Assignment 7
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UNIVERSITY OF TORONTO

Question 1

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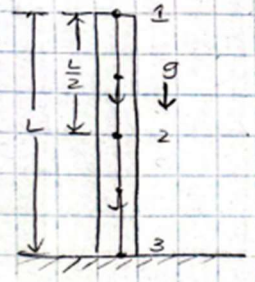

$d_3 = 0$

$$K_1 = K_2 = \frac{AE}{(L/2)} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$K_G = \frac{AE}{(L/2)} \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$$

Forces $\Rightarrow$

$$\begin{cases} F_1 = F_2 = \frac{mg}{2} \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix} = \frac{mg}{4} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \\ F_G = \frac{mg}{4} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \end{cases}$$

$K_G u = F$

$$\frac{2AE}{L} \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \frac{mg}{4} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$u_1 - u_2 = \frac{mgL}{8AE}$$

$$u_1 = u_2 + \frac{mgL}{8AE}$$

$m = \rho AL$

$$2u_2 - u_1 = \frac{2mgL}{4AE}$$

$$2u_2 - \left(u_2 - \frac{mgL}{8AE}\right) = \frac{mgL}{4AE}$$

displacements $\Rightarrow$

$$\begin{cases} u_1 = \frac{mgL}{2AE} = \frac{\rho g L^2}{2E} \\ u_2 = \frac{3mgL}{8AE} = \frac{3\rho g L^2}{8E} \\ u_3 = 0 \end{cases}$$

continuous system

$T = \rho A g (L - x)$

$$u|_L \Rightarrow \int_0^L \frac{T\,dx}{AE}$$

$$\int_0^L \frac{\rho A g}{AE} (L - x)\,dx$$

$$= \frac{\rho g}{E} \int_0^L (L - x)\,dx$$

$$= \frac{\rho g}{E} \left(Lx - \frac{x^2}{2}\right)\Big|_0^L$$

$$= \frac{\rho g}{E} \left(\frac{L^2}{2}\right) = \frac{\rho g L^2}{2E}$$

$$u|_{L/2} = \frac{\rho g}{E} \left(Lx - \frac{x^2}{2}\right)\Big|_0^{L/2}$$

$$= \frac{3\rho g L^2}{8E}$$