7.1 Given the I/O equation

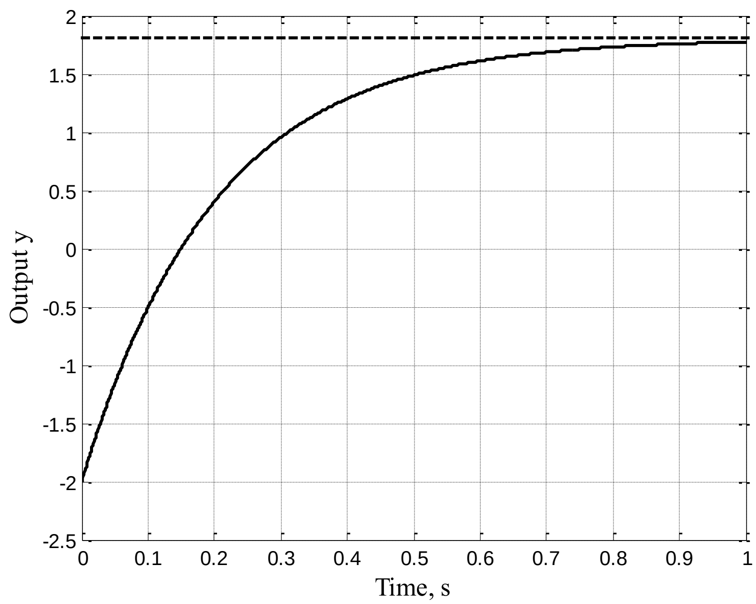
$$2\dot{y} + 10y = 3u(t)$$

Sketch the response $y(t)$ for a step input $u(t) = 6U(t)$ and the initial condition $y(0) = -2$.

7.1 Re-write the first-order ODE in standard form (divide all terms by 10):

$$0.2\dot{y} + y = 0.3u$$

The single root is $r = -1/0.2 = -5$ and therefore the response is a stable exponential function. The time constant is $\tau = 0.2$ s and hence the system reaches steady state in $t_S = 4\tau = 0.8$ s. The steady-state value for a constant input $u = 6$ is $y_{SS} = (0.3)(6) = 1.8$. A *hand-drawn sketch* of the response would match the response plot (below) and include labels for the initial condition $y(0) = -2$, settling time $t_S = 0.8$ s, and steady-state response $y_{SS} = 1.8$.



7.2 Given the following homogeneous ODE

$$2\ddot{y} + 12\dot{y} + 68y = 0 \quad \text{with initial conditions} \quad y(0) = 3, \dot{y}(0) = 0$$

a. Does the homogeneous response exhibit oscillations?
b. Estimate the time to reach steady state.
c. Describe the nature of the homogeneous response (a sketch may help).

7.2 a) First, we obtain the roots of the characteristic equation (below):

$$2r^2 + 12r + 68 = 0$$

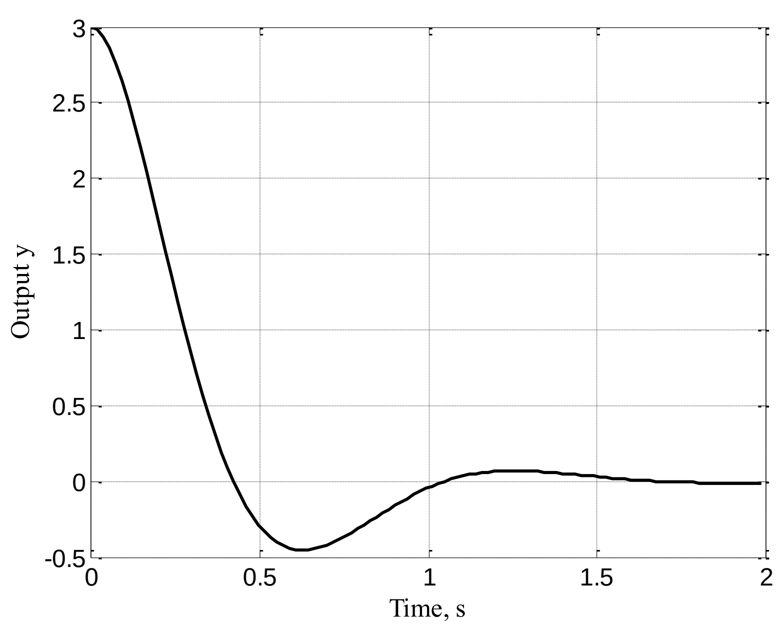
The two roots are complex: $r_{1,2} = -3 \pm j5$. Hence, the homogeneous response **does** exhibit oscillations. Another way to show this is to re-write the I/O equation in the standard form for a second-order system:

$$\ddot{y} + 6\dot{y} + 34y = 0 \quad \text{or} \quad \ddot{y} + 2\zeta\omega_n\dot{y} + \omega_n^2 y = 0 \quad \text{Hence } \zeta = 0.514 < 1 \text{ (underdamped)}$$

b) For an underdamped second-order system the settling time is approximately

$$t_S = \frac{4}{\zeta\omega_n} = \frac{4}{0.514\sqrt{34}} = 1.3333 \text{ s}$$

c) Because the second-order system is underdamped, the homogeneous response will exhibit decaying oscillations at frequency ω_d = 5 rad/s (or, period = 1.257 s). The homogeneous response will begin at the initial condition $y(0) = 3$ with "zero slope" and exhibit a damped sinusoidal response that decays to zero in about 1.3333 s (a bit longer than one period). A sketch would match the plot:



7.3 Given the following homogeneous ODE

$$4\ddot{y} + 22\dot{y} + 18y = 0 \quad \text{with initial conditions} \quad y(0) = 3, \dot{y}(0) = 0$$

a. Does the homogeneous response exhibit oscillations?
b. Estimate the time to reach steady state.
c. Describe the nature of the homogeneous response (a sketch may help).

7.3 a) First, we obtain the roots of the characteristic equation (below):

$$4r^2 + 22r + 18 = 4(r+1)(r+4.5) = 0$$

The two roots are real and negative: $r_1 = -1$ and $r_2 = -4.5$ and hence the homogeneous response is comprised of two decaying exponential functions (no oscillations).

b) The homogeneous response has the form $y_H(t) = c_1e^{-t} + c_2e^{-4.5t}$ and the *slowest* exponential mode "dies out" at time $t_S = 4$ s. Hence the settling time for the system is 4 s.

c) The homogeneous response will begin at the initial condition $y(0) = 3$ with "zero slope" and then exponentially decay to zero in about 4 s. A sketch would match the plot below.