

#### Mechanical, Automotive, & Materials Engineering

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# Report 3: Vehicle Ride Models

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#### Introduction

The analysis of various car models to see realistic results and the chose of parameters that are available to use. This is important when analysing the car behaviour under rough road conditions and to design to be better adaptive for the consumer experience.

### 1 Quarter Car Model

Variables	Value
ms	200kg
mu	36kg
kt	119921 N/m
ks	17658 N/m
cs	3750Ns/m

Table 1.1: Variables of the Quarter Model

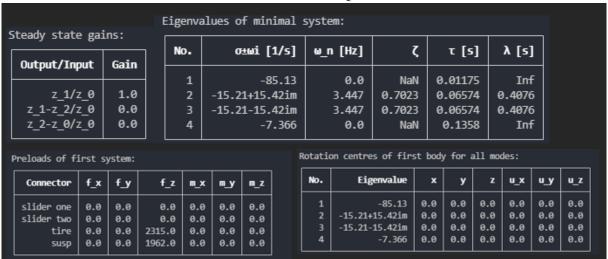


Figure 1.1: Eigenvalues for the Quarter Car Model

When viewing eigenvalues of minimal systems in figure 1.1, there is only one natural frequency with complex eigenvalues and 2 real eigenvalues. The natural frequency is much higher than the region of 1-2 hz meaning it is running at a higher frequency meaning tires will be more at use to supress with a higher natural frequency. Eigenvalues as seen in figure 1.1, within the rotation centres of first body for all modes there is not any rotation in any direction, with the all the eigenvalues. The chassis of the car does not rotate at the given eigenvalues of the minimal system. Preloads of the first system is showing the weight system acting upon the tire and suspension. Using the eigenvalues can find the wavelengths and the damping. At 10hz where your tires are excited before the suspension, while at 1hz is where the suspension and chassis excites before the tire which is the preferable, so the worst frequency is at 10hz.

After viewing all the 4 modes using EOM XD software, I was able to better understand each of the eigenvalue points. The common factor of all modes is that they are all in compression with a negative notation. With mode 1, it experiences a shock with a heavy bump to all systems with the later frequency parts were dealt with by the suspension and the tires movement. With the second mode, a lot of the movement occurs with the chassis meaning that where most of the absorption would be. The third mode simulation looked more like the load was dropped with most of the movement between the chassis and

suspension. The fourth mode had the chassis and the suspension moving therefore it shows that the tire is not in effect here. Overall mode 1,2 and 3,4 are similar with which system is in motion with the eigenvalues at that mode.

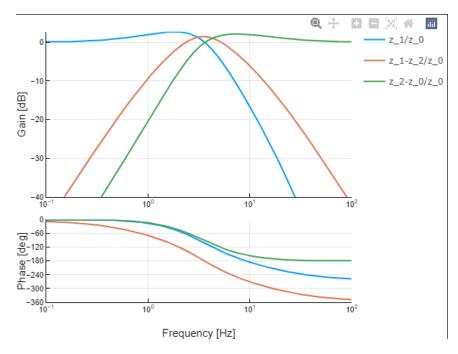


Figure 1.2: Gain and Phase vs Frequency for Quarter Car Model

The blue line in figure 1.2 is the frequency of the chassis lifting up and down, orange being absorbed by the suspension of the vehicle and green being the compression of the tires of the vehicles. Analysing the highest gains of each section it is evident that in low frequency the cabin of the vehicles is dealing with, medium level of frequency is dealt with by the suspension for a short duration while the high frequency is being handled by the tires in majority in each section as all three are continuously working together for ride quality.

When analysing the frequency where the disturbances would amplify the before they reach the vehicle it can be seen from the blue line representing the cabin in figure 1.2, with an amplification of 134%, which is generally not as bad as many systems. With other data that has been run previously at 184% amplified. 34% was the highest amplitude however it does occur in all three sections. As evidently seen, this would be the worst frequency at 2.3HZ which is low frequency disturbance.

The minimum road amplitude required to cause the tire to lose contact with the ground would be at the peak of the green curve of the tire in figure 1.2. The size of the tire compression will be 127% times the size of the road, which could be solved for by realizing the correlation of the amplitude of the oscillatory motion is equal to the static compression of the tire. The frequency which this occurs is at 7.4HZ which is at high frequency. The amplitude of the bump is 0.01288m will cause the tire to lose contact with the ground. This was calculated from static compression of the tire which is the stiffness multiplied by the deflection equaled to mass force. After finding the gravity and having the relation between the road size and tire size dividing the ratio to the deflection got the amplitude of the bump. I would not think a 1.29cm bump to cause the car to lose traction.

When understanding the wavelength of the highway speeds, it would de amplify the gains a lot as inertial forces would allow for high frequency in response. With higher speeds causes lower frequency, a lot of

the frequency absorption will be done by the chassis as the chassis is the best at absorbing low frequency. With the vehicle going at a higher speed, it would create a shock type force which would occur at lower frequency leading to chassis absorption. It would happen so quick that the overall wavelength would be smaller.

### 2 Random Road Simulation

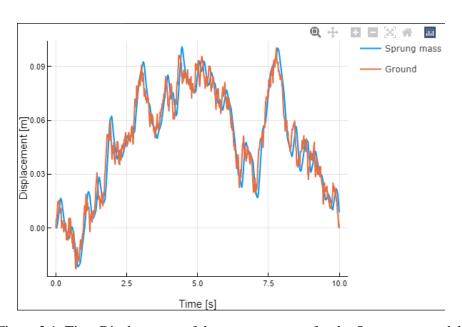


Figure 2.1: Time Displacement of the unsprung mass for the Quarter car model.

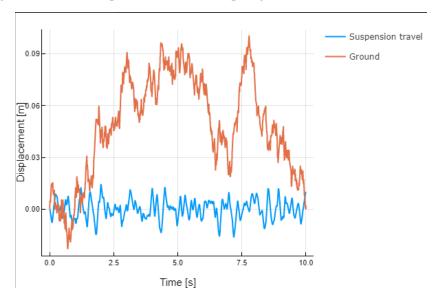


Figure 2.2: Time Displacement of the suspension for the Quarter car model.

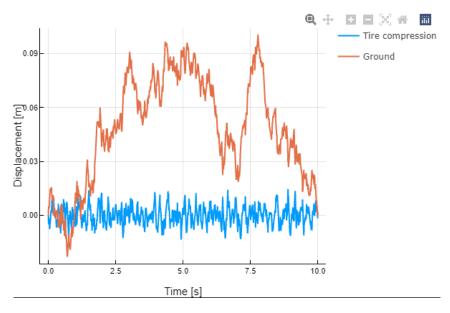


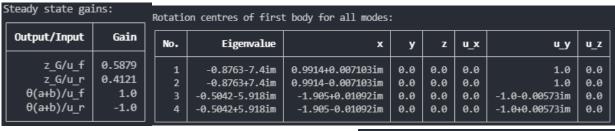
Figure 2.3: Time Displacement of the tire compression for the Quarter car model.

After altering the random road generator to a roughness factor of 3, figure 2.1 to 2.3 were formed. Claims made in section 1 of the report are further supported through the time displacement as when viewing the unsprung mass graph in figure 2.1, it is seen that the displacements are longer, and smoother compared to the other graphs. This is due to the fact that the chassis is absorbing mostly the low frequency bumps when riding along the road. The chassis displacement is following the ground curve without having every small bump showing that the chassis is moving with the longer bigger bumps proving section 1 more. When reviewing figure 2.2, it is seen that with the suspension it deals with the medium frequency as the displacement is with shorter waves than unsprung mass and but bigger waves than tire compression. This is further supported with the initial graph of figure 2.3, with how the suspension would handle all the medium frequencies between the chassis and tire. The tire compression displacement time graph in it is more violent looking displacement with straight up and down with small amplitudes alluding to higher frequency being transmitted to the tire. With all 3 of the graphs presented in this section added would equal to the orange line of the road as every bump needs to go somewhere showing that all three systems are working together to dampen the road amplitudes into the cabin of the vehicle.

### 3 Bounce Pitch Model

Variables	Value
m	1730kg
a	1.189m
b	1.696m
kf	35000N/m
kr	38000N/m
cf	1000Ns/m
cr	1200Ns/m
Iy	3267

Table 3.1: Parameters for the bounce pitch model.



Eigenvalues of minimal system:						Preloads of f	irst sy	ystem:				
No.	σ±wi [1/s]	ω_n [Hz]	ζ	τ [s]	λ [s]	Connector	f_x	f_y	f_z	m_x	m_y	m_z
1 2 3 4	-0.8763-7.4im -0.8763+7.4im -0.5042-5.918im -0.5042+5.918im	1.186 1.186 0.9453 0.9453	0.1176 0.1176 0.08489 0.08489	1.141 1.141 1.983 1.983	0.8491 0.8491 1.062 1.062	bounce pitch front susp rear susp	0.0 0.0 0.0 0.0	0.0 0.0 0.0 0.0	0.0 0.0 0.0 0.0	0.0 0.0 0.0 0.0	0.0 0.0 0.0 0.0	0.0 0.0 0.0 0.0

Figure 3.1: Eigenvalues for the bounce pitch model.

When viewing the eigenvalues in the 3d simulation, for the first and second mode is experiencing more pitch motion with the front bouncing at a higher amplitude than the rear. The third and fourth eigenvalue modes experience more bounce but do exhibit some pitch with this time more in the rear.

Further analyzing figure [...], there is a rotation of the y in eigenvalues 1 and 2 in the pitch direction of y, while the other two eigen values experience a negative pitch with complex values. Preloads are none as there nothing that was preloaded on the vehicle. Viewing the natural frequency with the range of 1-1.2hz which is low. Damping is higher on the first two eigenvalue modes.

$$c = \frac{bk_r - ak_f}{k_f + k_r}$$

$$c = 0.3127m, \frac{c}{a+b} = 10.8\%$$

Figure 3.2: Distance of the spring center to the mass centre with the mass being behind the mass center.

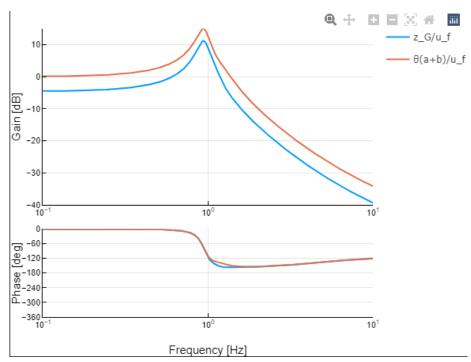


Figure 3.3: Initial parameters for the bounce pitch model

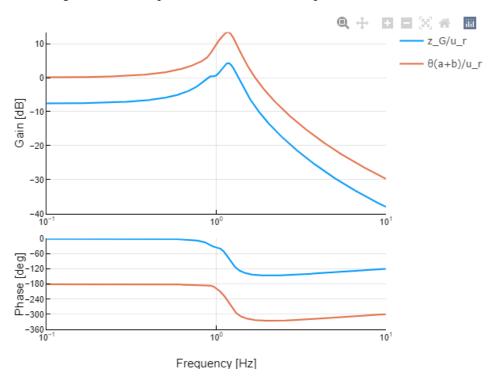


Figure 3.4: Initial parameters for the bounce pitch model.

When analyzing flat ride properties, it is common to have b length larger than a, with the rear stiffness also being larger than the frontal stiffness. These properties will lead to more improved ride quality, therefore having the higher rear frequency allows for the motion to catch up the front overall reducing the pitch effect. This is evidently seen in figure 3.3 and 3.4, as seen by the phase angle is offset with comparing the rear and front and in turn has a more separation of the gain. When understanding Olley's criteria, the first is that the rear is at least 30% greater than the front which is seen with the 200Ns/m difference between the dampening and the tire stiffness is similar. This will show how beneficial it is compared to in the next section of without any dampening. The bounce and pitch frequency should be close together as seen in figure 3.1, they are both relatively at 1hz. These criteria better understand flat ride properties which this vehicle in the current parameters has but will better understand why in the following section. The third criteria of the natural frequency should not be above 1.28hz which the values. Center of oscillation is not fixed any more with the damping added. The second criteria also hold as seen in figure 3.2, where it is above 6%.

Then the special case was considered without any dampening which is what the following results are.

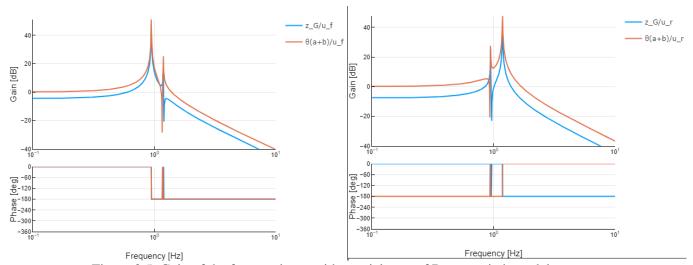


Figure 3.5: Gain of the front and rear with special case of Bounce pitch model

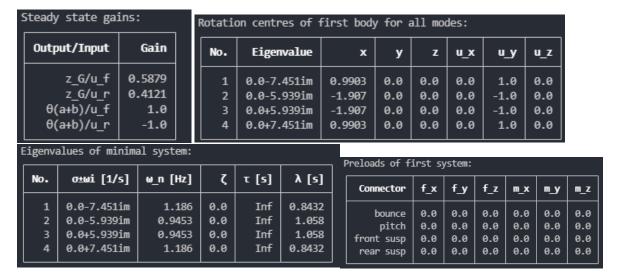


Figure 3.6: Eigenvalues of special case of bounce pitch model with no damping.

When reviewing the same parameters without the dampening it is seen in figure 3.6, the rotation is different with the mode 2 and 3 being -1 in the y direction and other two being positive one, however with dampening it is a pairing of mode 1,2 and 3,4. The eigenvalues themselves are 0 with a complex number involved. When viewing the 3d simulations all the modes are experiencing heavier pitch than bounce compared to with damping where two modes still experienced bounce. This is important in terms of ride quality as a passenger would prefer bounce more than pitch which is what the dampers are helping to produce. Natural frequency is the same as the previous parameters. With the dampened scenario the rotation is complex with non proportionally damped system do not have real results. When also comparing the oscillation, center of oscillation is not fixed any more with the damping added this is seen once comparing without damping. This is due to the additive complex numbers once the damping is added. Frequency stays the same but the complex of center of rotation changes the aspect of this simulation.

Therefore, the flat ride properties are fulfilled with the Olley's criteria when the damping is added. When viewing the gain frequency graphs, of figure 3.3, 3.4 and 3.5 it is evidently seen the effects of having and not having dampeners with sporadic gains without damping. This show how much of the cabin will feel the bump or disturbance that is occur compared to when damping is there it is a lot smoother and at a lower gain value.

### 4 Wheelbase Filtering

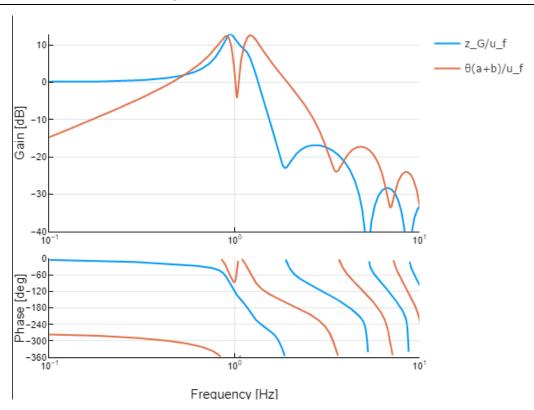


Figure 4.1: Input delay graph of bounce pitch model.

A lot is still happening near the natural frequency with the jump in the curve but looking past that there are holes that appear. When the pitch is going to zero, the vehicle is having more bounce properties. When analyzing figure 4.1, with time lag delay the orange curve of pitch reaches a minimum before jumping up it is accompanied by the lift of the blue curve at its peak which is correlated to the bounce of the vehicle. Bounce and pitch are inversely proportional in the sense that when pitch is at the minimum, bounce is at the maximum. This is also the case where the pitch is at its highest where bounce is its lowest. At the beginning of the graph in figure 4.1, a gain of 1 means the input equals the output meaning it is just lifting the vehicle up and down even which is reassured to be bounce based on wheelbase filtering. Wheelbase filtering is the assumption of the front and rear ground motions to be the same but only shifted by the time difference to see that both act in the same way which is why only front is being analysed in the graph. The time shift causes either the bounce or pitch motion to be alternately excited depending on the wavelength. When wheelbase happens to be close to the wavelength bounce is initiated, but if wavelength is half the wavelength, it is pitch. The lag effects become lower when frequency becomes low it is not going to be exciting pitch motion.

The plots did give the expected behaviour as when the tires both experience the bump it is clearer of the motion of pitch and bounce. When bounce occurs, the pitch is very little and the opposite when pitch is maximum. This is reasonable when understanding the overall motion of the vehicle and analysis of the previous sections.

#### 5 Half Car Model

-1.016+81.88im

-1-015+82-25im

-1.015-82.25im

-0.07848-7.985im

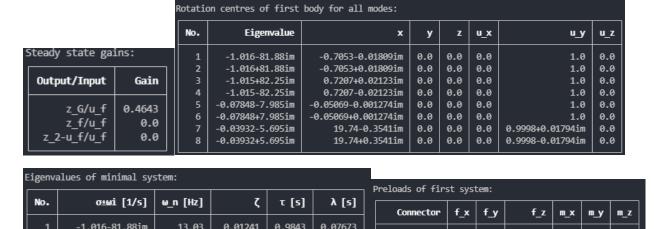
-0.07848+7.985im

-0.03932-5.695im

-0.03932+5.695im

3

The half car model is the combination of two quarter car and one bounce pitch model to get a better understanding of the behaviour of the vehicle.



0.01241

0.01234

0.01234

0.009827

0.009827

0.006904

0.006904

13.03

13.09

13.09

1.271

1.271

0.9064

0.9064

0.9843

0.9855

0.9855

12.74

12.74

25.43

25.43

Figure 5.1: Eigenvalues of Half car model.

0.07673

0.07639

0.07639

0.7868

0.7868

1.103

1.103

slider front

slider rear

road frc

road mmt

front susp

rear susp

tire

tire

0.0

0.0

0.0

0.0

0.0

0.0

0.0

0.0

0.0

0.0

0.0

0.0

0.0

0.0

0.0

0.0

0.0

0.0

0.0

9109.0

10510.0

9600.0

11000.0

0.0

0.0

0.0

0.0

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0.0

0.0

0.0

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0.0

0.0

0.0

0.0

0.0

0.0

0.0

0.0

0.0

0.0

0.0

Parameter	Value
m	2000kg
a	1.5m
b	1.3m
Cf	100Ns/m
Cr	100Ns/m
Iy	2000

Table 5.2: Values used for the half car model.

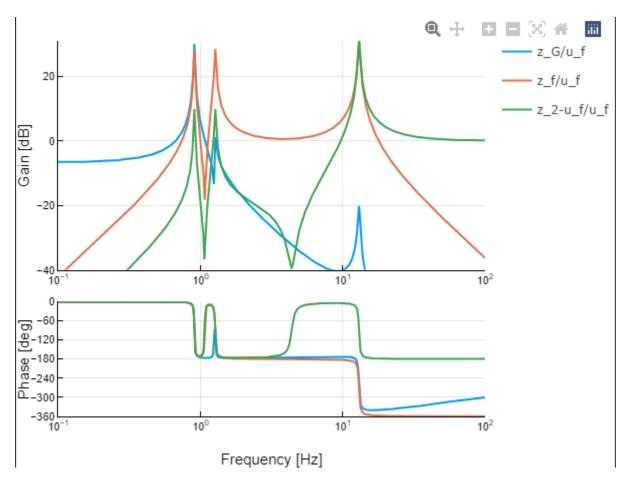


Figure 5.3: Gain Graph of the half car model.

When analysing the half car model, it had a lot of characteristics of both car models as all were being represented in the modes. When viewing figure 5.1, with all the eigenvalues it is evident that there are two sets of high frequency at 13hz and two sets of low frequency at 1hz. When reviewing rotation centers in figure 5.1, it is evident that all modes are in pitch inertia while the fourth mode is pitch however with complex meaning the oscillation is not fixed at a point allowing for the bounce to occur. The system is incurred with preloads for the suspension and tires through the actual weight of the vehicle.

When analyzing the first mode it has a high oscillation of the front axle while the second mode is the high oscillation of the rear axle. This is representative of the natural frequency which these modes are occurring at high frequency. The third mode is the low frequency affects of pitch that occur on the vehicle chassis with the representation of the unsprung mass from the quarter car model. Mode 4 is the representation of the low frequency bounce that occurs on the chassis, but the XD program the study of how it effects the quarter car model can be analysed with a simple up and down oscillation equally on both sides.

The results are expected as all four modes represented all the motions that would occur, but when analysing the actual motion some did not make sense whether due to parameter values or functionality. This is mostly evident in the first two modes as the tires and unsprung mass would be below the ground which is not physically possible making for an unrealistic simulation and results. There is a potential error due to parameter ratios however even if the scenario should not be like that so either over damping or underdamping creating higher oscillation for high frequency region.

Reviewing the gain graph in figure 5.3, it can be seen how sporadic it can be based on the how aggressive gain is without smooth lines like in the first section. The overall frequency is higher with very drastic deeps causing the everything to deep. When analysing at the two frequency ranges where the eigenvalues are it is evident that at the low natural frequency tires and suspension are at the lowest leading to chassis to be the main absorption. This is clear when viewing the modes as the low frequency is the bounce pitch motions. Further showing at the high natural frequency the tire and suspension at the peak further supporting the fact the high frequency is supported from the suspension and tires.

### 6 Full Car Model

Parameters	Value
m	1730kg
a	1.189m
b	1.696m
tf	1.595m
tr	1.631m
cf	1000Ns/m
cr	1200Ns/m
Iy	3267kg m^2

Table 6.1: Parameters of Full car model.

The full car model has 4 quarter car models and two bounce pitch models for a more realistic approach on the vehicle analysis with the parameter in table 6.1.

Eigenv	Eigenvalues of minimal system:									
No.	σ±wi [1/s]	ω_n [Hz]	ζ	τ [s]	λ [s]					
1	-20.3-78.15im	12.85	0.2514	0.04927	0.0804					
2	-20.3+78.15im	12.85	0.2514	0.04927	0.0804					
3	-20.15-78.51im	12.9	0.2486	0.04963	0.08003					
4	-20.15+78.51im	12.9	0.2486	0.04963	0.08003					
5	-14.45+73.3im	11.89	0.1934	0.06922	0.08571					
6	-14.45-73.3im	11.89	0.1934	0.06922	0.08571					
7	-14.42+73.38im	11.9	0.1929	0.06933	0.08562					
8	-14.42-73.38im	11.9	0.1929	0.06933	0.08562					
9	-1.461+6.971im	1.134	0.2051	0.6846	0.9013					
10	-1.461-6.971im	1.134	0.2051	0.6846	0.9013					
11	-1.453-6.973im	1.134	0.204	0.6881	0.9011					
12	-1.453+6.973im	1.134	0.204	0.6881	0.9011					
13	-0.8469-5.538im	0.8916	0.1512	1.181	1.135					
14	-0.8469+5.538im	0.8916	0.1512	1.181	1.135					

Steady state gains:

Output/Input	Gain
z_G/u_lf	0.2946
θ(a+b)/u_lf	-0.5061
φ(t_f)/u_lf	0.4821

Rotation centres of first body for all modes:

No.	Eigenvalue	x	у	z	u_x	u_y	u_z			
1	-20.3-78.15im	1.115-0.004998im	0.0	0.0	0.0	1.0+0.004484im	0.0			
2	-20.3+78.15im	1.115+0.004998im	0.0	0.0	0.0	1.0-0.004484im	0.0			
3	-20.15-78.51im	0.0	0.0	0.0	1.0	0.0	0.0			
4	-20.15+78.51im	0.0	0.0	0.0	1.0	0.0	0.0			
5	-14.45+73.3im	-1.583+0.006912im	0.0	0.0	0.0	-1.0-0.004365im	0.0			
6	-14.45-73.3im	-1.583-0.006912im	0.0	0.0	0.0	-1.0+0.004365im	0.0			
7	-14.42+73.38im	0.0	0.0	0.0	1.0	0.0	0.0			
8	-14.42-73.38im	0.0	0.0	0.0	1.0	0.0	0.0			
9	-1.461+6.971im	0.0	0.0	0.0	1.0	0.0	0.0			
10	-1.461-6.971im	0.0	0.0	0.0	1.0	0.0	0.0			
11	-1.453-6.973im	1.025-0.004662im	0.0	0.0	0.0	1.0+0.004549im	0.0			
12	-1.453+6.973im	1.025+0.004662im	0.0	0.0	0.0	1.0-0.004549im	0.0			
13	-0.8469-5.538im	-1.844-0.006707im	0.0	0.0	0.0	-1.0+0.003637im	0.0			
14	-0.8469+5.538im	-1.844+0.006707im	0.0	0.0	0.0	-1.0-0.003637im	0.0			

Preloads of first system:

Connector	f_x	f_y	f_z	m_x	m_y	m_z
left front susp	0.0	0.0	0.0	0.0	0.0	0.0
right front susp	0.0	0.0	0.0	0.0	0.0	0.0
left rear susp	0.0	0.0	0.0	0.0	0.0	0.0
right rear susp	0.0	0.0	0.0	0.0	0.0	0.0
road frc	0.0	0.0	0.0	0.0	0.0	0.0
road mmt	0.0	0.0	0.0	0.0	0.0	0.0
left front spring	0.0	0.0	4988.0	0.0	0.0	0.0
right front spring	0.0	0.0	4988.0	0.0	0.0	0.0
left rear spring	0.0	0.0	3497.0	0.0	0.0	0.0
right rear spring	0.0	0.0	3497.0	0.0	0.0	0.0
left front tire	0.0	0.0	5332.0	0.0	0.0	0.0
right front tire	0.0	0.0	5332.0	0.0	0.0	0.0
left rear tire	0.0	0.0	3792.0	0.0	0.0	0.0
right rear tire	0.0	0.0	3792.0	0.0	0.0	0.0

Figure 6.2: Eigenvalues of full car model.

When reviewing the full car model, the eigenvalue there are four modes of high frequency, with the three modes of low frequency. The low frequency is occurring at 1hz while the high frequency is occurring at 12hz. All eigenvalues of the minimal system are complex making the range of motion more. When analysing figure 6.2, the rotation of motion is occurring in the pitch and roll moments creating a more realistic representation. This leads to the representation of bounce pitch and roll motions.

With a total of 7 modes analysing the motion of the vehicle it is evident within the first mode and second mode is the front axles are coupled with the same amplitude oscillation within a high frequency. When reviewing the 3d model for modes three and four, it is the opposite of the first two modes as the front axle is bouncing but not coupled together but oscillating at an offset phase while also occurring at high frequency. With modes 5 and 6 it is the same motion as modes 1 and 2 however for the rear axles within the high frequency range. With modes 7 and 8 it is the same motions as 3 and 4 however for the rear axles in the high frequency range. Modes 9 and 10 are very small bounce motion within the low frequency but the main forces represented in this mode is the roll of the vehicle that occurs as seen in the table of eigenvalues. When analysing mode 11 and 12, it is both pitch and bounce motion however with a more prominence of pitch on the front axles occurring at low frequency. Mode 13 and 14 has similar motion to mode 11 and 12, however the pitch motion is more emphasized in the rear axles.

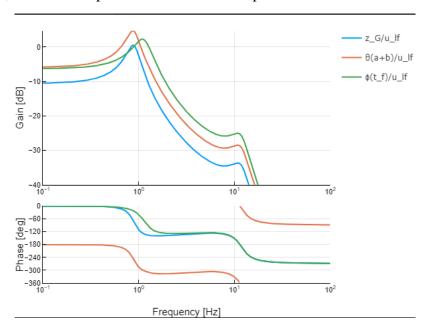


Figure 6.3: Gain graph of the full car model.

When investigating figure [...], it shows a more realistic and feasible result compared to that of the half car model. With the irregularities of the half car not being seen in the full car as all the systems are uniformly working together for a more realistic result. In the gain graph the frequencies associated with the type of motion can be seen. At low frequency the peak of the gain is high showing that the frequency can be felt in the cabin leading for the roll, bounce, and pitch motions to occur. Later at the low natural frequency there is another bump with a negative gain which is representative of the absorption through suspension and tires.

# Conclusion

In conclusion when analysing each of the models separately the motions are limited in a one-dimensional entity causing it not to be a good representation of vehicle motion. However, when combining to the full car model it allows for more representation of motion which is important when completing vehicle analysis. When considering Olley's criteria, it further benefits for a flat ride and with damping this allows for the frequency to be absorbed by the appropriate system based on the wavelength of the frequency.

## References

- [1] "Catalog the Catalog of Cars, Car Specs Database." *Automobile*, https://www.automobilecatalog.com/.
- [4] Minaker, Bruce P. Fundamentals of Vehicle Dynamics and Modelling: A Textbook for Engineers with Illustrations and Examples. Wiley, 2020.