5.9 Given the SSR

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 \\ -20 & -4 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0.3 \\ 1 \end{bmatrix}u \qquad y = \begin{bmatrix} 1 & 0 \end{bmatrix}\mathbf{x}$$

a. Obtain the I/O equation for this system where y is the output and u is the input.
b. Obtain the transfer function.

5.9 a) The two state equations are

$$\dot{x}_1 = x_2 + 0.3u \quad \text{and} \quad \dot{x}_2 = -20x_1 - 4x_2 + u$$

Apply the *D*-operator to both state equations and substitute the output $y = x_1$ to obtain

$$Dy = x_2 + 0.3u \quad \text{and} \quad Dx_2 = -20y - 4x_2 + u$$

Solve the first equation for x_2 ($x_2 = Dy - 0.3u$) and substitute this result in the second equation

$$D(Dy - 0.3u) = -20y - 4(Dy - 0.3u) + u$$

or, $(D^2 + 4D + 20)y = (0.3D + 2.2)u$

Finally, replace $Dy = \dot{y}$, $D^2 y = \ddot{y}$, and $Du = \dot{u}$ to obtain the I/O equation

$$\boxed{\ddot{y} + 4\dot{y} + 20y = 0.3\dot{u} + 2.2u} \quad \text{I/O equation}$$

b) Apply the *D*-operator to the I/O equation:

$$(D^2 + 4D + 20)y = (0.3D + 2.2)u$$

Form the output/input ratio *y*/*u* and replace *D* with Laplace variable *s*

$$\boxed{\frac{Y(s)}{U(s)} = \frac{0.3s + 2.2}{s^2 + 4s + 20}} \quad \text{Transfer function}$$

5.13 The vibration isolation system is shown in Fig. P5.13. Damper b_1 connects mass m to the fixed overhead horizontal surface. The vibration mounts that support mass on the moving base are modeled by lumped stiffness k and viscous friction b_2. The vertical displacement of mass m is measured from the static equilibrium position.

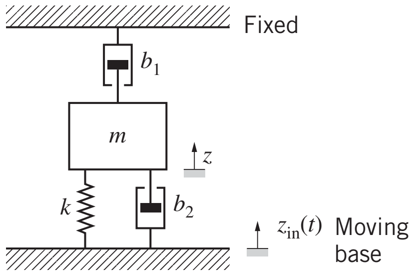
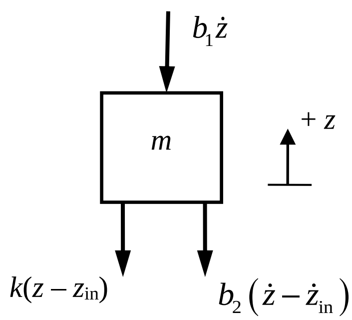


Figure P5.13

a. Obtain a complete SSR of this mechanical system where position of the mass is the system output and the two inputs are displacement and velocity of the base, $z_{in}(t)$ and $\dot{z}_{in}(t)$.

b. Derive the transfer function $G(s) = Z(s)/Z_{in}(s)$ for this system.

5.13 The free-body diagram (FBD) is shown below, assuming $z > z_{in}(t)$ and $\dot{z} > \dot{z}_{in}(t)$:



Applying Newton's second law (summing positive upward):

$$+\uparrow \sum F = -b_1\dot{z} - k(z - z_{in}) - b_2(\dot{z} - \dot{z}_{in}) = m\ddot{z}$$

Rearrange and put all dynamic variables (z and $\dot{z}$) on the left–hand side and input variables (z_{in} and $\dot{z}_{in}$) on the right-hand side.

$$m\ddot{z} + (b_1 + b_2)\dot{z} + kz = b_2\dot{z}_{in}(t) + kz_{in}(t) \qquad \text{Mathematical model}$$

With the mathematical model of the vibration isolation system as

$$m\ddot{z} + (b_1 + b_2)\dot{z} + kz = b_2\dot{z}_{\text{in}}(t) + kz_{\text{in}}(t)$$

a) Because we have a second-order ODE we require two states: choose $x_1 = z$ and $x_2 = \dot{z}$. The two inputs are $u_1 = z_{\text{in}}(t)$ and $u_2 = \dot{z}_{\text{in}}(t)$. The two first-order state equations (substituting the modeling equations and state and input variables) are

$$\dot{x}_1 = \dot{z} = x_2$$
$$\dot{x}_2 = \ddot{z} = \frac{1}{m}\left(-(b_1 + b_2)\dot{z} - kz + b_2\dot{z}_{\text{in}}(t) + kz_{\text{in}}(t)\right) = \frac{-(b_1 + b_2)}{m}x_2 - \frac{k}{m}x_1 + \frac{b_2}{m}u_2 + \frac{k}{m}u_1$$

The output is $y = z = x_1$. Assembling the state and output equations in their matrix-vector format yields

$$\boxed{\begin{aligned} &\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 \\ -k/m & -(b_1 + b_2)/m \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 & 0 \\ k/m & b_2/m \end{bmatrix}\mathbf{u} \qquad \text{State equation} \\ &y = \begin{bmatrix} 1 & 0 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 & 0 \end{bmatrix}\mathbf{u} \qquad \text{Output equation} \end{aligned}}$$

b) Apply the *D*-operator to the modeling equation

$$(mD^2 + b_1D + b_2D + k)z = (b_2D + k)z_{\text{in}}(t)$$

Form the output/input ratio z/z_{in} and replace D with Laplace variable s

$$\boxed{\frac{Z(s)}{Z_{\text{in}}(s)} = \frac{b_2s + k}{ms^2 + (b_1 + b_2)s + k}} \qquad \text{Transfer function for vibration isolation system}$$

5.18 Sketch the block diagram for the third-order system below using the integrator-block method. Label all blocks and signal-path variables.

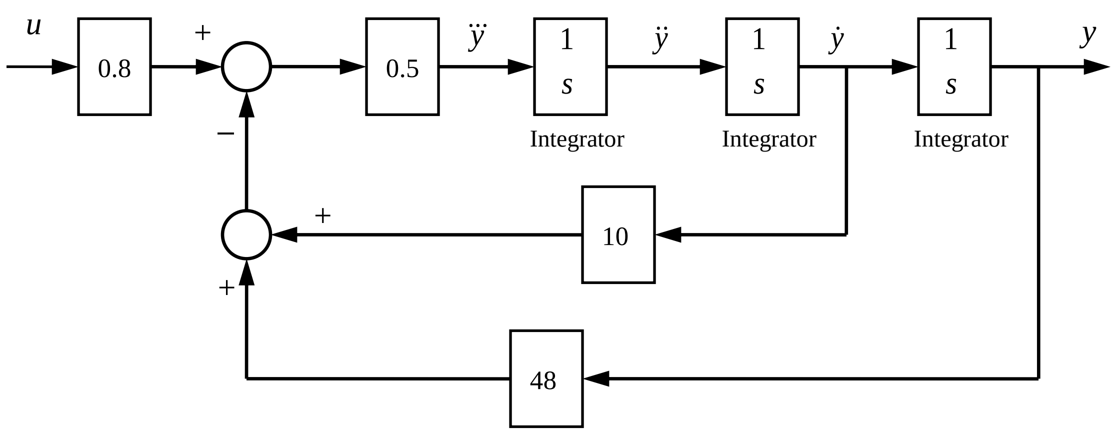
$$2\dddot{y} + 10\dot{y} + 48y = 0.8u$$

5.18 The third-order I/O equation (see Problem 5.6) is

Because the I/O equation is third order we need three integrator blocks. The primary part of the block diagram includes a "chain" of three integrators in series in order to integrate $\dddot{y}$, $\ddot{y}$, and $\dot{y}$ in succession. The input to the first integrator must be $\dddot{y}$, which we can obtain from the I/O equation:

$$\dddot{y} = \frac{1}{2}\left(-10\dot{y} - 48y + 0.8u\right)$$

A sketch of the block diagram would match the diagram shown below:



5.19 The modeling equations of a linear system are given below. The overall system input and output variables are u and y, respectively.

$$2\dot{z} + z = u$$

$$\dot{y} + 6y = 4z$$

a. Sketch the simplest possible block diagram for the case where all variables have zero initial conditions. Label all blocks and signal-path variables.

b. Sketch the block diagram using integrator blocks for the case where nonzero initial conditions are present; that is, $z(0) = 2$ and $y(0) = -3$. Label all blocks and signal-path variables.

5.19 The second-order system is

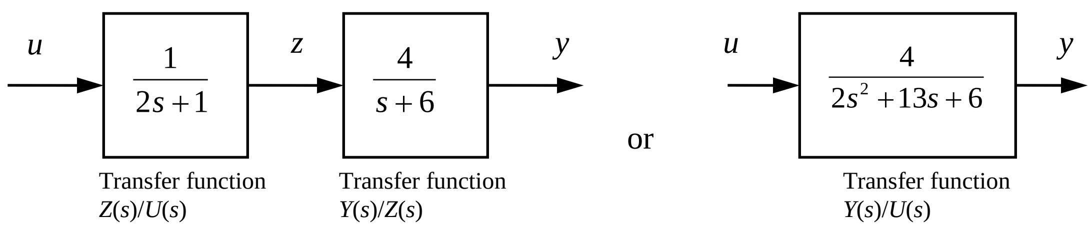
$$2\dot{z} + z = u$$

$$\dot{y} + 6y = 4z$$

a) Because each I/O equation is linear and the dynamic variables have zero initial conditions we can use transfer functions in the block diagram. The transfer functions for the I/O equations are

$$\frac{Z(s)}{U(s)} = \frac{1}{2s+1} \qquad \frac{Y(s)}{Z(s)} = \frac{4}{s+6}$$

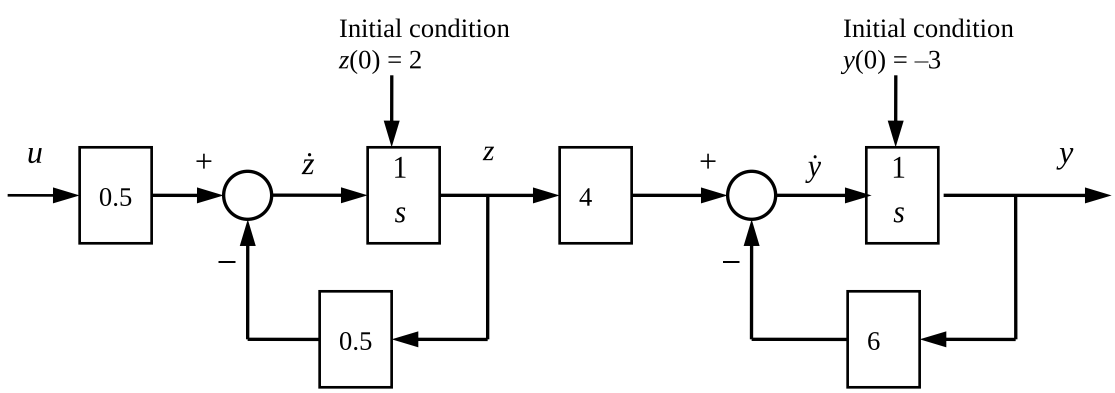
We see that u is the input and z is the output for the first transfer function, while z is the input to the second transfer function. The overall input and output are u and y. The simplest block diagrams could consist of a series of the two transfer functions, or a single transfer function $Y(s)/U(s)$ which is the product of $Z(s)/U(s)$ and $Y(s)/Z(s)$. If a single transfer function is used then the intermediate variable z is "lost" and not available in the simulation.



b) Because the system has non-zero initial conditions we cannot use transfer functions and therefore the integrator-block method is employed. The block diagram includes two integrators in order to integrate $\dot{z}$ and $\dot{y}$. The inputs to each respective integrator block must be

$$\dot{z} = -0.5z + 0.5u \quad \text{and} \quad \dot{y} = -6y + 4z$$

A sketch of the block diagram would match the diagram shown below:



5.20 Sketch the simplest possible block diagram for the following system:

$$w = K(u - y)$$

$$\ddot{y} + 6\dot{y} + 20y = 3w$$

All dynamic variables have zero initial conditions. The parameter K is a constant or "gain." Label all signal-path variables.

5.20 The system model is

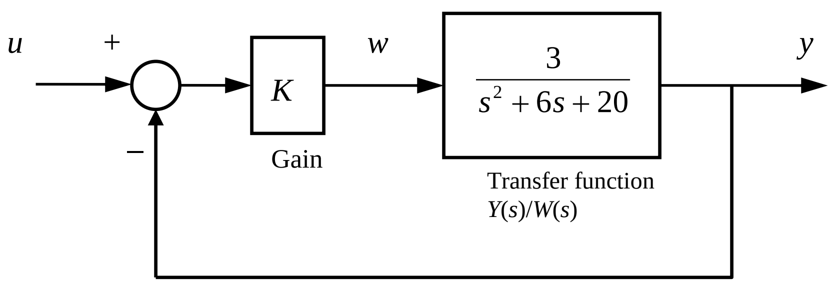
$$w = K(u - y)$$

$$\ddot{y} + 6\dot{y} + 20y = 3w$$

Because the second-order I/O equation is linear and the dynamic variables have zero initial conditions we can use a transfer function in the block diagram. The transfer function for the I/O equation has input w and output y :

$$\frac{Y(s)}{W(s)} = \frac{3}{s^2 + 6s + 20}$$

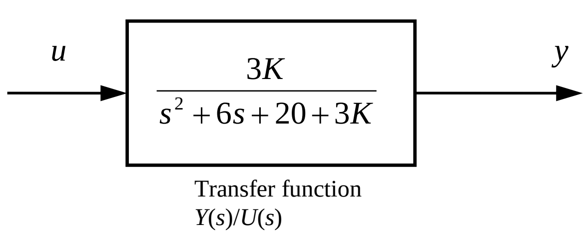
The algebraic equation shows that transfer-function input w is the difference between system input u and output y multiplied by "gain" (constant) K. The block diagram is



A second option is to substitute $w = K(u - y)$ into the second-order ODE to yield

$$\ddot{y} + 6\dot{y} + (20 + 3K)y = 3Ku$$

Hence the transfer function is $\dfrac{Y(s)}{U(s)} = \dfrac{3K}{s^2 + 6s + 20 + 3K}$ and the block diagram is



5.27 A simplified, linear representation of an electrohydraulic actuator (EHA) consists of a power amplifier, a solenoid model, and a mechanical valve model. The linear I/O equations for each subsystem are

$$\text{Power amplifier: } \tau_a \dot{e}_0 + e_0 = K_a e_{\text{in}}(t)$$

$$\text{Solenoid actuator: } \tau_s \dot{f} + f = K_s e_0$$

$$\text{Spool valve: } m\ddot{z} + b\dot{z} + kz = f$$

where $e_{\text{in}}(t)$ is the low-power voltage input to the amplifier, e_0 is the amplifier voltage output, f is the output force of the solenoid actuator (in N), and z is the position of the spool-valve mass m (in m).

a. Obtain a complete SSR with valve position z as the single output variable.

b. Derive the transfer functions for each subsystem of the EHA. Sketch a block diagram of the complete EHA system. Assume that all dynamic variables have zero initial conditions. Label all blocks and signal-path variables (with units).

5.27 The EHA system model is

Power amplifier: $\tau_a \dot{e}_0 + e_0 = K_a e_{\text{in}}$

Solenoid actuator: $\tau_S \dot{f} + f = K_S e_0$

Spool valve: $m\ddot{z} + b\dot{z} + kz = f$

a) The system order is $n = 4$. Hence, let $x_1 = e_0$, $x_2 = f$, $x_3 = z$, and $x_4 = \dot{z}$. The system input is $u = e_{\text{in}}$. The four first-order modeling equations are

$$\dot{x}_1 = \dot{e}_0 = \frac{1}{\tau_a}(-x_1 + K_a u) \qquad \dot{x}_2 = \dot{f} = \frac{1}{\tau_S}(-x_2 + K_S x_1)$$

$$\dot{x}_3 = \dot{z} = x_4 \qquad \dot{x}_4 = \ddot{z} = \frac{1}{m}(-bx_4 - kx_3 + x_2)$$

The output is $y = z = x_3$. Assembling the state and output equations in their matrix-vector format yields

$$\boxed{\begin{array}{ll} \dot{\mathbf{x}} = \begin{bmatrix} -1/\tau_a & 0 & 0 & 0 \\ K_S/\tau_S & -1/\tau_S & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1/m & -k/m & -b/m \end{bmatrix}\mathbf{x} + \begin{bmatrix} K_a/\tau_a \\ 0 \\ 0 \\ 0 \end{bmatrix}u & \text{State equation} \\ y = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}\mathbf{x} + [0]u & \text{Output equation} \end{array}}$$

b) Apply the D-operator to each modeling equation:

Power amp: $(\tau_a D + 1)e_0 = K_a e_{\text{in}}$

Solenoid actuator: $(\tau_S D + 1) f = K_S e_0$

Spool valve: $(mD^2 + bD + k)z = f$

Form the output/input ratios and replace D with Laplace variable s

$$\boxed{\frac{E_0(s)}{E_{\text{in}}(s)} = \frac{K_a}{\tau_a s + 1}}$$ Transfer function for power amplifier

$$\boxed{\frac{F(s)}{E_0(s)} = \frac{K_S}{\tau_S s + 1}}$$ Transfer function for solenoid actuator

$$\boxed{\frac{Z(s)}{F(s)} = \frac{1}{ms^2 + bs + k}}$$ Transfer function for spool valve

Block diagram of complete EHA system: chain together the three transfer functions