

Measurement Tutorial 2

Arjun Posarajah 104980541

PART 1:

Problem 1 PART 1

$$T = 2 \text{ sec}$$
$$m = 1 \text{ kg}$$
$$\omega_n = \sqrt{\frac{k}{m}}$$
$$\omega_n^2 m = k$$
$$k = (3.14)^2 (1)$$
$$= 9.87 \text{ N/m}$$
$$\omega_n = 2\pi f$$
$$= 2\pi (0.5)$$
$$= 3.14 \text{ rad/s}$$
$$f = \frac{1}{T}$$
$$= \frac{1}{2} = 0.5 \text{ Hz}$$

Problem 2

a) $\sin(10\pi t/5)$

$$\omega = \frac{10\pi}{5} = 2\pi \text{ rad/s}$$
$$\omega = 2\pi f = 2\pi$$
$$f = 1 \text{ Hz}$$
$$T = \frac{1}{f} = 1 \text{ sec}$$

b) $\cos 8t$

$$\omega = 8 \text{ rad/s}$$
$$f = \frac{\omega}{2\pi} = \frac{4}{\pi} \text{ Hz}$$
$$T = \frac{1}{f} = \frac{\pi}{4} \text{ sec}$$

c) $\sin 5n\pi t$ $n = 1 \text{ to } \infty$

$$\omega = 5n\pi \text{ rad/s}$$
$$f = \frac{\omega}{2\pi} = \frac{5n\pi}{2\pi} = \frac{5}{2} n \text{ Hz}$$
$$T = \frac{1}{f} = \frac{2}{5} n \text{ sec}$$

Problem 3

$$\begin{aligned} a) \quad y(t) &= 2\sin 4\pi t + 4\cos 4\pi t \\ &= 2\sin 2(2\pi t) + 4\cos 2(2\pi t) \\ &= 2(2)\sin 2\pi t \cdot \cos 2\pi t + 4[\cos^2(2\pi t) - \sin^2 2\pi t] \\ &= 4\sin 2\pi t \cdot \cos 2\pi t + 4\cos^2(2\pi t) - 4\sin^2 2\pi t \\ &= 4\sin 2\pi t [1 - 2\sin^2 \pi t] + 4(1 - \sin^2 2\pi t) - 4\sin^2 2\pi t \\ &= 4\sin 2\pi t - 8\sin^4 \pi t + 4 - 4\sin^2 2\pi t - 4\sin^2 2\pi t \\ &= 4\sin 2\pi t - 16\sin^2 \pi t + 4 \end{aligned}$$

$$b) \quad y(t) = \sqrt{2} \cos(8t - 45^\circ) \quad \text{Already in cosine}$$

Problem 3 cont'd

$$\begin{aligned} c) \quad y(t) &= 2\cos 3t + 5\sin 3t \\ &= 2[4\cos^3 A - 3\cos A] + 5[3\sin A - 4\sin^3 A] \\ &= 8\cos^3 A - 6\cos A + 15\sin A - 20\sin^3 A \\ &= 2\cos A [4 - 4\sin^2 A - 3] + (5\sin A [3 - 4\sin^2 A]) \\ &= 2\sqrt{1 - \sin^2 A} (1 - 4\sin^2 A) + 5(\sin A)(3 - 4\sin^2 A) \end{aligned}$$

Problem 4

$$m = \frac{1}{2} \text{ kg} \quad k = 5 \text{ N/m} \quad c = 1 \text{ Ns/m}$$

$$x(0) = 1 \quad v(0) = 1$$

$$\lambda = -\frac{c}{2m} \pm \frac{1}{2m} \sqrt{c^2 - 4km}$$

$$= -\frac{1}{2 \times \frac{1}{2}} \pm \frac{1}{2 \times (\frac{1}{2})} \sqrt{1^2 - 4 \times 5 \times \frac{1}{2}}$$

$$= -1 \pm \sqrt{-9} = -1 \pm 3i$$

$$x(t) = \left(c_1 e^{it} + c_2 e^{-3it} \right) e^{-t}$$

$$= e^{-t} (c_1 e^{3it} + c_2 e^{-3it})$$

$$x(0) = 1 \quad v(0) = 1$$

use stow

$$c_1 = \frac{3+i}{10}, \quad c_2 = \frac{3-i}{10}$$

Apply Euler's Formula:

$$\begin{aligned} x(t) &= e^{-t} [C_1 (\cos 3t - j \sin 3t) + C_2 (\cos 3t + j \sin 3t)] \\ &= e^{-t} [(C_1 + C_2) \cos 3t + j(-C_1 + C_2) \sin 3t] \\ &= e^{-t} (A \cos 3t + B \sin 3t) \end{aligned}$$

Apply BC

$$x(0) = 1 \times (A \cos 0 + B \sin 0) \Rightarrow A = 1$$

$$\begin{aligned} x'(t) &= -e^{-t} (A \cos 3t + B \sin 3t) \\ &\quad + e^{-t} (-3A \sin 3t + 3B \cos 3t) \end{aligned}$$

$$x'(0) = -(A + 0) + (3B) = 1$$

$$-1 + 3B = 1$$

$$B = \frac{2}{3}$$

$$\therefore x(t) = e^{-t} (\cos 3t + \frac{2}{3} \sin 3t)$$

$$\begin{aligned} A \cos \omega t + B \sin \omega t &= C \cos(\omega t - \phi) \\ &= C \sin(\omega t + \phi') \end{aligned}$$

$$x(t) = \frac{\sqrt{3}}{3} \sin(3t + \tan^{-1}(\frac{2}{1}))$$

$$\begin{aligned} C &= \sqrt{A^2 + B^2} \\ \phi &= \tan^{-1} \frac{B/A}{1} \quad \phi' = \tan^{-1} \frac{A}{B} \\ C &= \sqrt{1 + (\frac{2}{3})^2} = \sqrt{\frac{13}{9}} \\ &= \frac{\sqrt{13}}{3} \end{aligned}$$

$$\phi = \tan^{-1}(\frac{2/3}{1})$$

$$\phi = \tan^{-1}(\frac{2}{3}) = \tan^{-1}(\frac{2}{3})$$

$$\text{phase form } \phi(t) = \frac{\sqrt{13}}{3} \cos(3t - \phi)$$

PART 2:

```

%%problem 5
%%parta)
syms x y;
y = x^3 - 3*x^2 + x;

%%partb)
syms a b;
b= sin(a) + tan(a);

%%partc)
syms c d;
d=(2*c^2 -3*c -2)/(c^2 - 5*c);

%%partd)
syms e f;
f=(e^2-9)/(e+3);

disp(y)
disp(b)
disp(d)
disp(f)

```

```

>> Problem5
x^3 - 3*x^2 + x

sin(a) + tan(a)

(- 2*c^2 + 3*c + 2)/(- c^2 + 5*c)

(e^2 - 9)/(e + 3)

```

```

%%Problem 6
%%First eqn part a and b
syms y x;
y = x^3 - 3*x^2 + x;
expand(y)
factor(y)

%%Second eqn part a and b
syms a b;
b=((2*a^2) - (3*a)-2)/(a^2 - 5*a);
expand(b)
factor(b)

```

```

>> Problem6

ans =

x^3 - 3*x^2 + x

ans =

[ x, x^2 - 3*x + 1]

```

```
ans =
```

```
(3*a)/(- a^2 + 5*a) + 2/(- a^2 + 5*a) - (2*a^2)/(- a^2 + 5*a)
```

```
ans =
```

```
[ 2*a + 1, a - 2, 1/a, 1/(a - 5)]
```

```
%Problem 7
```

```
syms x y z;
```

```
one=2*x +y -2*z == 3;
```

```
two=x-y-z==0;
```

```
three=x+y+3*z ==12;
```

```
%parta
```

```
[x,y,z]=solve(one,two,three)
```

```
%partb
```

```
E1=subs(2*x +y -2*z,[x,y,z],[7/2,1,5/2])
```

```
E2=subs(x-y-z,[x,y,z],[7/2,1,5/2])
```

```
E3=subs(x+y+3*z,[x,y,z],[7/2,1,5/2])
```

```
%partc
```

```
symN=sym([7/2,1,5/2]);
```

```
doubleN=double(symN)
```

```
>> Problem7
```

```
x =
```

```
7/2
```

```
y =
```

```
1
```

```
z =
```

```
5/2
```

```
E1 =
```

```
3
```

```
E2 =
```

```
0
```

```
E3 =
```

```
12
```

```
doubleN =
```

```
3.5000 1.0000 2.5000
```

Problem 8:

```
>> Problem8
```

```
A =
```

```
    2    1   -2  
    1   -1   -1  
    1    1    3
```

```
B =
```

```
    3  
    0  
   12
```

```
X =
```

```
A=[ 2 1 -2; 1 -1 -1; 1 1 3]    3.5000  
B=[3;0;12]                    1.0000  
X=linsolve(A,B)               2.5000
```

PART 3:

