Assignment 6

Arjun Posarajah 104980541

Problem 2

Problem 2

a) $5\dot{y} + 5y = U(t)$

$\dot{y} + y = \frac{1}{5}U(t) \implies \tau = 1 s$

$r = 1 - e^{-t/\tau} = 1 - e^{-t}$

$t = -\ln(1-r)$

$t_{75} = -\ln(1-0.75) = 1.39 s$

$t_{90} = -\ln(1-0.9) = 2.30 s$

$t_{95} = -\ln(1-0.95) = 3.00 s$

b) $2\ddot{P} + 8\dot{P} + 8P = 2U(t)$

$\frac{1}{4}\ddot{P} + \dot{P} + P = \frac{1}{4}U(t) \implies \frac{1}{w_n^2} = \frac{1}{4}, \quad \frac{2\zeta}{w_n} = 1$

$w_n = 2 \qquad \zeta = 1$

```
%Assignment 6
%%%Arjun Posarajah 104980541
%%Problem 2
t=linspace(0,2.5,100)
r=1-(1+2*t).*exp(-2*t);
plot(t,r); grid on
xticks([0:0.1:2.5])
yticks([0:0.5:1])
```

r75 =

    1.3463

r90 =
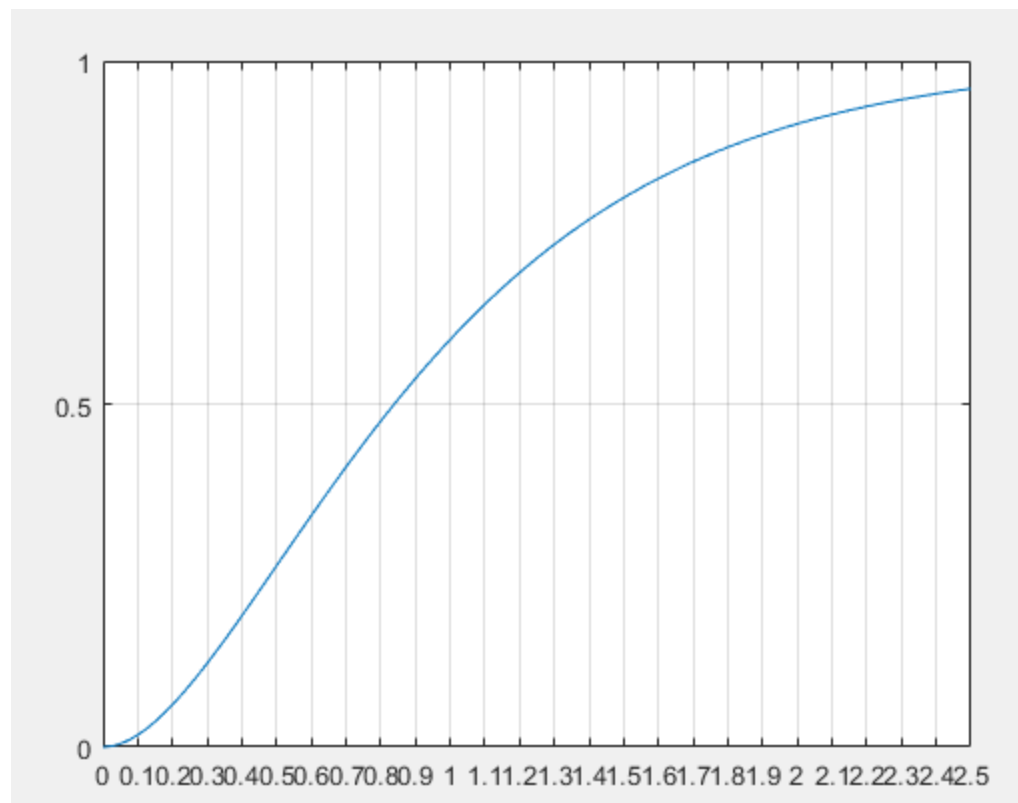
    1.9449

```
foo75 = @(t) 0.75 - 1 + (1+2*t)*exp(-2*t);
foo90 = @(t) 0.90 - 1 + (1+2*t)*exp(-2*t);
foo95 = @(t) 0.95 - 1 + (1+2*t)*exp(-2*t);
r75 = fzero(foo75, 1.3)
r90 = fzero(foo90, 2.0)
r95 = fzero(foo95, 2.3)
```

r95 =

    2.3719

Problem 3

Problem 3

$$mc_v \frac{dT}{dt} = hA_s(T_\infty - T)$$

$$T_\infty = T_\infty(t) = 0.065t + 25$$

$$\frac{mc_v}{hA_s} \frac{dT}{dt} + T = 0.065t + 25$$

$$e^{At} \qquad A = \frac{hA}{mc_v}$$

$$\frac{d(e^{At}T)}{dt} = (0.065t + 25)Ae^{At}$$

$$e^{At}T = 0.065A \int t\,e^{At}dt + 25A \int e^{At}dt \rightarrow c$$

$$= 0.065A \left[\frac{e^{At}}{A}\left(t - \frac{1}{A}\right)\right] + 25A \left[\frac{e^{At}}{A}\right] + c$$

$$T = 0.065\left(t - \frac{1}{A}\right) + 25 + Ce^{-At}$$

$$T(0) = 25 \qquad 25 = 0.065\left(-\frac{1}{A}\right) + 25 + c$$

$$C = \frac{0.065}{A}$$

$$T = 0.065\left(t - \frac{1}{A}\right) + 25 + \frac{0.065}{A}e^{-At}$$

$$A = \frac{hA_s}{m_c C_v} = \frac{h(4\pi r^2)}{\int \frac{4}{3}\pi r^3 C_v} = \frac{3(1000)}{8900(400)(0.065)}$$

$$= 0.1685$$

$$\frac{1}{A} = 5.93$$

$$T(t) = 0.065t + 0.385e^{-t/5.93} + 24.6$$

Problem 4

$$y(t) = cA - cAe^{-3\omega_n t}\left[\frac{5}{\sqrt{1-5^2}}\sin\left(\omega_n t\sqrt{1-5^2}\right) + \cos\left(\omega_n t\sqrt{1-5^2}\right)\right]$$

$$y(t) = 1 - e^{-t/4}\left[0.577\sin(0.433t) + \cos(0.433t)\right]$$

cont'd on matlab

```
t=linspace(0,20,2000);
yt=1-exp(-t/4).*(0.577*sin(0.433*t)+cos(0.433*t));
plot(t,yt); grid on
xticks([0:0.5:20])
yticks([0:0.05:1.2])
```

Problem 5

Problem 5

$$M(\omega) = \frac{1}{\sqrt{[1-(\omega/\omega_n)^2]^2 + [2\zeta\omega/\omega_n]^2}}$$

$$= \frac{1}{\sqrt{[1-(0.25)^2]^2 + [(6)(0.8)(0.25)]^2}}$$

$$= 0.98$$

$$\delta(\omega) = 0.98 \cdot 1 = -0.02$$

$$\phi(\omega) = -\tan^{-1}\left(\frac{2\zeta\omega/\omega_n}{1-(\omega/\omega_n)^2}\right)$$

$$= -\tan^{-1}\left(\frac{2(0.8)(0.25)}{1-(0.25)^2}\right) = -0.4 \text{ rad}$$
$$= -23°$$

$$\frac{1}{0.707^2} = [1-(f_{max}/f_n)^2]^2 + [2\cdot3 f_{max}/f_n]^2$$
$$\frac{1}{0.707^2} = (1-y^2) + 2.56y$$
$$y^2 + 0.56y - 1 = 0$$
$$3.56 (f_{max}/f_n)^2 - 2(f_{max}/f_n) - 1 = 0$$

using online calculator
$$\sqrt{y} = f_{max}/f_n \leq 0.8709$$
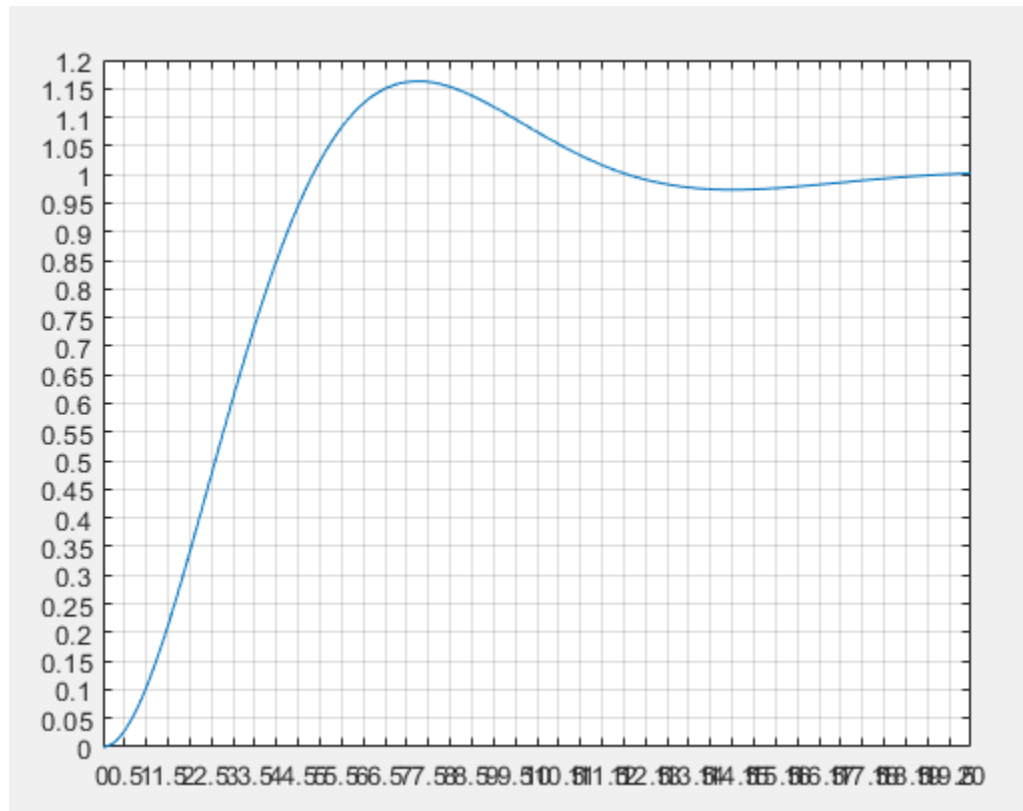$$f_{max} \leq 0.8807 \quad f_n = 0.8709 \text{ (2000)}$$
$$= 1742 Hz$$

$$y =$$

$$-1.3185$$
$$0.7585$$

%Assignment 6

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%%Problem 5

$$ans =$$

```
y = roots([1 0.56 -1])
sqrt(y(2))
```

$$0.8709$$

Problem 6

$$T_d = \frac{2\pi}{\omega_d} = \frac{1}{f_d}$$

$$= \frac{2\pi}{T_d} = \frac{2\pi}{0.572 \cdot (10^{-3})} = 10899 \ rad/s$$

$$f_d = \frac{1}{T_d} = \frac{1}{0.572 \times 10^{-3}}$$

$$= 1733 \ Hz$$

Problem 7

Problem 7

Dynamic Error

$$M(\omega) = \frac{1}{\sqrt{[1 - (\omega/\omega_n)^2]^2 + [2\zeta\omega/\omega_n]^2}}$$

$$= \frac{1}{[1 - (0.25)^2]^2 + [2(0.4)(0.25)]^2}$$

$$= 1.04$$

$$\delta(\omega) = 1.04 - 1 = 0.04$$

phase shift

$$\phi f(\omega) = -\tan^{-1}\left(\frac{2\ \zeta\omega/\omega_n}{1 - (\omega/\omega_n)^2}\right)$$

$$= -\tan^{-1}\left(\frac{2(0.4)(0.25)}{1 - (0.25)^2}\right)$$

$$= -0.210 \text{ rad}$$

Resonance Freq

$$\omega_R = \omega_n \sqrt{1 - 2\zeta^2}$$

$$f_R = f_n \sqrt{1 - 2\zeta^2} = 18000 \sqrt{1 - 2(0.4)^2}$$

$$= 14843 \text{ Hz}$$

Problem 8

```
%Assignment 6
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%%Problem 8

xbar1 = mean(FNSet1)
xbar2 = mean(FNSet2)
xbar33 = mean(FNSet3)
s1 = std(FNSet1)
s2 = std(FNSet2)
s3 = std(FNSet3)
stdMean1 = s1/sqrt(length(FNSet1))
stdMean2 = s2/sqrt(length(FNSet2))
stdMean3 = s3/sqrt(length(FNSet3))
nu1 = length(FNSet1) - 1
nu2 = length(FNSet2) - 1
nu3 = length(FNSet3) - 1
```

```
xbar1 =        xbar2 =         xbar3 =
   50.4650        50.6800          50.6400


s1 =           s2 =            s3 =
    0.9719         1.1768          0.9816

stdMean1 =     stdMean2 =      stdMean3 =
    0.2173         0.2631          0.2195


nu1 =      nu2 =      nu3 =
    19         19         19
```

Problem 9

```
function F=func(t)
F=5*sin(10*t)+10*sin(5*t)+1.5*sin(2.5*t)+2.0*sin(1.25*t);
```