Assignment 6
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Question 2

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$$J[y] = \int_0^1 \left( \frac{(y')^2}{6} + xy \right) dx$$

$y(0) = 0$

$y(1) = 1$

$y \Rightarrow u$

$$F = \frac{(u')^2}{6} + xu$$

Euler $\quad \frac{dF}{du} - \frac{d}{dx}\left(\frac{dF}{du'}\right) = 0$

$$x - \frac{d}{dx}\left(\frac{u'}{3}\right) = 0$$

$$x - \frac{u''}{3} = 0$$

$$x - \frac{y''}{3} = 0$$

$$y'' = 3x$$

$$y' = \frac{3x^2}{2} + c_1$$

$$y = \frac{x^3}{2} + c_1 x + c_2$$

sub Boundary conditions

$y(0) = 0 \qquad y(1) = 1$

$0 = c_2 \qquad 1 = \frac{1}{2} + c_1$

$c_1 = 1/2$

$$y = \frac{x^3}{2} + \frac{x}{2} \qquad y' = \frac{3x^2}{2} + \frac{1}{2}$$

$$J[y]_{opt} = \int_0^1 \left( \frac{\left(\frac{3x^2}{2} + \frac{1}{2}\right)^2}{6} + x\left(\frac{x^3}{2} + \frac{x}{2}\right)\right) dx$$

$$= \int_0^1 \frac{3}{8}x^4 + \frac{1}{4}x^2 + \frac{1}{24} + \frac{1}{2}x^4 + \frac{1}{2}x^2$$

$$= \int_0^1 \frac{7}{8}x^4 + \frac{3}{4}x^2 + \frac{1}{24}$$

$$= \left[ \frac{7}{40}x^5 + \frac{1}{4}x^3 + \frac{x}{24} \right]_0^1$$

$$= \frac{7}{40} + \frac{1}{4} + \frac{1}{24}$$

$$= \frac{7}{15} = 0.467$$

Arbitrary function $\Rightarrow \quad y = \frac{x}{3}(x^2+2) \Rightarrow y(0) = 0 \quad y(1) = 1$

$$y' = x^2 + \frac{2}{3}$$

$$J[y]_{new} = \frac{127}{270} = 0.47037$$

$\therefore \quad J[y]_{new} > J[y]_{opt} \qquad$ (MIN)