Assignment 6
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Question 1

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$f(0) = 0$

$f(2) = 2$

$f \Rightarrow u$

$$J[f] = \int_0^2 \left[\left(\frac{df}{dx}\right)^2 + f\right] dx$$

$$F = \left(\frac{du}{dx}\right)^2 + u$$

$$\frac{dF}{du} - \frac{d}{dx}\left(\frac{dF}{du'}\right) = 0$$

$$1 - 2u'' = 0$$

$$y'' = \frac{1}{2}$$

$$y' = \frac{1}{2}x + c_1$$

$$y = x^2 + c_1 x + c_2 \qquad y(2) = 2^2 + c_1(2) + 0 = 2$$

$$y(0) = 0 + 0 + c_2 \qquad 2 = 4 + c_1(2)$$

$$c_2 = 0 \qquad c_1 = -1$$

$$f = x^2 - x \qquad f' = 2x - 1$$

$$J[F]_{opt} = \int_0^2 (2x-1)^2 + x^2 - x$$

$$= \int_0^2 4x^2 - 4x + 1 + x^2 - x$$

$$= \int_0^2 5x^2 - 5x + 1$$

$$= \left[\frac{5}{3}x^3 - \frac{5}{2}x^2 + x\right]_0^2$$

$$= \frac{5}{3}(2)^3 - \frac{5}{2}(2)^2 + 2$$

$$= 13.33 - 10 + 2 = 5.33$$

(ARB) $y = \frac{1}{2}x^2 \qquad y' = x \Rightarrow$ same Boundary conditions

$$J[F]_{new} = \int_0^2 x^2 + \frac{1}{2}x^2$$

$$= \int_0^2 \frac{3}{2}x^2$$

$$= \left[\frac{1}{2}x^3\right]_0^2$$

$$= 4$$

$$J[F]_{new} < J[F]_{opt}$$

$\therefore$ Max