Assignment 7

Question 1)

1. a) Excel

b) $\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1}$

$x = 10 \quad @ \; y = 13.07$

$x = 5 \quad @ \; y = 1.519$

$$\frac{10-7.5}{10-5} = \frac{1.307-\mu_{7.5}}{1.307-1.519}$$

$$\mu_{7.5} = 1.413 \times 10^{-3} \; Ns/m^2$$

c)

x_i	y_i	$(y_i-\bar{y})^2$	$(y_i-a_0-a_1x_i-a_2x_i^2)^2$
0	1.787	27.862	
5	1.519	30.7626	
10	1.307	33.1592	
20	1.002	36.7648	
30	0.7975	39.2866	
40	0.6579	41.1202	
Σ	7.0654	208.9554	

$n = 6$ $\quad \Sigma x_i = 105$ $\quad \Sigma x_i^4 = 3540625$

$n = 6$ $\quad \Sigma y_i = 7.0654$ $\quad \Sigma x_i y_i = 90.746$

$\bar{x} = 17.5$ $\quad \Sigma x_i^2 = 3025$ $\quad \Sigma x_i^2 y_i = 2331.865$

$\bar{y} = 1.178$ $\quad \Sigma x_i^3 = 100125$

$$\begin{bmatrix} 6 & 105 & 3025 \\ 105 & 3025 & 100125 \\ 3025 & 100125 & 3540625 \end{bmatrix} \begin{Bmatrix} a_0 \\ a_1 \\ a_2 \end{Bmatrix} = \begin{Bmatrix} 7.0654 \\ 90.746 \\ 2331.865 \end{Bmatrix}$$

$$y = 1.7672 \times 10^{-3} - 0.0495 \times 10^{-3} x + 0.005 \times 10^{-3} x^2$$

d) $f(7.5) = 1.7672 \times 10^{-3} - 0.0495 \times 10^{-3}(7.5) + 0.005 \times 10^{-3}(7.5)^2$

$= 1.43 \times 10^{-3} \; Ns/m^2$

Assignment 7

Question 2)

$$-\log 10.\ K_w = \frac{a}{T_a} + b.\log 10.T_a + C.T_a + al$$

matlab

Question 3)

a) Estimate temp at $x = 4$, $y = 3.2$

Extrapolation $\quad \frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1}$

at $x = y$

$y = 2$ $\qquad$ $y = 4$

$T = 53.50$ $\qquad$ $T = 38.43$

$\therefore$ at $y = 3.2 \Rightarrow \frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1}$

$$\frac{T - 53.50}{38.43 - 53.50} = \frac{3.2 - 2}{4 - 2}$$

$$2T - 107 = -18.054$$

$$T = 44.458\,^{\circ}C$$

b) $x = 4.3 \qquad y = 2.7$

$x = 4 \qquad x = 6$

$y = 2 \qquad y = 4$

At $x = 4$: $\qquad y_1 = 2,\ y = 4$ set $y = 2.7$

$T_1 = 53.50 \qquad T_2 = 38.43$

$$\frac{y - y_1}{y_2 - y_1} = \frac{T - T_1}{T_2 - T_1} \qquad \frac{2.7 - 2}{4 - 2} = \frac{T - 53.50}{38.43 - 53.50}$$

$$0.7(38.43 - 53.50) = 2(T - 53.50)$$

$$T = 48.2255$$

Question 3):

At $x = 6$ ($y_1 = 2$, $y_2 = 4$) set $y = 2.7$

$T_1 = 48.15$ $T_2 = 35.03$

$$\frac{2.7-2}{4-2} = \frac{T - 48.15}{48.15 - 35.03}$$

$$0.7(48.15 - 35.03) = 2(T - 48.15)$$

$$T = 43.558\,^{\circ}C$$

At $y = 2.7$ $x_1 = 4$ $x_2 = 6$ set $x = 4.3$

$T_1 = 48.225$ $T_2 = 43.558$

$$\frac{4.3-4}{6-4} = \frac{T - 48.225}{43.558 - 48.225}$$

$$T = 47.525\,^{\circ}C$$

Assignment 7

Question 4):

ORDER 1

$f(x) = b_1 + b_2(x - x_1)$

$b_1 = f(x_1)$

$b_2 = \dfrac{f(x_2) - f(x_1)}{x_2 - x_1} = f[x_2, x_1]$

$x = 3.5 = b_1$

$b_2 = \dfrac{2.375 - 3.5}{4.5 - 3} = -0.75$

$f(x) = 3.5 - 0.75(x - 3)$

$f(3.5) = 3.125$

ORDER 2

$x = 2.5, 3, 4, 5$

$b_1 = 5.375$

$b_2 = f[x_2, x_1]$

$= \dfrac{3.5 - 5.375}{3 - 2.5} = -3.75$

$f[x_3, x_2] = \dfrac{2.375 - 3.5}{4.5 - 3} = -0.75$

$b_3 = \dfrac{-0.75 + 3.75}{4.5 - 2.5} = 1.5$

$f(x) = 5.375 - 3.75(x - 2.5) + 1.5(x - 2.5)(x - 3)$

$f(3.5) = 2.375$

Question 4) :

Order 3

$f(x) = b_1 + b_2(x - x_1) + b_3(x - x_1)(x - x_2) + b_4(x - x_1)(x - x_2)(x - x_3)$

$x = 2.5, 3, 4.5, 5$

$[x_2, x_1] = -3.75$

$[x_3, x_2] = -0.75$

$[x_4, x_3] = \frac{3.5 - 2.375}{5 - 4.5} = 2.25$

$[x_3, x_2, x_1] = \frac{-0.75 + 3.75}{4.5 - 2.5} = 1.5$

$[x_4, x_3, x_2] = \frac{0.75 + 0.75}{5 - 3} = 0.75$

Third order Difference

$[x_4, x_3, x_2, x_1] = \frac{0.75 - 1.5}{5 - 2.5} = -0.3$

$f(3.5) = 2.525$

ORDER 4

$f(x) = b_1 + b_2(x - x_1) + b_3(x - x_1)(x - x_2) + b_4(x - x_1)(x - x_2)(x - x_3) + b_5(x - x_1)(x - x_2)(x - x_3)(x - x_4)$

$x = 1, 2.5, 3, 4, 5.5$

$[x_2, x_1] = -6.75$

$[x_3, x_2] = -3.75$

$[x_4, x_3] = -0.75$

$[x_5, x_4] = 2.25$

$[x_3, x_2, x_1] = 1.5$

$[x_4, x_3, x_2] = 1.5$

$[x_5, x_4, x_3] = 0.75$

$[x_4, x_3, x_2, x_1] = 0$

$[x_5, x_4, x_3, x_2] = -0.3$

$\Rightarrow [x_5, x_4, x_3, x_2, x_1] = -0.075$

$f(4) = 2.328125$

$\therefore 2.328$