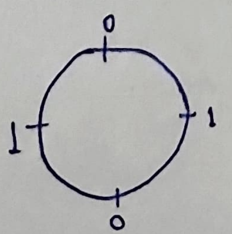
$$\boxed{\therefore y_1(n) = \{2, 2, 2, 2\}}$$

d) Circular convolution of $x(n)$ and $h(n)$

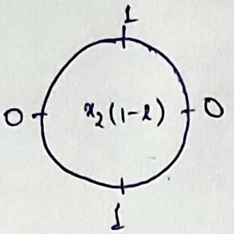
Let $x(n) = x_1(n)$, $h(n) = x_2(n)$



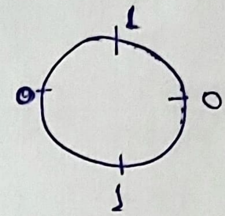
$\sum x_1(l)\, x_2(-l) =$



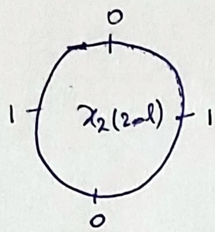
$= 2$



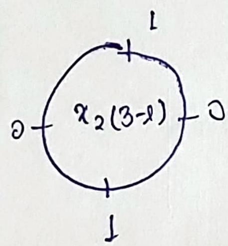
$\rightarrow \sum x_1(l)\, x_2(1-l) =$



$= 2$



$\longrightarrow \sum x_1(l)\, x_2(2-l) = 2$



$\rightarrow \sum x_1(l)\, x_2(3-l) = 2$

$\therefore\ y_2(n) \Rightarrow \{$ output $= x_1(n) \circledN x_2(n)$

$y_2(n) = \{2, 2, 2, 2\}$

$\therefore\ y_1(n) = y_2(n)$

Q2. –

a.) Linear Convolution of $x(n)$ & $h(n)$.

2) Let $N = 4$, $a_0 = a_1 = a_2 = 0.25$

$$x(n) = 0.25 - 0.25\cos\left(\frac{2n\pi}{4}\right) + 0.25\cos\left(\frac{4n\pi}{4}\right)$$

$$\therefore\ x(n) = \{\underset{\uparrow}{0.25}, 0, 0.75, 0\}$$

Now, $h(n) = 1 - \left|\frac{n - 3/2}{4}\right|$

$$\boxed{\therefore\ h(n) = \{0.625, 0.875, 0.875, 0.625\}}$$

Now we need to perform linear convolution,

		0.625	0.825	0.875	0.625
	0.25	0.15625	0.21875	0.21875	0.15625
	0	0	0	0	0
$x(n)$	0.75	0.46875	0.65265	0.65265	0.468
	0	0	0	0	0

$y_1[n] = \{0.15625, 0.21875, 0.6875, 0.8125, 0.6562,$
$0.4688, 0\}$

b) Circular convolution to determine linear convolution

$N = N_1 + N_2 - 1 = 4 + 4 - 1 = 7$

$\therefore \hat{x}(n) = \{0.25, 0, 0.75, 0, 0, 0, 0\}$

$\hat{h}(n) = \{0.625, 0.825, 0.875, 0.625, 0, 0, 0\}$

We will solve this using matrix method,

$x(1-n)$

$$
\begin{array}{l}
x(-n) \\ x(1-n) \\ x(2-n) \\ x(3-n) \\ x(4-n) \\ x(5-n) \\ x(6-n)
\end{array}
\begin{bmatrix}
0.25 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0.25 & 0 & 0 & 0 & 0 & 0.25 \\
0.25 & 0 & 0.25 & 0 & 0 & 0 & 0 \\
0 & 0.25 & 0 & 0.25 & 0 & 0 & 0 \\
0 & 0 & 0.25 & 0 & 0.25 & 0 & 0 \\
0 & 0 & 0 & 0.25 & 0 & 0.25 & 0 \\
0 & 0 & 0 & 0 & 0.25 & 0 & 0.25
\end{bmatrix}
\begin{array}{c} h(n) \\ \begin{bmatrix} 0.625 \\ 0.825 \\ 0.875 \\ 0.625 \\ 0 \\ 0 \\ 0 \end{bmatrix} \end{array}
$$

$$
= \begin{bmatrix} 0.625 \times 0.25 \\ 0.25 \times 0.875 \\ 0.75 \times 0.625 + 0.25 \times 0.825 \\ 0.75 \times 0.825 + 0.625 \times 0.25 \\ 0.75 \times 0.825 \\ 0.75 \times 0.625 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.15625 \\ 0.21875 \\ 0.6875 \\ 0.8125 \\ 0.6562 \\ 0.4688 \\ 0 \end{bmatrix}
$$

$\therefore y_2(n) = \{0.15625, 0.21875, 0.6875, 0.8125, 0.6562, 0.4688, 0\}$

Hence both $y_1(n)$ & $y_2(n)$ match //

Q3. –

a) $N = 7$

$K_0 = 1, \; K_1 = 2, \; K_2 = 3$

$M = 3$

$L = N - M + 1 = 5$

$$\therefore x(n) = \frac{\cos\left(\frac{2\pi n}{7}\right)}{\cos\left(\frac{4\pi n}{7}\right) + 0.5 \sin\left(\frac{6\pi n}{7}\right)} \; ; \; 0 \le n \le 6$$

$x(n) = \{1, -111.755, 0.1722, -0.81, -6.62, 0.486, -1.418\}$

$h(n) = \{0.0001, 0.0183, 0.3679\}$

$= \exp\{-(n-3)^2\}$

number of segments $= \left\lceil \frac{N+M-2}{L} \right\rceil + 1$

$= \left\lceil \frac{7+3-1}{5} \right\rceil + 1 = 1 + 1 = 2$

Now, the blocks are :-

$x_1(n) = \{0, 0, 1, -111.75, 0.172, -0.811, -6.6235\}$

$x_2(n) = \{-0.311, -6.6235, 0.4363, -1.418, 0, 0, 0\}$

Now, DFT,

$$\therefore X_1(k) = \sum_{n=0}^{N-1} x_1(n)\, e^{\frac{-j2\pi k n}{N}}$$

$$= \{-118,\ 96.352 + 41.6j,\ -68.26 - 93.18j,\ 30.9 + 107.6j,\ 30.9 - 107.6j,\ -68.26 + 93.18j,\ 96.352 - 41.6j\}$$

$$X_2(k) = \sum_{n=0}^{N-1} x_2(n)\, e^{\frac{-j2\pi k n}{N}}$$

$$= \{-8.416,\ -3.76 + 5.368j,\ -0.614 + 5.538j,\ 5.74 + 4.597j,\ 5.74 - 4.59j,\ -0.614 - 5.538j,\ -3.76 - 5.368j\}$$

$$H(k) = \sum_{n=0}^{N-1} h(n)\, e^{\frac{-j2\pi k n}{N}}$$

$$= \{0.386,\ -0.07 - 0.373j,\ -0.335 + 0.142j,\ 0.213 + 0.28j,\ 0.213 - 0.28j,\ -0.335 - 0.142j,\ -0.07 + 0.373j\}$$

$Y_1(k) = X_1(k)\, H(k)$

$Y_2(k) = X_2(k)\, H(k)$

$y_1(n) = \text{IDFT}\{Y_1(k)\}$

= ~~[illegible]~~,

$= \{-0.4197, -2.4367, 0.0001, 0.0045, -1.679, -41.1, 0.0477\}$

$y_2(n) = \text{IDFT}\{Y_2(k)\}$

$= \{-0.0001, -0.0157, -0.4196, -2.428, -0.134, -0.5219, 0\}$

$\therefore\ y(n) = y_1(n)\Big|_2^6 + y_2(n)\Big|_2^6$

$y(n) = \{0.0001, 0.0045, -1.679, -41.1, 0.0477, -0.4196, -2.48, 0.1345, -0.52, 0\}$

$x(l-n)$

b)

$$\begin{array}{c} x(-n) \\ x(1-n) \\ x(2-n) \\ x(3-n) \\ x(4-n) \\ x(5-n) \\ x(6-n) \end{array}\begin{bmatrix} 1 & -1.418 & 0.44 & -6.6 & -0.8 & 0.17 & -111.8 \\ -111.8 & 1 & -1.42 & 0.44 & -6.6 & -0.8 & 0.17 \\ 0.17 & -111.8 & 1 & -1.42 & 0.44 & -6.6 & -0.8 \\ -0.8 & 0.17 & -111.8 & 1 & -1.42 & 0.44 & -6.6 \\ -6.6 & -0.8 & 0.128 & -111.8 & 1 & -1.42 & 0.44 \\ 0.44 & -6.6 & -0.8 & 0.17 & -111.8 & 1 & -1.42 \\ -1.42 & 0.44 & -6.6 & -0.8 & 0.17 & -111.8 & 1 \end{bmatrix}$$

$$\times \underbrace{\begin{bmatrix} 0.0001 \\ 0.0183 \\ 0.3679 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}}_{h(n)} = \begin{bmatrix} 0.0001 - 1.418 \times 0.0183 + 0.3679 \times 0.44 \\ 0.1118 + 0.0183 - 1.42 \times 0.3629 \\ 0.17 \times 0.0001 + 0.0183 \times (-111.8) + 0.3629 \\ -0.8 \times 0.0001 + 0.17 \times 0.0183 + 0.3629 \times (-111.8) \\ -6.6 \times 0.0001 - 0.8 \times 0.183 + 0.3679 \times 0.17 \\ 0.44 \times 0.0001 + 0.0183 \times (-6.6) - 0.8 \times 0.3629 \\ -0.000142 + 0.44 \times 0.0183 - 66 \times 0.3629 \end{bmatrix}$$

$$= \begin{bmatrix} 0.0001 \\ 0.0045 \\ -1.679 \\ -41.1 \\ 0.0477 \\ -0.4196 \\ -2.429 \\ 0.1345 \end{bmatrix} = y(n)$$

$\therefore y(n) = \{0.0001, 0.0045, -1.679, -41.1, 0.0477, -0.4196, -2.48, 0.13, -0.5129\}$

$\therefore$ hence, the results from linear convolution and overlap method coincide.