Project 1: Optimal Design of Composite Sandwich Beams
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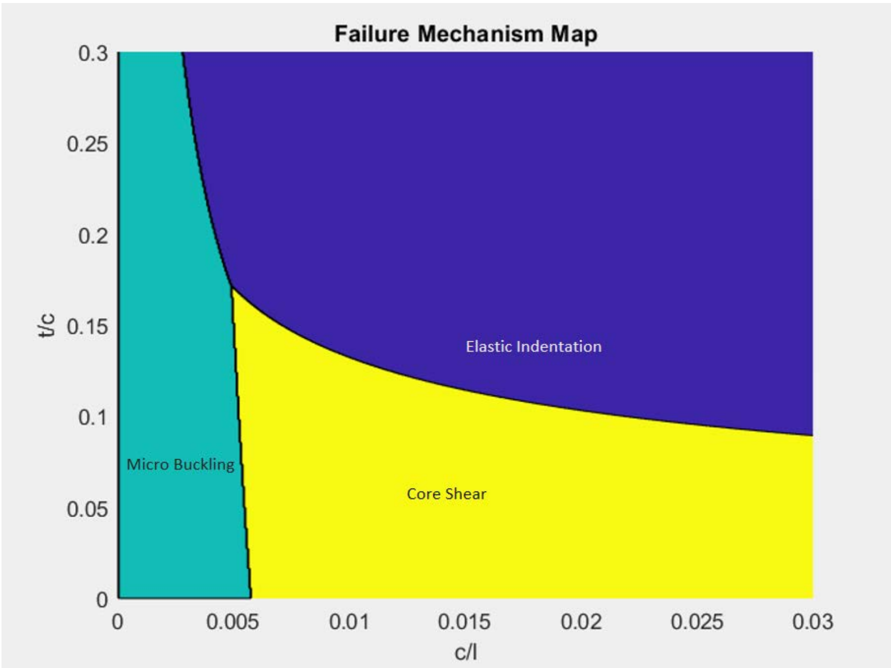
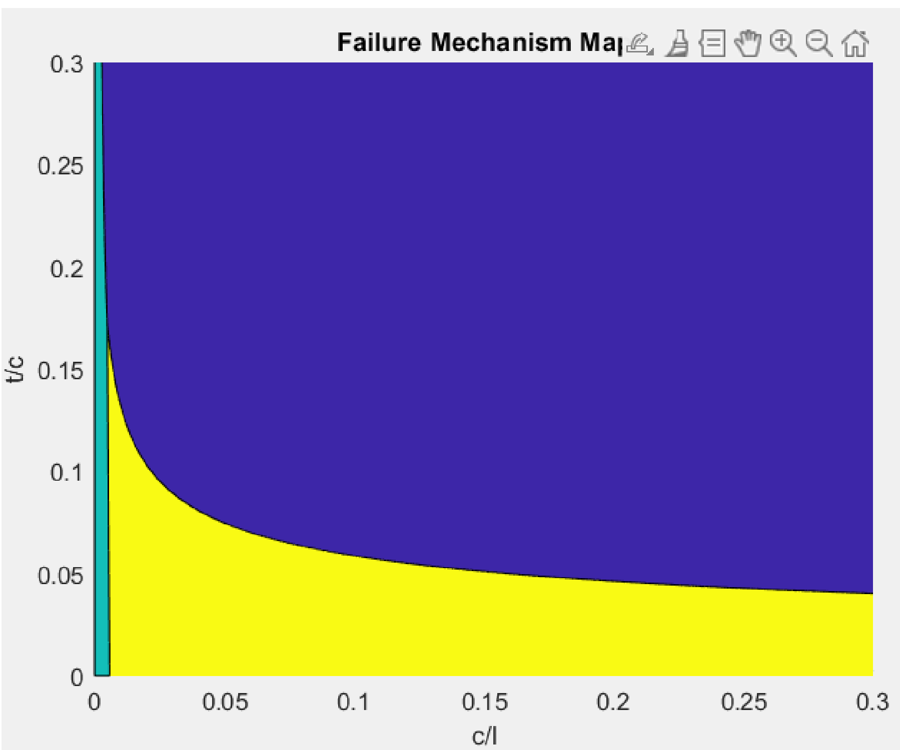
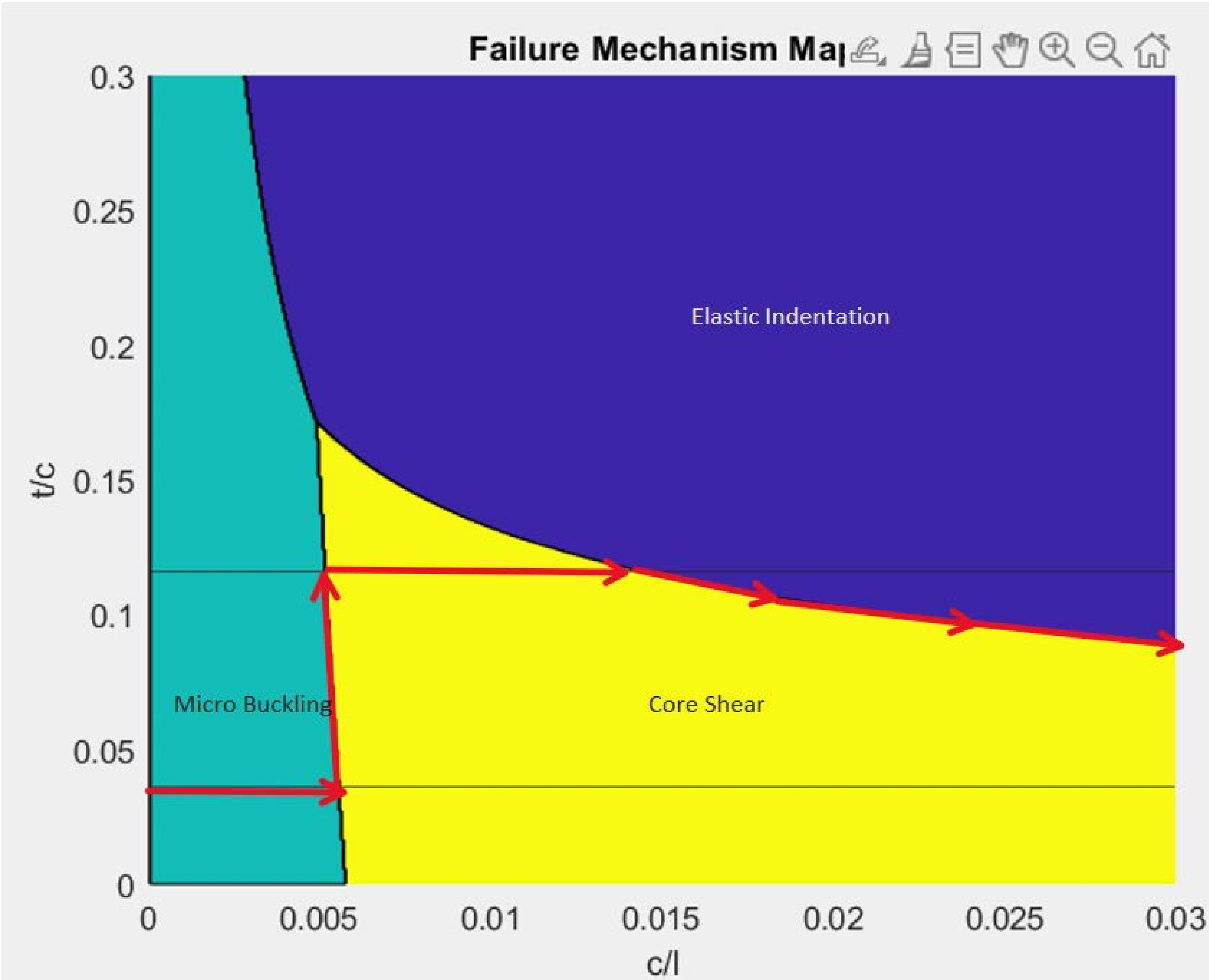
<u>Question 1:</u>



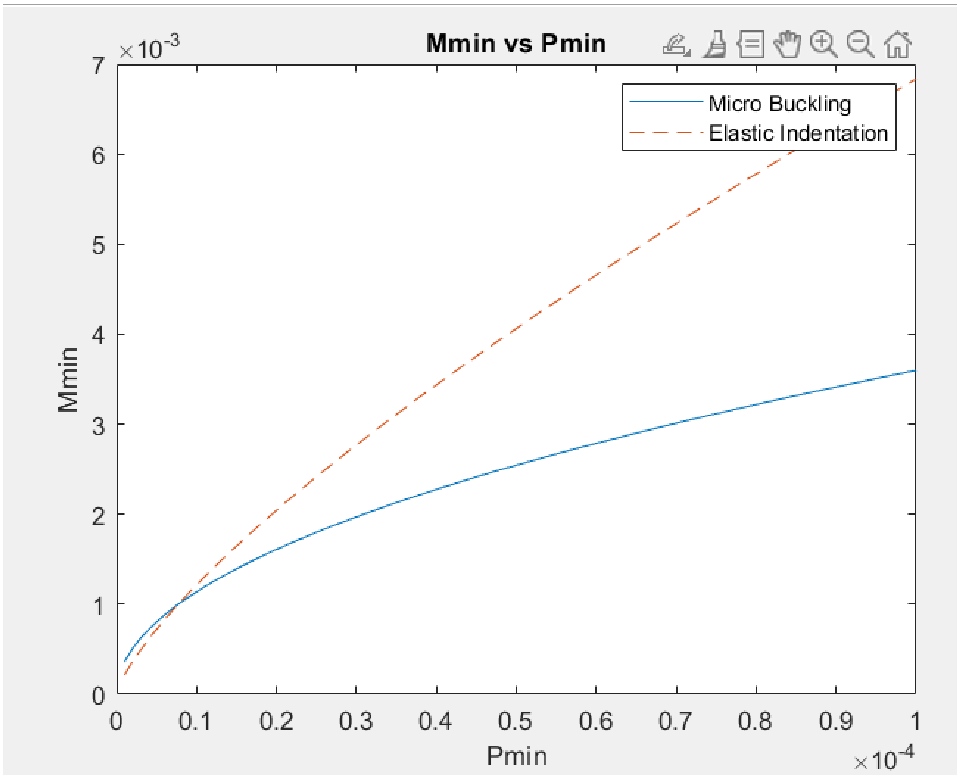


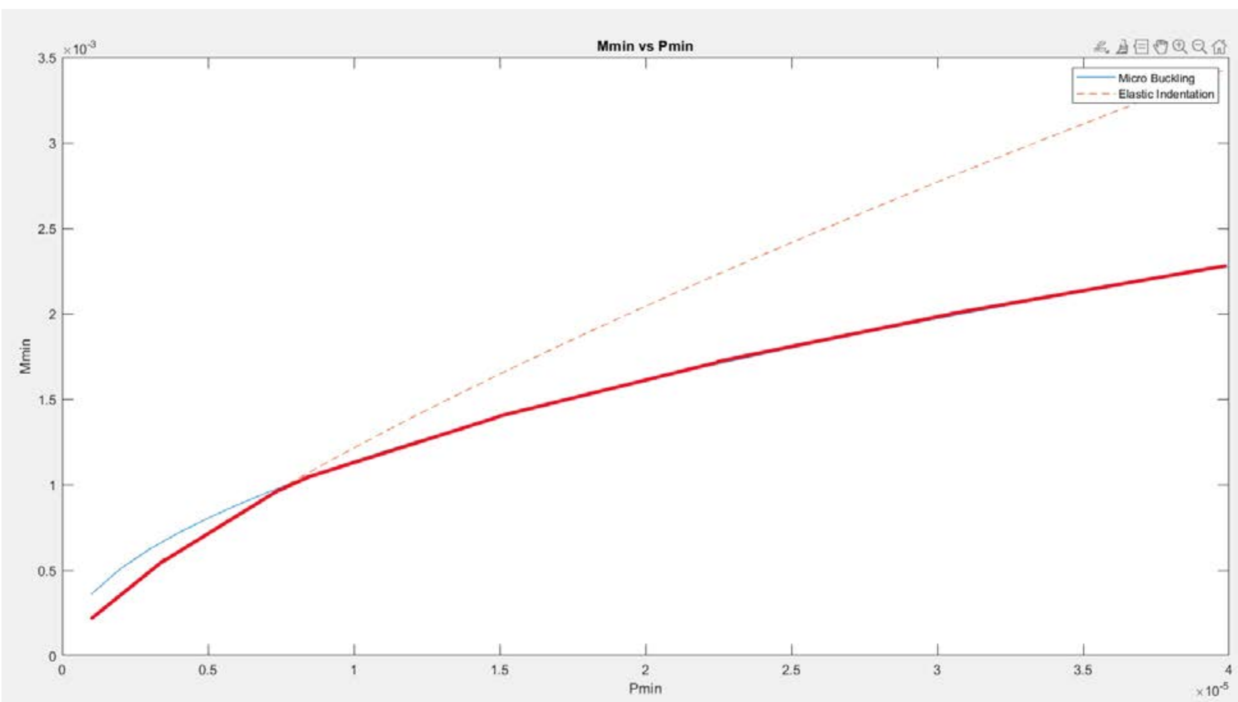
<u>Question 2:</u>



The red line represents the trajectory of the optimal design on the mechanism map.

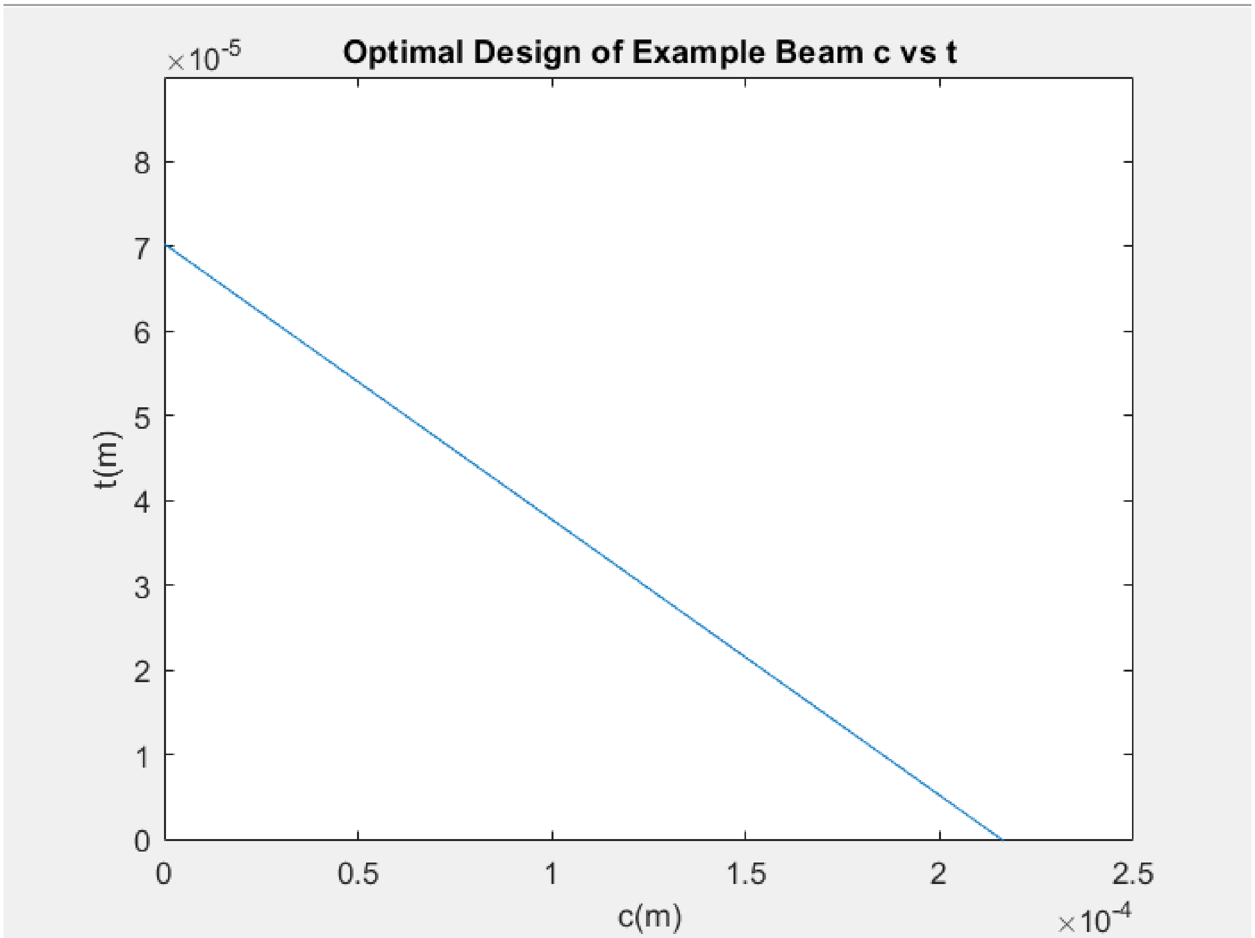
Question 3:





The red line represents the best minimum mass as a function of the applied load.

Question 4:



Question 5:

Question 1 thought process:

When trying to plot for the failure mechanism map, I first made all the equations in term of t/c and c/l. After that I made three different equations equal each other, core shear=micro buckling, elastic indentation=micro buckling, and core shear= elastic indentation. Simplifying these equations down, I got 1 normal function and two implicit functions which I plotted. To know if the values were real or not, I created a mesh grid and ran all the values through the three different failure equations to find the lowest one in each grid. After making an index I ran the contour function which showed the regions of the failure mechanism.

$$P_{core\ shear} = P_{Microbuckling}$$
$$\cancel{2}\cancel{b}\cancel{d}\,\tau_c = \frac{\cancel{2}\cancel{b}\cancel{d}\,t\,\sigma_f}{L}$$
$$\tau_c = \frac{2t\sigma_f}{L}$$
$$\tau_c = 2\sigma_f\left(\frac{t}{c}\right)\left(\frac{c}{L}\right)$$
$$t_{bar} = \frac{\tau_c}{2\sigma_f(C_{bar})}$$

$t_{bar} = \frac{t}{c}$ $\quad c_{bar} = \frac{c}{L}$

$\rho_f = 3040$ $\quad \rho_c = 205$

$E_f = 121000$

$\sigma_f = 900000000$ $\quad \tau_c = 1.5128\times10^6$

$\sigma_c = 1.9629\times10^6$

$$P_{elastic\ Identation} = P_{Microbuckling}$$
$$\cancel{b}\cancel{t}\left(\frac{\pi^2 d E_f \sigma_c^2}{3L}\right)^{1/3} = \frac{4\cancel{b}d\cancel{t}\sigma_f}{L}$$
$$\frac{d^2}{L^2} = \frac{\pi^2 E_f \sigma_c^2}{64(3)\sigma_f^3}$$
$$\frac{c^2}{L^2} + \frac{2ct}{L^2} + \frac{t^2}{L^2} = \frac{\pi^2 E_f \sigma_c^2}{192\sigma_f^3}$$
$$(c_{bar})(c_{bar}) + (c_{bar})^2(t_{bar}) + (c_{bar})^2(t_{bar})^2 = \frac{\pi^2 E_f \sigma_c^2}{192\sigma_f^3}$$
$$(c_{bar})^2\left(t_{bar}^2 + t_{bar} + 1\right) = \frac{\pi^2 E_f \sigma_c^2}{192\sigma_f^3} \qquad (y+1)^2 = 3.2874\times10^{-2}$$
$$\cancel{*} \quad (t_{bar}+1)^2 = \frac{\pi^2 E_f \sigma_c^2}{192\sigma_f^3(c_{bar})^2} \qquad \frac{3.2874\times10^{-11}}{c_{bar}}$$

$$P_{core\ shear} = P_{elastic\ Identation}$$
$$2bd\tau_c = bt\left(\frac{\pi^2 d E_f \sigma_c^2}{3L}\right)^{1/3}$$
$$8d^{\cancel{3}2}\tau_c^3 = \frac{t^3\pi^2 \cancel{d} E_f \sigma_c^2}{3L}$$
$$c^2 + 2ct + t^2 = \frac{t^3\pi^2 E_f \sigma_c^2}{24\tau_c^3 L}$$
$$\left(\frac{c}{L}\right)^2 + 2\left(\frac{c}{L}\right)\left(\frac{t}{L}\right) + \left(\frac{t}{L}\right)^2 = \frac{t^3\pi^2 E_f \sigma_c^2}{24\tau_c^3 L^3}$$
$$-\left(\frac{t}{L}\right)^3\left(\frac{\pi^2 E_f \sigma_c^2}{24\tau_c^3}\right) + \left(\frac{t}{L}\right)^2 + \left(\frac{2c}{L}\right)\left(\frac{t}{L}\right) + \left(\frac{c}{L}\right)^2 = 0$$

$(t_{bar}+1)(t_{bar}+1)$

$t_{bar}^2 + 2$

$$\frac{t}{c} = \frac{\pi^2 E_f \sigma_c^2}{24\tau_c^3} - \left(\frac{t}{c}\right)^2\left(\frac{c}{L}\right) - \frac{1}{(t/c)} - 2$$

0.055376522

$$\left(\frac{t}{c}\right)^3\left(\frac{t}{L}\right)(c) - \left(\frac{t}{c}\right)^2 - 2\left(\frac{t}{c}\right) - 1 = 0$$

$$P_{core\,shear} = P_{elastic\ identation} \qquad A = \frac{\pi^2 E_f \delta_c^2}{24\,\tau_c^3}$$

$$2bd\tau_c = bt\left(\frac{\pi^2 d E_f \delta_c^2}{3L}\right)^{1/3}$$

$$\left(\frac{c}{L}\right)^2 + 2\left(\frac{c}{L}\right)\left(\frac{t}{L}\right) + \left(\frac{t}{L}\right)^2 = A\left(\frac{t}{L}\right)^3$$

$$0 = A\left(\frac{t}{L}\right)^3 - \left(\frac{c}{L}\right)^2 - 2\left(\frac{c}{L}\right)\left(\frac{t}{L}\right) - \left(\frac{t}{L}\right)^2$$

$$0 = A\left(\frac{t}{c}\right)^3\left(\frac{c}{L}\right) - \left(\frac{t}{c}\right)^2 - 2\left(\frac{t}{c}\right) - 1$$

Question 2 thought process:

For obtaining the trajectory of optimal design, I plugged all the equations from before into the Mhat and Phat equations to simplify it as much in terms of t/c and t/l. After that I partially derived for t/c=tbar and c/l=cbar. With the equation Mhat=lambda*Phat, I had two systems of the equations and I first isolated for lambda. I took that equation and subbed it into the other one and simplified it down to a constant value. The trajectory of core shear was a negative value which means it won't show up on the graph and can be discarded. When plotting these horizontal lines, you must consider the lowest lines trajectory to the nearest failure line and follow that up to the next horizontal line through to the next failure mechanism on. This guided line will show the trajectory of the optimal design on the mechanism map.

Trajectory core shear

$$\frac{t}{c} = y \qquad \frac{c}{L} = x \qquad \hat{P} = \frac{2\tau_c}{\delta_f}\left(\frac{c}{L}\right) + \frac{2\tau_c}{\delta_f}\left(\frac{t}{c}\right)\left(\frac{c}{L}\right)$$

$$= \frac{2\tau_c}{\delta_f}\left(x + xy\right)$$

$$\nabla M = \begin{bmatrix} y + \frac{\rho_c}{\rho_f} \\ x \end{bmatrix}$$

$$\nabla P = \begin{bmatrix} \frac{2\tau_c}{\delta_f}(y+1) \\ \frac{2\tau_c}{\delta_f}(x) \end{bmatrix}$$

$$\cancel{x} = \lambda \frac{2\tau_c}{\delta_f}\cancel{x}$$

$$\lambda = \frac{\delta_f}{2\tau_c}$$

$$y + \frac{\rho_c}{\rho_f} = \left(\frac{2\tau_c}{\delta_f}(y+1)\right)\frac{2\tau_c}{\delta_f}$$

$$y + \frac{\rho_c}{\rho_f} = \frac{4\tau_c^2}{\delta_f^2}y + \frac{4\tau_c^2}{\delta_f^2}$$

$$y = -0.064 \qquad \therefore \text{ Not usable}$$

Trajectory Micro Buckling

$\frac{t}{c} = y \quad \frac{c}{L} = x$

$(y)(x) + \frac{\rho_c}{\rho_f}(x) - \lambda\left(4(x)^2(y) + 4(x)^2(y)^2\right)$

$\nabla \hat{M} = \begin{bmatrix} y + \frac{\rho_c}{\rho_f} \\ x \end{bmatrix}$

$\nabla \hat{P} = \begin{bmatrix} 8xy + 8y^2x \\ 4x^2 + 8x^2y \end{bmatrix}$

$x = \lambda\left(4x^2 + 8x^2y\right)$

$\lambda = \frac{1}{4(x) + 8(xy)}$

$y + \frac{\rho_c}{\rho_f} = \frac{\cancel{8}^{4}y + \cancel{8}y^2}{2\cancel{4} + \cancel{8}y}$

$\frac{2y + 2y^2}{1 + 2y} - y - \frac{\rho_c}{\rho_f} = 0$

$\frac{2y + \cancel{2y^2} - y - \cancel{2y^2}}{1 + 2y} - \frac{\rho_c}{\rho_f} = 0$

$\frac{y}{1 + 2y} - \frac{\rho_c}{\rho_f} = 0$

$y = \frac{\rho_c}{\rho_f} + 2y\left(\frac{\rho_c}{\rho_f}\right)$

$2y\left(1 - \frac{\rho_c}{\rho_f}\right) = \frac{\rho_c}{\rho_f}$

$y = \frac{\frac{\rho_c}{\rho_f}}{2\left(1 - \frac{\rho_c}{\rho_f}\right)} = \frac{0.067}{2(1 - 0.067)} = 0.0359$

Trajectory Elastic Indentation

$$x = \frac{c}{l} \qquad y = \frac{t}{c}$$

$$\frac{\partial P}{\partial x} = k \cdot \frac{4}{3} y \left(x(y+1) \right)^{1/3} \qquad \frac{\partial M}{\partial y} = 2x$$

$$\frac{\partial P}{\partial x} = k \cdot \frac{x^2(4y+3)}{3(x(y+1))^{2/3}} \qquad \bar{\rho} = \frac{\rho_c}{\rho_f}$$

$$\frac{\partial M}{\partial x} = 2y + \frac{\rho_c}{\rho_f} = 2y + \bar{\rho}$$

$$2y + \bar{\rho} = \lambda \left(k \cdot \frac{4}{3} y \left(x(y+1) \right)^{1/3} \right)$$

$$\lambda = \frac{2y + \bar{\rho}}{k\left(\frac{4}{3} y \left(x(y+1)\right)^{1/3}\right)}$$

$$2x = \lambda \left(\frac{k x^2 (4y+3)}{3(x(y+1))^{2/3}} \right)$$

$$2x = \frac{2y + \bar{\rho}}{\cancel{K}\left(\frac{4}{3} y \left(x(y+1)\right)^{1/3}\right)} \left(\frac{\cancel{k} \cdot x^2 (4y+3)}{3(x(y+1))^{2/3}} \right)$$

$$2\cancel{x} = \frac{(2y+\bar{\rho})(\cancel{x^2}(4y+3))}{4y(\cancel{x}(y+1))}$$

$$8y(y+1) = (2y+\bar{\rho})(4y+3)$$

$$\cancel{8y^2} + 8y = \cancel{8y^2} + 6y + 4y\bar{\rho} + 3\bar{\rho}$$

$$2y - 4\bar{\rho}y = 3\bar{\rho}$$

$$y = \frac{3\bar{\rho}}{2 - 4\bar{\rho}} \qquad \bar{\rho} = \frac{\rho_c}{\rho_f} = 0.067$$

$$y = 0.116$$

Question 3 thought process:

For plotting the minimum mass as a function of the applied load, I first took the Phat equation for each respective failure mechanism and isolated for cbar leaving the Phat in the equation as a variable. I then inputted this cbar into Mhat where I derive in terms of tbar and make equal to zero and isolate for y which is a constant. I then sub that tbar back into the main Mhat equation and simplify so that Mhat and Phat are the only variable. I then graph in terms of these variables and the lowest sections of the graph will give the minimum mass, which is mostly micro buckling.

$\hat{M}$ vs $\hat{P}$ core shear

$x = \frac{c}{L} \quad y = \frac{t}{c}$

$\bar{\rho} = \frac{\rho_c}{\rho_f}$

$$\hat{P} = \frac{2\tau_c}{\sigma_f}\left(\frac{c}{L} + \left(\frac{t}{c}\right)\left(\frac{c}{L}\right)\right)$$

$$= \frac{2\tau_c}{\sigma_f}\left(x + yx\right)$$

$\frac{2\tau_c}{\sigma_f} = k$

$$\hat{P} = kx + kyx$$

$$x = \frac{\hat{P}}{k} - yx$$

$\hat{M} = x(2y + \bar{\rho})$

$$\hat{M} = \left(\frac{\hat{P}}{k} - yx\right)\left(2y + \bar{\rho}\right)$$

Not derivable ∴ no real roots

Since Trajectory is bad aswell

Question 4 thought process:

I took the example values and plugged it into the initially given Phat formula, where I found my value to be in the elastic Indentation region of the Mhat vs Phat graph from the previous question. I then solve for Mhat with all given. Mhat of elastic indentation in terms of c/l and t/c which was derived in the previous questions to isolate for c and t. Once in terms of c and t we have a graph that gives the range of values that can be used for the most optimal design for the example beam.

Optimal Design with given Values

$$\hat{P} = \frac{P}{b\,L\,\sigma_f}$$

$$= \frac{75}{(0.2)(2.5)(900000000)}$$

$$= 0.000060166$$

$$= 1.66 \times 10^{-7} \quad \text{(elastic Identation region)}$$

$$\hat{M} = 4\,\hat{P}^{3/4}\left(\frac{\bar{\rho}(2-\bar{\rho})^3}{9\pi^2\bar{\sigma}E}\right)^{1/4}$$

$$= 4\,(1.66\times10^{-7})^{3/4}\,(1.7084)$$

$$= 0.000056199$$

$$= 5.6199\times10^{-5}$$

$$\hat{M} = \frac{2\cancel{b}Lt\rho_f + \cancel{b}Lc\rho_c}{\cancel{b}L^2\rho_f}$$

$$= \frac{2t}{L} + \frac{\rho_c}{\rho_f}\frac{c}{L}$$

$$5.6199\times10^{-5} = \frac{2t}{2.5} + \frac{0.065}{2.5}c$$

$$5.6199\times10^{-5} = 0.8t + 0.026c$$

$$\frac{5.6199\times10^{-5}}{0.8} - \frac{0.026c}{0.8} = t$$

$$t = 0.000070248 - 0.0325c$$

Question 6:

Part1,2

```
%%%Project 1 Arjun Posarajah
%%Optimal Design of Composite Sandwich Beams
%Student Number: 1004881737
%Last 7 digits: 4881737
%               tuvwxyz
pf= 1600+(30*48)
Ef= (40+81)*10^9
SigmaF= (200+(100*7))*10^6
pc= 20+(5*37)
SigmaC= (0.5+(6.5*((37/100)^1.5)))*10^6
Tc= (0.5+(4.5*((37/100)^1.5)))*10^6

%tbar=0:0.001:0.5;
cbar=0:0.001:0.5;


%CoreShear and Microbuckling
tbar= Tc./(2*SigmaF.*cbar);
plot(cbar,tbar)
hold on

%%Elastic Indentation and Microbuckling
fimplicit(@(cbar,tbar)((tbar.^2)+tbar+1)-(((pi^2)*Ef*(SigmaC^2))/(192*(SigmaF^3)*(cbar.^2))))

%Coreshear and Elastic Indentation
fimplicit(@(cbar,tbar)((((pi^2)*Ef*(SigmaC^2))/(24*(Tc^3))).*(tbar.^3).*cbar)-(tbar.^2)-(2.*tbar)-1)
axis([0 0.03 0 0.3])
xlabel c/l
ylabel t/c
%hold off

%%Meshgrid system
% Create a 100x100 meshgrid
cbar = linspace(0, 0.3, 5000);
tbar = linspace(0, 0.3, 5000);
[X, Y] = meshgrid(cbar, tbar);

% Define three equations
cs= ((2*Tc)/SigmaF).*(X)+((2*Tc)/SigmaF).*X.*Y;
mb= 4*(X.^2).*Y+4.*(Y.^2).*(X.^2);
ei=(1/SigmaF).*(Y.*X).*(((((pi^2)*Ef*(SigmaC^2))/3).*X.*(1+Y)).^(1/3));

index = zeros(5000);

for i=1:5000
    for j=1:5000
        if cs(i,j)<mb(i,j) && cs(i,j)<ei(i,j)
            index(i,j)=1;
        elseif mb(i,j)<cs(i,j) && mb(i,j)<ei(i,j)
                index(i,j)=2;
        else
                    index(i,j)=3;

          end
     end
end
contourf(X,Y,index)

%Trajectory of Optimal Design on the mechanism map
yline([0.0359 0.116])

title('Failure Mechanism Map')
hold off
```

Part3

```
pf= 1600+(30*48);
Ef= (40+81)*10^9;
SigmaF= (200+(100*7))*10^6;
pc= 20+(5*37);
SigmaC= (0.5+(6.5*((37/100)^1.5)))*10^6;
Tc= (0.5+(4.5*((37/100)^1.5)))*10^6;
pbar=pc/pf;

P=0.000001:0.000001:0.00004;

Mm =(0.1295.*P).^(1/2);
plot(P,Mm)
hold on

b=(((0.4839/(9*(pi^2)*((SigmaC/SigmaF)^2)*(Ef/SigmaF)))).^(1/4));

Me= 4*(((0.4839/(9*(pi^2)*((SigmaC/SigmaF)^2)*(Ef/SigmaF))))^(1/4)).*(P.^(3/4));
plot(P,Me,'--')
%axis([0 0.00001 0 0.001])
xlabel Pmin
ylabel Mmin
title ('Mmin vs Pmin')
legend('Micro Buckling','Elastic Indentation')
hold off
```

```
%%Question 4
c=0:0.000001:0.000225;
t=0.000070248-(0.325.*c);
plot(c,t)
title('Optimal Design of Example Beam c vs t')
xlabel c(m)
ylabel t(m)
```