Chapter 4 – Quadratic Equations

Basic Formulas & Facts

- A quadratic equation is of the form

$ax^2 + bx + c = 0$, where **$a \neq 0$**

- Here **a**, **b**, **c** are real numbers.
- The solutions of a quadratic equation are called its **roots**.
- Quadratic equations can be solved by:

- **Factorization Method**
- **Quadratic Formula Method**
- **Quadratic Formula**:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- **Discriminant (D)**:

$$D = b^2 - 4ac$$

- Nature of roots:

- **D > 0** → Roots are real and distinct
- **D = 0** → Roots are real and equal
- **D < 0** → Roots are not real

- Ranker Academy – Prepared by Arjun

1–5 : Very Short / 1–2 Mark Type

Q1. Write the standard form of a quadratic equation.

Q2. Is $x^2 - 7x = 0$ a quadratic equation? Give reason.

Q3. Find the value of discriminant of the equation $x^2 - 4x + 4 = 0$.

Q4. How many roots does a quadratic equation have if $D < 0$?

Q5. Write the quadratic formula.

Q6. Find the value of k for which the equation $x^2 + (k - 2)x + k = 0$ has equal roots

Q7. Solve $x(x - 3) = 10$.

Q8. Solve $(x - 2)(x + 3) = 0$.

Q9. Find the value of *k* for which the equation

$$x^2 + (k - 1)x + k = 0$$

has **equal roots**.

Q10. Find the value of *k* for which the equation

$$(k + 1)x^2 - 6x + 9 = 0$$

has **no real roots**.

Q 11

If one root of the quadratic equation

$$x^2 + px + 9 = 0$$

is double the other, find the value of **p**.

Q 12

Solve

$$(2x - 1)(x + 3) = 7$$

Q.13. Solve

$$\frac{1}{x}+\frac{1}{x-2}=1$$

Q14. Find the roots of

$$(x+1)^2=5x$$

Q15. Solve

$$x^2-\frac{5}{2}x+1=0$$

Q16. Find the roots of

$$\frac{x^2}{4}-\frac{3x}{2}+2=0$$

Q17. Find the value of *k* if the equation

$$x^2+(2k-1)x+k=0$$

Q18

Find the roots of the equation

$$(x+1)^2=5x$$

Q19

Solve the quadratic equation

$$\frac{1}{x-1}+\frac{1}{x-2}=3$$

Q 20 (Hidden Quadratic)

Solve

$$x+\frac{1}{x}=5$$

Q21. (Perfect Square Condition)
Find the value of **k** such that the expression

$$x^2 + 2kx + (k^2 - 9)$$

has **equal roots**.

Q22 (Parameter + Equal Roots)
Find the value of **k** for which the equation

$$(k-1)x^2 - 2(k+1)x + (k+2) = 0$$

has **equal roots**.

Q23 **(Fractional Equation)**
Solve $= \frac{1}{x-1} + \frac{1}{x-3} = \frac{1}{2}$

Q24.The product of two consecutive positive integers is **380**.
Find the integers.

Q25.The sum of the squares of two consecutive positive integers is **365**.
Find the integers.

Q26.The difference of two natural numbers is **4** and the difference of their squares is **72**.
Find the numbers.

Q27.The product of two positive integers is **132** and their sum is **23**.
Find the numbers.

Q28.The sum of two numbers is **15** and the sum of their reciprocals is **3/10**.
Find the numbers.

Q29.A rectangular field has area **528 sq units**.
If its length is **4 units more** than its breadth, find its dimensions.

Q30.The product of two consecutive even positive integers is **288**.
Find the integers.

HINTS/SOLUTION

Solution 1:

$$ax^2 + bx + c = 0,\ a \neq 0$$

Solution 2:

Yes.

$$x^2 - 7x = x^2 - 7x + 0$$

Coefficient of $x^2 = 1 \neq 0$.

Solution 3:

$$a = 1,\ b = -4,\ c = 4$$
$$D = b^2 - 4ac$$
$$D = 16 - 16 = 0$$

Solution 4:

No real roots.

Solution 5:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Solution 6:

Equal roots $\Rightarrow D = 0$

$$(k-2)^2 - 4(1)(k) = 0$$
$$k^2 - 8k + 4 = 0$$
$$k = 4 \pm 2\sqrt{3}$$

Solution 7:

$$x(x-3) = 10$$
$$x^2 - 3x - 10 = 0$$
$$(x-5)(x+2) = 0$$
$$x = 5, \ -2$$

Solution 8:

$$(x-2)(x+3) = 0$$
$$x = 2, \ -3$$

Solution 9:

Equal roots $\Rightarrow D = 0$

$$(k-1)^2 - 4k = 0$$
$$k^2 - 6k + 1 = 0$$
$$k = 3 \pm 2\sqrt{2}$$

Solution 10:

No real roots $\Rightarrow D < 0$

$$36 - 36(k+1) < 0$$
$$-36k < 0$$
$$k > 0$$

Solution 11:

Roots: $a, 2a$

$$a + 2a = -p \Rightarrow 3a = -p$$
$$a \cdot 2a = 9 \Rightarrow 2a^2 = 9$$
$$a = \pm\frac{3}{\sqrt{2}}$$
$$p = \pm\frac{9}{\sqrt{2}}$$

Solution 12:

$$(2x-1)(x+3) = 7$$
$$2x^2 + 5x - 10 = 0$$
$$x = \frac{-5 \pm \sqrt{105}}{4}$$

Solution 13:

$$\frac{1}{x} + \frac{1}{x-2} = 1$$

LCM $= x(x-2)$

$$(x-2)+x=x(x-2)$$
$$2x-2=x^2-2x$$
$$x^2-4x+2=0$$
$$x=2\pm\sqrt{2}$$

Solution 14:

$$(x+1)^2=5x$$
$$x^2+2x+1-5x=0$$
$$x^2-3x+1=0$$
$$x=\frac{3\pm\sqrt{5}}{2}$$

Solution 15:

$$x^2-\frac{5}{2}x+1=0$$

Multiply by 2

$$2x^2-5x+2=0$$
$$(2x-1)(x-2)=0$$
$$x=\frac{1}{2},\ 2$$

Solution 16:

$$\frac{x^2}{4} - \frac{3x}{2} + 2 = 0$$

Multiply by 4

$$x^2 - 6x + 8 = 0$$
$$(x-2)(x-4) = 0$$
$$x = 2,\ 4$$

Solution 17:

$$x^2 + (2k-1)x + k = 0$$

Equal roots $\Rightarrow D = 0$

$$(2k-1)^2 - 4k = 0$$
$$4k^2 - 8k + 1 = 0$$
$$k = 1 \pm \frac{\sqrt{3}}{2}$$

Solution 18 (Proper):

$$(x+1)^2 = 5x$$
$$x^2 + 2x + 1 - 5x = 0$$
$$x^2 - 3x + 1 = 0$$
$$x = \frac{3 \pm \sqrt{5}}{2}$$

Solution 19:

$$\frac{1}{x-1}+\frac{1}{x-2}=3$$

LCM $=(x-1)(x-2)$

$$(x-2)+(x-1)=3(x-1)(x-2)$$
$$2x-3=3(x^2-3x+2)$$
$$3x^2-11x+9=0$$
$$x=\frac{11\pm\sqrt{13}}{6}$$

Solution 20

x +

1

— = 5

x

x se multiply karo

$x^2 + 1 = 5x$

$x^2 - 5x + 1 = 0$

Quadratic formula

$$x = \frac{5 \pm \sqrt{(25 - 4\times1\times1)}}{2}$$

Solution 21

Given expression

$x^2 + 2kx + (k^2 - 9)$

Equal roots ke liye

Discriminant = 0

$b^2 - 4ac = 0$

$(2k)^2 - 4(1)(k^2 - 9) = 0$

$4k^2 - 4k^2 + 36 = 0$

$36 = 0$ ❌

Conclusion:

Koi bhi real value of **k** possible **nahin** hai

(equal roots possible nahi)

Solution 22

Equation

$(k - 1)x^2 - 2(k + 1)x + (k + 2) = 0$

Equal roots ke liye

$b^2 - 4ac = 0$

$[-2(k + 1)]^2 - 4(k - 1)(k + 2) = 0$

$4(k + 1)^2 - 4(k^2 + k - 2) = 0$

$4(k^2 + 2k + 1 - k^2 - k + 2) = 0$

$4(k + 3) = 0$

$k + 3 = 0$

$k = -3$

Solution 23

$$\frac{1}{x-1} + \frac{}{x-3} = 1$$

$$\frac{2}{}$$

LCM lo

$$(x-1)(x-3)$$

$$\frac{(x-3)+(x-1)}{(x-1)(x-3)} = 1$$

$$\frac{2}{}$$

$$\frac{2x-4}{(x-1)(x-3)} = 1$$

$$\frac{2}{}$$

Cross multiply

$$4x - 8 = (x-1)(x-3)$$

$$4x - 8 = x^2 - 4x + 3$$

$$x^2 - 8x + 11 = 0$$

Factorization

$$(x-3)(x-5) = 0$$

x = 3 , 5

($x \neq 1, 3 \rightarrow$ so only **x = 5** valid)

Final Answer:

x = **5**

Solution 24

Let the consecutive positive integers be
x and $x + 1$
Given
$x(x + 1) = 380$
$x^2 + x - 380 = 0$
Factorization
$(x - 19)(x + 20) = 0$
$x = 19$ (positive value)
Integers are: 19 and 20

Solution 25

Let the consecutive positive integers be
x and $x + 1$
Given
$x^2 + (x + 1)^2 = 365$
$x^2 + x^2 + 2x + 1 = 365$
$2x^2 + 2x - 364 = 0$
Divide by 2
$x^2 + x - 182 = 0$
Factorization
$(x - 13)(x + 14) = 0$
$x = 13$
Integers are: 13 and 14

Solution 26

Let the two natural numbers be
x and $x + 4$
Given
Difference of squares = 72
$(x + 4)^2 - x^2 = 72$
$x^2 + 8x + 16 - x^2 = 72$
$8x + 16 = 72$
$8x = 56$
$x = 7$
Second number = $7 + 4 = 11$
Numbers are: 7 and 11

Solution 27

Let the two positive integers be
x and y
Given
$x + y = 23$
$xy = 132$
Factors of 132 whose sum is 23:
11 and 12
Numbers are: 11 and 12

Solution 28 (Correctly Formatted)

Let the two numbers be

x and y

Given

$x + y = 15$

Sum of their reciprocals

$$\frac{1}{x} + \frac{1}{y} = \frac{3}{10}$$

Now

$$\frac{x + y}{xy} = 3$$

$$\frac{15}{xy} = 3$$

$$xy = 10$$

Cross multiply

$3xy = 150$

$xy = 50$

Now

$x + y = 15$

$xy = 50$

So the quadratic equation is

$t^2 - 15t + 50 = 0$
Factorisation
$(t - 5)(t - 10) = 0$
$\therefore t = 5$ or $t = 10$
Numbers are: 5 and 10

Solution 29
Let breadth = x units
Length = x + 4 units
Area = 528 sq units
$x(x + 4) = 528$
$x^2 + 4x - 528 = 0$
Factorization
$(x - 22)(x + 24) = 0$
$x = 22$
Breadth = 22 units
Length = 26 units
Dimensions are: 22 units × 26 units

Solution 30

Let the consecutive even positive integers be
x and x + 2
Given
$x(x + 2) = 288$
$x^2 + 2x - 288 = 0$
Factorization
$(x - 16)(x + 18) = 0$
$x = 16$
Second integer = 18
Integers are: 16 and 18