Numerical Analysis for Engineering Final Submission

Question 1

$$\frac{dc}{dt} = -\exp\left(\frac{-10}{T+273}\right)c \quad \text{——— ①}$$

$$\frac{dT}{dt} = 1000\exp\left(\frac{-10}{T+273}\right)c - 10(T-20) \quad \text{——— ②}$$

Initial:

$t=0$, $T = 15°c$ $\quad C = 1$ gmol/L

step size $h = 0.0625$ s

(1) Euler's Method

$$\frac{dx_n}{dt} = \frac{x_{n+1} - x_n}{h}$$

$$\frac{C_{n+1} - C_n}{h} = -\exp\left(\frac{-10}{T_n + 273}\right)C_n$$

$$C_{n+1} = C_n - h\cdot\exp\left(\frac{-10}{T_n+273}\right)C_n$$

$$C_{n+1} = \left[1 - h\cdot\exp\left(\frac{-10}{T_n+273}\right)\right]C_n$$

$$\frac{T_{n+1} - T_n}{h} = 1000\exp\left(\frac{-10}{T_n+273}\right)C_n - 10(T_n - 20)$$

Iteration 1

$T_0 = 15°c \quad C_0 = 1$ gmol/L

$n=0$

$$C_1 = \left[1 - 0.0625\left(e\left(\frac{-10}{T_0+273}\right)\right)\right](1)$$

$$C_1 = 0.9396$$

$$T_1 = T_0 + h\left(1000\exp\left(\frac{-10}{T_0+273}\right)C_0 - 10(T_0-20)\right)$$

$$T_1 = 15 + 0.0625\left(1000\exp\left(-\frac{10}{15+273}\right)(1) - 10(15-20)\right)$$

$$T_1 = 78.4921$$

Iteration 2

$n=1$

$$C_2 = \left[1 - h\exp\left(\frac{-10}{T_1+273}\right)\right]C_1 = \left[1 - 0.0625\exp\left(\frac{-10}{78.492+273}\right)\right]\times 0.9396$$

$$= 0.8825$$

$$T_2 = 78.4921 + 0.0625\left[1000\exp\left(\frac{-10}{78.4921+273}\right)\times 0.9396 - 10(78.4921-20)\right]$$

$$T_2 = 99.0123$$

Question 1 cont'd

(2) Midpoint Method

$$\frac{dy}{dx} = f(x,y)$$

$$k_1 = h f(x_n, y_n)$$

$$k_2 = h f\left(x_n + \frac{h}{2}, y_n + \frac{k_1}{2}\right)$$

$$y_{n+1} = y_n + k_2$$

$$k_1 = \frac{h\,dc}{dt}$$

$$k_1 = h\left[-\exp\left(\frac{-10}{T_n + 273}\right)\right] C_n$$

$$k_2 = h\left[-\exp\left(\frac{-10}{(T_n + \frac{h}{2}) + 273}\right)\right]\left(C_n + \frac{k_1}{2}\right)$$

$$C_{n+1} = C_n + k_2$$

$$l_1 = \frac{h\,dr}{dt}$$

$$l_1 = h\left[1000 \exp\left(\frac{-10}{T_n + 273}\right) C_n - 10(T_n - 20)\right]$$

$$l_2 = h\left[1000 \exp\left(\frac{-10}{(T_n + l_1/2) + 273}\right)\left(C_n + \frac{k_1}{2}\right) - 10\left(\left(T_n + \frac{l_1}{2}\right) - 20\right)\right]$$

$$T_{n+1} = T_n + l_2$$

Iteration 1

$n = 0$

$$k_1 = h\left[-\exp\left(\frac{-10}{T_0 + 273}\right)\right] C_0 = 0.0625\left(-\exp\left(\frac{-10}{15 + 273}\right)\right) \times 1$$

$$k_1 = -0.0604$$

$$l_1 = 0.0625\left[1000 \exp\left(\frac{-10}{15 + 273}\right) - 10(15 - 20)\right)$$

$$l_1 = 63.4921$$

$$k_2 = h\left(-\exp\left(\frac{-10}{(T_0 + l_1/2) + 273}\right)\right)\left(C_0 + \frac{k_1}{2}\right)$$

$$k_2 = -0.0587$$

$$l_2 = h\left(1000 \exp\left(\frac{-10}{(T_0 + l_1/2) + 273}\right)\left(C_0 + \frac{k_1}{2}\right) - 10\left[\left(T_0 + \frac{l_1}{2}\right) - 20\right]\right)$$

$$l_2 = 42.0299$$

$$T_1 = T_0 + l_2 = 57.0299$$

$$C_1 = C_1 + k_2 = 0.9413$$

Question 1 cont'd

(2) Midpoint Method

Iteration 2

$n = 1 \quad k_1 = -0.0571 \quad l_1 = 33.9282$

$k_2 = -0.0554$

$l_2 = 21.6774$

$C_2 = C_1 + k_2 = 0.8858$

$T_2 = T_1 + l_2 = 78.7053$

(3) Runge Kutta Method

$\frac{dy}{dx} = f(x_n, y_n)$

$k_1 = hf(x_n, y_n)$

$k_2 = hf\left(x_n + \frac{h}{2}, y_n + \frac{k_1}{2}\right)$

$k_3 = hf\left(x_n + \frac{h}{2}, y_n + \frac{k_2}{2}\right)$

$k_4 = hf\left(x_n + h, y_n + k_3\right)$

$y_{n+1} = y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

$\frac{dc}{dt} = f(c_n, T_n)$

$k_1 = hf(c_n, T_n)$

$k_2 = hf\left(c_n + \frac{k_1}{2}, T_n + \frac{l_1}{2}\right)$

$k_3 = hf\left(c_n + \frac{k_2}{2}, T_n + \frac{l_2}{2}\right)$

$k_4 = hf\left(c_n + k_3, T_n + l_3\right)$

$\frac{dT}{dt} = g(c_n, T_n)$

$l_1 = hg(c_n, T_n)$

$l_2 = hg\left(c_n + \frac{k_1}{2}, T_n + \frac{l_1}{2}\right)$

$l_3 = hg\left(c_n + \frac{k_2}{2}, T_n + \frac{l_2}{2}\right)$

$l_4 = hg\left(c_4 + k_3, T_n + l_3\right)$

$c_{n+1} = c_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

$T_{n+1} = T_n + \frac{1}{6}(l_1 + 2l_2 + 2l_3 + l_4)$

Question 1 cont'd

(3) Runge Kutta cont'd

Iteration 1

$n = 0$

$k_1 = h\left(-\exp\left(\frac{-10}{T_0 + 273}\right)\right) c_0$

$k_1 = -0.0604$

$l_1 = h\left(1000\exp\left(\frac{-10}{T_0 + 273}\right)c_0 - 10(T_0 - 20)\right)$

$l_1 = 63.4921$

$k_2 = h\left(-\exp\left(\frac{-10}{T_0 + l_1/2 + 273}\right)\right)\left(c_0 + \frac{k_1}{2}\right)$

$k_2 = -0.0587$

$l_2 = h\left(1000\exp\left(\frac{-10}{T_0 + l_1/2 + 273}\right)\left(c_0 + \frac{k_1}{2}\right) - 10\left(T_0 + \frac{l_1}{2} - 20\right)\right)$

$l_2 = 42.0399$

$k_3 = h\left(-\exp\left(\frac{-10}{T_0 + l_2/2 + 273}\right)\right)\left(c_0 + \frac{k_2}{2}\right)$

$k_3 = -0.0587$

$l_3 = h\left(\left(1000\exp\left(\frac{-10}{T_0 + \frac{l_2}{2} + 273}\right)\left(c_0 + \frac{k_2}{2}\right) - 10\left(T_0 + \frac{l_2}{2} - 20\right)\right)\right)$

$l_3 = 48.7228$

$k_4 = h\left(-\exp\left(\frac{-10}{T_0 + l_3 + 273}\right)\right)\left(c_0 + k_3\right)$

$k_4 = -0.0571$

$l_4 = h\left(1000\exp\left(\frac{-10}{T_0 + l_3 + 273}\right)\left(c_0 + k_3\right) - 10\left(T_0 + l_3 - 20\right)\right)$

$l_4 = 29.7822$

$c_1 = c_0 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

$c_1 = 0.9413$

$T_1 = T_0 + \frac{1}{6}(l_1 + 2l_2 + 2l_3 + l_4)$

$T_1 = 60.7988$

Question 1 cont'd

(3) Runge Kutta cont'd

Iteration 2

$n = 1$

T_1 & C_1 is $\Rightarrow$

$R_1 = -0.0571$ $\quad l_1 = 31.59446$

R_1 and l_1 is $\Rightarrow$

$R_2 = -0.0554$ $\quad l_2 = 20.0648$

R_2 & l_2 is $\Rightarrow$

$R_3 = -0.0555$ $\quad l_3 = 23.6915$

R_3 & l_3 is $\Rightarrow$

$R_4 = -0.0538$ $\quad l_4 = 13.5302$

$$C_2 = C_1 + \frac{1}{6}(R_1 + 2R_2 + 2R_3 + R_4)$$

$C_2 = 0.8858$

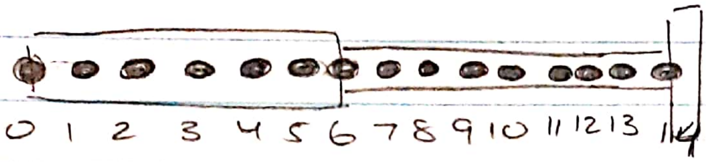
$$T_2 = T_1 + \frac{1}{6}(l_1 + 2l_2 + 2l_3 + l_4)$$

$T_2 = 82.9030$

Question 2

rod: $\frac{d^2T}{dx^2} = 0$

tube: $\frac{d^2T}{dx^2} + \frac{2h}{rK}(T_\infty - T)$



rod $i = 1 \to 5$

$$\frac{T_{i-1} - 2T_i + T_{i+1}}{\Delta x^2} = 0$$

Tube $i = 7 \to 13$

$$\frac{T_{i-1} - 2T_i + T_{i+1}}{\Delta x^2} + \frac{2h}{rK_2}(T_\infty - T) = 0$$

$T = T_0$ so for node 1

$$2T_1 - T_2 = T_0$$

For node $2 \to 5$:

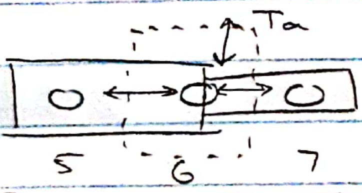
$$-T_1 + 2T_2 - T_3 = 0$$
$$-T_2 + 2T_3 - T_4 = 0$$
$$-T_3 + 2T_4 - T_5 = 0$$
$$-T_4 + 2T_5 - T_6 = 0$$

For nodes $7 \to 13$:

$$-T_{i+1} + \left(2 + \frac{2h\,\Delta x^2}{rK_2}\right)T_i - T_{i-1} = \frac{2h\,\Delta x^2}{rK_2}T_\infty$$

$$-T_{i+1} + \left(2 + \frac{2\times 3000\,(0.05)^2}{0.03\times 0.615}\right)T_i + T_{i+1} = \frac{2(3000)(0.05)^2}{(0.03)\ 0.615}T_\infty$$

$$-T_{i+1} + 3254\,T_i - T_{i+1} = 975610$$



$A_c = \pi r^2$

$A_s = \pi r \Delta x$

$$-k_1\frac{T_6 - T_5}{\Delta x}A_c - k_2\frac{T_6 - T_7}{\Delta x}A_c + h(T_\infty - T_6)A_s = 0$$

$$-k_1\frac{T_6 - T_5}{\Delta x}\pi r^2 - k_2\frac{T_6 - T_7}{\Delta x}\pi r^2 + h(T_\infty - T_6)\pi r\Delta x = 0$$

$$-k_1 T_5 + \left(k_1 + k_2 + \frac{h}{r}\Delta x^2\right)T_6 - k_2 T_7 = \frac{h}{r}\Delta x^2 T_\infty$$

$$80.2T_5 + 1080.8T_6 - 0.615T_7 = 300000$$

Question 2 cont'd

T_{13} $\Delta x/2$ T_{14} T_∞

$$0 = -k_2 \frac{T_{14} - T_{13}}{\Delta x} A_c + h(T_\infty - T_{14}) A_s$$

$$-T_{13} + \left(1 + \frac{h\Delta x^2}{k_2 r}\right) T_{14} = \frac{h \Delta x^2}{k_2 r} T_\infty$$

$$-T_{13} + 1627\, T_{14} = 487805$$

$T_1 = 383.553$

$T_2 = 367.106$

$T_3 = 350.659$

$T_4 = 334.212$

$T_5 = 301.318$

$T_6 = 300.0004$

$T_7 = 300$

$T_8 = 300$

$T_9 = 300$

$T_{10} = 300$

$T_{11} = 300$

$T_{12} = 300$

$T_{13} = 317.765$

$T_{14} = 300$

$[T_1 \; T_2 \; T_3 \; T_4 \; T_5 \; T_6 \; T_7 \; T_8 \; T_9 \; T_{10} \; T_{11} \; T_{12} \; T_{13} \; T_{14}]^T = [400 \; 0 \; 0 \; 0 \; 0 \; 300000 \; 975610 \; 975610 \; 97561 \; 97561 \; 97561 \; 97561 \; 97561 \; 487805]^T$