Utilizing Differentiated Kinematic Sensors to Assess the Accuracy of a Robot in an Autonomous State

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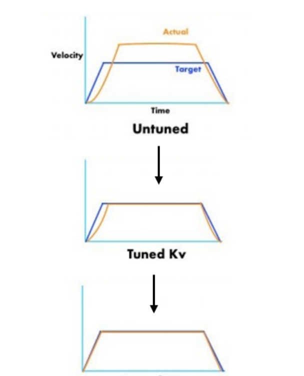
Abstract

As the field of automation, primarily kinematics, is increasing its prominence among automobiles and intelligent machines, this research paper conveys the influence of three primary kinematic sensor arrays: four wheel drive encoders, two wheel odometry and IMU Gyro, and three wheel odometry, on the accuracy of an autonomous robot to obtain its reference position. This was assessed through three primary kinematic procedures: a 360 rotational assessment, straight 152.4 cm line assessment, and a spline curve assessment, integrating both components into one primary nonlinear curvature trajectory resembling that of an "S" shape. These trajectories were pursued through two primary algorithms: feedforward projection and PID mitigation. In addition, calculating the alteration in position, or Pose, was determined through integrating differential equation models, referred to as Pose Exponentials. The data corresponded to the hypothesis, denoting the three-wheel odometry consisted of the greatest accuracy, followed by two wheel odometry and four drive encoders. This was primarily due to the ideals of traction and data reliance regarding the odometry wheels and its capability of pursuing these complex trajectories. Similarly, due to dead wheel documentation, the odometry modules consisted of the ability of course correct, indicating it retained its position regardless of external forces as a response to stimuli. Thus, this study has introduced significant insights in the field of control theory and kinematics and consists of a multitude of enhancements regarding AI and deep learning, external sensors, and algorithmic interfaces to advance automated kinematics within technological society.

***Keywords*:** Odometry, feedforward, kinematics, autonomous, encoders, PID

1 Introduction

As technological society is advancing through intelligent machines, exemplified through the automobile industry, this study attempts to maximize the accuracy of an autonomous robot through sensor integration and software enhancements. As progression occurs in automated machines, primarily through kinematics and motion profiling, this study assesses the kinematic foundations of these advanced robotics intelligent machines to ensure accuracy and safety in addition to enhancing the capabilities of this promising field. The primary **research inquiry** of this study denotes the influence of differentiated kinematic sensor on the accuracy of a robot pursuing a trajectory in an autonomous state. Furthermore, the primary objective of this study was to attain a mean absolute error, **MAE, below 3% of error** of both longitudinal and rotational errors in the spline trajectory. However, prior to stating the components of this study, the basis and foundations of kinematics must be addressed.**Kinematics** refers to the means of movement within objects. In this study, two primary algorithms are utilized in this kinematic application: feedforward and PID.



(Paper: Implementation of Adaptive Pursuit Controller, n.d.)

Figure 1: Feedforward Tune

Feedforward is a projection algorithm that profiles the velocity and acceleration to pursue tuning in conjunction with sensor data. This permits for a highly accurate trajectory sequence. This is an open loop controller, in which a power is applied to an actuator or motor and is utilized to obtain the specified position through the tuning of the power to the velocity and acceleration variables. In addition, this method is beneficial due to its lack of time delay and sensor noise, or error, that may skew its accuracy. The tuning procedure is depicted above. Furthermore, this is a **model-based feedforward** algorithm in which it calculates the mathematical model of the system regarding the value of the input that corresponds to the effective and accurate velocity and acceleration for the respective trajectory. However, though this algorithm is accurate to a great extent, once the robot pursues the trajectory and is in its final position, it is unable to determine its position relative to the reference position. This introduces the purpose and implementation of PID. **PID** is a feedback controller in which it is a closed loop algorithm, indicating that sensors determine the error of the true position to the reference position and converts this noise into a velocity to mitigate for the given error. This further contributes to autonomous accuracy through error mitigation within trajectory sequencing and vector positioning. This error conversion into a velocity occurs through the PID controller model depicted below.

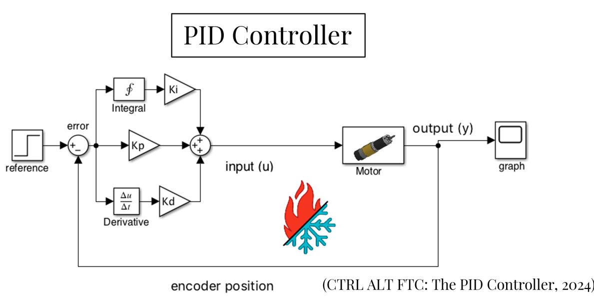


Figure 2: PID Simulink Model

PID itself refers to three variables: proportional, integral, and derivative. These variables are tuned to calculate the final conversion interval. This interval is essentially the value input applied to the associated actuators to mitigate for the given error and correct to the reference position. A Simulink model of the PID Controller is depicted to above. Kp, or the proportional term, is directly proportional to the error of the system in which it determines the speed of error mitigation. Ki, or the integral term, is directly proportional to the sum of all of errors over time to surpass non-linear disturbances, such as static friction. Kd, or the derivative term, is directly proportional to the change in error rate in which it calculates slope of error from previous update and applys it in the running loop. This is essentially a dampener as it decreases fast system response error through mitigating oscillations. However, though these PID and feedforward algorithms may be effective, the underlying calculation of positional difference must be assessed to determine the efficiency in field relative translation of the Pose2D, or position coordinates. Originally, the method regarding this calculation of position translation was referred to as Euler Integration. However, this assumes the trajectory occurring from the original to update position is linear, indicating its lack of accuracy as the concept of linearity is absent in society.

$$\begin{pmatrix} \Delta x \\ \Delta y \\ \varphi \end{pmatrix} = \begin{pmatrix} \Delta x_c(\theta_0) - \Delta x \perp sin(\theta_0) \\ \Delta x_c(\theta_0) + \Delta x \perp sin(\theta_0) \\ \varphi \end{pmatrix}$$

Figure 3: Mathematical Calculation of the Field-Relative Alteration in Position Through Euler Integration

This influenced the conversion of Euler Integration to Pose Exponentials, in which it integrates curvature trajectories in these computational algorithms through the utilization of differential equations as a method of modification to account for these complex and prominent trajectories, contributing to further accuracy within the internal kinematic profiling. This aids in the accuracy of determining robot and vector position through integrating these prevalent curvature trajectories as opposed to linear motions.

$$\begin{pmatrix} \Delta x \\ \Delta y \\ \varphi \end{pmatrix} = \begin{pmatrix} \cos\theta_0 & -\sin(\theta_0) & 0 \\ \sin\theta_0 & \cos\theta_0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{\sin\varphi}{\varphi} & \frac{\cos\varphi - 1}{\varphi} & 0 \\ \frac{1-\cos\varphi}{\varphi} & \frac{\sin\varphi}{\varphi} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \Delta x_c \\ \Delta x_\perp \\ \varphi \end{pmatrix}$$

Figure 4: Mathematical Calculation of the Field-Relative Alteration in Position Through Pose Exponentials

These components of feedforward, PID, and Pose Exponentials may be implemented in a custom algorithm. However, this may be further advanced by odometry. **Odometry** refers to a method of localization through utilizing the data of given sensors. The primary sensors implemented in this research are drive encoders, two-wheel odometry, and three-wheel odometry. Drive encoders refer to the documentation of ticks regarding the kinematic motor. Though this may appear feasible and effective, the probable slippage and lack of accuracy is believed to inhibit its localization ability. Furthermore, this is sensors is not a specified odometry sensors. Two and three-wheel odometry refers to the utilization of external sensors, consisting of wheels in conjunction with an encoder in a parallel and perpendicular position that is not powered. These entities are referred to as dead wheels in which they lack power though consist of internal encoders that track the rotational ticks. This is utilized in conjunction with the robot velocity and projection profiling to increase autonomous accuracy.

2 Materials & Procedures

There were a multitude of critical components within this study. These were primarily the subject of testing in which it was the robot, sensors regarding the drive encoders, two wheel odometry with the IMU gyro, and three wheel odometry, and a software interface to implement these algorithms and assess accuracy. Within robot kinematics, there are three primary components of movement: longitudinal, rotational, and curvature trajectories. Thus, within this study, three procedures were pursued: A rotational assessment in which the robot pursues a 360^0 rotation and the rotational discrepancy is calculated to the nearest 0.001 degree, a straight line assessment in which the robot pursues a 152.4 cm straight line and the longitudinal discrepancy is calculated to the nearest 0.001 centimeter, and a spline trajectory assessment in which a robot pursues a spline curve, resembling that of an "S," consisting a 76.2 cm longitudinal extent in addition to rotation throughout the path. The rotational procedure assesses the foundational rotational accuracy of the robot. Similarly, the longitudinal procedure assesses the linear accuracy of the robot. Furthermore, the spline procedure integrates both rotational and longitudinal components into a singular, non-linear curvature trajectory to assess its true accuracy through societal application in addition sensor compatibility and capacity. Each procedure was repeated three times and the mean absolute error, MAE, was calculated to increase validity. Critical **constants** within study was the robot, location of odometry modules on the robot, testing environment, and similar ideals. Through extensive evaluation, the **hypothesis** was determined: if three-wheel odometry is implemented, then the accuracy of the robot will be the greatest due to the individual dead wheel documentation assessing critical data to contribute to an advanced feedforward and PID algorithm in addition to its sole reliance on data as opposed to the two wheel odometry due to the assumptions of the IMU Gyro in addition to the drive encoders due to the lack of traction and greater error susceptibility.

3 Data Results

The data obtained through this study validated the hypothesis in which the three wheel odometry would be the most efficient, followed by two wheel odometry & IMU Gyro, and the four wheel drive encoders. A compiled data of the mean absolute error, MAE, is depicted below for the individual sensor arrays.

Table 1: Mean Absolute Error of Sensor Arrays

Sensor & Procedure:	**Rotation**	**Linear**	**Spline**
Robot Encoders	2.252°	11.533 cm	1.579°& 11.125 cm
Two Wheel Odometry & IMU Gyro	1.283°	3.877 cm	1.33°& 2.301 cm
Three Wheel Odometry	1.098°	2.367 cm	1.023°& 2.119 cm

Table 2: Mean Absolute Error Percentage of Sensor Arrays

Sensor & Procedure:	**Rotation**	**Linear**	**Spline °& cm**
Robot Encoders	0.625%	7.567%	0.439% & 14.5%
Two Wheel Odometry & IMU Gyro	0.356%	2.544%	0.369% & 3.02%
Three Wheel Odometry	0.305%	1.533%	0.284% & 2.781%

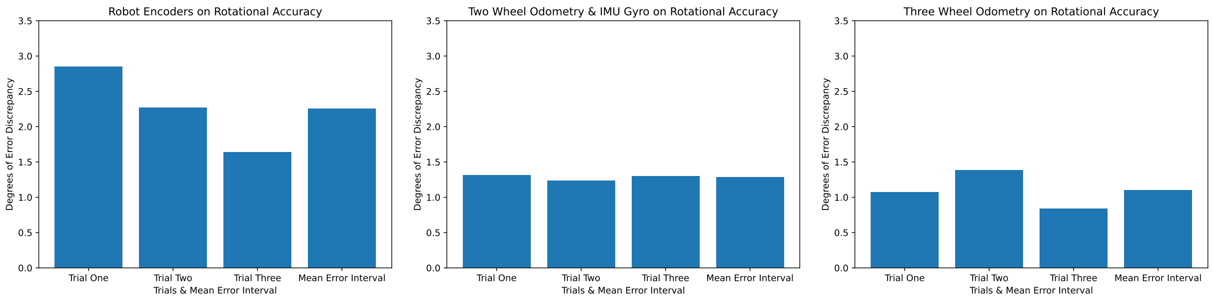


Figure 5: Rotational Accuracy Among Sensor Arrays

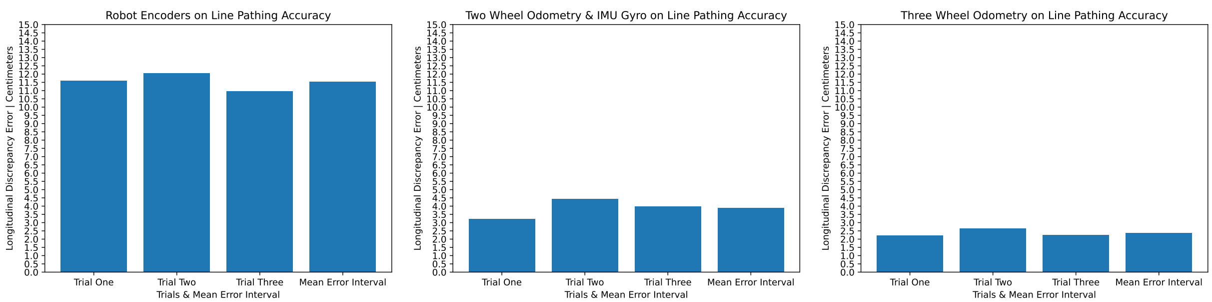


Figure 6: Linear & Longitudinal Accuracy Among Sensor Arrays

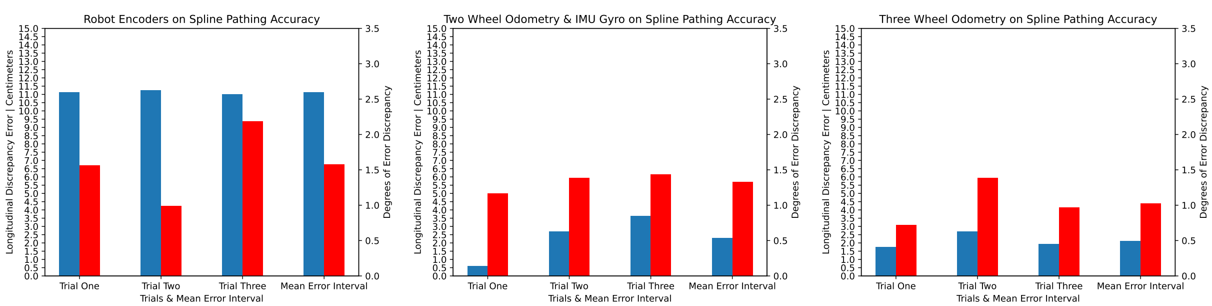


Figure 7: Spline Trajectory Accuracy Among Sensor Arrays — **Blue**: Longitudinal & **Red**: Rotational

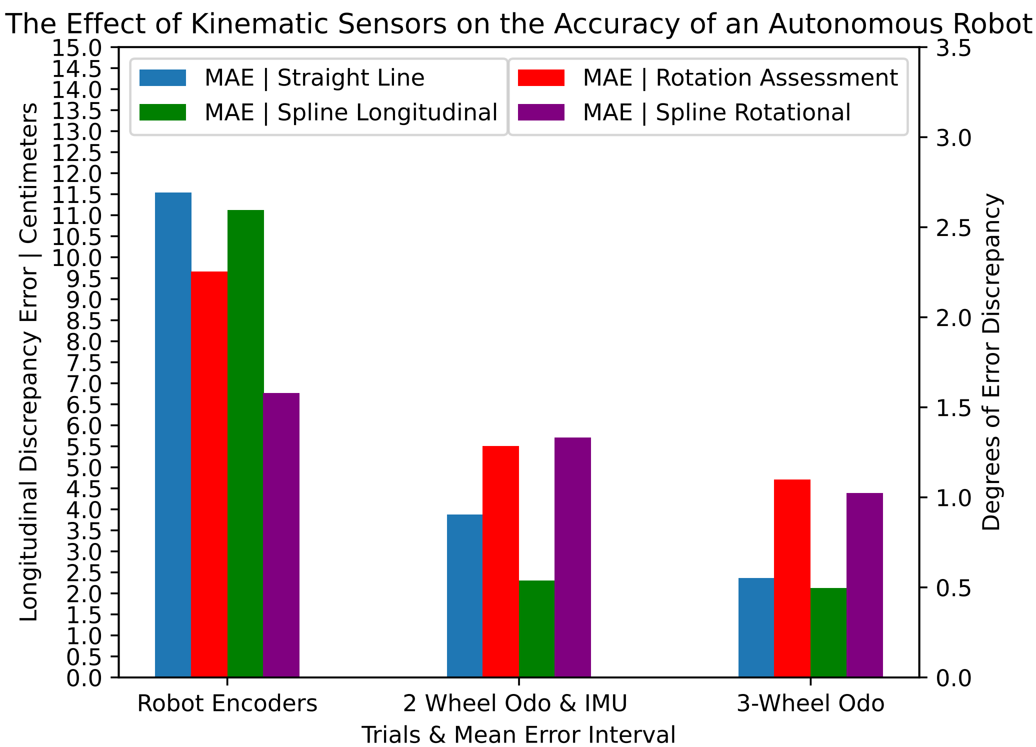


Figure 8: Compiled Data: The Effect of Kinematic Sensors on the Accuracy of an Autonomous Robot

4 Assessment & Interpretation of Data

In the **rotational assessment**, the three-wheel odometry consisted of the greatest accuracy, followed by the two-wheel odometry, and the robot encoders. However, the two-wheel odometry & IMU Gyro consisted of the greatest consistency. This may have been due to the utilization of the gyroscope within the IMU as it is specialized towards rotation. Within the **linear assessment**, the three-wheel odometry consisted of the greatest accuracy, followed by two-wheel odometry & IMU, and the robot encoders once again. This projection accuracy for the longitudinal trajectory may have been due to the three-wheel odometry consisting of two parallel odometry pods obtaining longitudinal data. This increased the data volume to increase the localization accuracy. However, the two wheel odometry consisted of the IMU Gyro as opposed to a second parallel odometry pod, proposing a rationale for its decrease in accuracy. Similarly, the robot encoders may have experienced a decrease in accuracy due to the lack of odometry wheels and reliance on the wheel encoders in which the decreased traction increased the error interval. Similarly, within the **spline assessment**, the three-wheel odometry consisted of the greatest accuracy, followed by the two-wheel odometry & IMU Gyro, and robot encoders. This dominance in the three-wheel odometry accuracy may have been due to its **data reliance** through the pure utilization of dead wheel encoders as opposed to the two-wheel odometry due to the assumptions of IMU Gyro and drive encoders due to its lack of consistency through decreased **traction** and greater drift, substantiating and validating the hypothesis. Furthermore, due to the presence of individual dead wheels in odometry modules, the robot consisted of the ability to pursue **course correct.** As these odometry pods track the data movement of the robot, it was found that the robot was able to experience an external force that altered its position and correct for it through the PID to revert to its original position and trajectory. This indicates that these sensors consist of the capability to respond to external stimuli autonomously, contributing to its societal application. However, this was not present within the robot encoders due to the lack of odometry pods that may account for these forces. This course correct may be observed in Course Correct 1 and Course Correct 2 of the robot in a spline with the three wheel odometry. In addition, through observing the spline path, it was observed that the drive encoders consisted of greater individual **linear trajectories** as opposed to the **universal curvature**

trajectory of the three-wheel odometry. This indicates that, within the spline curve, the drive encoders were unable to pursue the critical trajectory to its position, further signifying that it lacked the capacity to render and pursue these complex trajectories. This may be observed in the Robot Encoder Spline and the Three Wheel Odometry Spline. In addition, one may believe these trajectories in the methodology consist of limitations due to the decreased length or metric extent of the procedures, in which all endeavors did not surpass five feet within the trajectory. This is observed through the **Second Law of Thermodynamics** in which it denotes entropy or disorder, represented as error in this study, increases as time progresses. This indicates the error obtained would have continued to increase at a constant interval in correspondence to the length of the trajectory and duration of movement. Within the drive encoders, this was present due to the greater drift and lack of course correct. However, due to the accuracy of these odometry modules in conjunction with the constant PID and course correction, this error would not only be in a constant, but may experience mitigation and regression to contribute to its universal accuracy. This mitigates the impositions of this law to a minuscule extent in which it may lack a significant influence on the error of the system.

5 Conclusion

Through extensive research and interpretation, the data and its assessment corresponded to the hypothesis in which the **three-wheel odometry was the most efficient**, due to its immense data reliance and societal application, followed by two-wheel odometry & IMU Gyro, due to its relatively less data reliance though consistency in rotational error, and the robot drive encoders, due to its lack of traction regarding the drive wheels to introduce drift and complications that were substantiated through the complex spline, indicating its lacked the capacity to project these curvature trajectories. This has introduced substantial insights to the field of kinematics through the utilization of feedforward projection and PID in conjunction with these kinematic sensors. In addition, due to the ability of odometry modules to pursue **course correct** and mitigate the impositions of the second law of thermodynamics, trajectories can significantly increase their accuracy regardless of external forces to further simulate ideals within society. In addition, the three-wheel odometry exemplified and surpassed the engineering and scientific objective of this research study, obtaining a rotational MAE of 0.284% and longitudinal MAE of 2.781% . Furthermore, the two wheel odometry and IMU gyro obtained a rotational MAE of 0.369% and longitudinal MAE of 3.02%, signifying its minuscule discrepancy to the research objective of a rotational and longitudinal MAE below 3%. Though this study has introduced a multitude of insights regarding the accuracy and application of kinematics in addition to control theory within robotic intelligent machines, there may have been apparent limitations and a plethora of corresponding advancements to enhance these robotic interfaces. The **feedforward and PID algorithms** consisted of robustness though these were relatively simple algorithms and may be enhanced through optimizing these algorithms in addition to integrating greater algorithms and software advancements. This may be imposed through utilizing the Kalman filter as a noise disturbance mitigation method and similar protocols to further contribute to localization accuracy. In addition, these algorithms consisted of **manual tuning procedures** due to its specificity in conjunction with the robot's mass, velocity, and acceleration. This introduces susceptibility to human error within the sensor accuracy and interpretation in which tuning may continue to occur further to contribute to the greater accuracy, proposing a limitation within the findings of this study. Furthermore, the electrical interfaces in which these algorithms and kinematic libraries were implemented to that correspond to the actuators of the robot were referred to as the **Control & Expansion Hub**. However, as these hubs or motherboards were specialized for a less robust purpose, these advanced algorithms and the mass of software implementation may have induced mechanical complications due to the capacity of these hubs in addition to the battery voltage. This may propose a complication in implementing the more advanced algorithms stated previously. Though the accuracy of this robot in an autonomous state proposes significant insights, it was unable to respond to various stimuli, necessitating the **integration of AI deep learning** within these interfaces to promote and enhance scientific society. This will contribute to greater industrial and societal application as observed within autonomous vehicles, biotechnological components, and similar purposes. Within this study, the independent variables were a specified sensor array. However, through assessing their individual abilities, these sensors arrays may be altered to ensure greater accuracy through **sensor integration**. This may be applied through utilizing the IMU Gyro with the three-wheel odometry to increase rotational accuracy and consistency in conjunction with the longitudinal accuracy of the three-wheel odometry. In addition, as observed through course correct, external forces upon the robot may be mitigated and accounted for to retain the position and trajectory of the robot. However, the robot must experience physical force for this to occur, indicating it is susceptible mechanical and electrical complications. Thus, **utilizing external sensors** regarding ultrasonic or similar sensors along the exterior components of the robot may be utilized to detect external entities prior to collision and evade this obstruction to enhance this accuracy in addition to safety of the robot. Though this study has introduced significant insights, it may not be directly applied to society. Thus, a significant future advancement may be oriented around specializing these robotic systems in **societal applications and industries** regarding automated vehicles, biotechnological and medical advancements, and aeronautical rovers. In addition, this is not confined to drive kinematics. This study essentially utilized drive kinematics to represent the influence of these algorithms though they may be applied to various subsystems as well that consist of a similar utilization of sensors. This is due to these algorithms and similar methods inducing mechanical actuator movement as opposed to the wheel movement itself. Holistically, this study has introduced significant insights regarding the influence of sensors on the extent of accuracy in robot kinematics and its future enhancements may advance the field of kinematics and society itself to its greatest extent.

6 References

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