7.14 Given the system I/O equation

$$2\ddot{y} + 8\dot{y} + 6y = 3u$$

- **a.** Compute the characteristic roots.
- **b.** Compute the poles of the transfer function.
- **c.** Compute the eigenvalues of the system matrix from an SSR.
- **d.** Qualitatively describe the complete system response when initial conditions are $y(0) = 5$, $\dot{y}(0) = 0$, and $u(t) = 0$ (no input).

7.14 The I/O equation is $2\ddot{y} + 8\dot{y} + 6y = 3u$

a) The characteristic equation can be written from inspection:

$$2r^2 + 8r + 6 = 0 \quad \text{or} \quad 2(r+1)(r+3) = 0$$

The two characteristic roots are $r_1 = -1$ and $r_2 = -3$.

b) The following transfer function can be derived from the given I/O equation:

$$G(s) = \frac{Y(s)}{U(s)} = \frac{3}{2s^2 + 8s + 6}$$

The poles of $G(s)$ are determined by setting the denominator polynomial to zero:

$$2s^2 + 8s + 6 = 0$$

Therefore the two poles of $G(s)$ are $s_1 = -1$ and $s_2 = -3$, which are identical to the roots in (a).

c) We can obtain a SSR for the states $x_1 = y$ and $x_2 = \dot{y}$. The resulting state equation is

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 \\ -3 & -4 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 1.5 \end{bmatrix} u$$

The eigenvalues are computed from the determinant $|\lambda \mathbf{I} - \mathbf{A}| = 0$

$$|\lambda \mathbf{I} - \mathbf{A}| = \begin{vmatrix} \lambda & -1 \\ 3 & \lambda + 4 \end{vmatrix} = \lambda^2 + 4\lambda + 3 = 0$$

The eigenvalues are $\lambda_1 = -1, \lambda_2 = -3$ which are identical to the roots and poles in (a) and (b).

d) Because the two roots are real and negative the zero-input response consists of two decaying exponential functions

$$y(t) = c_1 e^{-t} + c_2 e^{-3t}$$

The response starts at $y(0) = 5$ with "zero slope" due to $\dot{y}(0) = 0$ and eventually decays to zero. The slowest exponential decay mode is e^{-t} which takes about 4 s to reach zero.

7.16 Given the SSR

$$\dot{\mathbf{x}} = \begin{bmatrix} -0.2 & -0.6 \\ 2 & -4 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 1.5 \end{bmatrix}u \qquad y = \begin{bmatrix} 1 & 0 \end{bmatrix}\mathbf{x}$$

a. Compute the eigenvalues "by hand."
b. Use MATLAB to verify your answer in part (a).
c. Describe the *free* response of the output $y(t)$ given an arbitrary initial state $\mathbf{x}(0)$.
d. Use MATLAB or Simulink to verify your answer in part (c). The initial state vector is $\mathbf{x}(0) = \begin{bmatrix} x_1(0) & x_2(0) \end{bmatrix}^T = \begin{bmatrix} -2 & -1 \end{bmatrix}^T$.

7.16 a) The eigenvalues are computed from the determinant $|\lambda\mathbf{I} - \mathbf{A}| = 0$

$$|\lambda\mathbf{I} - \mathbf{A}| = \begin{vmatrix} \lambda + 0.2 & 0.6 \\ -2 & \lambda + 4 \end{vmatrix} = \lambda^2 + 4.2\lambda + 2 = 0$$

The eigenvalues are $\lambda_1 = -0.5476$ and $\lambda_2 = -3.6524$.

b) The eigenvalues can be computed using the simple MATLAB commands; matches part (a)

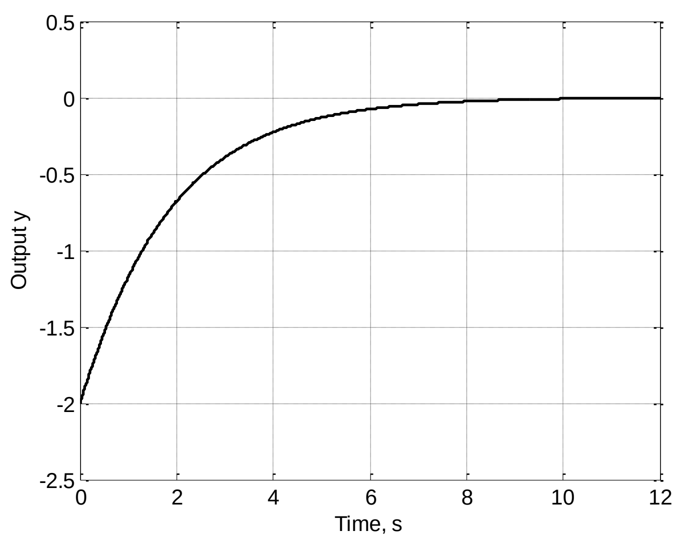
```
>   A = [-0.2 -0.6 ; 2 -4];           % define the state matrix A
>   eig(A)
```

c) Because the two roots are real and negative, the free (zero-input) response will consist of two decaying exponential functions, or $y(t) = c_1e^{-0.5476t} + c_2e^{-3.6524t}$. The "slow" exponential mode is $e^{-0.5476t}$ which takes about 7.3 s to decay to zero.

d) MATLAB commands are probably the easiest method: use the command `initial`

```
>> A = [-0.2 -0.6 ; 2 -4];       % define the state matrix A
>> B = [0 ; 1.5                  % define the input matrix B
>> C = [1 0                      % define the input matrix C
>> D =                           % define the direct-link matrix D
>> sys = ss(A,B,C,D              % define sys as the SSR
>> x0 =  -2 ; -1];               % define the initial state vector
>> t = 0:0.01  2;                % define the time vector
>> [y,t]=initial(s  ,x0,t);      % obtain the free response to the initial conditions
>> plot(t,                       % plot the free response
```

The plot (below) verifies the free response described in part (c).



7.17 Given the SSR

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -12 & -20 & -9 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ 0.4 \end{bmatrix}u \qquad y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}\mathbf{x}$$

a. Use MATLAB to determine the eigenvalues.

b. Describe the *free* response of the output $y(t)$ given an arbitrary initial state $\mathbf{x}(0)$.

c. Use MATLAB or Simulink to verify your answer in part (b). The initial state vector is $\mathbf{x}(0) = [x_1(0)\ \ x_2(0)\ \ x_3(0)]^T = \begin{bmatrix} 2 & -0.5 & 0 \end{bmatrix}^T$.

7.17 a) The MATLAB commands define state matrix **A** and determine its eigenvalues:

```
>> A = [0 1 0 ; 0 0 1 ; -12 -20 -9];          % define the state matrix A
>> eig(A)
```

The three eigenvalues are –1, –2, and –6.

b) Because the three eigenvalues (roots) are real and negative, the free (zero-input) response will consist of three decaying exponential functions, or $y(t) = c_1e^{-t} + c_2e^{-2t} + c_3e^{-6t}$. The "slow" exponential mode is e^{-t} which takes about 4 s to decay to zero.

c) MATLAB commands are probably the easiest method: use the command `initial`

```
>> A = [0 1 0 ; 0 0 1 ; -12 -20 -9];          % define the state matrix A
>> B = [0 ; 0 ; 0.4 ];                         % define the input matrix B
>> C = [1 0 0];                                % define the input matrix C
>> D = 0;                                      % define the direct-link matrix D
>> sys = ss(A,B,C,D);                          % define sys as the SSR
>> x0 = [2 ; -0.5 ; 0];                        % define the initial state vector
>> t = 0:0.01:5;                               % define the time vector
>> [y,t]=initial(sys,x0,t);                    % obtain the free response
>> plot(t,y)                                   % plot the free response
```

The plot below verifies the free response described in part (b).