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56 EA
47.

Part - A

2.a) The poynting vector is given as:

$\bar{P} = \bar{E} \times \bar{H}$

Taking only the magnitudes

$$P = E \times H = \frac{V}{r \ln(b/a)} \times \frac{I}{2\pi r} = \frac{VI}{2\pi \ln(b/a) r^2}$$

Total power flow along the cable is

$$W = \int_s (E \times H) ds = \int_a^b \frac{VI}{2\pi \ln(b/a) r^2} \times 2\pi r dr$$

$$= \frac{VI}{\ln(b/a)} \int_a^b \frac{dr}{r} = \frac{VI}{\ln(b/a)} \times \ln(b/a) = VI$$

6.a)

$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)}$

$\gamma = \alpha + j\beta$

$\gamma = \sqrt{j(2\pi f)(\mu_0\mu_r)(\sigma + j(2\pi f)\epsilon_0\epsilon_r)}$

$\gamma = \sqrt{j(2\pi \times 10 \times 10^6)(4\pi \times 10^{-7} \times 4)(10^{-3} + j(2\pi \times 10 \times 10^6)(8.854 \times 10^{-12} \times 2.5)}$

$= \sqrt{j(315.82)(10^{-3} + j1.3907 \times 10^{-3}}$

$\gamma = 0.7355 \angle 72.14 = 0.2255 + j0.7$

$\gamma = \alpha + j\beta$

$\alpha = 0.2255$ Np/m

$\beta = 0.7$ rad/m.

wavelength

$\lambda = \frac{2\pi}{\beta} = 2\pi / 0.7$

$\lambda = 8.975$ m

velocity of propogation

$V_p = \omega / \beta$

$= 2\pi f / \beta = 2\pi \times 10^6 / 0.7 = 8.976 \times 10^7$ m/s

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$$e - \eta = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{4\pi \times 10^{-7} \times 4}{2.5 \times 8.85 \times 10^{-12}}} = \underline{\underline{476.54\,\Omega}}$$

2b:) $\nabla \times H = J_c = \frac{dD}{dt}$

since the field is time invariant,

$$\nabla \times \bar{H} = J_c$$

$$\vec{H} = (6x\sin\alpha + 2y^2\cos\beta)\,a_z$$

$$\nabla \times H = \begin{vmatrix} a_x & a_y & a_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 0 & 0 & H_z \end{vmatrix}$$

$$= \frac{dH_z}{dy} a_x - \frac{dH_z}{dx} a_y$$

$$J_c = \underline{\underline{4y\cos\beta\,\bar{a_x}}} - 6\sin\alpha\,\bar{a_y}$$

1b:) $J_c = 200000000\sin\theta\,\bar{a_r}$

~~Area = $4\pi r^2$ = 0.314 m²~~

~~$I_c = J_c A = 628000\,0\sin\theta\,\bar{a_r}$~~

$$I = \oint_s 1 \cdot ds = ds = r^2 \sin\theta\, d\theta\, d\phi\ \bar{a_r}$$

$$\oint_s J \cdot ds$$

$$\oint 200000000\sin\theta\,\bar{a_r} \cdot r^2\sin\theta\, d\theta\, d\phi\, a_r$$

$$2 \times 10^8 \int_0^{2\pi}\int_0^{2\pi} \sin\theta\, d\theta\, d\phi \times r^2$$

$$2 \times 10^8 \times (0.05)^2 \int_0^{2\pi}\int_0^{\pi} \frac{1-\cos 2\theta}{2}\, d\theta\, a\phi$$

$$= \left[\frac{\theta}{2} - \frac{\sin\theta}{4}\right]_0^{\pi} d\theta$$

$$\pi/2 \times \int_0^{2\pi} \phi\, d\phi$$

$$= \pi/2 \times 2\pi \times 2 \times 10^8 \times (0.05)^2$$

$$= \underline{4.93\,MA}$$

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1a) $E = 9a_x - 4a_y + 1.5a_z$

magnitude of tangential component of E at the surface boundary b/w conductor & free space $= 0$

$E_{Tan} = 0 \ \frac{N}{C}$

normal component

$D_N = \rho_s$

$E_N = |E|$

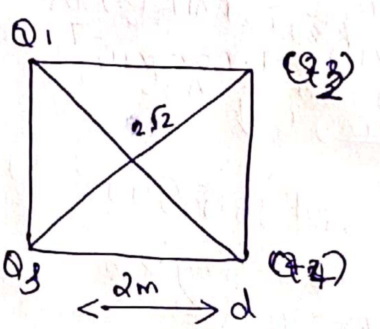
$E_N = \frac{\rho_s}{\varepsilon_0} \Rightarrow \sqrt{9^2+4^2+1.5^2} = \underline{\underline{9.96 \ V/m}}$

$\rho_s = E_N \times \varepsilon_0 = 9.96 \times 8.85 \times 10^{-12} = 8.76 \times 10^{-11} \ c/m^2$

3a) $\delta = 1/\alpha = \frac{1}{\beta} = \frac{1}{\sqrt{\pi F \mu \sigma}} = \frac{1}{\sqrt{3.14 \times 2 \times 10^6 \times 1.5 \times 10^{-6} \times 4.8 \times 10^{-6}}}$

$= \frac{1}{0.0212} = \underline{\underline{47.169}}$

4)



Let the work done to place $Q_1 = 0$, since $E = 0$, initially, $W_1 = 0$

The work done to place Q_2 in the electric field of Q_1 be

$W_2 = Q_2 V_{21}$ $\quad V_{21} = \frac{Q_1}{4\pi\varepsilon_0 \times R_{21}}$

$= \frac{1 \times 10^{-9}}{4 \times \pi \times 8.85 \times 10^{-12} \times 2}$

$W_2 = \frac{2 \times 10^{-9} \times 1 \times 10^{-9}}{4 \times \pi \times 8.85 \times 10^{-12} \times 2}$

$= 8.98 \times 10^{-9} J$

work done to place Q_3 in the electric field exerted by Q_2 & Q_1

$W_3 = Q_3 V_{31} + Q_3 V_{32}$

$$w_3 = 3\times10^{-4}\left[\frac{1\times10^{-9}}{4\pi\times\varepsilon_0\times2} + \frac{2\times10^{-9}}{4\pi\times\omega_0\times2\sqrt{2}}\right] \qquad V_{31} = \frac{Q_1}{4\pi\omega_0\times R_{31}}$$

$$= 3.254\times10^{-8}\,J \qquad V_{32} = \frac{Q_2}{4\pi\omega_0\times R_{32}}$$

workdone in bringing Q_4 to the field of Q_1, Q_2, Q_3

$$W_4 = Q_4 V_{41} + Q_4 V_{42} + Q_4 V_{43} \qquad V_{41} = \frac{Q_1}{4\pi\omega_0\times R_{41}}$$

$$W_4 = 4\times10^{-4}\left(\frac{1\times10^{-9}}{4\pi\omega_0\times2\sqrt{2}} + \frac{2\times10^{-9}}{4\pi\omega_0\times2} + \frac{3\times10^{-9}}{4\pi\omega_0\times2}\right] \qquad V_{42} = \frac{Q_2}{4\pi\omega_0\times R_{42}}$$

$$V_{43} = \frac{Q_3}{4\pi\omega_0\times R_{43}}$$

$$W_4 = \frac{4\times10^{-9}\times10^{-9}}{4\pi\omega_0\times2}\left(\frac{1}{\sqrt{2}} + \frac{2}{1} + \frac{3}{1}\right)$$

$$= 1.025\times10^{-7}\,J$$

Total energy stored in the system

$$W = W_1 + W_2 + W_3 + W_4$$

$$= 0.899\times10^{-9} + 3.254\times10^{-8} + 1.025\times10^{-7}$$

$$W = 1.4398\times10^{-7}\,J$$

5.) By maxwells equation:

$$\nabla\times E = \frac{-\partial B}{\partial t} = -\mu\cdot\frac{\partial H}{dt}$$

$$\therefore H = \frac{-1}{\mu}\int\nabla\times E \qquad \bar{E} = 60\sin(10^6 t)\sin(0.01z)$$

$$\nabla\times E = \begin{vmatrix} a_x & a_y & a_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ E_x & 0 & 0 \end{vmatrix}$$

$$= +a_y\frac{dE_x}{dz} + a_z\left(\frac{-dE_x}{dy}\right)$$

$$= a_y\left(60\sin(10^6 t)\times\cos(0.012)\times0.01\right]$$

$$= 0.6\sin(10^6 t)\cos(0.012)a_y$$

$\therefore \quad H = -\frac{1}{\mu}\int \nabla \times E = -\frac{1}{b}\cos(0.012) \times 0.6 + -\frac{\cos 10^6 t}{10^6}\, a_y$

$= \frac{0.6}{10^6 b}\cos(0.12)\cos 10^6 t\, a_y$

Comparing H obtained,

$\frac{0.6}{10^6 b} = 0.6$

$\Rightarrow 10^6 b = 1 \Rightarrow b = \frac{1}{10^6} \Rightarrow$

$\mu = 10^{-6}$ H/m

3b) $f = 2$ GHz, $H = 10\, a_y$ A/m

$v_p = 3 \times 10^8$

$\lambda = \frac{v_p}{f} = \frac{3 \times 10^8}{2 \times 10^9} = 1.5 \times 10^{-1}$ m

$= 0.15$ m