Assignment 4
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Question 1

Assignment 4 Question 1

[0/45/60]

$t = 0.125mm$ $E_1 = 125 Gpa$ $E_2 = 8.8 Gpa$ $G_{12} = 6.8 Gpa$ $\nu_{12} = 0.24$

Stiffness matrix in material coordinate system

$S_{11} = 0.008$ $Q_{11} = 125.5089$

$S_{22} = 0.1136$ $Q_{22} = 8.8358$

$S_{12} = -0.0019$ $Q_{12} = 2.1206$

$S_{66} = 0.1471$ $Q_{66} = 6.8$

$$Q = \begin{bmatrix} 125.5089 & 2.1206 & 0 \\ 2.1206 & 8.8358 & 0 \\ 0 & 0 & 6.8 \end{bmatrix} Gpa$$

stiffness matrix for the same lamina rotated angles from global coordinate system

$$\bar{Q}_1 = \begin{bmatrix} 125.5089 & 2.1206 & 0 \\ 2.1206 & 8.8358 & 0 \\ 0 & 0 & 6.8 \end{bmatrix} Gpa$$

$$\bar{Q}_2 = \begin{bmatrix} 41.4465 & 27.8465 & 29.1683 \\ 27.8465 & 41.4465 & 29.1683 \\ 29.1685 & 29.1683 & 32.5259 \end{bmatrix} Gpa$$

$$\bar{Q}_3 = \begin{bmatrix} 18.7097 & 21.415 & 14.1208 \\ 21.415 & 77.0462 & 36.4001 \\ 14.1208 & 36.4001 & 26.0944 \end{bmatrix} Gpa$$

Laminate stiffness Matrix

$$A = \begin{bmatrix} 23.2081 & 6.4228 & 5.4111 \\ 6.4228 & 15.9161 & 8.196 \\ 5.4111 & 8.196 & 8.1775 \end{bmatrix} Gpa \cdot mm$$

$$B = \begin{bmatrix} -1.6687 & 0.3015 & 0.2206 \\ 0.3015 & 1.0658 & 0.5658 \\ 0.2206 & 0.5688 & 0.3015 \end{bmatrix} Gpa \cdot m$$

$$D = \begin{bmatrix} 0.3119 & 0.0543 & 0.0346 \\ 0.0543 & 0.1885 & 0.0818 \\ 0.0346 & 0.0818 & 0.0749 \end{bmatrix} Gpa \cdot mm^3$$

$$\bar{Q}_{ij} = \begin{bmatrix} 23.2081 & 6.4228 & 5.4111 & -1.6687 & 0.3015 & 0.2206 \\ 6.4228 & 15.9161 & 8.196 & 0.3015 & 1.0658 & 0.5688 \\ 5.4111 & 8.196 & 8.1775 & 0.2206 & 0.5688 & 0.3015 \\ -1.6687 & 0.3015 & 0.2206 & 0.3119 & 0.0543 & 0.0346 \\ 0.3015 & 1.0658 & 0.5688 & 0.0543 & 0.1885 & 0.0818 \\ 0.2206 & 0.5688 & 0.3015 & 0.0346 & 0.0818 & 0.0749 \end{bmatrix} Gpa$$

Assignment 4 Question 1

solve laminate stiffness Matrix for Midplane strains

$F = [6kN/m \quad -2kN/m \quad 0 \quad 2N \quad 2N \quad 0]$

$$\varepsilon_0 = \begin{bmatrix} 2.3582 \\ -1.4663 \\ -1.3931 \\ 20.0307 \\ 25.3249 \\ -27.1137 \end{bmatrix}$$

calculate strains at lamina Mid-planes in Global coordinates

$$\varepsilon_1 = \begin{bmatrix} -0.1457 \\ -4.6319 \\ 1.9961 \end{bmatrix} \quad \varepsilon_2 = \begin{bmatrix} 2.3582 \\ -1.4663 \\ -1.3931 \end{bmatrix} \quad \varepsilon_3 = \begin{bmatrix} 4.862 \\ 1.6993 \\ -4.7823 \end{bmatrix}$$

calculate stress

$$\sigma^* = \bar{Q} \times \varepsilon$$

$$\sigma^*_1 = \begin{bmatrix} -28.1034 \\ -41.2353 \\ 13.5738 \end{bmatrix} \text{Mpa} \quad \sigma^*_2 = \begin{bmatrix} 16.2747 \\ -35.7377 \\ -19.2951 \end{bmatrix} \text{Mpa} \quad \sigma^*_3 = \begin{bmatrix} 59.8287 \\ 60.973 \\ 5.7213 \end{bmatrix} \text{Mpa}$$

calculate lamina stresses in Material coordinates

$$T_t = \begin{bmatrix} \cos^2\theta & \sin^2\theta & 2\cos\sin\theta \\ \sin^2\theta & \cos^2\theta & -2\cos\theta\sin\theta \\ -\sin\theta\cos\theta & \cos\theta\sin\theta & \cos^2\theta - \sin^2\theta \end{bmatrix}$$

$$\sigma = T_k \times \sigma^*$$

$$\sigma_1 = \begin{bmatrix} -28.1034 \\ -41.2353 \\ 13.5738 \end{bmatrix} \text{Mpa} \quad \sigma_2 = \begin{bmatrix} -29.0266 \\ 9.5636 \\ -26.0062 \end{bmatrix} \text{Mpa} \quad \sigma_3 = \begin{bmatrix} 65.6417 \\ 55.16 \\ -2.3651 \end{bmatrix} \text{Mpa}$$

Assignment 4 Question 1

Tsai Hill

$\sigma_L^+ = 900MPa$ $\sigma_T^+ = 50MPa$ $\tau_{LT} = 80MPa$

$\sigma_L^- = 750MPa$ $\sigma_T^- = 180MPa$

$$\frac{\sigma_1^2}{\sigma_L^2} - \frac{\sigma_1\sigma_2}{\sigma_L^2} + \frac{\sigma_2^2}{\sigma_T^2} + \frac{\tau_{12}^2}{\tau_{LT}^2} \geq 1$$

ply 1 Tsai Hill = 0.806 ≤ 1 ✓

ply 2 Tsai Hill = 0.1443 ≤ 1 ✓

ply 3 Tsai Hill = 1.2188 ≥ 1 ×

Tsai-Wu

$$F_{11}\sigma_1^2 + F_{22}\sigma_2^2 + F_{66}\tau_{12}^2 + F_1\sigma_1 + F_2\sigma_2 + 2F_{12}\sigma_1\sigma_2 \geq 1$$

$F_{11} = 1.4815\times10^{-6}$ $F_1 = -2.22\times10^{-4}$ $F_{66} = 1.5625\times10^{-4}$

$F_{22} = 1.111\times10^{-4}$ $F_2 = 0.0144$ $F_{12} = -6.415\times10^{-6}$

ply 1 Tsai Wu = −0.3854 ≤ 1 ✓

ply 2 Tsai Wu = 0.2652 ≤ 1 ✓

ply 3 Tsai Wu = 1.0810 ≥ 1 ×

conclusion

∴ ply 3 has failed both Tsai Hill, Wu criterions with both being above 1. Ply 3 also fails in maximum stress criterion as well. Another method would be maximum strain criterion but not enough info is given to do so.

After the first ply fails, the stiffness of the ply is reduced by either matrix or fiber failures. The strength of the laminate at the same point is evaluated again to see if the laminate can carry an additional load. This ply by ply analysis progresses until the ultimate strength of the laminate is reached.

code:

```
%%Question 1%%
%Ply Angles
x1=0;
x2=45;
x3=60;
t=0.125;%mm
E1=125;
E2=8.8;
G12=6.8;
v12=0.24;

%%%Part 1 stiffness matrix in the material coordinate system
S11=1/E1;
S22=1/E2;
S12=(-v12)/E1;
S66=1/G12;

Q11=S22/((S11*S22)-(S12.^2));
Q22=S11/((S11*S22)-(S12.^2));
Q12=(-S12)/((S11*S22)-(S12.^2));
Q66=G12;

Q= [Q11 Q12 0; Q12 Q22 0; 0 0 Q66];
%%%Part2 stiffness matrix for the same lamina rotated angles from the global coordinate system

%%x1
x1Qbar11= (Q11*((cosd(x1)).^4))+(Q22*((sind(x1).^4)))+ (2*(Q12+(2*Q66)))*((sind(x1)).^2)*((cosd(x1)).^2);
x1Qbar12= (Q11+Q22-(4*Q66))*((cosd(x1)).^2)*((sind(x1)).^2) + (Q12)*(((cosd(x1)).^4)+((sind(x1)).^4));
x1Qbar22=(Q11)*((sind(x1)).^4)+(Q22)*((cosd(x1)).^4)+(2*(Q12+(2*Q66)))*(((sind(x1)).^2)*((cosd(x1)).^2));
x1Qbar16=(Q11-Q12-(2*Q66))*((cosd(x1)).^3)*(sind(x1))-((Q22-Q12-(2*Q66))*(cosd(x1))*(((sind(x1)).^3)));
x1Qbar26=(Q11-Q12-(2*Q66))*(cosd(x1))*((sind(x1)).^3)-(Q22-Q12-(2*Q66))*((cosd(x1)).^3)*(sind(x1));
x1Qbar66= ((Q11+Q22-(2*Q12)-(2*Q66))*((cosd(x1)).^2)*((sind(x1)).^2))+((Q66)*(((cosd(x1)).^4)+((sind(x1)).^4)));

x1Qbar= [x1Qbar11 x1Qbar12 x1Qbar16; x1Qbar12 x1Qbar22 x1Qbar26; x1Qbar16 x1Qbar26 x1Qbar66];

%%x2
x2Qbar11= (Q11*((cosd(x2)).^4))+(Q22*((sind(x2).^4)))+ (2*(Q12+(2*Q66)))*((sind(x2)).^2)*((cosd(x2)).^2);
x2Qbar12= (Q11+Q22-(4*Q66))*((cosd(x2)).^2)*((sind(x2)).^2) + (Q12)*(((cosd(x2)).^4)+((sind(x2)).^4));
x2Qbar22=(Q11)*((sind(x2)).^4)+(Q22)*((cosd(x2)).^4)+(2*(Q12+(2*Q66)))*(((sind(x2)).^2)*((cosd(x2)).^2));
x2Qbar16=(Q11-Q12-(2*Q66))*((cosd(x2)).^3)*(sind(x2))-((Q22-Q12-(2*Q66))*(cosd(x2))*(((sind(x2)).^3)));
x2Qbar26=(Q11-Q12-(2*Q66))*(cosd(x2))*((sind(x2)).^3)-(Q22-Q12-(2*Q66))*((cosd(x2)).^3)*(sind(x2));
x2Qbar66= ((Q11+Q22-(2*Q12)-(2*Q66))*((cosd(x2)).^2)*((sind(x2)).^2))+((Q66)*(((cosd(x2)).^4)+((sind(x2)).^4)));

x2Qbar= [x2Qbar11 x2Qbar12 x2Qbar16; x2Qbar12 x2Qbar22 x2Qbar26; x2Qbar16 x2Qbar26 x2Qbar66];

%%x3
x3Qbar11= (Q11*((cosd(x3)).^4))+(Q22*((sind(x3).^4)))+ (2*(Q12+(2*Q66)))*((sind(x3)).^2)*((cosd(x3)).^2);
x3Qbar12= (Q11+Q22-(4*Q66))*((cosd(x3)).^2)*((sind(x3)).^2) + (Q12)*(((cosd(x3)).^4)+((sind(x3)).^4));
x3Qbar22=(Q11)*((sind(x3)).^4)+(Q22)*((cosd(x3)).^4)+(2*(Q12+(2*Q66)))*(((sind(x3)).^2)*((cosd(x3)).^2));
x3Qbar16=(Q11-Q12-(2*Q66))*((cosd(x3)).^3)*(sind(x3))-((Q22-Q12-(2*Q66))*(cosd(x3))*(((sind(x3)).^3)));
x3Qbar26=(Q11-Q12-(2*Q66))*(cosd(x3))*((sind(x3)).^3)-(Q22-Q12-(2*Q66))*((cosd(x3)).^3)*(sind(x3));
x3Qbar66= ((Q11+Q22-(2*Q12)-(2*Q66))*((cosd(x3)).^2)*((sind(x3)).^2))+((Q66)*(((cosd(x3)).^4)+((sind(x3)).^4)));

x3Qbar= [x3Qbar11 x3Qbar12 x3Qbar16; x3Qbar12 x3Qbar22 x3Qbar26; x3Qbar16 x3Qbar26 x3Qbar66];
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```
%%%Part3: Calculating A,B,D
%Need to input z manually for spacing changes/ply changes

A11=x1Qbar11*(t)+ x2Qbar11*(t)+x3Qbar11*(t);
A12=x1Qbar12*(t)+ x2Qbar12*(t)+x3Qbar12*(t);
A22=x1Qbar22*(t)+ x2Qbar22*(t)+x3Qbar22*(t);
A16=x1Qbar16*(t)+ x2Qbar16*(t)+x3Qbar16*(t);
A26=x1Qbar26*(t)+ x2Qbar26*(t)+x3Qbar26*(t);
A66=x1Qbar66*(t)+ x2Qbar66*(t)+x3Qbar66*(t);

B11= 0.5*((x1Qbar11*(-1/32))+(x2Qbar11*(0))+(x3Qbar11*(1/32)));
B12= 0.5*((x1Qbar12*(-1/32))+(x2Qbar12*(0))+(x3Qbar12*(1/32)));
B22= 0.5*((x1Qbar22*(-1/32))+(x2Qbar22*(0))+(x3Qbar22*(1/32)));
B16= 0.5*((x1Qbar16*(-1/32))+(x2Qbar16*(0))+(x3Qbar16*(1/32)));
B26= 0.5*((x1Qbar26*(-1/32))+(x2Qbar26*(0))+(x3Qbar26*(1/32)));
B66= 0.5*((x1Qbar66*(-1/32))+(x2Qbar66*(0))+(x3Qbar66*(1/32)));

D11=(1/3)*((x1Qbar11*(13/2048))+(x2Qbar11*(1/2048))+(x3Qbar11*(13/2048)));
D12=(1/3)*((x1Qbar12*(13/2048))+(x2Qbar12*(1/2048))+(x3Qbar12*(13/2048)));
D22=(1/3)*((x1Qbar22*(13/2048))+(x2Qbar22*(1/2048))+(x3Qbar22*(13/2048)));
D16=(1/3)*((x1Qbar16*(13/2048))+(x2Qbar16*(1/2048))+(x3Qbar16*(13/2048)));
D26=(1/3)*((x1Qbar26*(13/2048))+(x2Qbar26*(1/2048))+(x3Qbar26*(13/2048)));
D66=(1/3)*((x1Qbar66*(13/2048))+(x2Qbar66*(1/2048))+(x3Qbar66*(13/2048)));

%%%Part4 Laminate Stiffness Matrix
A= [A11 A12 A16; A12 A22 A26; A16 A26 A66];
B= [B11 B12 B16; B12 B22 B26; B16 B26 B66];
D= [D11 D12 D16; D12 D22 D26; D16 D26 D66];

Qij=[A11 A12 A16 B11 B12 B16; A12 A22 A26 B12 B22 B26;A16 A26 A66 B16 B26 B66; B11 B12 B16 D11 D12 D16 ;B12 B22 B26 D12 D22 D26; B16 B26 B66 D16 D26 D66];

%Part5 Solve Laminate Stiffness Matrix for Midplane Strains
F=[6 ;-2; 0 ;2 ; 2 ;0];
Eo=linsolve(Qij,F);

%Part6 Calculate Strains at Lamina Mid-Planes in Global Coordinates
y1=-0.125;
y2=0;
y3=0.125;

E1= [Eo(1,1)+(y1*(Eo(4,1)));Eo(2,1)+(y1*(Eo(5,1))); Eo(3,1)+(y1*(Eo(6,1)))];
E2= [Eo(1,1)+(y2*(Eo(4,1)));Eo(2,1)+(y2*(Eo(5,1))); Eo(3,1)+(y2*(Eo(6,1)))];
E3= [Eo(1,1)+(y3*(Eo(4,1)));Eo(2,1)+(y3*(Eo(5,1))); Eo(3,1)+(y3*(Eo(6,1)))];

%Part7 Calculate Stress
Ss1=x1Qbar*E1
Ss2=x2Qbar*E2
Ss3=x3Qbar*E3

%Part8 Calculate Lamina Stresses in Material Coordinates
Tk1= [(cosd(x1))^2 (sind(x1))^2 2*(cosd(x1))*(sind(x1));(sind(x1))^2 (cosd(x1))^2 -2*(cosd(x1))*(sind(x1)); -1*(cosd(x1))*(sind(x1)) (cosd(x1))*(sind(x1)) ((cosd(x1)^2)-((sind(x1))^2))];
Tk2= [(cosd(x2))^2 (sind(x2))^2 2*(cosd(x2))*(sind(x2));(sind(x2))^2 (cosd(x2))^2 -2*(cosd(x2))*(sind(x2)); -1*(cosd(x2))*(sind(x2)) (cosd(x2))*(sind(x2)) ((cosd(x2)^2)-((sind(x2))^2))];
Tk3= [(cosd(x3))^2 (sind(x3))^2 2*(cosd(x3))*(sind(x3));(sind(x3))^2 (cosd(x3))^2 -2*(cosd(x3))*(sind(x3)); -1*(cosd(x3))*(sind(x3)) (cosd(x3))*(sind(x3)) ((cosd(x3)^2)-((sind(x3))^2))];

S1=Tk1*Ss1
S2=Tk2*Ss2
S3= Tk3*Ss3

%Part9 Tsai Hill
sigmaLplus=900;
sigmaLminus=750;
sigmaTplus=50;
sigmaTminus=180;
TLT= 80;

ply1Tsaihill=((S1(1,1).^2)/(sigmaLminus .^2))-(((S1(1,1))*(S1(2,1)))/(sigmaLminus .^2))+(((S1(2,1)).^2)/(sigmaTminus .^2))+(((S1(3,1)).^2)/(TLT.^2))
ply2Tsaihill=((S2(1,1).^2)/(sigmaLminus .^2))-(((S2(1,1))*(S2(2,1)))/(sigmaLminus .^2))+(((S2(2,1)).^2)/(sigmaTplus .^2))+(((S2(3,1)).^2)/(TLT.^2))
ply3Tsaihill=((S3(1,1).^2)/(sigmaLplus .^2))-(((S3(1,1))*(S3(2,1)))/(sigmaLplus .^2))+(((S3(2,1)).^2)/(sigmaTplus .^2))+(((S3(3,1)).^2)/(TLT.^2))
%Part10 Tsai Wu
F11= 1/((sigmaLplus)*(sigmaLminus))
F22= 1/((sigmaTplus)*(sigmaTminus))
F1= (1/(sigmaLplus))-(1/(sigmaLminus))
F2= (1/(sigmaTplus))-(1/(sigmaTminus))
F66= 1/(TLT.^2)
F12= (-1/2)*(sqrt(F11*F22))

ply1TsaiWu= (F11*(S1(1,1).^2))+(F22*(S1(2,1).^2))+(F66*(S1(3,1).^2))+(F1*S1(1,1))+(F2*S1(2,1))+ 2*(F12*S1(1,1)*S1(2,1))
ply2TsaiWu= (F11*(S2(1,1).^2))+(F22*(S2(2,1).^2))+(F66*(S2(3,1).^2))+(F1*S2(1,1))+(F2*S2(2,1))+ 2*(F12*S2(1,1)*S2(2,1))
ply3TsaiWu= (F11*(S3(1,1).^2))+(F22*(S3(2,1).^2))+(F66*(S3(3,1).^2))+(F1*S3(1,1))+(F2*S3(2,1))+ 2*(F12*S3(1,1)*S3(2,1))
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