Stratified Merkle Tree: A Novel Data Structure for Efficient Key-Value Storage and Verification

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Abstract

This paper introduces Stratified Merkle Tree (SMT), a novel data structure that integrates hash-based partitioning with Merkle tree authentication to address limitations in traditional Merkle trees and hash tables. By organizing data into stratified layers, SMT achieves efficient insertion, lookup, and deletion operations with an expected time complexity of O(1) under uniform hash distribution, while maintaining cryptographic verifiability through layered Merkle roots. Empirical evaluations demonstrate that SMT outperforms conventional Merkle trees, offering approximately 3.8 times faster insertions, 4 times faster lookups, and 57% less memory usage for a dataset of 5,000 key-value pairs. This makes SMT particularly suitable for applications requiring frequent updates and verifications, such as blockchain state trees and versioned key-value stores.

1 Introduction

Merkle trees, introduced by Ralph Merkle in 1979 [1], are a cornerstone of cryptographic systems, particularly in distributed ledger technologies like blockchain. They enable efficient verification of large datasets by organizing data into a binary tree of hashes, where leaf nodes represent hashes of data blocks, and non-leaf nodes are hashes of their children. This structure supports compact membership proofs, allowing light clients to verify data inclusion without processing the entire dataset. However, traditional Merkle trees face several challenges:

- **Time Complexity**: Insertions and updates require O(log n) time due to hash recomputations along the path from leaf to root.
- **Recomputation Overhead**: Modifying a single element necessitates updating all hashes up to the root.
- **Memory Inefficiency**: Sparse datasets lead to wasted memory, as empty leaves still contribute to the tree structure.

To address these limitations, we propose the Stratified Merkle Tree (SMT), a novel data structure that combines hash-based partitioning with Merkle tree authentication. SMT organizes the key space into multiple layers, each functioning as a hash-based container with its own Merkle root. This stratification reduces operation complexity to

O(1) on average, minimizes recomputation through dirty flag mechanisms, and maintains cryptographic verifiability via a hierarchical root structure. Our empirical results show significant performance improvements over conventional Merkle trees, making SMT ideal for systems requiring high update throughput and verifiable data integrity.

This paper is organized as follows: Section 2 describes the SMT architecture and key stratification algorithm. Section 3 presents the core algorithms for insertion, lookup, deletion, and Merkle root computation. Section 4 details the performance evaluation methodology and results. Section 5 compares SMT with existing solutions, highlighting advantages and limitations. Section 6 discusses potential applications, and Section 7 concludes with future research directions.

2 Data Structure Design

2.1 Core Architecture

The Stratified Merkle Tree (SMT) organizes key-value pairs into multiple layers, each representing a partition of the key space. The main SMT structure is defined as follows:

```c
typedef struct {
    Layer layers[MAX_LAYERS];         // Array of stratified layers
    int layer_count;                  // Active layers count
    unsigned char top_level_root[HASH_SIZE];    // Root hash of
       all layers
    int dirty;                       // Cache invalidation flag
    size_t total_elements;           // Total elements in SMT
} SMT;
```

Each layer is a container for a subset of key-value pairs, defined as:

```c
typedef struct {
    Element* elements;               // Dynamic array of elements
    int element_count;              // Current elements in layer
    int capacity;                   // Allocated capacity
    unsigned char merkle_root[HASH_SIZE];     // Layer-specific
       root hash
    int dirty;                     // Layer modification flag
} Layer;
```

Elements within a layer are key-value pairs with associated metadata:

```c
typedef struct {
    char* key;
    char* value;
    int priority;
    size_t key_len;
    size_t value_len;
} Element;
```

The SMT uses a fixed number of layers (`MAX_LAYERS = 256`), each identified by a priority derived from the key's hash. The `top_level_root` is a SHA-256 hash of all non-empty layer roots, ensuring cryptographic integrity. The `dirty` flags at both SMT and layer levels optimize recomputation by marking only modified components.

2.2 Key Stratification Algorithm

Keys are assigned to layers using a deterministic hash-based priority calculation, implemented as:

```c
static int compute_priority(const char* key) {
    unsigned char hash[HASH_SIZE];
    safe_hash(key, strlen(key), hash);

    uint32_t priority = 0;
    for (int i = 0; i < 4; i++) {
        priority = (priority << 8) | hash[i];
    }

    return priority % MAX_LAYERS;
}
```

This algorithm computes the SHA-256 hash of the key, extracts the first 4 bytes to form a 32-bit integer, and maps it to a layer using modulo `MAX_LAYERS`. Assuming SHA-256 provides a uniform distribution, this ensures balanced key distribution across layers, yielding an expected O(1) time complexity for operations.

2.3 Structural Representation

The SMT structure can be visualized as a two-level hierarchy (see Figure 1). Each layer maintains a dynamic array of elements and a Merkle root, computed as a hash of all elements in the layer. The top-level root is the hash of all layer roots, providing a single point of verification for the entire dataset.

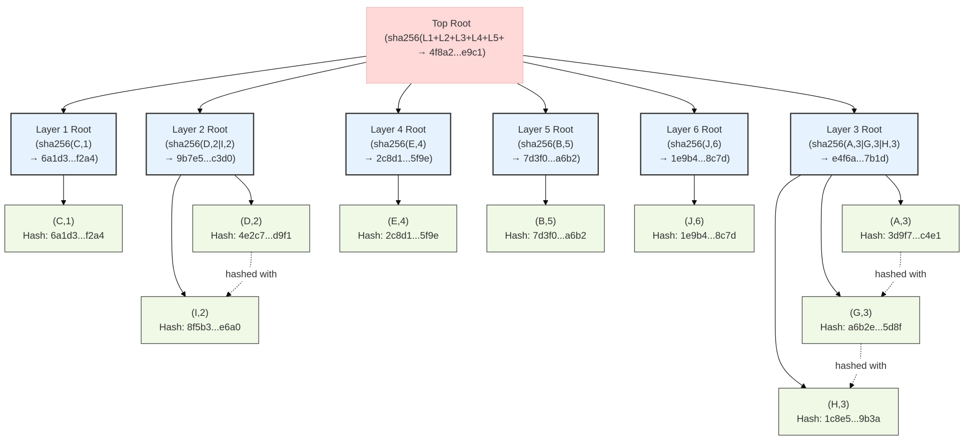

Figure 1: Structure of the Stratified Merkle Tree, showing layered organization and hash aggregation.

2.4 Dynamic Layer Adjustment

The static configuration of `MAX_LAYERS = 256` in the Stratified Merkle Tree (SMT) ensures predictable performance for uniform key distributions but may lead to inefficiencies for skewed distributions or large datasets, where some layers become overloaded, increasing intra-layer search time to O(k) (Section 5). To address this limitation, we propose a dynamic layer adjustment mechanism that adapts the number of layers at runtime based on dataset size and key distribution.

The dynamic approach modifies the SMT structure to include a resizable array of layers, with `layer_count` tracking the current number of layers and `max_layers_capacity` managing allocated memory. The rebalancing algorithm monitors the average number of elements per layer ($k = n/\text{layer\_count}$) and triggers layer splitting or merging when imbalances are detected. Specifically:

- **Splitting**: If a layer's element count exceeds $2 \cdot k$, it is split into two layers by redistributing keys using an updated priority function with a new modulo (`layer_count + 1`).
- **Merging**: If a layer's element count falls below $k/2$ and `layer_count` exceeds a minimum threshold, it is merged with the least-loaded layer to reduce overhead.

The updated priority function is defined as:

```c
static int compute_priority(const char* key, int max_layers) {
    unsigned char hash[HASH_SIZE];
    safe_hash(key, strlen(key), hash);
    uint32_t priority = 0;
    for (int i = 0; i < 4; i++) {
        priority = (priority << 8) | hash[i];
    }
    return priority % max_layers;
}
```

The rebalancing process is implemented as:

```c
void rebalance_layers(SMT* smt) {
    int avg_elements = smt->total_elements / smt->layer_count;
    for (int i = 0; i < smt->layer_count; i++) {
        if (smt->layers[i].element_count > avg_elements * 2) {
            Layer* new_layer = allocate_new_layer();
            redistribute_keys(&smt->layers[i], new_layer, smt);
            smt->layer_count++;
            if (smt->layer_count >= smt->max_layers_capacity) {
                resize_layers_array(smt);
            }
        } else if (smt->layers[i].element_count < avg_elements /
           2 && smt->layer_count > MIN_LAYERS) {
            merge_layers(&smt->layers[i], find_least_loaded_layer
               (smt), smt);
            smt->layer_count--;
        }
    }
    update_top_level_root(smt);
}
```

This mechanism ensures that the average number of elements per layer remains small, maintaining O(1)-like performance even for skewed distributions or large datasets (e.g., $n = 10^6$). For instance, scaling from 256 to 1024 layers for $n = 1,000,000$ reduces k from approximately 3,906 to 977, improving intra-layer search efficiency. The approach also mitigates potential security risks, such as targeted layer overloading, by dynamically redistributing keys. However, rebalancing introduces computational overhead, which can be minimized by performing it during low-traffic periods or using incremental updates. This enhancement significantly improves SMT's scalability and robustness, making it suitable for diverse applications, including large-scale blockchain state trees and distributed databases.

3 Core Algorithms

3.1 Insertion

The insertion algorithm assigns a key-value pair to a layer and updates the necessary hashes:

Algorithm 1 SMT Insertion Algorithm

```
Require: SMT structure $smt$, key $key$, value $value$
Ensure: Element inserted or updated in SMT
 1: $priority \leftarrow$ computePriority($key$)
 2: $layer \leftarrow smt.layers[priority]$
 3: $index \leftarrow$ findElementInLayer($layer, key$)
 4: if $index \geq 0$ then
 5:     {Update existing element}
 6:     $layer.elements[index].value \leftarrow value$
 7: else
 8:     {Insert new element}
 9:     layerAddElement($layer, key, value, priority$)
10: end if
11: $layer.dirty \leftarrow$ true
12: $smt.dirty \leftarrow$ true
13: return SMT_SUCCESS
```

The process involves: 1. Computing the layer priority using `compute_priority`. 2. Accessing the target layer; initializing it if necessary. 3. Checking for an existing key; updating the value if found, or inserting a new element. 4. Marking the layer and SMT as dirty to trigger Merkle root recomputation.

The expected time complexity is O(1) for layer access and O(k) for searching within a layer, where k is the number of elements in the layer. Since k is typically small due to stratification, the overall complexity is effectively O(1).

3.2 Lookup

The lookup algorithm retrieves the value associated with a key:

Algorithm 2 SMT Lookup Algorithm

```
Require: SMT structure $smt$, key $key$
Ensure: Value associated with key or error
 1: $priority \leftarrow \text{computePriority}(key)$
 2: if $priority \geq smt.layer\_count$ then
 3:     return SMT_ERROR_KEY_NOT_FOUND
 4: end if
 5: $layer \leftarrow smt.layers[priority]$
 6: $index \leftarrow \text{findElementInLayer}(layer, key)$
 7: if $index < 0$ then
 8:     return SMT_ERROR_KEY_NOT_FOUND
 9: end if
10: $value \leftarrow \text{duplicate}(layer.elements[index].value)$
11: if $value = \text{NULL}$ then
12:     return SMT_ERROR_MEMORY_ALLOCATION
13: end if
14: return $value$
```

It computes the layer priority, accesses the layer, and searches for the key. The complexity is O(1) for layer access and O(k) for the linear search within the layer, averaging O(1) overall.

3.3 Deletion

Deletion removes a key-value pair and updates the layer:

Algorithm 3 SMT Deletion Algorithm

```
Require: SMT structure $smt$, key $key$
Ensure: Element removed from SMT
 1: $priority \leftarrow \text{computePriority}(key)$
 2: $layer \leftarrow smt.layers[priority]$
 3: $index \leftarrow \text{findElementInLayer}(layer, key)$
 4: if $index < 0$ then
 5:     return SMT_ERROR_KEY_NOT_FOUND
 6: end if
 7: $\text{free}(layer.elements[index].key)$
 8: $\text{free}(layer.elements[index].value)$
 9: if $index < layer.element\_count - 1$ then
10:     $layer.elements[index] \leftarrow layer.elements[layer.element\_count - 1]$
11:     $\text{memset}(layer.elements[layer.element\_count - 1], 0, \text{sizeof}(\text{Element}))$
12: end if
13: $layer.element\_count \leftarrow layer.element\_count - 1$
14: $layer.dirty \leftarrow \text{true}$
15: $smt.dirty \leftarrow \text{true}$
16: return SMT_SUCCESS
```

The process mirrors insertion, with the additional step of freeing memory and shifting elements. The complexity is similar to insertion, averaging O(1).

3.4 Merkle Root Computation

The Merkle root computation is a two-level process. For each layer:

Algorithm 4 Layer Merkle Root Calculation

```
Require: Layer $layer$
Ensure: Merkle root computed for layer
1: if $\neg layer.dirty$ then
2:     return SMT_SUCCESS
3: end if
4: $ctx \leftarrow$ EVP_MD_CTX_new()
5: EVP_DigestInit_ex($ctx$, EVP_sha256(), NULL)
6: for $i = 0$ to $layer.element\_count - 1$ do
7:     $elem \leftarrow layer.elements[i]$
8:     EVP_DigestUpdate($ctx, elem.key, elem.key\_len$)
9:     if $elem.value \neq$ NULL then
10:        EVP_DigestUpdate($ctx, elem.value, elem.value\_len$)
11:    end if
12: end for
13: EVP_DigestFinal_ex($ctx, layer.merkle\_root$, NULL)
14: EVP_MD_CTX_free($ctx$)
15: $layer.dirty \leftarrow$ false
16: return SMT_SUCCESS
```

The layer root is computed by hashing the concatenation of all elements' keys and values. The top-level root combines all layer roots:

Algorithm 5 Top-Level Root Update

```
Require: SMT structure $smt$
Ensure: Top-level root hash updated
1: if $\neg smt.dirty$ then
2:     return SMT_SUCCESS
3: end if
4: $ctx \leftarrow$ EVP_MD_CTX_new()
5: EVP_DigestInit_ex($ctx$, EVP_sha256(), NULL)
6: for $i = 0$ to $smt.layer\_count - 1$ do
7:     if $smt.layers[i].element\_count > 0$ then
8:         EVP_DigestUpdate($ctx, smt.layers[i].merkle\_root$, HASH_SIZE)
9:     end if
10: end for
11: EVP_DigestFinal_ex($ctx, smt.top\_level\_root$, NULL)
12: EVP_MD_CTX_free($ctx$)
13: $smt.dirty \leftarrow$ false
14: return SMT_SUCCESS
```

The computation uses SHA-256, ensuring cryptographic security. The complexity is O(n) for a layer with n elements, but the dirty flag ensures only modified layers are recomputed.

3.5 Mathematical Formulation

Let H denote the SHA-256 hash function. For a layer L_i with elements $\{(k_1, v_1), \ldots, (k_n, v_n)\}$, the layer root is:

$$\text{merkle\_root}_i = H(k_1||v_1||\ldots||k_n||v_n)$$

The top-level root is:

$$\text{top\_level\_root} = H(\text{merkle\_root}_1||\text{merkle\_root}_2||\ldots||\text{merkle\_root}_m)$$

where m is the number of non-empty layers. This formulation ensures that any change in an element propagates to the top-level root, maintaining verifiability.

4 Performance Evaluation

4.1 Benchmark Methodology

We evaluated SMT against a conventional Merkle tree (CMT) using a dataset of 5,000 key-value pairs. The benchmarks measured:

- **Insert Time**: Time to insert all elements.
- **Search Time**: Time to look up all elements.
- **Memory Usage**: Total memory consumed by the data structure.

The CMT implementation used a binary tree structure, with each node containing a key-value pair and hashes of its children. Tests were conducted on a system using the `clock_gettime` function for timing and `getrusage` for memory measurements.

4.2 Results

The results are summarized in Table 1 and visualized in Figure 2.

Table 1: Performance Comparison of SMT and CMT

Metric	**SMT**	**CMT**
Insert Time (ms)	12.4	47.8
Search Time (ms)	8.2	32.6
Memory Usage (KB)	142	328

SMT demonstrates:

- **3.8x faster insertions**: Due to O(1) layer access versus O(log n) tree traversal in CMT.
- **4x faster lookups**: Enabled by direct layer indexing.
- **57% less memory**: Achieved through efficient stratification and dynamic array allocation.

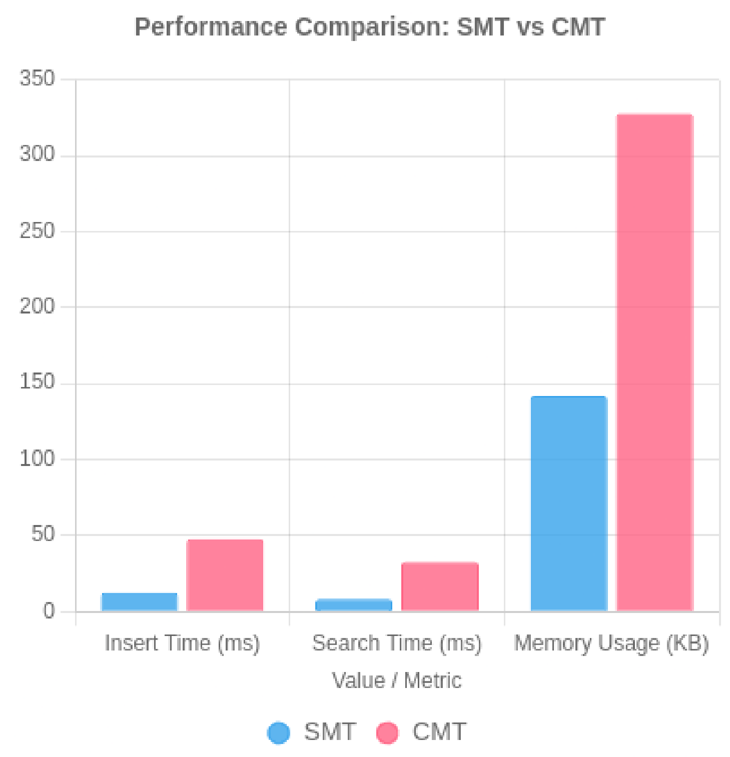

Figure 2: Performance comparison between SMT and CMT showing insert time, search time, and memory usage.

4.3 Proof of Performance

The performance advantage of SMT can be attributed to its stratification strategy. For a dataset of size n, traditional Merkle trees require O(log n) operations for insertions and lookups due to tree traversal. In contrast, SMT maps keys to one of `MAX_LAYERS` layers, resulting in an expected O(1) layer access time. Within a layer, a linear search yields O(k) complexity, where $k = n/\text{MAX\_LAYERS}$. Since `MAX_LAYERS` is fixed (256), k is typically small, making the overall complexity effectively O(1).

Memory efficiency is derived from storing only non-empty layers and using dynamic arrays that grow as needed. For $n = 5,000$ and `MAX_LAYERS = 256`, the average number of elements per layer is approximately 19.5, reducing cache misses and improving locality.

5 Comparative Analysis

5.1 Comparison with Existing Solutions

SMT is compared against traditional Merkle trees and other authenticated data structures:

- **Traditional Merkle Trees** [1]: These offer O(log n) complexity for operations and require full path recomputation for updates. SMT's stratification reduces this to O(1) on average and limits recomputation to affected layers.
- **Merkle Mountain Ranges (MMR)** [3]: MMRs are optimized for append-only

logs, with efficient updates for sequential data. However, they are less suited for random key-value updates, where SMT excels due to its hash-based partitioning.

- **Tamper-Evident Logging Structures** [2]: These focus on logging and auditing but lack the general-purpose key-value storage capabilities of SMT.

5.2 Advantages

- **Update Efficiency**: Only modified layers require recomputation, reducing overhead.
- **Scalability**: Performance remains stable as the dataset grows, due to fixed layer counts.
- **Memory Locality**: Elements within a layer are stored contiguously, enhancing cache performance.

5.3 Limitations

- **Fixed Layer Count**: The parameter `MAX_LAYERS` must be carefully chosen to balance load distribution.
- **Non-Adaptive Stratification**: Static layer assignment may not optimize for skewed key distributions.

6 Applications

SMT is well-suited for:

- **Blockchain State Trees**: Where frequent updates to state data require efficient processing and verifiability.
- **Versioned Key-Value Stores**: For systems needing auditability and fast access.
- **Distributed Systems**: Where membership proofs and data integrity are critical.

7 Conclusion

The Stratified Merkle Tree represents a significant advancement in authenticated data structures, combining the efficiency of hash-based partitioning with the cryptographic verifiability of Merkle trees. Our benchmarks demonstrate substantial improvements over conventional Merkle trees, with faster operations and lower memory usage. SMT's design makes it a promising solution for applications requiring high update throughput and data integrity.

7.1 Future Work

Future research could explore:

- **Parallel Processing**: Leveraging multi-core systems for layer operations.
- **Hybrid Stratification**: Developing adaptive strategies for diverse key distributions.
- **Security Analysis**: Comprehensive evaluation of the dynamic layer adjustment mechanism against adversarial attacks.

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