Quantum Computing Group Open Project-2023

Building a basic Quantum Adder using Toffoli Gates

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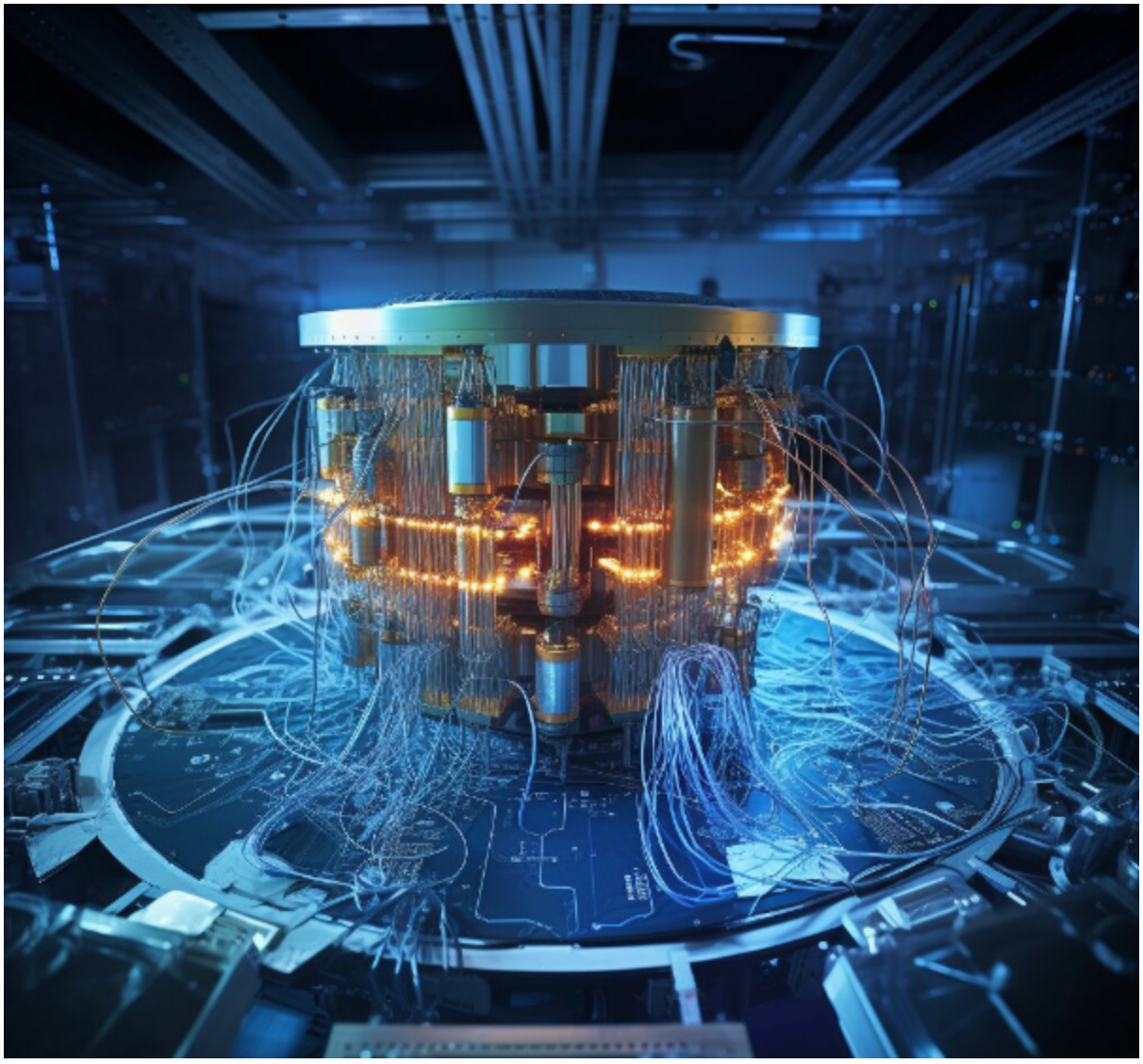


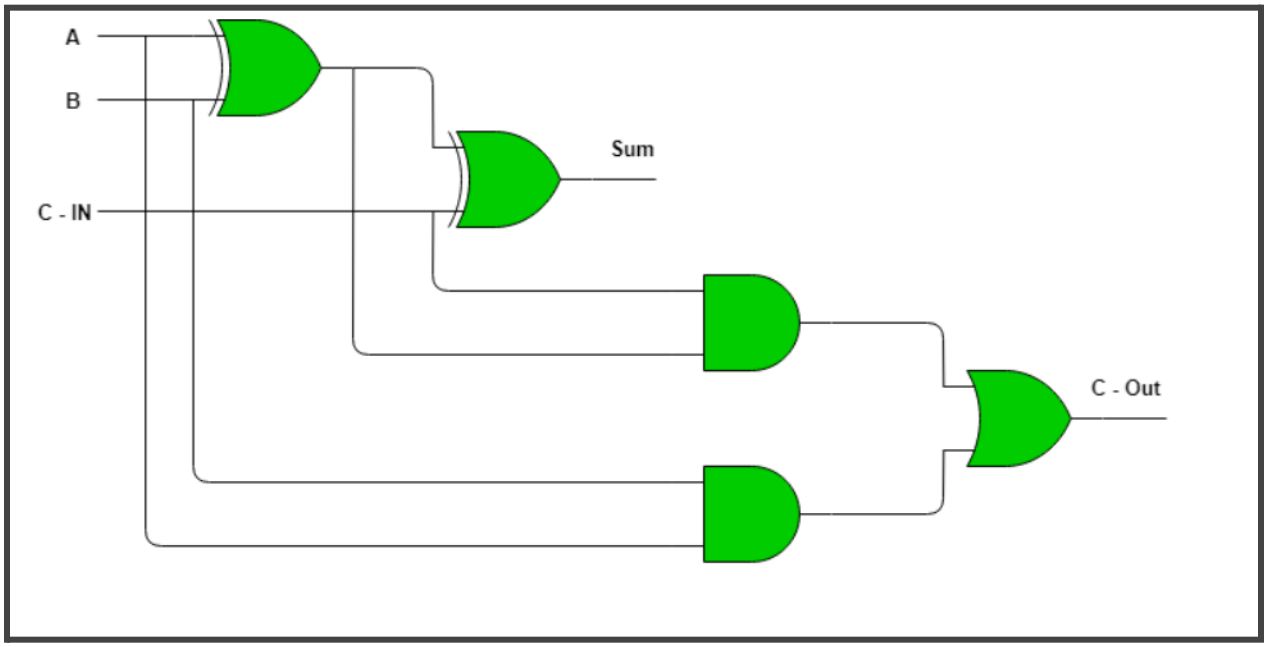
Problem Statement

Alice wishes to perform the addition of two 4-bit numbers using Quantum Circuits. Let those two numbers be a and b. In classical digital electronics, this is achieved using the full-adder circuit, which inputs the bits a_i,b_i and carry bit C_(i-1) and returns the sum bit S_i and carry bit C_i. Your task is to help Alice design a fully functional Quantum Adder using just CNOT and Toffoli Gates.

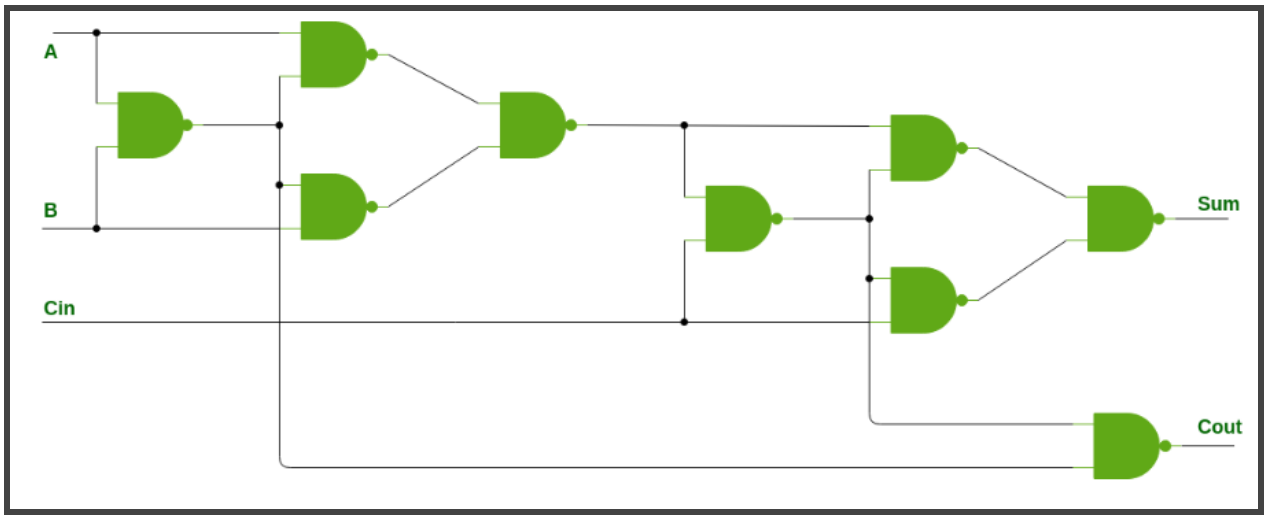
Technical Methodology Used

Classical Digital Electronics Way

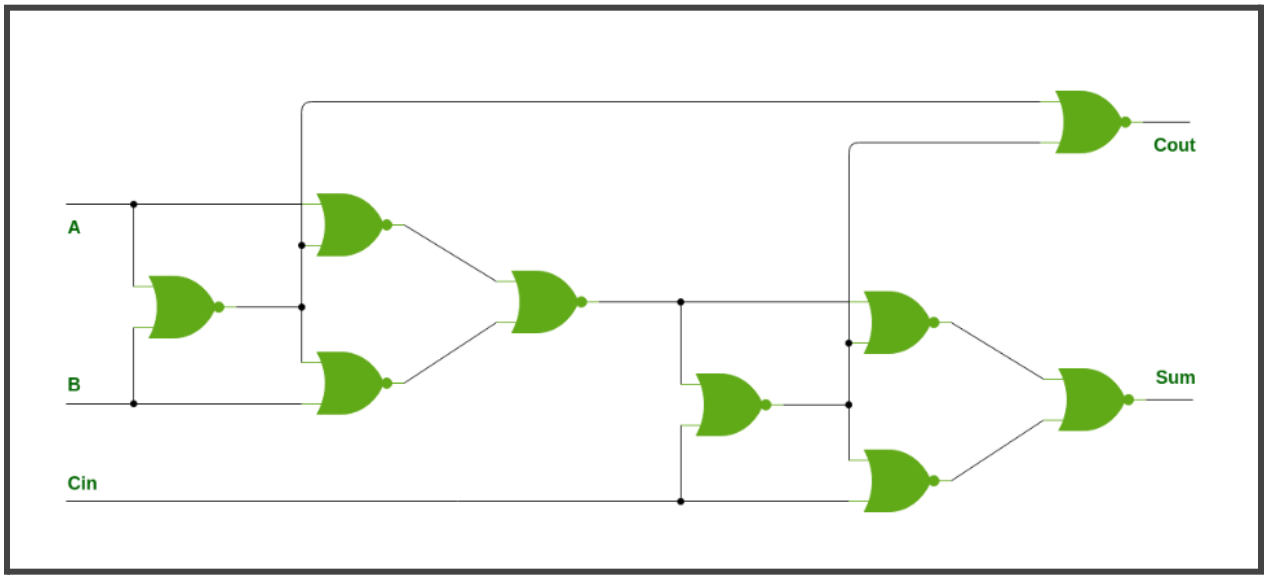
The classical way involves using XOR, OR and AND gates.



An another way consists of using only NAND gates



NOR gates can also be used for the same

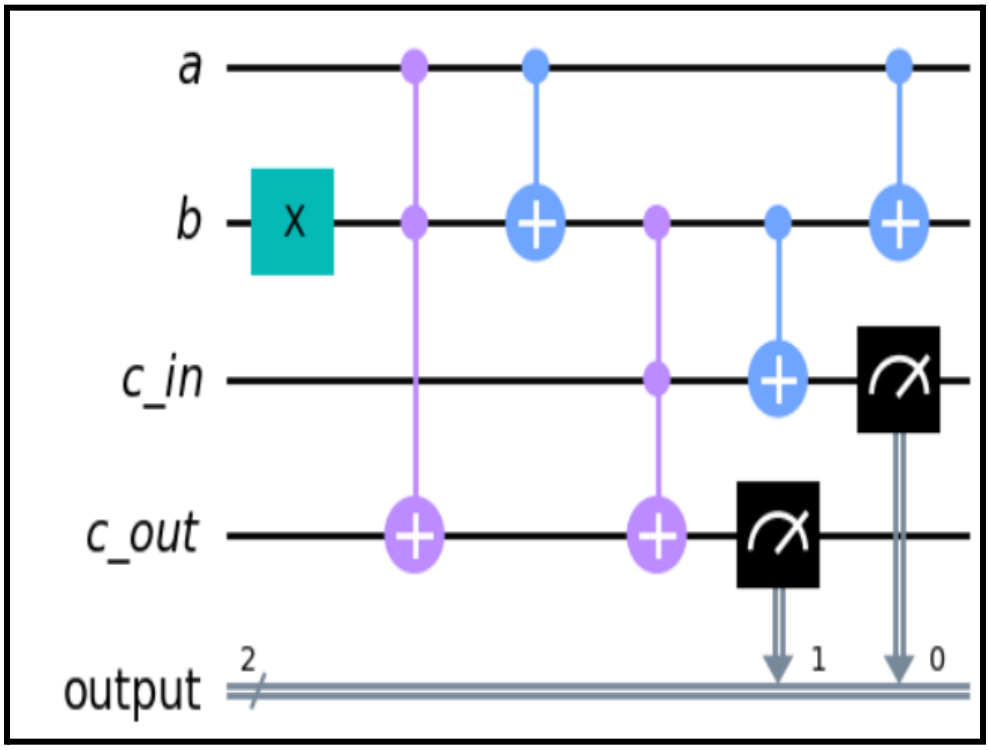


Quantum Circuit Method

For replicating the full adder behavior, quantum gates are used. For input, X and H gates were mainly used. For the logic and processing , Controlled-Not (CNOT) and Toffoli gates

are used. CNOT works quite similar to XOR logic gate with the difference that the first bit is used to decide whether to reverse the second bit or not. CCNOT works similar to CNOT but there are two control bits and can be used to replicate the behavior of AND logic.

To make the 4 bit adder, first we made a single bit adder using CNOT and CCNOT gates.



Then we combine these single bit adder circuits to create a 4 bit adder circuit based on 4 bits of A and B working as the principle input , 4 carry in bits for interim calculations and outputs consisting of Carry Out BIT and 4 sum bits all of which are made using classical registers.

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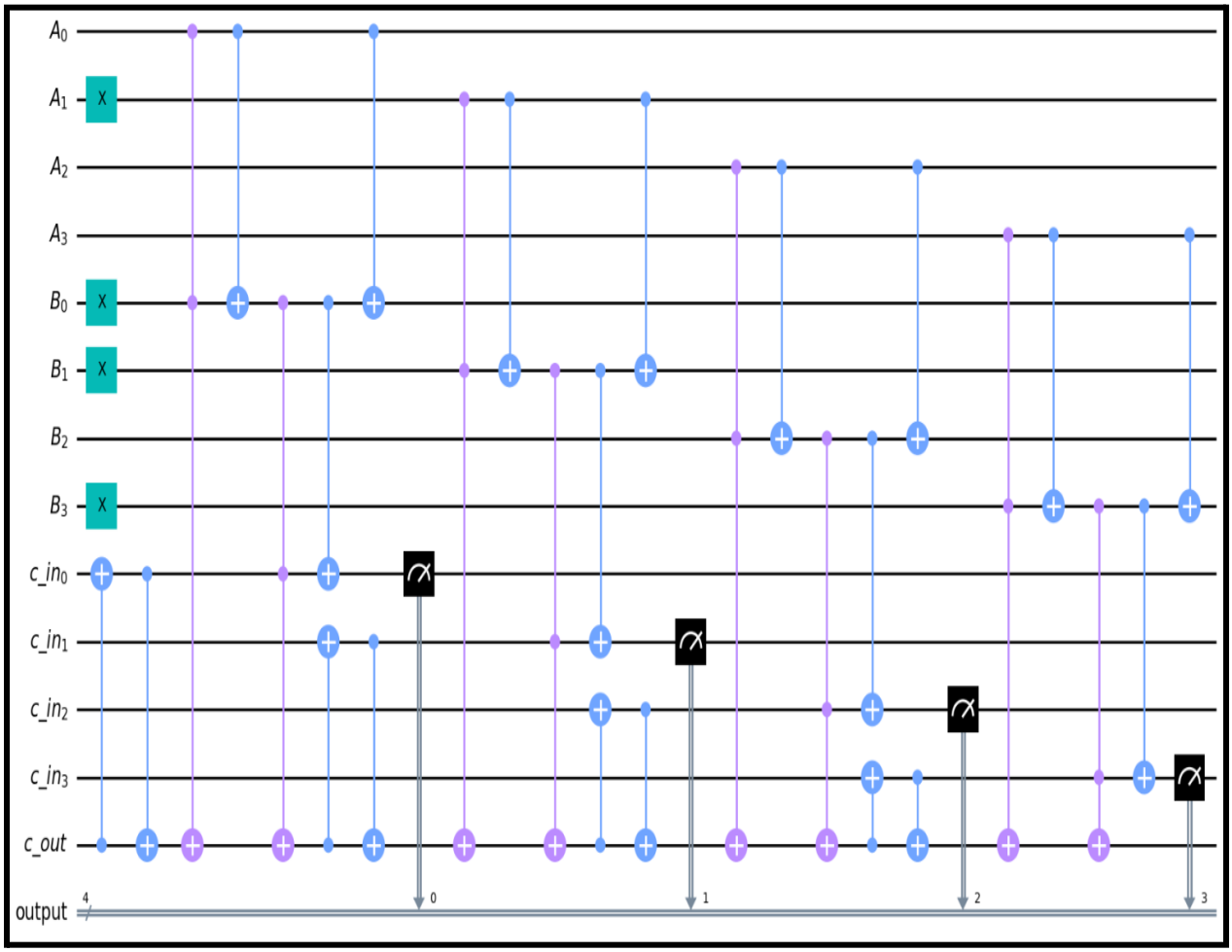

A0
A1
A2
A3
B0
B1
B2
B3
c_in0
c_in1
c_in2
c_in3
c_out
output
4
0
1
2
3

Testing and Simulations

Simulations were done using QASM simulator. QASM Simulator is a simulator provided by Qiskit that runs the Quantum Circuit and measures its results. It is useful when the Quantum Circuit contains measurement instructions for Qubits into bits. The simulations were ran for 2048 shots.4 test cases were ran as provided in the problem statement:-

- $|a\rangle = |0010\rangle \quad , \quad |b\rangle = |1011\rangle$
- $|a\rangle = |0001\rangle \quad , \quad |b\rangle = |0011\rangle$
- $|a\rangle = \frac{|0010\rangle + |0100\rangle}{\sqrt{2}}, \; |b\rangle = \frac{|1011\rangle + |0001\rangle}{\sqrt{2}}$
- $|a\rangle = \frac{|0000\rangle + |0111\rangle}{\sqrt{2}}, \; |b\rangle = \frac{|0111\rangle + |1000\rangle}{\sqrt{2}}$

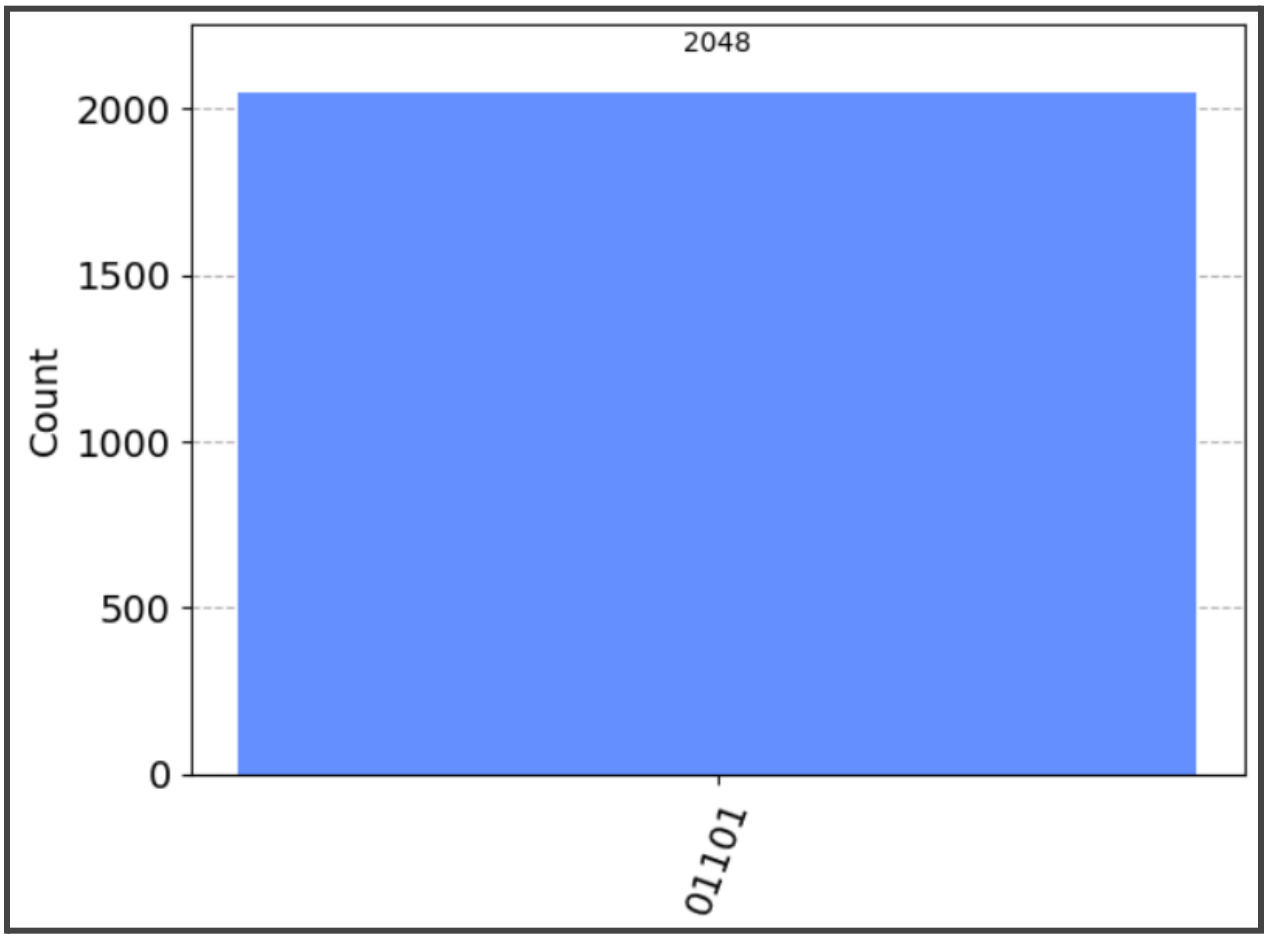
For example:-

Test Case 1 consists of:-

A: ‘0010’

B: ‘1011’

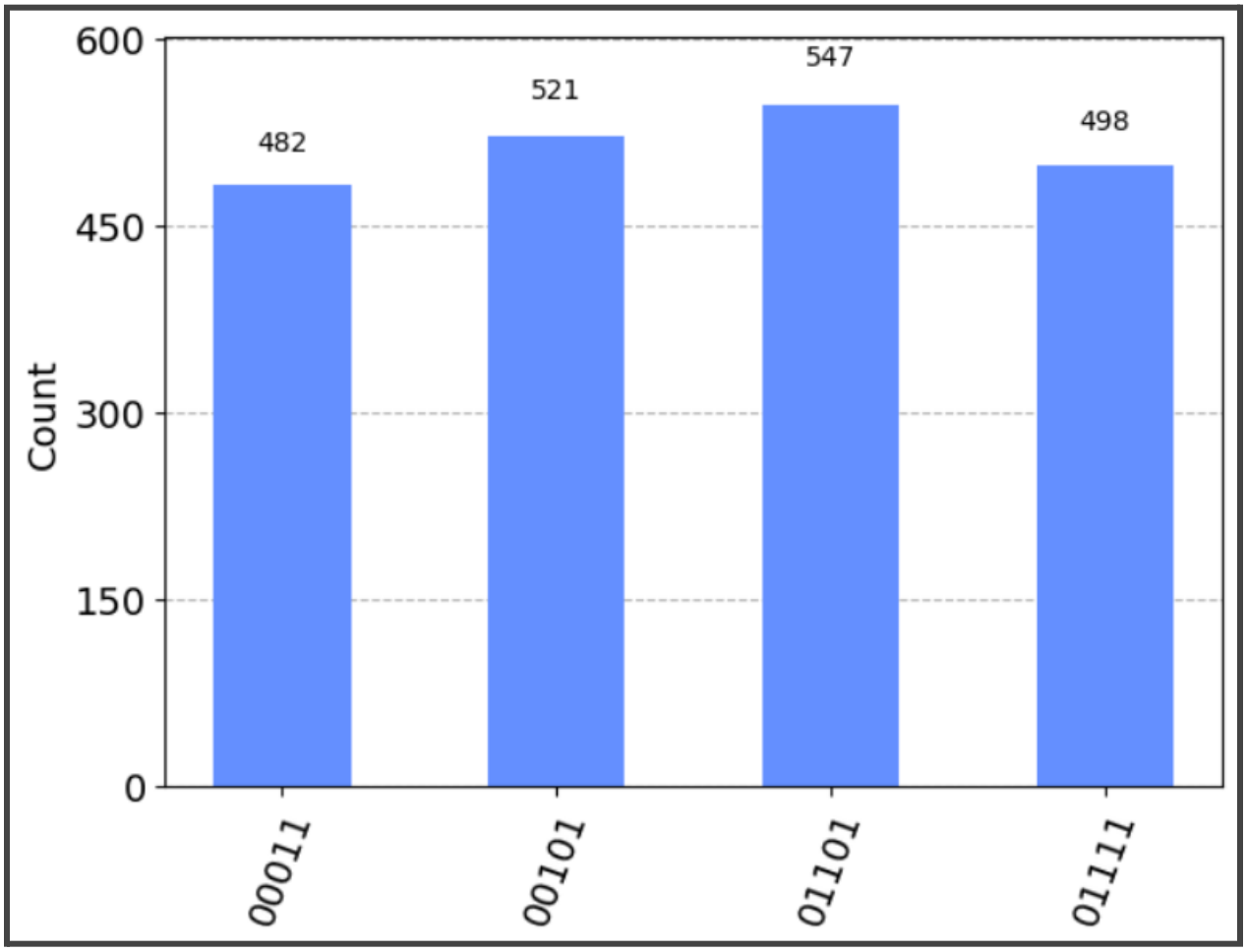
The simulations resulted into :-



Let us check simulations of some other test cases with transposed states. Let's take test case 3 which consists of :-

$$\circ\ |a\rangle = \frac{|0010\rangle + |0100\rangle}{\sqrt{2}},\ |b\rangle = \frac{|1011\rangle + |0001\rangle}{\sqrt{2}}$$

The simulations resulted into :-



Conclusion

The 4 bit adder was successfully made using toffoli gates and controlled not gates which were then tested on test cases through QASM simulators where they gave satisfactory results which were then displayed on a histogram.