CHAPTER 1: REAL NUMBER

- **1 MARK QUESTIONS**
- 1️⃣ Write the smallest prime number.
  2️⃣ Write the HCF of two consecutive natural numbers.
  3️⃣ Is $\sqrt{7}$ a rational or irrational number?
  4️⃣ Write the decimal expansion of a rational number of the form p/q where $q = 2^n \times 5^m$.
  5️⃣ If HCF(a, b) = 1, what is HCF(a^2, b^2)?.
- **2 MARK QUESTIONS**
- 6️⃣ Find whether the decimal expansion of 13/125 is terminating or not.
  7️⃣ Write whether 0.375 is terminating or non-terminating.
  8️⃣ Find the LCM of 12 and 18 using prime Factorization.
  9️⃣ Find the HCF of 24 and 60 using prime Factorization.
  🔟 If p and q are prime numbers, find HCF(p, q).

3 MARK QUESTIONS

1️⃣1️⃣ Find the largest number that divides 130 and 235 leaving remainders 7 and 11 respectively.

1️⃣2️⃣ Find the largest number that divides 398 and 436 leaving remainders 7 and 11.

1️⃣3️⃣ Find whether the decimal expansion of 7/40 is terminating or non-terminating.

1️⃣4️⃣ Express 0.0008 in the form p/q and find whether its decimal expansion terminates.

1️⃣5️⃣ Find the HCF and LCM of 18 and 48 using prime factorization.

5 MARK QUESTIONS (VERY IMPORTANT 🔥)

1️⃣6️⃣ Prove that $\sqrt{5}$ is irrational.

1️⃣7️⃣ Prove that $\sqrt{3}$ is irrational.

1️⃣8️⃣ Prove that $3 + \sqrt{5}$ is irrational.

1️⃣9️⃣ Prove that the product of a non-zero rational number and an irrational number is irrational.

2️⃣0️⃣ Find the largest number which divides 2053 and 967 leaving remainders 5 and 7 respectively.

PYQ / HIGH WEIGHTAGE QUESTIONS

- 2️⃣1️⃣ Find the largest number that divides 520 and 640 leaving remainders 4 and 8 respectively.
2️⃣2️⃣ Find whether the decimal expansion of 17/500 is terminating or not.
2️⃣3️⃣ Express 0.125 in the form p/q and check the nature of its decimal expansion.
2️⃣4️⃣ Prove that $7\sqrt{5}$ is irrational.
2️⃣5️⃣ If HCF(a, b) = 12 and a = 180, find b.
2️⃣6️⃣ Find the HCF of 72 and 120 using prime factorisation.
2️⃣7️⃣ Find the LCM of 45 and 90 using prime factorisation.
2️⃣8️⃣ Find whether 6/15 has terminating decimal expansion or not.
2️⃣9️⃣ Prove that $2 + \sqrt{7}$ is irrational.
3️⃣0️⃣ Find the decimal expansion of 7/125.

HINTS/SOLUTION

- **Solution 1:**
  2
  (2 has exactly two factors: 1 and 2, so it is the smallest prime number.)
- **Solution 2:**
  1
  (Any two consecutive natural numbers are co-prime, so their HCF is 1.)
- **Solution 3:**
  $\sqrt{7}$ is an irrational number.
  (7 is not a perfect square, so $\sqrt{7}$ cannot be expressed in the form p/q.)
- **Solution 4:**
  Terminating decimal expansion.
  (When the denominator is of the form $2^n \times 5^m$, the decimal expansion always terminates.)
- **Solution 5:**
  1
  (If HCF(a, b) = 1, then a and b have no common factors; squaring them does not create any common factor, so HCF(a^2, b^2) = 1.)

Solution 6:

$13/125 = 13 / (5^3)$

Since the denominator has only prime factor **5**, the decimal expansion is **terminating**.

Solution 7:

$0.375 = 375/1000 = 3/8$

The denominator has only prime factor **2**, so the decimal expansion is **terminating**.

Solution 8:

$12 = 2^2 \times 3$

$18 = 2 \times 3^2$

LCM $= 2^2 \times 3^2 =$ **36**

(LCM is found by taking the **highest powers** of common prime factors.)

Solution 9:

$24 = 2^3 \times 3$

$60 = 2^2 \times 3 \times 5$

HCF $= 2^2 \times 3 =$ **12**

(HCF is found by taking the **lowest powers** of common prime factors.)

Solution 10:

HCF(p, q) = **1**

(Any two different prime numbers have no common factor except 1, so their HCF is 1.)

Solution 11:

Required number = HCF of (130 − 7) and (235 − 11)

= HCF of 123 and 224

$123 = 3 \times 41$

$224 = 2^5 \times 7$

HCF = **1**

👉 Largest required number = **1**

(We subtract the remainders first, then find HCF.)

Solution 12:

Required number = HCF of (398 − 7) and (436 − 11)

= HCF of 391 and 425

$391 = 17 \times 23$

$425 = 5^2 \times 17$

HCF = **17**

👉 Largest required number = **1**

Solution 13:

$7/40 = 7 / (2^3 \times 5)$

The denominator has only prime factors **2 and 5**,

so the decimal expansion is **terminating**.

Solution 14:

$0.0008 = 8/10000$

$= 1/125$

$125 = 5^3$

Since the denominator has only prime factor **5**,

the decimal expansion is **terminating.**

Solution 15:

$18 = 2 \times 3^2$

$48 = 2^4 \times 3$

HCF = $2^1 \times 3^1 = \mathbf{6}$

LCM = $2^4 \times 3^2 = \mathbf{144}$

Solution 16:

Prove that √5 is irrational.

Proof:

Assume √5 is rational. Then

$$\frac{p}{q}$$

where p, q are co-prime integers.

Squaring both sides:

$$\frac{p^2}{q^2} = 5$$

$\rightarrow p^2 = 5 \times q^2$

$\therefore$ p^2 divisible by 5 → p divisible by 5 → p = 5k

Then

$$\frac{p^2}{q^2}$$

$= 25k^2 = 5q^2 \rightarrow q^2 = 5k^2 \rightarrow q$ divisible by 5
∴ p and q have a common factor 5 → contradiction.
Hence, √5 is irrational.

Solution 17:
Prove that √3 is irrational.
Assume √3 is rational. Then

$$\frac{p}{q}$$

where p, q are co-prime.
Squaring both sides:

$$\frac{p^2}{q^2} = 3$$

$\rightarrow p^2 = 3 \times q^2 \rightarrow p$ divisible by 3 $\rightarrow p = 3k$
Then

$$\frac{p^2}{q^2}$$

$= 9k^2 = 3q^2 \rightarrow q^2 = 3k^2 \rightarrow q$ divisible by 3
Contradiction.
Hence, √3 is irrational.

Solution 18:

Prove that $3 + \sqrt{5}$ is irrational.

Assume $3 + \sqrt{5}$ is rational. Then

$$\frac{p}{q}$$

where p, q are integers, $q \neq 0$.

Then

$$\frac{p - 3q}{q}$$

$= \sqrt{5} \rightarrow$ rational? ❌ False

Hence, $3 + \sqrt{5}$ is irrational.

Solution 19:

Prove that the product of a non-zero rational number and an irrational number is irrational.

Let r be a non-zero rational number and x be irrational.

Assume $r \times x$ is rational. Then

$$x = \frac{r \times x}{r}$$

Since $r \neq 0$, x would be rational $\rightarrow$ Contradiction

Hence, the product is irrational.

Solution 20:

Find the largest number which divides 2053 and 967 leaving remainders 5 and 7 respectively.

Subtract remainders:

$2053 - 5 = 2048$

$967 - 7 = 960$

Find HCF of 2048 and 960 using prime factorization:

$2048 = 2^{11}$

$960 = 2^6 \times 3 \times 5$

$HCF = 2^6 = 64$

Largest required number = 64

Solution 21:

Largest number dividing 520 and 640 leaving remainders 4 and 8:

$520 - 4 = 516$

$640 - 8 = 632$

HCF(516, 632)

$516 = 2^2 \times 3 \times 43$

$632 = 2^3 \times 79$

$HCF = 2^2 = \mathbf{4}$

Answer: 4

Solution 22:

Decimal expansion of 17/500:

$500 = 2^2 \times 5^3$ → only 2 and 5 as prime factors

∴ Decimal expansion is **terminating**

Answer: Terminating

Solution 23:

0.125 = 125 / 1000

Simplify: 125 ÷ 1000 = 1 ÷ 8

$8 = 2^3$ → only prime factor 2

∴ Decimal expansion is **terminating**

Answer: 0.125 = 1/8 → Terminating

Solution 24:

Prove that $7\sqrt{5}$ is irrational.

Assume $7\sqrt{5}$ is rational. Then $\sqrt{5} = (7\sqrt{5}) \div 7$ → $\sqrt{5}$ is rational ❌

Contradiction, since $\sqrt{5}$ is irrational.

Hence, $7\sqrt{5}$ is irrational.

Solution 25:

HCF(a, b) = 12, a = 180

HCF × LCM = a × b → Not needed, simple method:

b must be divisible by HCF:

Let b = 12 × k, HCF(180, 12k) = 12

$180 \div 12 = 15 \rightarrow 15$ and k must be co-prime
Choose smallest possible $k = 1 \rightarrow b = 12 \times 1 = 12$
Other possibilities: $k = 1,2,\ldots$ (co-prime with 15)
Answer: $b = 12, 24, 36, \ldots$

Solution 26:
HCF of 72 and 120 using prime factorization:
$72 = 2^3 \times 3^2$
$120 = 2^3 \times 3 \times 5$
HCF = $2^3 \times 3 =$ **24**

Solution 27:
LCM of 45 and 90 using prime factorization:
$45 = 3^2 \times 5$
$90 = 2 \times 3^2 \times 5$
LCM = $2 \times 3^2 \times 5 = 90$

Solution 28:
$6/15 = 2 \times 3 / (3 \times 5) = 2 / 5$
Denominator = 5 → only prime factor 5 → Decimal expansion **terminating**
Answer: Terminating

Solution 29:

Prove that 2 + √7 is irrational.

Assume 2 + √7 is rational. Then √7 = (2 + √7) – 2 → √7 is rational ❌

Contradiction.

Hence, 2 + √7 is irrational.

Solution 30:

Decimal expansion of 7/125:

125 = 5^3 → only prime factor 5 → decimal expansion **terminating**

7 ÷ 125 = 0.056

Answer: 0.056 → Terminating