Assignment 5
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Nov 12, 2023

UNIVERSITY OF TORONTO

Question 3

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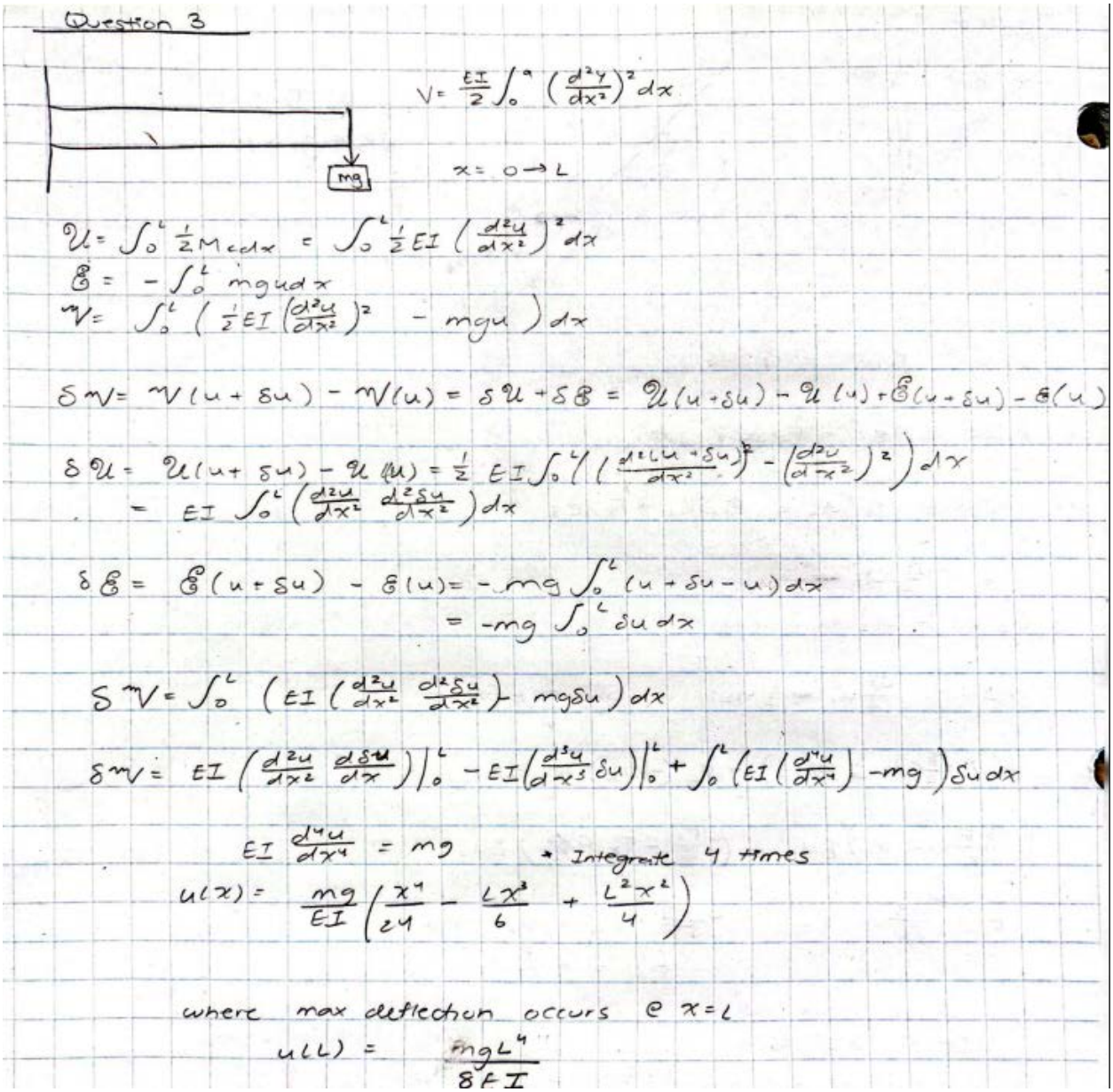

$$V = \frac{EI}{2}\int_0^a \left(\frac{d^2y}{dx^2}\right)^2 dx$$

$x = 0 \rightarrow L$

$$\mathcal{U} = \int_0^L \frac{1}{2} M_c \, d\kappa = \int_0^L \frac{1}{2} EI \left(\frac{d^2u}{dx^2}\right)^2 dx$$

$$\mathcal{E} = -\int_0^L mgu\,dx$$

$$\mathcal{V} = \int_0^L \left(\frac{1}{2} EI \left(\frac{d^2u}{dx^2}\right)^2 - mgu\right) dx$$

$$\delta\mathcal{V} = \mathcal{V}(u+\delta u) - \mathcal{V}(u) = \delta\mathcal{U} + \delta\mathcal{E} = \mathcal{U}(u+\delta u) - \mathcal{U}(u) + \mathcal{E}(u+\delta u) - \mathcal{E}(u)$$

$$\delta\mathcal{U} = \mathcal{U}(u+\delta u) - \mathcal{U}(u) = \frac{1}{2} EI \int_0^L \left(\left(\frac{d^2(u+\delta u)}{dx^2}\right)^2 - \left(\frac{d^2u}{dx^2}\right)^2\right) dx$$

$$= EI \int_0^L \left(\frac{d^2u}{dx^2}\frac{d^2\delta u}{dx^2}\right) dx$$

$$\delta\mathcal{E} = \mathcal{E}(u+\delta u) - \mathcal{E}(u) = -mg\int_0^L (u+\delta u - u)\,dx$$

$$= -mg\int_0^L \delta u\,dx$$

$$\delta\mathcal{V} = \int_0^L \left(EI\left(\frac{d^2u}{dx^2}\frac{d^2\delta u}{dx^2}\right) - mg\delta u\right) dx$$

$$\delta\mathcal{V} = EI\left(\frac{d^2u}{dx^2}\frac{d\delta u}{dx}\right)\Big|_0^L - EI\left(\frac{d^3u}{dx^3}\delta u\right)\Big|_0^L + \int_0^L \left(EI\left(\frac{d^4u}{dx^4}\right) - mg\right)\delta u\,dx$$

$$EI\frac{d^4u}{dx^4} = mg \quad \text{* Integrate 4 times}$$

$$u(x) = \frac{mg}{EI}\left(\frac{x^4}{24} - \frac{Lx^3}{6} + \frac{L^2x^2}{4}\right)$$

where max deflection occurs @ $x = L$

$$u(L) = \frac{mgL^4}{8EI}$$