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# MECH-4463 Vehicle Dynamics MECH-4463 Course Requirements

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## SECTION 1

# INTRODUCTION

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This document has been prepared to explain the requirements for the Winter 2022 offering of MECH-4463/MECH-8290-36 Vehicle Dynamics (formerly 94-463). It will outline the learning exercises, design goals and milestones that will be required; however, supplementary information may be provided over the course of the semester as needed.

### 1.1 Reports and Project

#### Topics, Deadlines, Weighing

The course requirements include both individual lab reports and a competitive group design project. The lab reports are intended to familiarize you with some important aspects of vehicle dynamics and modelling. The project consists of the development of a virtual race car that will compete against your classmates in an event termed 'Formula 463'. The list of deliverables is given in Table 1.1, along with the due date, and the fraction of your grade that is associated with the item. The requirements for the individual reports are explained in the following chapters.

**Table 1.1:** Course Deliverables

Deliverable	Due Date	Grade Weight
Lab Report – Drag Race Simulation (individual)	Week 4	10%
Lab Report – Handling Simulation (individual)	Week 6	10%
Lab Report – Ride Quality Simulation (individual)	Week 8	10%
*Design Project Data Set (group)	Week 10	10%
*Design Project Report (group)	Week 11	10%

\*Only for MECH-4463 students. Students in MECH-8290 will complete an alternate project.

## Format

You are expected to prepare the reports in standard engineering format, with a proper Table of Contents, List of Figures, etc. The reports will normally be in the neighbourhood of ten pages of content (not counting the previously listed front matter, Appendices, etc.). Make sure to include a List of References if you have consulted outside sources. Recognize the distinction between a List of References and a Bibliography; you *must* refer to the References in the text of the report. There is no need to reproduce this document in your report.

The reports will require that you perform some numerical simulations, and you will often be required to include a significant number of data plots in your reports. However, simply including a plot in the report is *insufficient*; a collection of plots does not make a report. It is the text that references the plots that should make up the substance of your report; a figure should never appear in a report without some accompanying text. The text should provide an *explanation* of the data rather than a *description*. Any figures must have descriptive captions that are sufficient to describe the content of the figure without relying on the surrounding text. Always ensure that your units and scaling of any plot are appropriate. Results that are peripheral to the discussion should be included in an Appendix. There is no page limit on the Appendices.

For 2022, the reports will be completed with the assistance of a software tool known as Julia ([www.julialang.org](http://www.julialang.org)). Julia is a free, open-source computing language designed for high-speed numerical computing. The Equations of Motion library (EoM), a software package written in Julia by the University of Windsor Vehicle Dynamics and Control Research Group, will prove very helpful in assisting you to generate and analyze the models that are covered in the course. It can be found online here: [www.github.com](http://www.github.com)

## Grading

The reports will be graded on the following criteria:

- Correctness and completeness of analysis and simulation (4/10, Learning Outcome 6)
- Thorough discussion of results, indicating a clear understanding of the theory (4/10, Learning Outcome 2, 3, or 4)
- Appropriate incorporation of graphics (1/10, Learning Outcome 7)
- Overall report quality, including format, style, grammar, and spelling (1/10, Learning Outcome 7)

## 1.2 Partnering

You are encouraged complete the design project with a partner, subject to the following restrictions:

- A maximum of two members per group are permitted (no exceptions)
- You are responsible for finding your own partners
- You may complete the project on your own if you wish, or if you are unable to locate a partner

- It is assumed that both partners will contribute equally to the project, and so each will receive the same grade (regardless of their actual contribution, so choose wisely!)
- You and your partner will submit only one data set, together
- If your partnership is dissolved at any time before the deadline, you are each responsible for submitting a report, and should be prepared to do so

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## SECTION 2

# REPORT #1

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In the first report, your task is to perform a thorough analysis of the longitudinal dynamics capability of a road vehicle of your choice. You are free to choose any production vehicle for which you have reliable specifications, including vehicle size and mass, engine torque vs rpm, transmission ratios, wheelbase, etc., and you are encouraged to be creative in your selection. Modelling of ‘supercars’ is discouraged. They tend to be somewhat more challenging to model than something more modest, as their performance is significantly affected by the tire limits, which are notoriously tough to model accurately. In this case, you will have a hard time to draw meaningful conclusions from your simulations. Also, you should select a vehicle with a manual or semi-automatic transmission. The large majority of automatic transmissions are combined with a torque converter, (a fluid-based coupler) to allow the engine to idle while the vehicle is stationary. There will always be some slippage that occurs in the torque converter that is very challenging to model. (Note that most torque converters actually have three sets of blades internally, not two, with the middle set fixed to the transmission case by a one-way clutch. As a surprising result, they can actually increase torque output at low speeds, much like a gear reduction.)

Obviously, the more complete your specification data, the more accurate your model will be. Your data should also include measured performance information, ideally vehicle speed vs. time, and distance vs. time. Most popular automotive magazine tests will provide most of the information you need. Below are a series of tasks and questions that should assist you in preparing your report. Note that the sections of your report need not be numbered as indicated in the points below; section your report with meaningful headings as you feel most appropriate.

1. Calculate the vehicle’s theoretical top speed, using a maximum horsepower criterion (i.e., equate the maximum power that your engine produces to the power loss (total drag force times velocity)). If necessary, estimate the frontal area and drag coefficient of your vehicle. Assume that while the aerodynamic forces increase as a square of velocity, rolling resistance forces  $X_{\text{roll}}$  are linearly proportional to normal load at the tire, i.e.,

$$X_{\text{roll}} = k_{\text{roll}}Z$$



and that the rolling loss coefficient  $k_{\text{roll}}$  can be found from

$$k_{\text{roll}} = (6.5 \times 10^{-6})u^2 + 0.013$$

where  $u$  is the forward speed of the vehicle in m/s. Is the maximum speed that you compute a reasonable estimate? Explain why or why not. Compare it to the measured top speed, and explain any differences.

2. Consider the engine data for your vehicle. Your simulations will use an interpolation to predict the engine torque at any engine speed, so you will need data from several different engine speeds; ideally every 500 or 1000 rpm, from idle to redline. Use a linear interpolation to plot your fitted engine torque data as a function of engine speed. Do the curve fits accurately describe your engine over the entire range of speeds? What are the differences? What effect do you believe this will have on your calculations?
3. Consider the transmission and final drive ratios of your vehicle. Using a software tool of your choice, plot the engine speed (in rpm) as a function of vehicle speed (in km/h), showing each gear. Examine the range of speeds where the various ratios are useful. Is the ratio progression a geometric one? What does this tell you about the choice of ratios? Is the vehicle ‘undergeared’ or ‘overgeared’, i.e., when the vehicle is at its maximum speed, is the engine speed above or below the speed at which it makes maximum power? Why do you suppose this is?
4. Using the combined engine and transmission data, plot the maximum traction force available vs vehicle speed (in km/h), showing each gear. Show the total resistance force, and the tire limit traction force on the same plot as well. In which gear does the vehicle reach its maximum speed? Hint: it is not necessarily top gear. What is the maximum acceleration available at 100 km/h? Do you think this is sufficient for overtaking, or are downshifts necessary?
5. Download and install the EoM Julia library. Using the template provided (`drag_race.jl`), enter your vehicle and engine data, and simulate the drag strip performance of your vehicle. Note that Julia template solves the equations of motion by rewriting  $\Sigma \mathbf{X} = m\dot{\mathbf{u}}$  in the form  $\dot{\mathbf{u}} = f(\mathbf{u})$  and using a numerical differential equation solver.  
Use EoM to obtain plots of the vehicle’s performance, including distance vs time, velocity vs time, acceleration vs time, and axle normal force vs time. From these results, determine the 0 – 60 mile/h time, the 1/4 mile time, and speed at the 1/4 mile in mile/h. Compare the results with the real vehicle data, and comment. Do you over or under predict the actual performance? Where are the inaccuracies in your model? Explain how you might overcome some of these shortcomings with a more sophisticated model.
6. Modify the front/rear drive torque distribution of your parameter set, and repeat the simulations above. Try a few different choices. What affect do you see? What is the optimum torque distribution, and why is this the case?
7. Investigate the effect of modifying the height of the mass centre of your vehicle, and repeat the simulations above. Explore this effect for both front and rear wheel drive configurations. What affect do you see? Can you explain it?

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## SECTION 3

# REPORT #2

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Your task in this report is to evaluate the results of a simple vehicle handling simulation. Choose a set of parameters that describe a vehicle of your choice, ideally one which understeers. Reliable inertia and tire property data will be challenging to find; use typical values if necessary (see Jazar<sup>1</sup> or Heydinger et al<sup>2</sup>)

Below are a series of tasks and questions that should assist you in preparing your report. Note that the sections of your report should not be numbered as indicated in the points below; section your report with meaningful headings as you feel most appropriate.

1. Download and install the EoM Julia library. The EoM software will automatically generate and analyze the linearized equations of motion of a mechanical system. A number of sample input systems have been included to help you. In the list of example systems, you will find a yaw plane (bicycle) model that has been developed (yaw\_plane.jl). Use the EoM software to analyze the dynamics of the yaw plane model, with the vehicle specifications of your choice.
2. Consider the eigenvalue data that is presented. Describe what it tells you about your specific vehicle's behaviour. Can you determine the speed where the transient response of the vehicle shifts from pure decay to an oscillatory decay? Is this in the feasible speed range? What about the natural frequency? Is it in a range where it could be excited by the steering, or other external forces? Is there sufficient damping to prevent excessive yaw motions? Recall the relationship between eigenvalues, natural frequencies, damping ratios, and time constants.

$$s = \sigma \pm \omega_d i$$

$$\omega_n = \sqrt{\sigma^2 + \omega_d^2}$$

$$\zeta = -\sigma/\omega_n$$

$$\tau = -1/\sigma = 1/\zeta\omega_n$$

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<sup>1</sup>Jazar, R.N., 2013. Vehicle dynamics: theory and application. Springer Science & Business Media.

<sup>2</sup>Heydinger, G.J. et al, 1999. Measured Vehicle Inertial Parameters – NHTSA Data Through November 1998, SAE paper no 1999-01-1336

3. Consider the steady state response data that is presented. Determine the characteristic or critical speed of your vehicle. What is the maximum yaw rate sensitivity?
4. Select a forward speed of 30 m/s (108 km/h), and using the `splsim()` function, generate the time history of the vehicle's response. Use a sine wave steering input that lasts 2 seconds to change lanes to the left, and following a 5 second delay, a second similar sinewave to return to the original path. Adjust the amplitude of the sine wave to get an approximately 3.5 m lane offset. Be sure to use a sufficiently small time step, on the order of 0.01 seconds. Note that the function `EoM.pulse(t, t1, t2)` is provided for your convenience. This function is zero everywhere, except at times between  $t_1$  and  $t_2$ , where it is one.

From the simulations, obtain the following data plots:

- a) the yaw rate  $r$  vs time
- b) the body slip angle  $\beta$  vs time
- c) the lateral acceleration vs time
- d) the path of the vehicle

Comment on the physical significance of the results. Do they make sense? Are the yaw rate and slip angle results during the peaks of the steer input in agreement with the steady state gain expressions? Do you believe this is an accurate prediction of the vehicle's motion? What do you believe is the largest source of error in the model, and why? Look very closely at the transient behaviour of the slip angle immediately after the steer angle begins to change. You are likely to see some unexpected behaviour. Can you explain it?

5. Adjust the location of the centre of mass of the vehicle rearward (or forward, if it already oversteers), and repeat the simulations. Comment on the physical significance of the results. Are they what you would expect?
6. In the example input files, you will also find a planar truck and trailer model (`truck_trailer.jl`). Adjust the parameters of the model to represent a pickup truck and small trailer, and use it to investigate the effect of the location of the centre of mass of the trailer. Can you draw some conclusions on trailer safety from the results of this model? Pay special attention to the eigenvalue results.

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## SECTION 4

# REPORT #3

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Your task in this report is to evaluate the results of a number of vehicle ride models. Choose a set of parameters that describe a vehicle of your choice. Reliable inertia and tire property data will be challenging to find; use typical values if necessary (see Jazar<sup>1</sup> or Heydinger et al<sup>2</sup>)

1. If you have not done so already, download and install the EoM Julia library. The EoM software will automatically generate and analyze the linearized equations of motion of a mechanical system. A number of sample input systems have been included to help you. In the list of example systems, you will find a quarter-car model that has been developed (quarter\_car.jl). Use the EoM software to analyze the dynamics of the quarter-car model, with the vehicle specifications of your choice. Comment on the results, discussing the eigenvalues, and mode shapes. Hint: the eigenvector results are shown in the resulting animations. Consider the frequency response resulting from ground excitation, simulated by placing an actuator between the unsprung mass and the ground. Note that the ‘gain’ on the actuator matches the vertical stiffness of the tire, so the input is equivalent to vertical disturbance of the road. Comment on the responses, given what you know about the behaviour from your previous eigen-analysis. Are there any frequencies where the disturbances are actually amplified before they reach the vehicle? What is the worst frequency, in terms of ride isolation? What is the minimum road amplitude required to cause the tire to lose contact with the ground, and what frequency would be required for this to occur? Is this feasible? What would the resulting wavelength be at highway speed?
2. In the EoM code, you will find the `random_road()` function that, as the name implies, generates a random road profile. The profile is formed by a summation of a series of sinusoids where the amplitudes are linearly related to the wavelength, and the phase angles are chosen randomly. The function scales the amplitudes according

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<sup>1</sup>Jazar, R.N., 2013. Vehicle dynamics: theory and application. Springer Science & Business Media.

<sup>2</sup>Heydinger, G.J. et al, 1999. Measured Vehicle Inertial Parameters – NHTSA Data Through November 1998, SAE paper no 1999-01-1336

to the the ISO 8608 standard, such that the power spectral density (PSD) is linear with wavelength. (The PSD is measure of how the ‘power’ in the signal is distributed across the frequencies. Here power is a bit of a misnomer; the PSD is really only power if the signal is a voltage, and in this case it is just the square of the amplitude of the displacement signal. The concept of the PSD originates from circuit analysis.) The ‘class’ argument to the function is the roughness index, an integer ranging from 3 to 9, where 3 corresponds to the A-B class transition (very smooth) and 9 corresponds to the G-H class transition (very rough). The argument ‘L’ is the length of the road profile, which is also the longest wavelength used in the sum. The argument ‘B’ is the shortest wavelength used. The function defaults to a class A-B road of 100 m in length, with the shortest wavelength of 5 cm, requiring a sum of 2000 sinusoids. The output of the function is a function handle that will return a z coordinate when given the x coordinate. Using this function, generate a random road to act as the input to the quarter car model. Solve the time history of the sprung mass motion, the suspension compression, and the tire compression. Note that the `random_road()` function gives the disturbance as a function of location, where the `lsim()` differential equation solver requires the disturbance as a function of time, so a conversion based on the forward speed is required. Comment on the results.

3. You will also find a bounce-pitch model that has been developed. Use the EoM software to analyze the dynamics of the bounce-pitch model, with the vehicle specifications of your choice. Comment on the results, focusing on the eigen-analysis. Can you make any conclusions regarding the ‘flat-ride’ properties? Consider varying the properties to examine some special cases. Hint: the eigenvector results are shown in the resulting animations. Alternatively, you can consult the summary data, which contains the location of the centres of rotation, and the unit axis of rotation. Note that the values are presented as complex numbers, and in the event of a non-proportionally damped system, they may not give a purely real result.
4. Using the function `input_delay!`, you can force two inputs to a system to be identical, but shifted in time, as you might see in a vehicle driving over a disturbance. Use the `input_delay!` function to explore the ‘wheelbase filtering’ effect. Do the plots give the expected behaviour?
5. You will also find a four degree-of-freedom ‘half-car’ ride model has been developed, by merging a bounce-pitch model with two quarter-car models, i.e., the vehicle is a rigid body with vertical translation and pitch rotation as in the bounce-pitch model, and the front and rear suspension are two point masses, each with vertical translation only. Use the EoM software to analyze the half-car ride model. Comment on the results, again focusing on the eigen-analysis. Given what you know about the results of the bounce-pitch model, and the quarter-car model, are the results what you would expect? Describe the frequencies and motions associated with each mode that are identified.
6. You will also find a simple five-body full car model that has been developed. Use the EoM software to analyze it, with the vehicle specifications of your choice. Comment on the results, again focusing on the eigen-analysis. Describe the frequencies and motions associated with each mode that are identified.

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## APPENDIX A

# SYNTAX AND STYLE

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The following are some tips for writing technical documents.

**Fonts:** Choose a nice serif style font, e.g., Times, Cambria, or Garamond. For a printed document, where high resolution is typical (2400+ dpi), serifs aid in reading comprehension. Use a sans font, e.g., Arial or Calibri, for slide shows, where it will be seen with much lower resolution. A screen is on the order of 200 dpi, on a projector at a distance, even lower. Avoid handwriting or display style fonts. Avoid underlines; they are a leftover from the typewriter era, and do not belong in a modern digital document. Boldface is fine for section or subsection titles, or to label a figure or table, but just the words ‘Table’ or ‘Figure’ and the number, not the entire label. Use italics sparingly, to draw attention to words of special interest in the document, e.g., when introducing a new term or topic.

**Equations:** Avoid using whole words to stand for mathematical symbols. Each symbol should be a single character, to avoid confusion with multiplication. Use italics for variables, upright for mathematical constants ( $\pi$ ,  $e$ ,  $i$ ), and boldface for vectors. A variable vector would use bold italics. Greek letters are fine, apply rules consistent with Latin. Avoid uppercase, except for matrices, which are usually bold upright uppercase. When using subscripts, use upright if the subscript is a name or a word, and italics if the subscript is a variable. Equations should be numbered flush right in brackets. When defining the symbols that appear in an equation, do not start a sentence with a symbol.

**Units:** There is no need to apply special formatting, e.g., italics, to units. Choose a consistent set, preferably SI, for the entire document. If imperial units are appropriate in a particular instance, give the converted SI value as well, in round brackets, e.g., the tube diameter was 1 in (25.4 mm). There should be a single space between the value and the unit, e.g., 7 kg. Lowercase is standard; the exception is units containing proper names, e.g., kiloNewtons are denoted as kN.

**Headings:** When referencing a chapter or section, there is no need to capitalize, e.g., in the next chapter..., unless referencing a particular chapter, e.g., in Chapter 1. When

referencing an equation, the whole word 'Equation' should be used, rather than 'Eqn'. Table headings should be above the table, figure headings should be below the figure. A chapter heading requires a page break, and should start approximately 1/3 of the way down the page. Chapter headings should be left justified, rather than centred.

**Language:** Use Canadian spellings, e.g. centre, neighbour, etc.

**Latin contractions:** The contraction 'i.e.' stands for 'id est' and translates roughly as 'in other words', while 'e.g.' is short for 'exempli gratia', and means 'for example'. Make sure to choose correctly when rephrasing or giving examples. A comma following a Latin contraction is good practice, but not required as long as you are consistent.

**Spacing:** Use two spaces after periods. Single spaces after periods ('French spacing') makes it harder to detect sentence breaks. Learn the difference between a hyphen, an n-dash, and an m-dash, and use them appropriately. Line spacing of 1–1.15 is usually fine, but some documents such as theses often specify more.

**Referencing:** Usually other sources are referenced in the text by author name, rather than just the reference number, but it is not mandatory, especially if multiple references are given in series. It is standard to place the reference number in square brackets, immediately following the author's name, without a space, e.g., see the results from Smith[1]. For references with multiple authors, the contraction 'et al.', meaning 'and others', may be used in the text, but in the List of References, all authors' names should appear. (As an aside, 'et' is Latin, not French, and so the 't' is not silent.) Finally, when referring to multiple works of other authors, note that the word 'research' is like the word 'water'; it is measured, not counted, so there is no plural form. The word 'researches' is a verb, and not a plural noun.

**Commas:** There is some flexibility in comma frequency, but their use is not optional. Omitting the Oxford comma, i.e., the one following the 'and' when listing three or more items can have a significant effect on the meaning. As an example, consider the following: I must thank my parents, Sir Isaac Newton and Queen Elizabeth II. This may lead the reader to believe that your parents are Newton and the Queen; a comma after Newton would imply that you are thanking Newton and the Queen, in addition to your parents.

**Apostrophes:** The apostrophe is used to indicate a possessive, or a contraction, but is frequently misused to indicate a plural. Contractions should be avoided in technical writing, e.g., use 'did not' instead of 'didn't'. Tip: if in doubt, you probably don't need an apostrophe; they are placed incorrectly more often than they are missed. If you can replace the word 'its', with 'his' or 'hers' and the sentence still reads correctly, you don't need the apostrophe. If you have a plural possessive, the apostrophe is placed after the 's', e.g., if we have multiple students, we might say that the students' exams were very well done. On a related note, quotation marks should generally be avoided when indicating emphasis, and should not be used in addition to italics.

**Slang** Avoid slang words, e.g., the result was messed up, or common nicknames, e.g. 'chromoly' in place of AISI 4130 alloy steel.

**Grammatical person** Avoid first person, i.e., the use of ‘I’ or ‘we’, and instead favour third person passive, e.g., ‘I took measurements...’ should instead read ‘Measurements were taken...’

**Phrases/clauses** Learn when to use ‘that’ as opposed to ‘which’. While ‘that’ indicates the start of a phrase, ‘which’ indicates the start of a clause. As a result, there should usually be a comma before ‘which’, and not before ‘that’. The difference between a phrase and a clause is that a phrase does not express a complete thought on its own and so would not stand as a complete sentence, while a clause will. Examples: ‘I went to the store that was on the corner’ vs ‘I went to the store, which is why I was late for school.’

**Homophones** English is full of words that sound the same, but have very different meanings and spellings. Just put in the effort and learn them. Fairly or unfairly, your work will be immediately dismissed by many readers when making these kinds of mistakes. Common examples include using the wrong choice of ‘there, their, they’re’, and ‘your, you’re’. Less common examples are ‘brake vs break’, and ‘steel vs steal’. Finally, note the difference between ‘then’ and ‘than’; the first is frequently misused where the second is proper. If you are discussing amounts or choices, the proper use is ‘more than’ or ‘rather this than that’, where ‘then’ is used to indicate a sequence or a decision, e.g., ‘do this first, then that later’ or ‘if you do this, then that happens’.