Institute for Aerospace Studies
UNIVERSITY OF TORONTO

AER 1403: Project 1
Due: 29 October 2023

Optimal Design of Composite Sandwich Beams

Consider a design problem wherein a gap of length L and width b is to be spanned with a simply-supported sandwich beam. A concentrated load P is to be applied to the centre of the beam; this is a case of three-point bending. The design goal is to determine the beam with the least mass that will satisfy the design requirements.

The materials for the design have already been chosen: a strong, stiff composite to serve as the face material and a polymer foam as the core. The material properties of these constituents will be determined by your student number. Take the last seven digits of your student number. These digits are represented here by $TUVWXYZ$, where each letter corresponds to one digit. The material properties are given by the following table. Note that, for example, if $T = 6$ and $U = 2$, $TU = 62$, not 12.

Face material	Density (kg / m^3)	ρ_f	$1600 + 30(TU)$
	Young's modulus (GPa)	E_f	$40 + (VW)$
	Compressive strength (MPa)	σ_f	$200 + 100(X)$
Core material	Density (kg / m^3)	ρ_c	$20 + 5(YZ)$
	Compressive strength (MPa)	σ_c	$0.5 + 6.5\left(\frac{YZ}{100}\right)^{\frac{3}{2}}$
	Shear strength (MPa)	τ_c	$0.5 + 4.5\left(\frac{YZ}{100}\right)^{\frac{3}{2}}$

The beam is subject to three mechanisms of failure: elastic indentation, core shear and face microbuckling; the expressions provided in class govern these mechanisms. Calculate and plot a failure mechanism map[1] in the design space determined by the non-dimensional terms face thickness / core thickness (t/c) and core thickness / length (c/L), dividing the design space into the appropriate regions. Calculate the trajectory of optimal design in this design space with your material properties. Use the non-dimensional performance indices:

$$\hat{M} = \frac{M}{bL^2\rho_f}; \quad \hat{P} = \frac{P}{bL\sigma_f};$$

[1]For a detailed explanation of sandwich beam failure mechanism maps, see, amongst others, Gibson and Ashby, *Cellular Solids*.

where M is the mass of the beam:

$$M = 2bLt\rho_f + bLc\rho_c.$$

Recall that the condition for optimality is:

$$\nabla\hat{M} = \lambda\nabla\hat{P},$$

if the optimal occurs when one failure mechanism is active; otherwise the optimal occurs on the boundary between two failure mechanisms. Draw the trajectory of optimal design on the failure mechanism map. On a different graph, plot the relationship between $\hat{M}$ and $\hat{P}$ for the optimal trajectory; that is, what is the minimum mass index for a required load index? Finally, if $P = 75$ kN, $L = 2.5$ m and $b = 200$ mm, what is the optimal choice of face thickness and core thickness for your material combination?

To complete this design project, submit six things:

1. a failure mechanism map for these materials,
2. the trajectory of optimal design on the mechanism map,
3. a plot of the minimum mass as a function of the applied load: $\hat{M}$ vs $\hat{P}$,
4. the optimal design for the example beam,
5. an explanation of your work with enough equations and description to show how you arrived at your answers, and
6. any codes you use to generate your results.