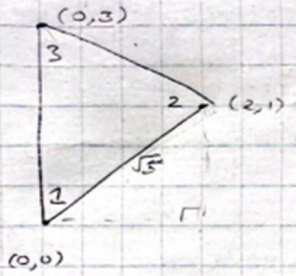


Question 2

Assignment 7  
Question 2



$E = 72 \text{ GPa}$   
 $\nu = 0.3$   
 plane stress  
 $D = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix}$   
 $= \frac{72 \times 10^9}{1 - (0.3)^2} \begin{bmatrix} 1 & 0.3 & 0 \\ 0.3 & 1 & 0 \\ 0 & 0 & 0.35 \end{bmatrix} = 7.912 \times 10^{10} \begin{bmatrix} 1 & 0.3 & 0 \\ 0.3 & 1 & 0 \\ 0 & 0 & 0.35 \end{bmatrix}$

$A = \frac{1}{2}bh$   
 $= \frac{1}{2}(2)(3)$   
 $= 3.354$

$(x_1, y_1) = (0, 0)$   
 $(x_2, y_2) = (2, 1)$   
 $(x_3, y_3) = (0, 3)$

$\epsilon(x, y) = \nabla_\epsilon u(x, y) \approx \nabla_\epsilon N(x, y)d = Hd$

shape functions:  $N_1 = \frac{1}{2A}((x_2 y_3 - y_2 x_3) + (y_2 - y_3)x + (x_3 - x_2)y)$   
 $= \frac{1}{2(3.354)}(((2)(3) - 1)(0) + (1 - 3)x + (0 - 2)y)$   
 $= 0.745 - 3.354x - 0.298y$

$N_2 = \frac{1}{2A}((x_3 y_1 - x_1 y_3) + (y_3 - y_1)x + (x_1 - x_3)y)$   
 $= \frac{250}{1677}((0)(0) - (0)(3) + (3 - 0)x + (0 - 0)y)$   
 $= 0.447x$

$N_3 = \frac{1}{2A}((x_1 y_2 - x_2 y_1) + (y_1 - y_2)x + (x_2 - x_1)y)$   
 $= \frac{250}{1677}((0)(1) - (2)(0) + (0 - 1)x + (2 - 0)y)$   
 $= -0.149x + 0.298y$

$H = \begin{bmatrix} \frac{\partial N_1}{\partial x} & 0 & \frac{\partial N_2}{\partial x} & 0 & \frac{\partial N_3}{\partial x} & 0 \\ 0 & \frac{\partial N_1}{\partial y} & 0 & \frac{\partial N_2}{\partial y} & 0 & \frac{\partial N_3}{\partial y} \\ \frac{\partial N_1}{\partial y} & \frac{\partial N_1}{\partial x} & \frac{\partial N_2}{\partial y} & \frac{\partial N_2}{\partial x} & \frac{\partial N_3}{\partial y} & \frac{\partial N_3}{\partial x} \end{bmatrix}$

$H = \begin{bmatrix} -3.354 & 0 & 0.447 & 0 & -0.149 & 0 \\ 0 & -0.298 & 0 & 0 & 0 & 0.298 \\ -0.298 & -3.354 & 0 & 0.447 & 0.298 & -0.149 \end{bmatrix}$

$k_e = \int_A H^T D H dA = \int_A H^T D H dA = H^T D H A$   
 \* solved on Matlab \*

Ke =

1.0e+12 \*

2.9935	0.1724	-0.3979	-0.0124	0.1244	-0.0754
0.1724	1.0684	-0.0106	-0.1392	-0.0893	0.0229
-0.3979	-0.0106	0.0530	0	-0.0177	0.0106
-0.0124	-0.1392	0	0.0186	0.0124	-0.0062
0.1244	-0.0893	-0.0177	0.0124	0.0141	-0.0077
-0.0754	0.0229	0.0106	-0.0062	-0.0077	0.0256

Pa\*m

MATLAB Code:

```
H= [-3.354 0 0.447 0 -0.149 0; 0 -0.298 0 0 0.298; -0.298 -3.354 0 0.447 0.298 -0.149];  
Ht=transpose(H);  
A= 3.354;  
D= (7.912*10^10) .* [1 0.3 0; 0.3 1 0; 0 0 0.35];  
  
Ke= Ht*D*H.*A %%in terms of (Pa*m)
```