Indian Institute of Technology Kharagpur
Department of Mathematics
MA11003 - Advanced Calculus
Tutorial Sheet - 6
Autumn 2025

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1. Expand $f(x,y) = e^{(2x+xy+y^2)}$ in powers of x and y upto second order term.

2. Expand $f(x,y) = \sin(xy)$ in powers of $(x-1)$ and $(y-\pi/2)$ up to second degree term, and then find the remainder term.

3. Expand $f(x,y) = e^y \sin x$ in Taylor's series upto second order term about $(\frac{\pi}{2}, 1)$. Also estimate the value of $f(x,y) = e^y \sin x$ when $x = \frac{51}{100}\pi,\ y = 0.99$.

4. Expand $f(x,y,z) = e^z \sin(x+y)$ in Taylor's series upto second order term about the point $(0,0,0)$.

5. Expand $f(x,y) = x^2y + \sin y + e^x$ in powers of $(x-1)$ and $(y-\pi)$ upto second order terms using taylor's theorem and find the remainder term.

6. Show that
$\sin x \sin y = xy - \dfrac{1}{6}[(x^3+3xy^2)\cos(\theta x)\sin(\theta y) + (y^3+3x^2y)\sin(\theta x)\cos(\theta y)]$, where $0 < \theta < 1$.

7. Classify the local extremum of the following functions:

   (a) $f(x,y) = x^2y - 2xy^2 + 3xy + 4$.

   (b) $f(x,y) = 2(x-y)^2 - x^4 - y^4$.

   (c) $f(x,y) = x^3 - 12x + y^3 + 3y^2 - 9y$.

   (d) $f(x,y) = x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$.

   (e) $f(x,y) = x^2y - xy^2 + 4xy - 4x^2 - 4y^2$.

8. Verify that $x^3y^2(1-x-y)$ has a maximum at $(\frac{1}{2}, \frac{1}{3})$.

9. Find the absolute maximum and minimum values of $f(x,y) = 4x^2+9y^2-8x-12y+4$ over thr rectangle in the first quadrant bounded by the lines $x = 2, y = 3$ and the co-ordinate axes.

10. Find the global extremum of $f(x,y) = x^2 + xy + y^2$ over the circular region $R = \{(x,y)/\ x^2 + y^2 \leq 1\}$.

11. Find the absolute maximum and minimum value of the function $f(x, y) = 3x^2 + y^2 - x$ over the region $2x^2 + y^2 \leq 1$.

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