9.3 Given the I/O equation

$$2\dot{y} + 10y = 3u$$

compute the frequency response $y_{ss}(t)$ for the input $u(t) = \ 18\sin 4t$.

9.3 First, we determine the sinusoidal transfer function of the given I/O equation:

$$G(s) = \frac{Y(s)}{U(s)} = \frac{3}{2s+10} \quad \rightarrow \text{ the sinusoidal transfer function is } G(j\omega) = \frac{3}{10 + j2\omega}$$

We know from Eq. (9-17) that the frequency response has the form

$$y_{ss}(t) = |G(j\omega)| U_0 \sin(\omega t + \phi) \qquad \text{where } \omega = 4 \text{ rad/s, } U_0 = 18$$

We use the sine function because the input is a sine function. Finally, we need the magnitude and phase angle of the sinusoidal transfer function $G(j\omega)$ at input frequency $\omega = 4$ rad/s

Magnitude: $|G(j4)| = \dfrac{\sqrt{3^2+0^2}}{\sqrt{10^2+8^2}} = \dfrac{3}{\sqrt{164}} = 0.2343$

Phase angle: $\phi = \arg[3] - \arg[10 + j8] = \tan^{-1}\left(\dfrac{0}{3}\right) - \tan^{-1}\left(\dfrac{8}{10}\right) = -38.66 \text{ deg or } -0.6747 \text{ rad}$

Because the input magnitude is $U_0 = 18$, the frequency response is

$$\boxed{y_{ss}(t) = 4.2167\sin(4t - 0.6747)}$$

9.8 Given the transfer function

$$G(s) = \frac{2s+6}{s^3+2s+26} = \frac{Y(s)}{U(s)}$$

compute the frequency response $y_{ss}(t)$ for the input $u(t) = 0.6\sin 3t$.

9.8 The sinusoidal transfer function is

$$G(j\omega) = \frac{2j\omega+6}{(j\omega)^3+2j\omega+26} \quad \text{or} \quad G(j\omega) = \frac{6+j2\omega}{26+j(2\omega-\omega^3)}$$

We know from Eq. (9-17) that the frequency response has the form

$$y_{ss}(t) = |G(j\omega)|U_0 \sin(\omega t+\phi) \qquad \text{where } \omega = 3 \text{ rad/s, } U_0 = 0.6$$

We use sine because the input is a sine function. We need the magnitude and phase angle:

Magnitude: $|G(j3)| = \dfrac{\sqrt{6^2+6^2}}{\sqrt{26^2+(-21)^2}} = \dfrac{\sqrt{72}}{\sqrt{1117}} = 0.2539$

Phase angle: $\phi = \dfrac{\arg[6+j6]}{\arg[26-j21]} = \tan^{-1}\left(\dfrac{6}{6}\right) - \tan^{-1}\left(\dfrac{-21}{26}\right) = 45 - -38.93 = \ 83.93 \text{ deg}$

Because the input magnitude is $U_0 = 0.6$, the frequency response is

$$\boxed{y_{ss}(t) = 0.1523\sin(3t+1.4648)}$$

9.11 Figure P9.11 shows a 1-DOF mechanical system driven by the displacement of the left end, $x_{in}(t)$, which could be supplied by a rotating cam and follower (see Problem 2.2). When displacements $x_{in}(t) = 0$ and $x = 0$ the spring k is neither compressed nor stretched. The system parameters are $m = 2$ kg, $k = 500$ N/m, and $b = 20$ N-s/m. Determine the frequency response if the position input is $x_{in}(t) = 0.04 \sin 50t$ m.

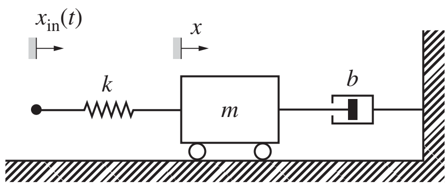

Figure P9.11

9.11 The mathematical model of the mechanical system is $m\ddot{x} + b\dot{x} + kx = kx_{in}(t)$

$$\rightarrow \text{ the transfer function is } G(s) = \frac{X(s)}{X_{in}(s)} = \frac{k}{ms^2 + bs + k}$$

Using m = 2 kg, b = 20 N-s/m and k = 500 N/m the sinusoidal transfer function becomes

$$G(j\omega) = \frac{500}{500 - 2\omega^2 + j20\omega}$$

We know from Eq. (9-17) that the frequency response has the form

$$x_{ss}(t) = |G(j\omega)| U_0 \sin(\omega t + \phi) \qquad \text{where } \omega = 50 \text{ rad/s}, U_0 = 0.04$$

We use sine because the input is a sine function. We need the magnitude and phase angle:

Magnitude: $$|G(j50)| = \frac{\sqrt{500^2 + 0^2}}{\sqrt{(-4500)^2 + 1000^2}} = \frac{500}{\sqrt{2.125(10^7)}} = 0.1085$$

Phase angle: $$\phi = \frac{\arg[500 + j0]}{\arg[-4500 + j1000]} = \tan^{-1}\left(\frac{0}{500}\right) - \tan^{-1}\left(\frac{1000}{-4500}\right) = -167.47 \text{ deg}$$

Because the input magnitude is $U_0 = 0.04$, the frequency response is

$$\boxed{x_{ss}(t) = 0.0043 \sin(50t - 2.9229) \quad \text{m}}$$

9.13 Figure P9.13 shows a vibration isolation system for a 1-DOF mechanical system. Displacement of the mass x is measured from the static equilibrium position and the system parameters are $m = 0.6$ kg, $k = 80$ N/m, $b_1 = 3$ N-s/m, and $b_2 = 0.4$ N-s/m.

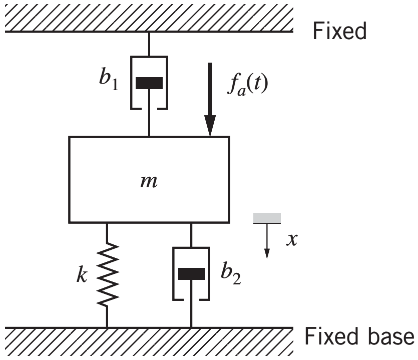

Figure P9.13

a. Compute the frequency response $x_{ss}(t)$ if the input force is $f_a(t) = 2 \sin 8t$ N.
b. Determine the input frequency ω that results in the largest steady-state amplitude of the mass displacement.

9.13 The mathematical model of the mechanical system is $m\ddot{x} + (b_1 + b_2)\dot{x} + kx = f_a(t)$

The transfer function and sinusoidal transfer function are

$$G(s) = \frac{X(s)}{F_a(s)} = \frac{1}{0.6s^2 + 3.4s + 80} \quad \text{and} \quad G(j\omega) = \frac{1}{80 - 0.6\omega^2 + j3.4\omega}$$

a) We know from Eq. (9-17) that the frequency response has the form

$$x_{ss}(t) = |G(j\omega)| U_0 \sin(\omega t + \phi) \qquad \text{where } \omega = 8 \text{ rad/s, } U_0 = 2$$

We use sine because the input is a sine function. We need the magnitude and phase angle:

Magnitude: $|G(j8)| = \dfrac{\sqrt{1^2 + 0^2}}{\sqrt{41.6^2 + 27.2^2}} = \dfrac{1}{\sqrt{2470.4}} = 0.0201$

Phase angle: $\phi = \dfrac{\arg[1 + j0]}{\arg[41.6 + j27.2]} = \tan^{-1}\left(\dfrac{0}{1}\right) - \tan^{-1}\left(\dfrac{27.2}{41.6}\right) = -33.1785$ deg

Because the input magnitude is $U_0 = 2$, the frequency response is

$$\boxed{x_{ss}(t) = 0.0402 \sin(8t - 0.5791) \quad \text{m}}$$

b) The **largest** output amplitude will occur when $\omega = \omega_r$ (resonant frequency). The resonant frequency is .

For the system we have 11.5470 rad/s and $\zeta = (b_1 + b_2)/2\sqrt{km} = 0.2454$

The resonant frequency is $\boldsymbol{\omega_r}$ **= 10.8295 rad/s.**