**BASICS OF CALCULUS**

**1) Sets**

Sets are an unorganised collection (of often related) data. It is represented in braces ‘{}’.

Eg:

A = {1,2,3}

**2) Relation**

A relation is a set of ordered pairs, indicating the connection (or relation) between two the elements of two or more sets. They are typically represented by R.

Eg:

A = {1,2,3}

B = {10, 20, 30}

Then R = {(1,10), (2,10), (3,10), (3,30)} is a relation between A and B.

A relation is defined to be a subset of AXB. Here, A is the domain and B is the co-domain.

**Domain:** Set of x-values

**Co-domain:** Set of y-values

**Range:** Set of y-values that have an image in the domain

**3) Function**

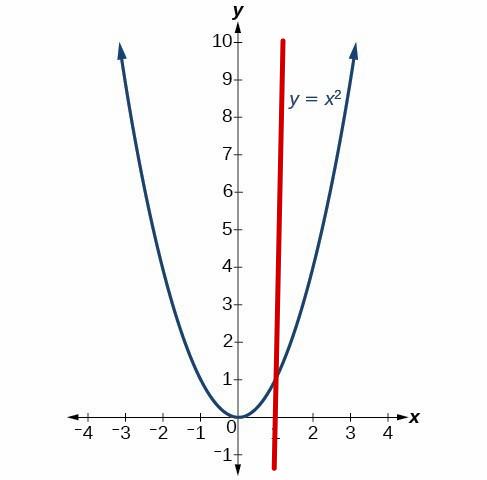
A relation becomes a function if every element in domain has a unique element in the co-domain. i.e,

1. Every x element must have an image

2. The y-value of an x element should be unique

A function **f** from a set A (domain) to a set B (codomain) is denoted as **f: A → B**, where for every element a in A, there exists a unique element b in B such that f(a) = b.

A function can be plotted graphically. All functions pass **vertical line test**.



The reason why functions are useful is because they can be thought of as machines, which takes in an element from the domain and converts it in to an element in range. They provide a way to model and analyze relationships, make predictions, solve equations, and understand the behavior and patterns in mathematical systems.It is important to note that every not every element in codomain has a pre-image in the domain. It is for this reason that we use ‘range’ instead of ‘co-domain’ for plotting graphs.

Functions can be represented in 4 formats:

* Algebraically
* Graphically
* Verbally
* Mathematically (as dataset)

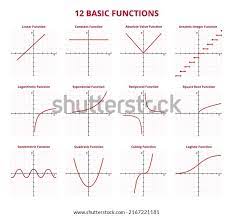
**Even and odd functions**

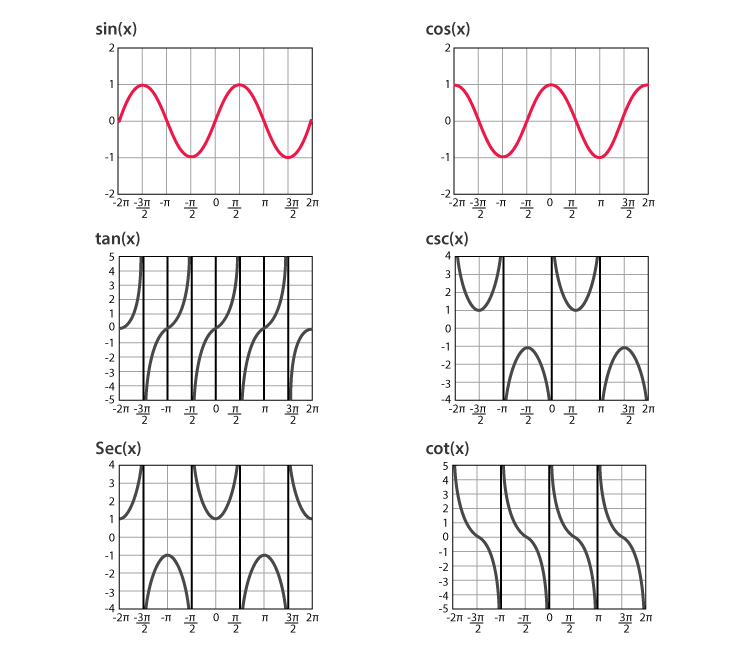
A function f(x) is called **odd** if f(-x) = -f(x). Its graph is symmetric about the origin.

A function f(x) is called **even** if f(-x) = f(x). Its graph is symmetric about the y-axis.

**Graph of a function**

The graphical method is a good way to visualize the function.





**Transformation of function**

Consider y = f(x) to be a function. Then the three main transformations on functions are:

1. **Translation**

y = f(x) + c :+ve y-axis

y = f(x) – c : -ve y-axis

y = f(x + c) : - ve x-axis

y = f(x – c) : +ve x-axis

1. **Stretching/ Shrinking**

y = c . f(x) : Stretch Vertically

y = 1/c . f(x) : ShrinkVertically When the change is made inside

y = f(c . x) : Shrink Horizontally f(x), the change appears

y = f(1/c . x) : Stretch Horizontally to misbehave

1. **Reflection**

y = - f(x) : Reflect vertically

y = f( - x) : Reflect horizontally

**Mathematical model**

A mathematical model is mathematical description of a real world phenomenon. We take in the data, and create a function that is a good enough idealization of the phenomenon. Its purpose is to help in studying and predicting the phenomenon.

**Composite function**

A function that composes another function. If f(x) and g(x) are two functions, then

**f o g = f(g(x))**

**Inverse of a function**

Inverse of a function can be thought of as a function that reverses what the original function does.i.e, **f-1(f(x)) = x**

* Also,If y = f-(x), then f-1(y) = x
* If a function has to have an inverse, it has to be one-one.
* Graphically, existance of inverse can be checked via horizontal line method
* Domain(f(x) = Range(f-1(x)) , and vice-versa
* Inverse functions have their graphs reflected by the line y = x
* For eg: The inverse of ex is ln x

**4) Limits**

Whenever a variable x approaches a number a, for a function f(x), we represent it in terms

of limits as **Limx->af(x) = L**

**Formally,** L is said to be the limit of a function f(x) as x approaches a, and is represented as

**Limx->a f(x) = L**

For every number **δ** > 0 there exists a number **ε** > 0

such that if **0 < | x – a | < δ** , then **0 < | f(x) – L | < ε**

Limits can be one-sided, as it approaches the graph only through one side.

**Infinite limits**

When limits approaches infinity. They cause vertical asymptotes in graph.

**Squeeze Theorem / Sandwich Theorem**

If **f(x)<= g(x) <= h(x)**  and  **Limx->a f(x) = L** and **Limx->a h(x) = L**

then **Limx->a g(x) = L**

The function in the middle is squeezed between the upper and lower.

**5) Continuity**

A function is said to be continuous on an interval, if there forms a continuous graph when forming the graph of the function. Formally, a function f(x) is said to be continuous at a point c in its domain if ,

The limit of the function as x approaches c is equal to the value of the function at c:

**Lim(x→c) f(x) = f(c).**

**Limits at infinity**

When we tend the limit of a number to ∞, some limit tends to L. They form horizontal asymptotes.

**Intermediate Value Theorem**

The IVT states that for a continuous function f(x) from interval a to b and a number N between f(a) and f(b) [f(a) ≠ f(b)], there exists a value c between a and b such that f(c) = N.

Differentiability