Assignment 3
Arjun Posarajah (1004881737)
Oct 8, 2023
Question 1

[60/45/0/30]

$t = 0.125mm$

$E_1 = 131\,GPa$ $\quad G_{12} = 5.8\,GPa$

$E_2 = 9.8\,GPa$ $\quad \nu_{12} = 0.22$

$Q_{11} = 131.48$

$Q_{22} = 9.8356$

$Q_{12} = 2.1638$

$Q_{66} = 5.8$

$$Q = \begin{bmatrix} 131.48 & 2.1638 & 0 \\ 2.1638 & 9.8356 & 0 \\ 0 & 0 & 5.8 \end{bmatrix} Gpa$$

ply 1 $\theta = 60°$

$$\bar{Q}_1 = \begin{bmatrix} 18.91 & 23.498 & 14.0184 \\ 23.498 & 79.7314 & 38.6534 \\ 14.084 & 38.6534 & 27.1345 \end{bmatrix} Gpa$$

ply 2 $\theta = 45°$

$$\bar{Q}_2 = \begin{bmatrix} 42.2098 & 30.6098 & 30.4101 \\ 30.6098 & 42.2098 & 30.4101 \\ 30.4101 & 30.4101 & 34.2460 \end{bmatrix} Gpa$$

ply 3 $\theta = 0°$

$$\bar{Q}_3 = \begin{bmatrix} 131.476 & 2.1638 & 0 \\ 2.1638 & 9.8356 & 0 \\ 0 & 0 & 5.8 \end{bmatrix} Gpa$$

ply 4 $\theta = 30°$

$$\bar{Q}_4 = \begin{bmatrix} 79.7314 & 23.4983 & 38.6534 \\ 23.4983 & 18.9112 & 14.0184 \\ 38.6534 & 14.0184 & 27.1345 \end{bmatrix} Gpa$$

Assignment 3 Question 1 cont'd

laminate stiffness Matrix

$$A_{ij} = \begin{bmatrix} 34.0411 & 9.9713 & 10.3852 \\ 9.9713 & 18.836 & 10.3852 \\ 10.3852 & 10.3852 & 11.7894 \end{bmatrix} \text{ Gpa} \cdot \text{mm}$$

$$B_{ij} = \begin{bmatrix} 3.6669 & 1.3575 & 1.4721 \\ 1.3575 & 2.7185 & 1.4721 \\ 1.4721 & 1.4721 & 1.5848 \end{bmatrix} \text{ Gpa} \cdot \text{mm}^2$$

$$D_{ij} = \begin{bmatrix} 0.5626 & 0.2355 & 0.2598 \\ 0.2355 & 0.4834 & 0.2598 \\ 0.2598 & 0.2598 & 0.2734 \end{bmatrix} \text{ Gpa} \cdot \text{mm}^3$$

$$Q_{ij} = \begin{bmatrix} 34.04 & 9.97 & 10.3852 & 3.6689 & 1.3575 & 1.4721 \\ 9.97 & 18.836 & 10.3852 & 1.3575 & 2.785 & 1.4721 \\ 10.385 & 10.385 & 11.789 & 1.4721 & 1.4721 & 1.5848 \\ 3.6659 & 1.3575 & 1.4721 & 0.5626 & 0.2355 & 0.2598 \\ 1.3575 & 2.7185 & 1.4721 & 0.2355 & 0.4834 & 0.2598 \\ 1.4721 & 1.4721 & 1.5848 & 0.2598 & 0.2598 & 0.2734 \end{bmatrix} \text{ Gpa}$$

Solve laminate stiffness Matrix for Midplane Strains

$F = [15\frac{GN}{m} \; 20\frac{kN}{m} \; 0 \quad 10N \; 5N \; 0]$

$F = Q_{ij} \times \varepsilon_0$

$\varepsilon_0 = [-0.0071 \quad -0.0035 \quad 0.0093 \quad 85234 \quad 4509 \quad -120.3244]$

calculate strains at Lamina Midplane in Global coordinate

$z_3 = 0.0625$ mm

$\varepsilon = [\varepsilon_x^0 + z k_x \quad \varepsilon_y^0 + z k_y \quad \varepsilon_{xy}^0 \; z k_{xy}]$

$$\varepsilon_3 = \begin{bmatrix} -1.7729 \\ -6.3184 \\ 1.7797 \end{bmatrix}$$

calculate stress

$\sigma_k^* = \bar{Q}_k \, \varepsilon_k$

$= \bar{Q}_3 \, \varepsilon_3$

$$\sigma_3^* = \begin{bmatrix} -246.7626 \\ -65.9815 \\ 10.3224 \end{bmatrix} \text{ Mpa}$$

Assignment 3 Question 1 cont'd

calculate lamina stresses in material coordinates

$\theta = 0°$ for ply 3

$\sigma_k = T_k \sigma^*_k$

$$T_k = \begin{bmatrix} \cos^2\theta & \sin^2\theta & 2\cos\theta\sin\theta \\ \sin^2\theta & \cos^2\theta & -2\cos\theta\sin\theta \\ -\sin\theta\cos\theta & \cos\theta\sin\theta & \cos^2\theta - \sin^2\theta \end{bmatrix}$$

$$\sigma_3 = \begin{bmatrix} -246.76 \\ -65.98 \\ -10.322 \end{bmatrix} \text{ Mpa}$$

· Maximum stress Failure criterion

$\sigma_L^+ = 850$ MPa $\sigma_T^+ = 40$ MPa $\tau_{LT} = 75$ MPa

$\sigma_L^- = 700$ MPa $\sigma_T^- = 160$ MPa

check

$\sigma_L^- = -700\text{MPa} < -246.76\text{MPa} < \sigma_L^+ = 850\text{MPa}$ ✓

$\sigma_T^- = -160\text{MPa} < -65\text{ MPa} < 40\text{MPa} = \sigma_T^+$ ✓

$-10.322\text{ MPa} < \tau_{LT} = 75\text{MPa}$ ✓

Tsai-Hill

$$\frac{\sigma_1^2}{\sigma_L^2} - \frac{\sigma_1\sigma_2}{\sigma_L^2} + \frac{\sigma_2^2}{\sigma_T^2} + \frac{\tau_{12}^2}{\tau_{LT}^2} \geq 1$$

$$TH = 0.516 \leq 1$$ ✓

Tsai-wu

$$F_{11} = \frac{1}{\sigma_L^+ \sigma_L^-} \qquad F_{22} = \frac{1}{\sigma_T^+ \sigma_T^-}$$

$$F_1 = \frac{1}{\sigma_L^+} - \frac{1}{\sigma_L^-} \qquad F_2 = \frac{1}{\sigma_T^+} - \frac{1}{\sigma_T^-} \qquad F_{66} = \frac{1}{\tau_{LT}^2}$$

$$F_{11}\sigma_1^2 + F_{22}\sigma_2^2 + F_{66}\tau_{12}^2 + F_1\sigma_1 + F_2\sigma_2 + 2F_{12}\sigma_1\sigma_2 \geq 1$$

$$TW = -0.0516 \leq 1$$ ✓

Code:

```
%%Question 1%%
%Ply Angles
x1=0;
x2=30;
x3=60;
x4=90;
t=0.125;%mm
E1=132;
E2=8.4;
G12=6;
v12=0.25;

%%%Part 1 stiffness matrix in the material coordinate system
S11=1/E1;
S22=1/E2;
S12=(-v12)/E1;
S66=1/G12;

Q11=S22/((S11*S22)-(S12.^2));
Q22=S11/((S11*S22)-(S12.^2));
Q12=(-S12)/((S11*S22)-(S12.^2));
Q66=G12;

Q= [Q11 Q12 0; Q12 Q22 0; 0 0 Q66];
```

```
%%%Part2 stiffness matrix for the same lamina rotated angles from the global coordinate system

%%x1
x1Qbar11= (Q11*((cosd(x1)).^4))+(Q22*((sind(x1).^4)))+ (2*(Q12+(2*Q66)))*((sind(x1)).^2)*((cosd(x1)).^2);
x1Qbar12= (Q11+Q22-(4*Q66))*((cosd(x1)).^2)*((sind(x1)).^2) + (Q12)*(((cosd(x1)).^4)+((sind(x1)).^4));
x1Qbar22=(Q11)*((sind(x1)).^4)+(Q22)*((cosd(x1)).^4)+(2*(Q12+(2*Q66)))*(((sind(x1)).^2)*((cosd(x1)).^2));
x1Qbar16=(Q11-Q12-(2*Q66))*((cosd(x1)).^3)*(sind(x1))-((Q22-Q12-(2*Q66))*(cosd(x1))*(((sind(x1)).^3)));
x1Qbar26=(Q11-Q12-(2*Q66))*(cosd(x1))*((sind(x1)).^3)-(Q22-Q12-(2*Q66))*((cosd(x1)).^3)*(sind(x1));
x1Qbar66= ((Q11+Q22-(2*Q12)-(2*Q66))*((cosd(x1)).^2)*((sind(x1)).^2))+((Q66)*(((cosd(x1)).^4)+((sind(x1)).^4)));

x1Qbar= [x1Qbar11 x1Qbar12 x1Qbar16; x1Qbar12 x1Qbar22 x1Qbar26; x1Qbar16 x1Qbar26 x1Qbar66];

%%x2
x2Qbar11= (Q11*((cosd(x2)).^4))+(Q22*((sind(x2).^4)))+ (2*(Q12+(2*Q66)))*((sind(x2)).^2)*((cosd(x2)).^2);
x2Qbar12= (Q11+Q22-(4*Q66))*((cosd(x2)).^2)*((sind(x2)).^2) + (Q12)*(((cosd(x2)).^4)+((sind(x2)).^4));
x2Qbar22=(Q11)*((sind(x2)).^4)+(Q22)*((cosd(x2)).^4)+(2*(Q12+(2*Q66)))*(((sind(x2)).^2)*((cosd(x2)).^2));
x2Qbar16=(Q11-Q12-(2*Q66))*((cosd(x2)).^3)*(sind(x2))-((Q22-Q12-(2*Q66))*(cosd(x2))*(((sind(x2)).^3)));
x2Qbar26=(Q11-Q12-(2*Q66))*(cosd(x2))*((sind(x2)).^3)-(Q22-Q12-(2*Q66))*((cosd(x2)).^3)*(sind(x2));
x2Qbar66= ((Q11+Q22-(2*Q12)-(2*Q66))*((cosd(x2)).^2)*((sind(x2)).^2))+((Q66)*(((cosd(x2)).^4)+((sind(x2)).^4)));

x2Qbar= [x2Qbar11 x2Qbar12 x2Qbar16; x2Qbar12 x2Qbar22 x2Qbar26; x2Qbar16 x2Qbar26 x2Qbar66];


%%x3
x3Qbar11= (Q11*((cosd(x3)).^4))+(Q22*((sind(x3).^4)))+ (2*(Q12+(2*Q66)))*((sind(x3)).^2)*((cosd(x3)).^2);
x3Qbar12= (Q11+Q22-(4*Q66))*((cosd(x3)).^2)*((sind(x3)).^2) + (Q12)*(((cosd(x3)).^4)+((sind(x3)).^4));
x3Qbar22=(Q11)*((sind(x3)).^4)+(Q22)*((cosd(x3)).^4)+(2*(Q12+(2*Q66)))*(((sind(x3)).^2)*((cosd(x3)).^2));
x3Qbar16=(Q11-Q12-(2*Q66))*((cosd(x3)).^3)*(sind(x3))-((Q22-Q12-(2*Q66))*(cosd(x3))*(((sind(x3)).^3)));
x3Qbar26=(Q11-Q12-(2*Q66))*(cosd(x3))*((sind(x3)).^3)-(Q22-Q12-(2*Q66))*((cosd(x3)).^3)*(sind(x3));
x3Qbar66= ((Q11+Q22-(2*Q12)-(2*Q66))*((cosd(x3)).^2)*((sind(x3)).^2))+((Q66)*(((cosd(x3)).^4)+((sind(x3)).^4)));

x3Qbar= [x3Qbar11 x3Qbar12 x3Qbar16; x3Qbar12 x3Qbar22 x3Qbar26; x3Qbar16 x3Qbar26 x3Qbar66];

%%x4
x4Qbar11= (Q11*((cosd(x4)).^4))+(Q22*((sind(x4).^4)))+ (2*(Q12+(2*Q66)))*((sind(x4)).^2)*((cosd(x4)).^2);
x4Qbar12= (Q11+Q22-(4*Q66))*((cosd(x4)).^2)*((sind(x4)).^2) + (Q12)*(((cosd(x4)).^4)+((sind(x4)).^4));
x4Qbar22=(Q11)*((sind(x4)).^4)+(Q22)*((cosd(x4)).^4)+(2*(Q12+(2*Q66)))*(((sind(x4)).^2)*((cosd(x4)).^2));
x4Qbar16=(Q11-Q12-(2*Q66))*((cosd(x4)).^3)*(sind(x4))-((Q22-Q12-(2*Q66))*(cosd(x4))*(((sind(x4)).^3)));
x4Qbar26=(Q11-Q12-(2*Q66))*(cosd(x4))*((sind(x4)).^3)-(Q22-Q12-(2*Q66))*((cosd(x4)).^3)*(sind(x4));
x4Qbar66= ((Q11+Q22-(2*Q12)-(2*Q66))*((cosd(x4)).^2)*((sind(x4)).^2))+((Q66)*(((cosd(x4)).^4)+((sind(x4)).^4)));

%%%Part3: Calculating A,B,D
A11=x1Qbar11*(t)+ x2Qbar11*(t)+x3Qbar11*(t)+ x4Qbar11*(t);
A12=x1Qbar12*(t)+ x2Qbar12*(t)+x3Qbar12*(t)+ x4Qbar12*(t);
A22=x1Qbar22*(t)+ x2Qbar22*(t)+x3Qbar22*(t)+ x4Qbar22*(t);
A16=x1Qbar16*(t)+ x2Qbar16*(t)+x3Qbar16*(t)+ x4Qbar16*(t);
A26=x1Qbar26*(t)+ x2Qbar26*(t)+x3Qbar26*(t)+ x4Qbar26*(t);
A66=x1Qbar66*(t)+ x2Qbar66*(t)+x3Qbar66*(t)+ x4Qbar66*(t);

B11= 0.5*((x1Qbar11*(-3/64))+(x2Qbar11*(-1/64))+(x3Qbar11*(1/64))+(x4Qbar11*(3/64)));
B12= 0.5*((x1Qbar12*(-3/64))+(x2Qbar12*(-1/64))+(x3Qbar12*(1/64))+(x4Qbar12*(3/64)));
B22= 0.5*((x1Qbar22*(-3/64))+(x2Qbar22*(-1/64))+(x3Qbar22*(1/64))+(x4Qbar22*(3/64)));
B16= 0.5*((x1Qbar16*(-3/64))+(x2Qbar16*(-1/64))+(x3Qbar16*(1/64))+(x4Qbar16*(3/64)));
B26= 0.5*((x1Qbar26*(-3/64))+(x2Qbar26*(-1/64))+(x3Qbar26*(1/64))+(x4Qbar26*(3/64)));
B66= 0.5*((x1Qbar66*(-3/64))+(x2Qbar66*(-1/64))+(x3Qbar66*(1/64))+(x4Qbar66*(3/64)));

D11=(1/3)*((x1Qbar11*(7/512))+(x2Qbar11*(1/512))+(x3Qbar11*(1/512))+(x4Qbar11*(7/512)));
D12=(1/3)*((x1Qbar12*(7/512))+(x2Qbar12*(1/512))+(x3Qbar12*(1/512))+(x4Qbar12*(7/512)));
D22=(1/3)*((x1Qbar22*(7/512))+(x2Qbar22*(1/512))+(x3Qbar22*(1/512))+(x4Qbar22*(7/512)));
D16=(1/3)*((x1Qbar16*(7/512))+(x2Qbar16*(1/512))+(x3Qbar16*(1/512))+(x4Qbar16*(7/512)));
D26=(1/3)*((x1Qbar26*(7/512))+(x2Qbar26*(1/512))+(x3Qbar26*(1/512))+(x4Qbar26*(7/512)));
D66=(1/3)*((x1Qbar66*(7/512))+(x2Qbar66*(1/512))+(x3Qbar66*(1/512))+(x4Qbar66*(7/512)));

%%%Part4 Laminate Stiffness Matrix
A= [A11 A12 A16; A12 A22 A26; A16 A26 A66];
B= [B11 B12 B16; B12 B22 B26; B16 B26 B66];
D= [D11 D12 D16; D12 D22 D26; D16 D26 D66];

Qij=[A11 A12 A16 B11 B12 B16; A12 A22 A26 B12 B22 B26;A16 A26 A66 B16 B26 B66; B11 B12 B16 D11 D12 D16 ;B12 B22 B26 D12 D22 D26; B16 B26 B66 D16 D26 D66]

%Part5 Solve Laminate Stiffness Matrix for Midplane Strains
F=[10 ;5; 0 ;10; 0 ;0];
Eo1=linsolve(Qij,F);
Eo= [.001; .001; 0.001; 1; 1; 1].* Eo1 %convert the KN/m to N/m
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%Part6 Calculate Strains at Lamina Mid-Planes in Global Coordinates
%%%%midplane is 0.0625mm

y3= 0.0625;

E3=[5.9+(y3*41.6103) ; (y3*-1.5344); -3.7+(y3*-3.0309)]


%Part7 Calculate Stress
Ss3= x3Qbar*E3

%Part8 Calculate Lamina Stresses in Material Coordinates
Tk3= [(cosd(x3))^2 (sind(x3))^2 2*(cosd(x3))*(sind(x3));(sind(x3))^2 (cosd(x3))^2 -2*(cosd(x3))*(sind(x3)); -1*(cosd(x3))*(sind(x3)) (cosd(x3))*(sind(x3)) ((sind(x3))^2
S3= Tk3*Ss3


%Part9 Maximum Stress Failure Criterion

%%Compare actual values no math needed on paper

%Part10 Tsai Hill
sigmaLplus=850;
sigmaLminus=700;
sigmaTplus=40;
sigmaTminus=100;
TLT= 75;

Tsaihill=((S3(1,1).^2)/(sigmaLplus .^2))-(((S3(1,1))*(S3(2,1)))/(sigmaLplus .^2))+(((S3(2,1)).^2)/(sigmaTminus .^2))+(((S3(3,1)).^2)/(TLT.^2))

%Part11 Tsai Wu
F11= 1/((sigmaLplus)*(sigmaLminus))
F22= 1/((sigmaTplus)*(sigmaTminus))
F1= (1/(sigmaLplus))-(1/(sigmaLminus))
F2= (1/(sigmaTplus))-(1/(sigmaTminus))
F66= 1/(TLT.^2);
F12= (-1/2)*(sqrt(1/(sigmaLplus*sigmaLminus*sigmaTplus*sigmaTminus)))

TsaiWu= (F11*(S3(1,1).^2))+(F22*(S3(2,1).^2))+(F66*(S3(3,1).^2))+(F1*S3(1,1))+(F2*S3(2,1))+ 2*(F12*S3(1,1)*S3(2,1))
```