CHAPTER 2 : POLYNOMIALS

- **Question 1:** Write the degree of the polynomial $x^2 - 5x + 6$.
  Question 2: Identify the coefficient of x in $3x^3 - 7x + 2$.
  Question 3: Determine the remainder when $x^3 - 2x^2 + 4$ is divided by $x - 1$.
  Question 4: Write a polynomial of degree 2.
  Question 5: Is $x^2 + 1$ a linear polynomial?
- **Question 6:** Check whether $x - 2$ is a factor of $x^3 - 3x^2 + 4x - 8$.
  Question 7: Find the remainder when $x^3 + x^2 - x + 2$ is divided by $x + 1$.
  Question 8: If α and β are zeroes of $x^2 - 5x + 6$, find $\alpha + \beta$.
  Question 9: Find the product of zeroes of $2x^2 - 3x + 1$.
  Question 10: Find the sum of zeroes of $3x^2 + 7x - 6$.

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Question 11: Verify that 1 and -2 are zeroes of $x^2 + x - 2$.
Question 12: If α and β are zeroes of $x^2 - 3x + 2$, form a quadratic polynomial whose zeroes are $\alpha + 1$ and $\beta + 1$.
Question 13: Find a quadratic polynomial whose sum of zeroes = 5 and product = 6.
Question 14: Find the zeroes of $2x^2 - 5x - 3$ and verify relationship with sum and product.
Question 15: If α and β are zeroes of $x^2 - x - 6$, find a polynomial whose zeroes are $1/\alpha$ and $1/\beta$.
Question 16: Verify that the zeroes of $x^2 - 5x + 6$ satisfy $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$.
Question 17: Find a quadratic polynomial whose zeroes are 2 more than the zeroes of $x^2 - 3x + 2$.
Question 18: Find a cubic polynomial with zeroes 1, -2, 3.
Question 19: If α and β are zeroes of $2x^2 - 5x + 3$, verify that $1/\alpha + 1/\beta = (\alpha + \beta)/(\alpha\beta)$.
Question 20: If α, β, γ are zeroes of $x^3 - 6x^2 + 11x - 6$, find $\alpha\beta + \beta\gamma + \gamma\alpha$.
Question 21: Check whether $x - 3$ is a factor of $x^3 - 4x^2 + x + 6$.
Question 22: Find the remainder when $x^3 + 2x^2 - 5x + 4$ is divided by $x - 2$.
Question 23: Find a quadratic polynomial whose zeroes are squares of zeroes of $x^2 - 3x + 2$.
Question 24: If α, β are zeroes of $x^2 - 2x - 3$, find a polynomial whose zeroes are $\alpha - 1$ and $\beta - 1$.
Question 25: Find the sum of cubes of zeroes of $x^2 - 5x + 6$.
Question 26: Find a quadratic polynomial whose zeroes are reciprocals of zeroes of $x^2 - x - 6$.
Question 27: If α, β, γ are zeroes of $x^3 - 3x^2 + 4x - 2$, find $\alpha^2 + \beta^2 + \gamma^2$.
Question 28: Verify that zeroes of $x^2 - 7x + 12$ satisfy $\alpha^2 - \beta^2 = (\alpha - \beta)(\alpha + \beta)$.
Question 29: Find the cubic polynomial whose zeroes are 1, -1, 2.
Question 30: Find the remainder when $2x^3 + 3x^2 - x + 5$ is divided by $x + 1$.

HINTS/SOLUTION

Solution 1:
Polynomial: $x^2 - 5x + 6$
Degree of a polynomial = highest power of x → 2
Answer: 2

Solution 2:
Polynomial: $3x^3 - 7x + 2$
Coefficient of x = number multiplying x → −7
Answer: −7

Solution 3:
Polynomial: $x^3 - 2x^2 + 4$
Divide by $x - 1$ → Use Remainder Theorem
Remainder = $f(1) = (1)^3 - 2(1)^2 + 4 = 1 - 2 + 4 = 3$
Answer: 3

Solution 4:

Polynomial of degree 2 → any polynomial where the highest power of x is 2

Example: $x^2 + 3x + 5$

Answer: $x^2 + 3x + 5$

Solution 5:

Polynomial: $x^2 + 1$

Linear polynomial → degree 1

Degree of $x^2 + 1 = 2$ → Not linear

Answer: No

Solution 6:

Check whether $x - 2$ is a factor of $x^3 - 3x^2 + 4x - 8$

Use **Remainder Theorem** → remainder = f(2)

$f(2) = (2)^3 - 3(2)^2 + 4(2) - 8 = 8 - 12 + 8 - 8 = -4$

Remainder $\neq 0$ → $x - 2$ is **not a factor**

Answer: Not a factor

Solution 7:

Find remainder when $x^3 + x^2 - x + 2$ is divided by $x + 1$

Remainder = $f(-1)$

$f(-1) = (-1)^3 + (-1)^2 - (-1) + 2 = -1 + 1 + 1 + 2 = 3$

Answer: 3

Solution 8:

If α, β are zeroes of $x^2 - 5x + 6$ → sum of zeroes = $\alpha + \beta = -$(coefficient of x)/(coefficient of x^2)

$\alpha + \beta = -(-5)/1 = 5$

Answer: 5

Solution 9:

Product of zeroes of $2x^2 - 3x + 1$ → product = constant term / coefficient of x^2

$\alpha\beta = 1 / 2$

Answer: ½

Solution 10:

Sum of zeroes of $3x^2 + 7x - 6$ → sum = −(coefficient of x)/(coefficient of x^2)

$\alpha + \beta = -(7)/3 = -7/3$

Answer: $-7/3$

Solution 11:

Verify that 1 and −2 are zeroes of $x^2 + x - 2$

$f(x) = x^2 + x - 2$

$f(1) = (1)^2 + 1 - 2 = 0$ ✅

$f(-2) = (-2)^2 + (-2) - 2 = 4 - 2 - 2 = 0$ ✅

Answer: Both 1 and −2 are zeroes

Solution 12:

Zeroes of $x^2 - 3x + 2 \rightarrow \alpha + \beta = -(-3)/1 = 3, \alpha\beta = 2$

New zeroes = $\alpha + 1, \beta + 1$

Sum of new zeroes = $(\alpha + 1) + (\beta + 1) = \alpha + \beta + 2 = 3 + 2 = 5$

Product of new zeroes = $(\alpha + 1)(\beta + 1) = \alpha\beta + \alpha + \beta + 1 = 2 + 3 + 1 = 6$

Quadratic polynomial = $x^2 - (\text{sum})x + (\text{product}) = x^2 - 5x + 6$

Answer: $x^2 - 5x + 6$

Solution 13:

Sum of zeroes = 5, Product = 6

Quadratic polynomial = $x^2 - (\text{sum})x + (\text{product}) = x^2 - 5x + 6$

Answer: $x^2 - 5x + 6$

Solution 14:

Find zeroes of $2x^2 - 5x - 3$

Use quadratic formula: $x = [5 \pm \sqrt{(25 - 4\times2\times(-3))}]/(2\times2) = [5 \pm \sqrt{(25 + 24)}]/4 = [5 \pm \sqrt{49}]/4 = [5 \pm 7]/4$

$x_1 = (5 + 7)/4 = 12/4 = 3$

$x_2 = (5 - 7)/4 = -2/4 = -1/2$

Check sum = $x_1 + x_2 = 3 - 1/2 = 5/2 = -(\text{coefficient of } x)/(\text{coefficient of } x^2) = -(-5)/2 = 5/2$ ✅

Check product = $x_1 \times x_2 = 3 \times (-1/2) = -3/2 = (\text{constant})/(\text{coefficient of } x^2) = -3/2$

Answer: Zeroes are 3 and $-1/2$

Solution 15:

Zeroes of $x^2 - x - 6 \rightarrow \alpha + \beta = 1, \alpha\beta = -6$

Zeroes of new polynomial = $1/\alpha$ and $1/\beta$

Sum = $1/\alpha + 1/\beta = (\alpha + \beta)/\alpha\beta = 1/(-6) = -1/6$ ❌ (check)

Wait step properly: Sum = $(1/\alpha + 1/\beta) = (\alpha + \beta)/(\alpha\beta) = 1/(-6) = -1/6$ ✅

Product = $(1/\alpha)(1/\beta) = 1/(\alpha\beta) = 1/(-6) = -1/6$

Quadratic polynomial = $x^2 - (\text{sum})x + (\text{product}) = x^2 - (-1/6)x + (-1/6) = x^2 + (1/6)x - 1/6$

Multiply by 6 to remove fraction $\rightarrow 6x^2 + x - 1$

Answer: $6x^2 + x - 1$

Question 17

Find a quadratic polynomial whose zeroes are 2 more than the zeroes of

$$x^2 - 3x + 2$$

Solution:

Given polynomial:

$$x^2 - 3x + 2 = 0$$

Let its zeroes be α and β

👉 From standard relations:

$$\alpha + \beta = \frac{3}{1} = 3$$

$$\alpha\beta = \frac{2}{1} = 2$$

New zeroes are:

$$\alpha + 2,\ \beta + 2$$

Sum of new zeroes:

$$(\alpha + 2) + (\beta + 2) = \alpha + \beta + 4 = 3 + 4 = 7$$

Product of new zeroes:

$$(\alpha + 2)(\beta + 2)$$
$$= \alpha\beta + 2(\alpha + \beta) + 4$$
$$= 2 + 2(3) + 4 = 12$$

Required quadratic polynomial:

$$x^2 - (sum)x + (product)$$
$$\boxed{x^2 - 7x + 12}$$

Question 18

Find a cubic polynomial with zeroes 1, −2, 3

Solution:

Zeroes given:

$$1,\ -2,\ 3$$

Polynomial:

$$(x - 1)(x + 2)(x - 3)$$

First multiply:

$$(x - 1)(x - 3) = x^2 - 4x + 3$$

Now multiply with $(x+2)$:

$$\left(x^2-4x+3\right)(x+2)$$
$$=x^3+2x^2-4x^2-8x+3x+6$$
$$=x^3-2x^2-5x+6$$
$$\boxed{x^3-2x^2-5x+6}$$

Question 19

If $\alpha$and $\beta$are zeroes of

$$2x^2-5x+3$$

verify that

$$\frac{1}{\alpha}+\frac{1}{\beta}=\frac{\alpha+\beta}{\alpha\beta}$$

Solution:

Given polynomial:

$$2x^2-5x+3=0$$
$$\alpha+\beta=\frac{5}{2}$$
$$\alpha\beta=\frac{3}{2}$$

LHS:

$$\frac{1}{\alpha}+\frac{1}{\beta}=\frac{\alpha+\beta}{\alpha\beta}$$
$$=\frac{5/2}{3/2}=\frac{5}{3}$$

RHS:

$$\frac{\alpha+\beta}{\alpha\beta}=\frac{5/2}{3/2}=\frac{5}{3}$$

$$\text{LHS} = RHS$$

$$\boxed{\text{Hence } \textit{verified}}$$

Question 20

If α, β, γ are zeroes of

$$x^3-6x^2+11x-6$$

find $\alpha\beta+\beta\gamma+\gamma\alpha$

Solution:

For cubic polynomial:

$$x^3+ax^2+bx+c$$

Sum of products of zeroes two at a time = b

Given:

$$x^3-6x^2+11x-6$$

Here,

$$\alpha\beta+\beta\gamma+\gamma\alpha=11$$

$$\boxed{11}$$

Question 21

Check whether $x-3$ is a factor of

$$x^3-4x^2+x+6$$

Solution (Remainder Theorem):

Let:

$$P(x) = x^3 - 4x^2 + x + 6$$

Substitute $x = 3$:

$$P(3) = 27 - 36 + 3 + 6 = 0$$

Since remainder = 0,

$$\boxed{x - 3 \ \textit{is a factor}}$$

Question 22

Find remainder when

$$x^3 + 2x^2 - 5x + 4$$

is divided by $x - 2$

Solution:

By Remainder Theorem:

$$P(2) = 8 + 8 - 10 + 4 = 10$$

$$\boxed{\text{Remainder} \ = 10}$$

Question 23

Find a quadratic polynomial whose zeroes are squares of zeroes of

$$x^2 - 3x + 2$$

Solution:

Let zeroes be α, β

$$\alpha + \beta = 3$$

$$\alpha\beta = 2$$

New zeroes:

$$\alpha^2,\ \beta^2$$

Sum:

$$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$$
$$=9-4=5$$

Product:

$$\alpha^2\beta^2=(\alpha\beta)^2=4$$

Polynomial:

$$x^2-5x+4$$
$$\boxed{x^2-5x+4}$$

Question 24

If α,β are zeroes of

$$x^2-2x-3$$

find polynomial whose zeroes are $\alpha-1,\beta-1$

Solution:

$$\alpha+\beta=2$$
$$\alpha\beta=-3$$

New zeroes:

$$\alpha-1,\ \beta-1$$

Sum:

$$(\alpha+\beta)-2=2-2=0$$

Product:

$$\alpha\beta - (\alpha + \beta) + 1$$
$$= -3 - 2 + 1 = -4$$

Polynomial:

$$x^2 - 4$$
$$\boxed{x^2 - 4}$$

Solution 25

Let the zeroes of the given polynomial be $\alpha$and β.
Given polynomial:

$$x^2 - 5x + 6$$
$$\alpha + \beta = 5, \alpha\beta = 6$$

We know,

$$\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$$
$$= 5^3 - 3(6)(5)$$
$$= 125 - 90$$
$$= \boxed{35}$$

Solution 26

Let the zeroes of the given polynomial be $\alpha$and β.
Given polynomial:

$$x^2 - x - 6$$
$$\alpha + \beta = 1, \alpha\beta = -6$$

Required zeroes:

$$\frac{1}{\alpha}, \frac{1}{\beta}$$

Sum of new zeroes:

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta} = \frac{1}{-6} = -\frac{1}{6}$$

Product of new zeroes:

$$\frac{1}{\alpha\beta} = -\frac{1}{6}$$

Required quadratic polynomial:

$$x^2 - (sum)x + (product)$$

$$x^2 + \frac{1}{6}x - \frac{1}{6}$$

Multiplying throughout by 6:

$$\boxed{6x^2 + x - 1}$$

Solution 27

Given cubic polynomial:

$$x^3 - 3x^2 + 4x - 2$$

Let zeroes be α, β, γ.

$$\alpha + \beta + \gamma = 3$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = 4$$

We know,

$$\alpha^2 + \beta^2 + \gamma^2 = (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \beta\gamma + \gamma\alpha)$$
$$= 3^2 - 2(4)$$
$$= 9 - 8$$
$$= \boxed{1}$$

Solution 28

Let the zeroes of the given polynomial be $\alpha$and β.

Given polynomial:

$$x^2 - 7x + 12$$
$$\alpha + \beta = 7$$

LHS:

$$\alpha^2 - \beta^2 = (\alpha - \beta)(\alpha + \beta)$$
$$= (\alpha - \beta)(7)$$

RHS:

$$(\alpha - \beta)(\alpha + \beta) = (\alpha - \beta)(7)$$
$$\text{LHS} = RHS$$
$$\boxed{\text{Hence } \textit{verified}}$$

Solution 29

Given zeroes:

$$1, \ -1, \ 2$$

Required cubic polynomial:

(x−1)(x+1)(x−2)

= (x2−1)(x−2)

$$= x^3 - 2x^2 - x + 2$$

$$\boxed{x^3 - 2x^2 - x + 2}$$

Solution 30

Given polynomial:

$$P(x) = 2x^3 + 3x^2 - x + 5$$

By Remainder Theorem:

$$\text{Remainder} = P(-1)$$

$$= 2\left(-1\right)^3 + 3(-1)^2 - (-1) + 5$$

$$= -2 + 3 + 1 + 5$$

$$= \boxed{7}$$