8.7 Figure P8.7 shows a simple 1-DOF rotational mechanical system with input torque $T_{in}(t)$. The Laplace transform of the angular velocity $\omega(t)$ of the rotational mechanical system is

$$\Omega(s) = \frac{0.15s + 0.4}{s(0.01s + 0.002)}$$

where $\mathscr{L}\{\omega(t)\} = \Omega(s)$. The initial angular velocity $\omega(0)$, constant input torque, and numerical values for inertia and friction have been accounted for in the above Laplace transform. The moment of inertia of the disk is $J = 0.01$ kg-m^2 and the viscous friction coefficient is $b = 0.002$ N-m-s/rad.

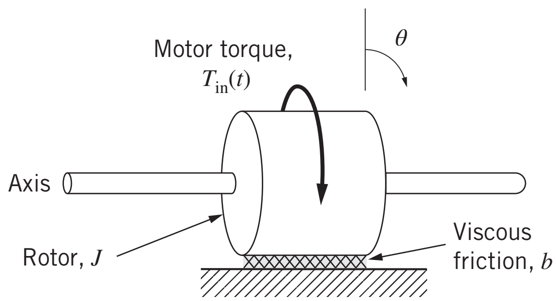


Figure P8.7

a. Compute the initial kinetic energy of the mechanical system.
b. Compute the steady-state kinetic energy of the mechanical system.
c. Determine the inverse Laplace transform to obtain the angular velocity $\omega(t)$ and use this solution to verify your answers in parts (a) and (b).

8.7 The given Laplace of the angular velocity is $\Omega(s) = \dfrac{0.15s + 0.4}{s(0.01s + 0.002)}$

which accounts for initial angular velocity and the input torque.

a) The initial angular velocity $\omega(0)$ can be computed from the initial value theorem (IVT):

$$\omega(0+) = \lim_{s\to\infty} s\Omega(s) = \lim_{s\to\infty} \frac{s(0.15s + 0.4)}{s(0.01s + 0.002)} = \lim_{s\to\infty} \frac{0.15s}{0.01s} = \mathbf{15\ rad/s}$$

Hence the initial kinetic energy is $J\omega_0^2/2$ = **1.125 J**

b) The steady-state angular velocity can be computed from the final value theorem (FVT):

$$\omega(\infty) = \lim_{s\to 0} s\Omega(s) = \lim_{s\to 0} \frac{s(0.15s + 0.4)}{s(0.01s + 0.002)} = \frac{0.4}{0.002} = \mathbf{200\ rad/s}$$

Hence the steady-state kinetic energy is $J\omega_{ss}^2/2$ = **200 J**

c) Using partial fractions, the Laplace transform is

$$\Omega(s) = \frac{0.15s + 0.4}{s(0.01s + 0.002)} = \frac{15s + 40}{s(s + 0.2)} = \frac{200}{s} + \frac{-185}{s + 0.2}$$

Hence the inverse Laplace transform yields the speed response: $\omega(t) = 200 - 185e^{-0.2t}$ rad/s

We see that the initial and final angular velocities are $\omega(0)$ = 15 rad/s and $\omega(\infty)$ = 200 rad/s as computed by the IVT and FVT.

8.17 Figure P8.17 shows a simple 1-DOF, frictionless, rotational mechanical system. The disk moment of inertia is $J = 0.2$ kg-m^2 and the torsional spring constant for the shaft is $k = 100$ N-m/rad. Angular displacement θ is zero when the shaft is untwisted. The disk is initially at rest (equilibrium) when the sinusoidal input torque $T_{in}(t) = 0.5 \sin 3t$ N-m is applied.

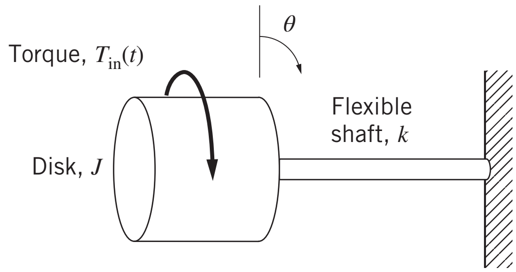


Figure P8.17

a. Using Laplace methods to determine the system response $\theta(t)$.

b. Use MATLAB or Simulink to obtain a numerical solution for the angular position response (use a simulation time of 10 s). Plot the analytical solution from part (a) and the numerical solution on the same figure.

8.17 The mathematical model of the mechanical system is $J\ddot{\theta} + k\theta = T_a(t)$

Substituting the parameters J and k and sinusoidal input $T_a(t)$ the I/O equation becomes

$$0.2\ddot{\theta} + 100\theta = 0.5 \sin 3t$$

a) Taking the Laplace transform (initial conditions are zero) yields

$$(0.2s^2 + 100)\Theta(s) = \frac{(0.5)(3)}{s^2 + 3^2} \quad \text{or} \quad \Theta(s) = \frac{1.5}{(0.2s^2 + 100)(s^2 + 9)}$$

Expanding in partial fractions:

$$\Theta(s) = \frac{7.5}{(s^2 + 500)(s^2 + 9)} = \frac{a_1 s}{s^2 + \sqrt{500}^2} + \frac{a_2 \sqrt{500}}{s^2 + \sqrt{500}^2} + \frac{a_3 s}{s^2 + 3^2} + \frac{a_4(3)}{s^2 + 3^2}$$

The residues for the first imaginary poles ($s = \pm j\sqrt{500}$) are

$$(s^2 + 500)Y(s)\Big|_{s=+j\sqrt{500}} = \frac{7.5}{s^2 + 9}\bigg|_{s=+j\sqrt{500}} = -0.01527 \quad = a_1 j\sqrt{500} + a_2\sqrt{500}$$

Equating real and imaginary parts we obtain the residues $a_1 = 0$ and $a_2 = -6.8312(10^{-4})$

The residues for the second imaginary poles ($s = \pm j3$) are

$$(s^2 + 9)Y(s)\Big|_{s=+j3} = \frac{7.5}{s^2 + 500}\bigg|_{s=+j3} = 0.01527 \quad = a_3 j3 + a_4(3)$$

Equating real and imaginary parts we obtain the residues $a_3 = 0$ and $a_4 = 0.0050916$.

Therefore, the partial-fraction expansion is

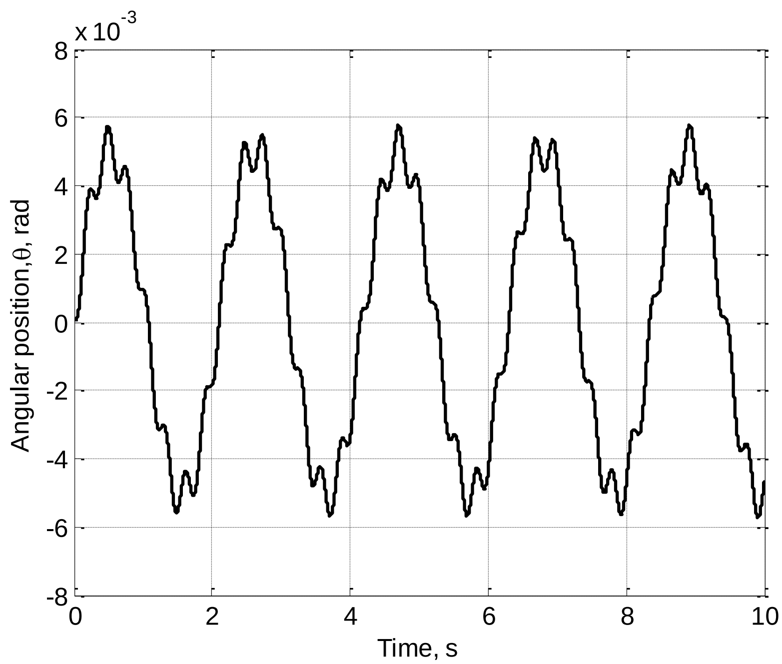
$$\Theta(s) = \frac{-6.8312(10^{-4})\sqrt{500}}{s^2 + \sqrt{500}^2} + \frac{(0.0050916)(3)}{s^2 + 3^2}$$

The inverse Laplace transform is: $\boxed{\theta(t) = -6.8312(10^{-4})\sin\sqrt{500}t + 0.0050916 \sin 3t \ \text{rad}}$

b) Because the system has zero initial conditions we can use a transfer function for the numerical solution. The MATLAB commands determine the response to a sinusoidal input.

```
>  sysG = tf(1,[0.2 0 100]);                  % Define system sysG
>  t = 0:0.001:1                              % Define time vector
>  a1 = -6.8312e-4                            % Analytic coefficient
>  a2 = 0.0050916                             % Analytic coefficient
>  th = a1*sin(sqrt(500)* t)+ a2*sin(3*t);    % Analytical response: part a
>  u = 0.5*sin(3*t);                          % Define input Ta = 0.5sin3t
 > [y,t] = lsim(sysG,u,t);                    % Numerical response x(t)
>  plot(t,th,t,y)                             % Plot both responses
```

The plot (below) shows both the analytical and numerical responses (they are the same).



Prob. 8.17: angular position vs. time

8.18 Figure P8.18 shows the simplified, linear electrohydraulic actuator (EHA) model. The voltage input is a step function $e_{in}(t) = 0.2U(t)$ V. The amplifier output is initially zero and the valve is initially at static equilibrium ($z = 0$).

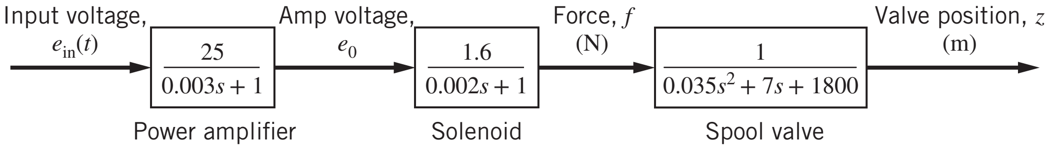


Figure P8.18

a. Use the final-value theorem to compute the steady-state position of the spool valve.

b. Use Laplace methods to determine the amplifier voltage response, $e_0(t)$, for the step input.

c. Use Laplace methods to determine the force response of the solenoid, $f(t)$, for the step input $e_{in}(t) = 0.2U(t)$ V.

d. Use Laplace methods to determine the spool-valve position, $z(t)$, for the step input $e_{in}(t) = 0.2U(t)$ V. Verify the steady-state position computed in part (a).

8.18 a) The transfer function relating the input voltage $e_{\text{in}}(t)$ to the spool-valve position z is

$$\frac{Z(s)}{E_{\text{in}}(s)} = \frac{E_0(s)}{E_{\text{in}}(s)}\frac{F(s)}{E_0(s)}\frac{Z(s)}{F(s)} = \left(\frac{25}{0.003s+1}\right)\left(\frac{1.6}{0.002s+1}\right)\left(\frac{1}{0.035s^2+7s+1800}\right)$$

Or, $$\frac{Z(s)}{E_{\text{in}}(s)} = \frac{40}{(0.003s+1)(0.002s+1)(0.035s^2+7s+1800)}$$

If $e_{\text{in}}(t) = 0.2U(t)$ V (step input), then $E_{\text{in}}(s) = 0.2/s$ and the Laplace transform of the position is

$$Z(s) = \frac{(40)(0.2)}{s(0.003s+1)(0.002s+1)(0.035s^2+7s+1800)}$$

The final value theorem yields

$$z(\infty) = \lim_{s\to 0} sZ(s) = \lim_{s\to 0}\frac{s(40)(0.2)}{s(0.003s+1)(0.002s+1)(0.035s^2+7s+1800)} = \frac{8}{1800} = \boxed{0.00444 \text{ m}}$$

b) The Laplace transform of the amplifier output is the product of the power amplifier transfer function and the Laplace transform of the step input, $E_{\text{in}}(s) = 0.2/s$

$$E_0(s) = \frac{(25)(0.2)}{s(0.003s+1)}$$ poles are $s = 0$ and $s = -333.333$

Partial-fraction expansion yields

$$E_0(s) = \frac{1666.667}{s(s+333.333)} = \frac{5}{s} + \frac{-5}{s+333.333}$$

The inverse Laplace transform yields the amp response: $\boxed{e_0(t) = 5(1-e^{-333.33t})\ \text{V}}$

c) The Laplace transform of the solenoid output is the product of the power amplifier and solenoid transfer functions and the Laplace transform of the step input, $E_{in}(s) = 0.2/s$

$$F(s) = \frac{(25)(1.6)(0.2)}{s(0.003s+1)(0.002s+1)}$$ poles are $s = 0$, –333.333, and –500

Partial-fraction expansion yields

$$F(s) = \frac{1.3333(10^6)}{s(s+333.333)(s+500)} = \frac{8}{s} + \frac{-24}{s+333.333} + \frac{16}{s+500}$$

The inverse Laplace transform yields the solenoid response:

$$\boxed{f(t) = 8 - 24e^{-333.33t} + 16e^{-500t} \text{ N}}$$

d) The Laplace transform of the valve output is the product of the power amp, solenoid, and valve transfer functions and the Laplace transform of the step input, $E_{in}(s) = 0.2/s$

$$Z(s) = \frac{(25)(1.6)(0.2)}{s(0.003s+1)(0.002s+1)(0.035s^2+7s+1800)}$$

poles are $s = 0$, –333.333, –500, and $s = -100 \pm j203.54$

Partial-fraction expansion yields

$$Z(s) = \frac{1.3333(10^6)}{s(s+333.333)(s+500)(0.035s^2+7s+1800)}$$

$$= \frac{0.00444}{s} + \frac{-0.00715}{s+333.333} + \frac{0.00227}{s+500} + \frac{4.384(10^{-4})(s+100)}{(s+100)^2+203.54^2} + \frac{-4.157(10^{-4})(203.54)}{(s+100)^2+203.54^2}$$

The inverse Laplace transform yields the spool-valve response (in m):

$$\boxed{\begin{aligned} z(t) &= 0.00444 - 0.00715e^{-333.33t} + 0.00227e^{-500t} + 4.384(10^{-4})e^{-100t}\cos 203.54t \\ &- 4.157(10^{-4})e^{-100t}\sin 203.54t \end{aligned}}$$

Note that at steady-state all exponential terms go to zero which leaves $z(\infty) = 0.00444$ m