

Engineering Measurement

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Question 1

Problem 1.

$$f(x) = x \quad (\pi, -\pi]$$

$$f(x) = 2 \sum_{n=1}^{\infty} \left\{ \frac{(-1)^n}{n} \sin(nx) \right\}$$

$$f(x) = a_0 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx) \quad l = \pi$$

$$f(-x) = -f(x)$$

$$a_n = 0 \quad \forall n$$

$$\begin{aligned} b_n &= \frac{1}{l} \int_{-l}^l f(x) \sin\left(\frac{n\pi x}{l}\right) dx = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin(nx) dx \\ &= \frac{1}{\pi} \int_{-\pi}^{\pi} x \sin(nx) dx \end{aligned}$$

$$\int u dv = uv - \int v du$$

$$dv = \sin(nx) dx$$

$$v = -\frac{1}{n} \cos(nx)$$

$$u = x$$

$$du = dx$$

$$\begin{aligned} b_n &= \frac{1}{\pi} \int_{-\pi}^{\pi} x \sin(nx) dx \\ &= \frac{1}{\pi} \left\{ -\frac{1}{n} x \cos(nx) + (-x) \frac{1}{n} \cos(nx) \right\} \\ &\quad + \frac{1}{\pi} \left\{ \frac{1}{n} \left(\frac{\sin(nx)}{n} \right) \right\} \Big|_{-\pi}^{\pi} \\ &= \frac{2}{n} (-1)^{n+1} \end{aligned}$$

$$f(x) = 2 \sum_{n=1}^{\infty} \left\{ \frac{(-1)^{n+1}}{n} \sin nx \right\}$$

Question 2

problem 2

$$f(x) = a_0 + \sum_{n=1}^{\infty} (a_n \cosh n\pi x + b_n \sin n\pi x)$$

$$\int_{-l}^l f(x) dx = \int_{-l+a}^{l+a} f(x) dx$$

$$= \int_{-l+l}^{l+l} f(x) dx = \int_0^{2l} f(x) dx$$

$$a_0 = \frac{1}{2l} \int_{-l}^l f(x) dx = \frac{1}{2} \int_0^2 e^{-x} dx = \frac{1}{2} (-e^{-x}) \Big|_0^2$$

$$= \frac{1 - e^{-2}}{2}$$

$$a_n = \frac{1}{2l} \int_0^{2l} f(x) \cos\left(\frac{n\pi x}{l}\right) dx = \int_0^2 e^{-x} \cos n\pi x dx$$

$$= \left(\frac{e^{-x} (-\cos n\pi x + n\pi \sin n\pi x)}{1 + n^2\pi^2} \right) \Big|_0^2$$

$$= \frac{e^{-2}(-1+0)}{n^2\pi^2+1} - \frac{(1)(-1+0)}{n^2\pi^2+1} = \frac{(1-e^{-2})}{n^2\pi^2+1}$$

$$b_n = \frac{1}{2l} \int_0^{2l} f(x) \sin \frac{n\pi x}{l} dx = \int_0^2 e^{-x} \sin n\pi x dx$$

$$= \left(\frac{e^{-x} (-\sin n\pi x - n\pi \cos n\pi x)}{1 + n^2\pi^2} \right) \Big|_0^2$$

$$= \frac{n\pi (1 - n\pi e^{-2})}{n^2\pi^2 + 1}$$

$$= \frac{n\pi (1 - e^{-2})}{n^2\pi^2 + 1}$$

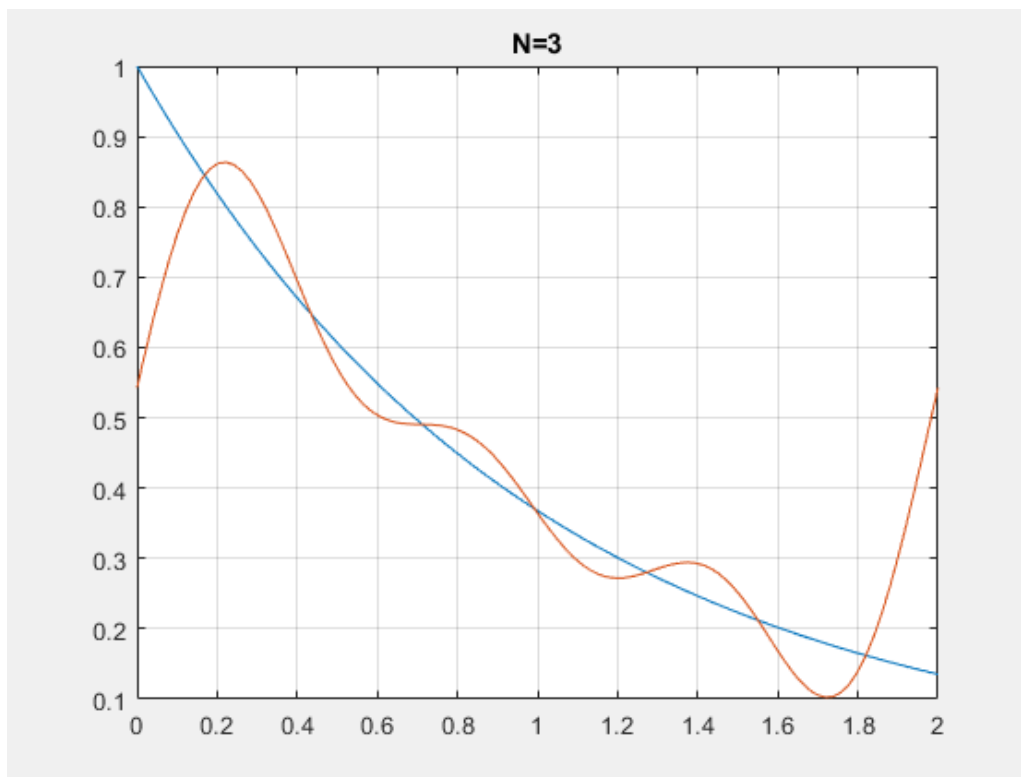
$$= \frac{1 - e^{-2}}{2} + (1 - e^{-2}) \sum_{n=1}^{\infty} \frac{1}{n^2\pi^2 + 1} (\cos n\pi x + n\pi \sin n\pi x)$$

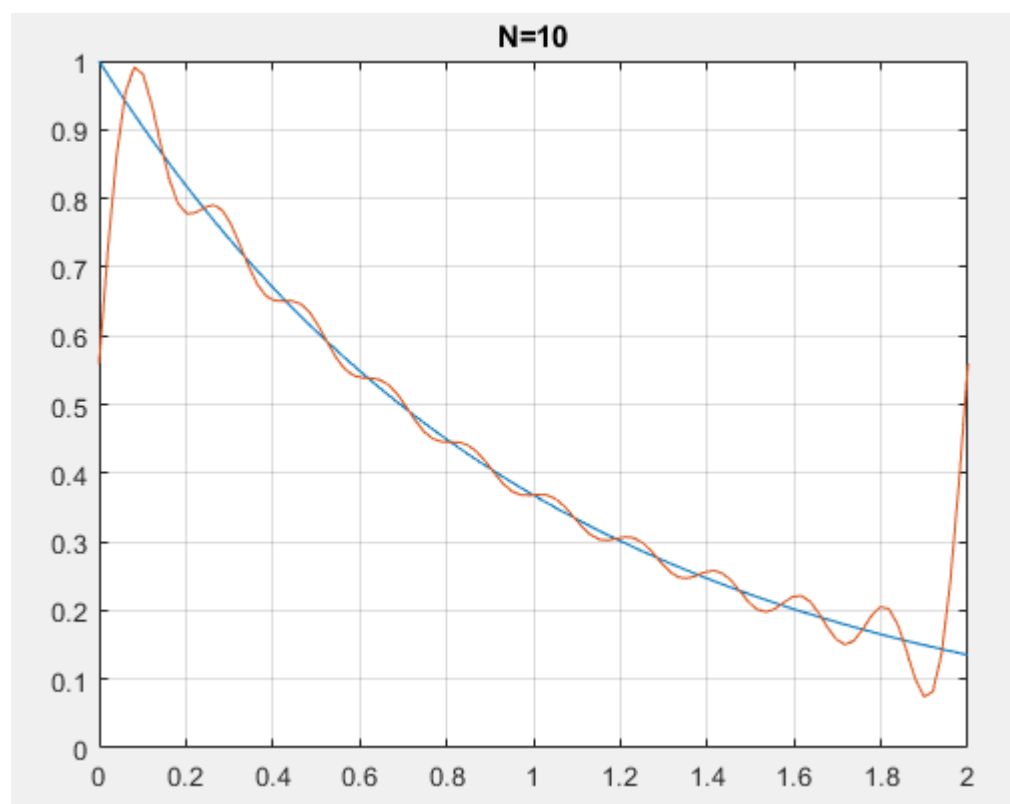
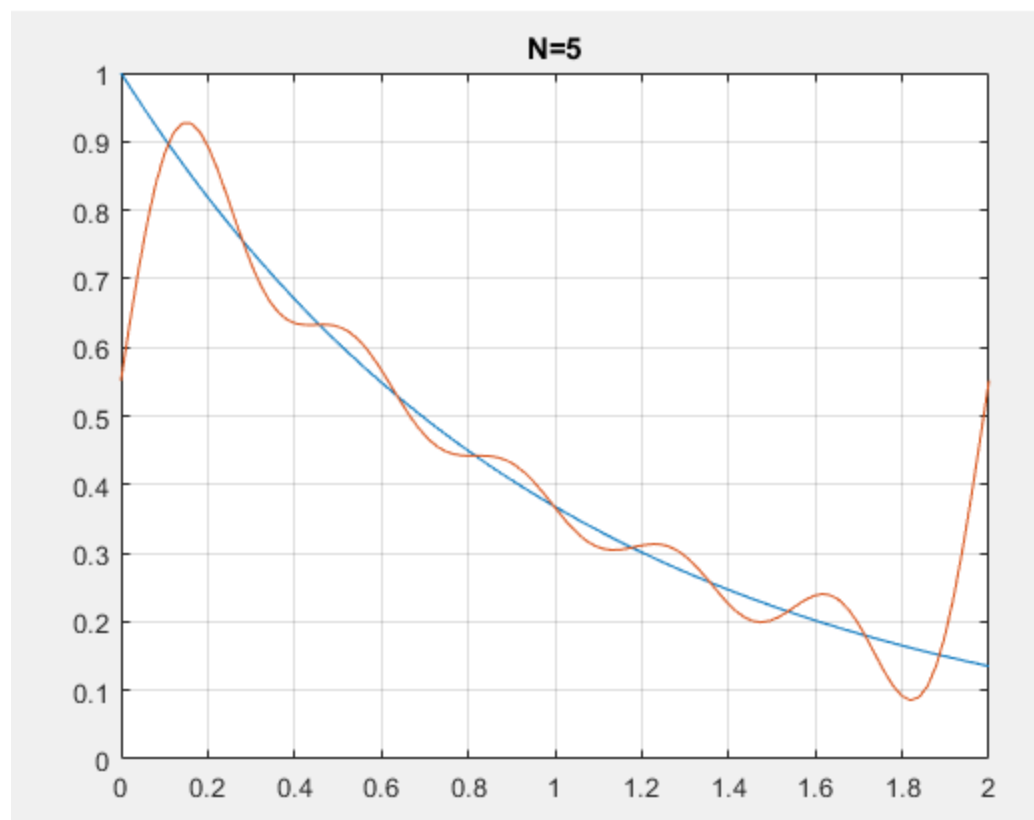
Question 3

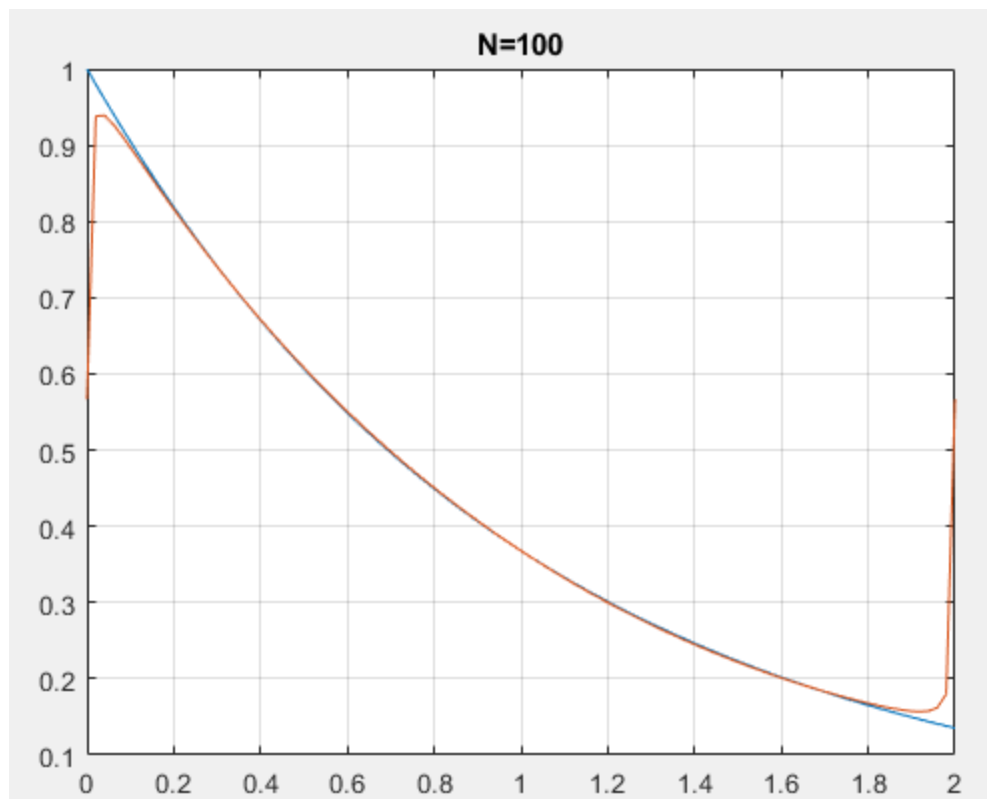
```
%%104980541 Arjun Posarajah
%Question 3
%%data points
N=3; %% replaced this value for all the parts of this question
x= linspace(0,2,100);
y=exp(-x);

%%Series to N
y2=0;
for n=1:N
    y2=y2+1/(n^2*pi^2+1)*(cos(n*pi*x)+n*pi*sin(n*pi*x));
end
y2=(1-exp(-2))*(1/2 + y2);

%%Graph
plot(x,y,x,y2);title('N=3');grid on
```







Question 4

Problem 4:

$$\begin{aligned}
 \mathcal{F}\{f(x)\} &= \hat{f}(\omega) = \int_{-\infty}^{\infty} f(x) e^{-i\omega x} dx \\
 &= \int_{-\infty}^{\infty} [H(x) e^{-ax}] e^{-i\omega x} dx \\
 &= \int_0^{\infty} e^{-(a+i\omega)x} dx \\
 &= \left. \frac{e^{-(a+i\omega)x}}{-(a+i\omega)} \right|_0^{\infty} \\
 &= \frac{1}{a+i\omega}
 \end{aligned}$$

Question 5

Problem 5

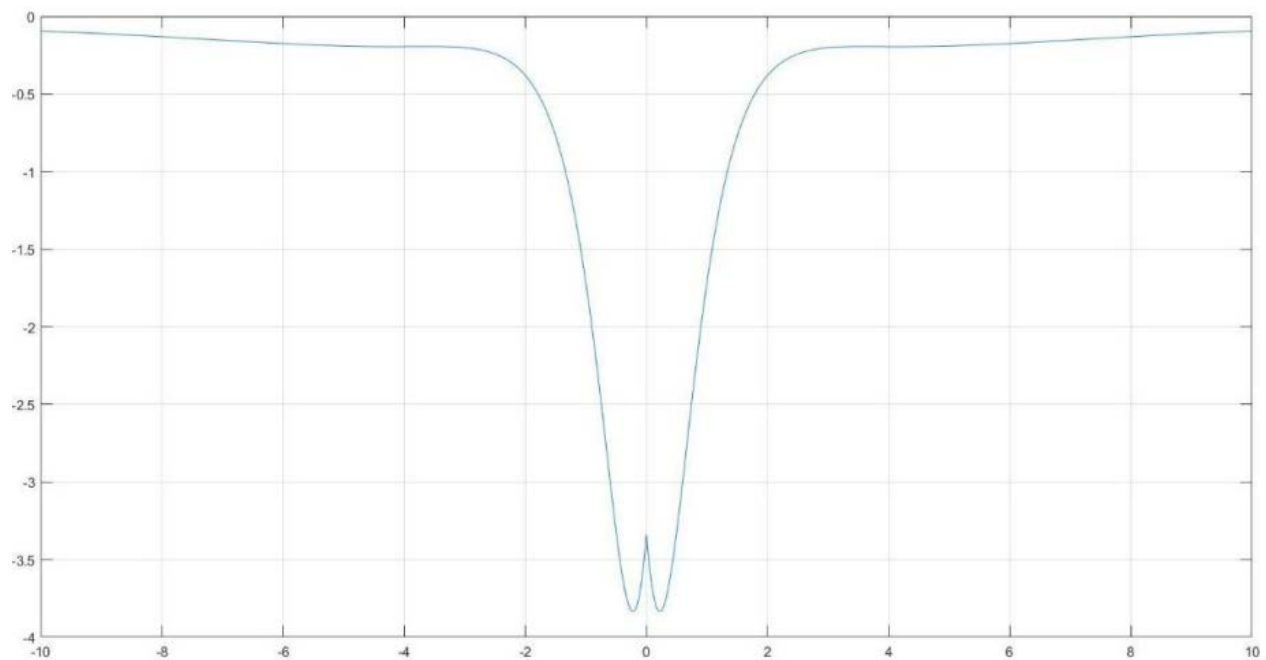
from table 18

$$\frac{3}{2} F \left\{ \frac{1}{x^2 + \frac{1}{2}} \right\} - 5 F \{ e^{-|x|} \}$$

table 1 and 4

$$F \left\{ \frac{1}{x^2 + (\sqrt{1/2})^2} \right\} = \frac{\pi}{\sqrt{1/2}} e^{-\sqrt{1/2} |\omega|}$$

$$F \left\{ \frac{3}{2x^2 + 1} - 5e^{-|x|} \right\} = \frac{3\pi}{\sqrt{2}} e^{-|\omega|/\sqrt{2}} - \frac{10}{\omega^2 + 1}$$



Question 6

Problem 6

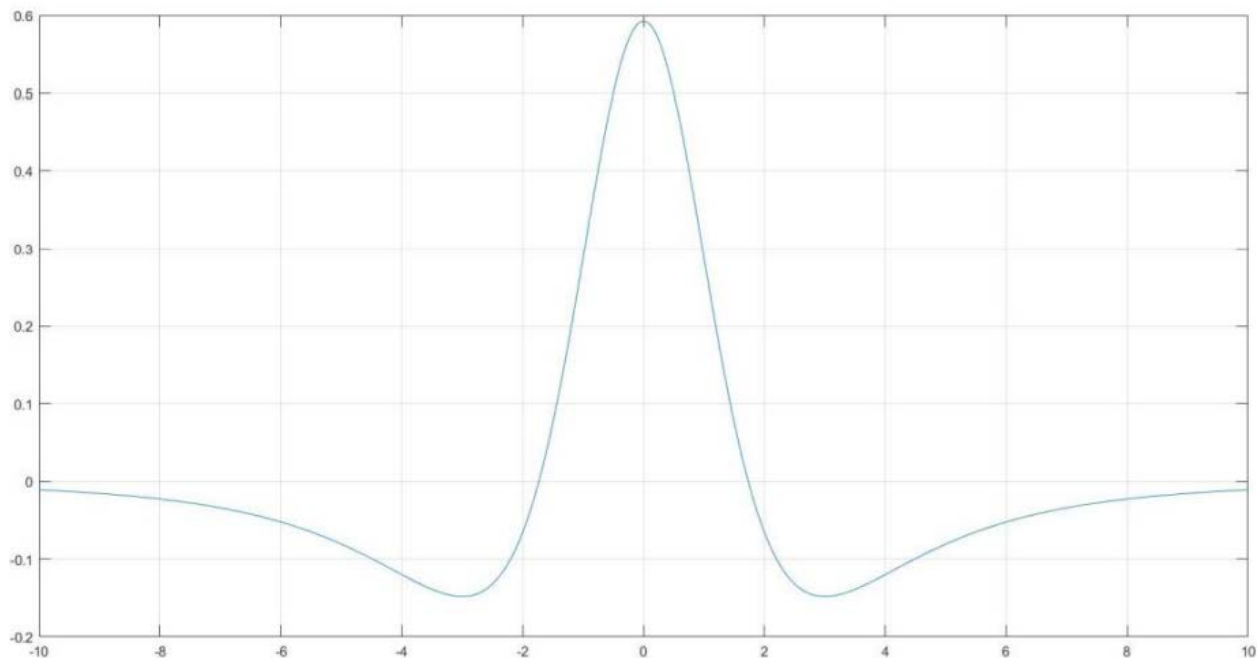
$$F\{f(x)\} = F\{4x^2 e^{-3|x|}\}$$

table 18 $\Rightarrow 4F\{x^2 e^{-3|x|}\}$

table 4 $\Rightarrow F\{e^{-3|x|}\} = \frac{6}{\omega^2 + 9}$

$$\begin{aligned} F\{x^2 e^{-3|x|}\} &= i^2 \frac{d^2}{d\omega^2} \left(\frac{6}{\omega^2 + 9} \right) = -6 \frac{d}{d\omega} \left(\frac{-2\omega}{(\omega^2 + 9)^2} \right) \\ &= 12 \left(\frac{1}{(\omega^2 + 9)^2} + \frac{2\omega^2}{(\omega^2 + 9)^3} \right) \\ &= 36 \frac{3 - \omega^2}{(\omega^2 + 9)^3} \end{aligned}$$

$$F\{4x^2 e^{-3|x|}\} = 144 \frac{3 - \omega^2}{(\omega^2 + 9)^3}$$



Question 7

