

# CHAPTER 3: PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

## 1-Mark Questions

**Question 1:** Write the general form of a pair of linear equations in two variables.

**Question 2:** How many solutions can a pair of linear equations have?

**Question 3:** What is meant by a consistent pair of linear equations?

**Question 4:** If two lines intersect at one point, how many solutions are possible?

**Question 5:** If two lines are parallel, how many solutions are possible?

## 2-Mark Questions

**Question 6:** Write the condition for a pair of linear equations to have

(a) unique solution

(b) no solution

**Question 7:** Check whether the pair of equations

$$2x + 3y = 11$$

$$4x + 6y = 22$$

has infinitely many solutions or not.

**Question 8:** Find the value of  $k$  for which the system

$$kx + y = 4$$

$$6x + 2y = 8$$

has infinitely many solutions.

**Question 9:** Write the condition for a pair of linear equations to be inconsistent.

**Question 10:** What is the graphical representation of two coincident lines?

**Question 11:** Solve using **substitution method**

$$x + y = 7$$

$$x - y = 1$$

**Question 12:** Solve using **elimination method**

$$2x + 3y = 13$$

$$x + y = 5$$

**Question 13:** Solve using **substitution method**

$$2x - y = 4$$

$$x + y = 5$$

**Question 14:** Find  $k$  for which the pair has **no solution**

$$kx + 2y = 5$$

$$3x + ky = 10$$

**Question 15:** Solve **graphically**

$$x + y = 6$$

$$x - y = 2$$



**Question 16:** Solve by **elimination method**

$$3x - 2y = 5$$

$$2x + y = 4$$

**Question 17:** Solve by **substitution method**

$$x = y + 3$$

$$2x + y = 11$$

**Question 18:** Solve by **elimination method**

$$5x - y = 9$$

$$3x + 2y = 7$$

**Question 19:** Solve

$$4x - 3y = -5$$

$$2x + y = 7$$

**Question 20:** Solve

$$x + 2y = 10$$

$$2x - y = 1$$

**PYQ / CASE-BASED / HIGH WEIGHTAGE**

**Question 21:** The sum of two numbers is 27 and their difference is 3. Find the numbers.

**Question 22:** The larger of two numbers is 5 more than the smaller. Their sum is 25. Find the numbers.

**Question 23:** Solve

$$2x + 5y = 14$$

$$4x + 10y = 28$$

**Question 24:** Solve

$$3x + y = 8$$

$$6x + 2y = 16$$

**Question 25:** Find k for which the pair has **no solution**

$$kx + y = 2$$

$$3x + 3y = 6$$

**Question 26:** Solve by elimination

$$7x - 3y = 5$$

$$3x + y = 11$$

**Question 27:** Solve by substitution

$$x - 2y = 4$$

$$2x + y = 1$$

**Question 28:** Solve graphically

$$2x + y = 6$$

$$x - y = 1$$

**Question 29:** Solve

$$3x + 4y = 10$$

$$2x - y = 1$$

**Question 30:** Solve

$$x + y = 8$$

$$2x - y = 1$$



**Question 31:** The sum of two numbers is 20 and their difference is 4. Find the numbers.

**Question 32:** Two numbers differ by 6 and their sum is 24. Find the numbers.

**Question 33:** Find the value of  $k$  for which the pair of equations has **infinitely many solutions**

$$kx + 4y = 8$$

$$2x + ky = 4$$

**Question 34:** Find the value of  $k$  for which the pair of equations has **no solution**

$$kx + y = 5$$

$$3x + 3y = 9$$

**Question 35**

(A) If  $\alpha, \beta$  are the zeroes of the polynomial

$3x^2 - 5x + 2$ , find the value of  $\alpha^2 + \beta^2$ . **(3)**

**OR**

(B) Find a quadratic polynomial whose sum of zeroes is 6 and product is 8. Also find its zeroes. **(3)**

**Question 36**

(A) If  $\alpha, \beta$  are the zeroes of  $4x^2 + 7x - 5$ , find  $\alpha^2 + \beta^2$ . (3)

**OR**

(B) Find a quadratic polynomial whose sum of zeroes is  $-1$  and product is  $-12$ . Also find its zeroes. (3)

**Question 37**

(A) If  $\alpha, \beta$  are the zeroes of  $2x^2 - 9x + 4$ , find the value of  $(\alpha + \beta)^2 - 2\alpha\beta$ . (3)

**OR**

(B) Find a quadratic polynomial whose zeroes are 3 and  $-5$ . (3)

**Question 38**

(A) If  $\alpha, \beta$  are the zeroes of  $5x^2 - 3x - 1$ , find  $\alpha^2 + \beta^2$ . (3)

**OR**

(B) Find a quadratic polynomial whose sum of zeroes is 2 and product is  $-15$ . Also find its zeroes. (3)

**Question 39**

(A) If  $\alpha, \beta$  are the zeroes of  $x^2 - 6x + 1$ , find the value of  $1/\alpha + 1/\beta$ . (3)



(B) Find a quadratic polynomial whose zeroes are  $-2$  and  $7$ . **(3)**

**Question 40**

(A) If  $\alpha, \beta$  are the zeroes of  $2x^2 - 5x - 7$ , find  $(\alpha - \beta)^2$ . **(3)**

**OR**

(B) Find a quadratic polynomial whose zeroes are  $1/2$  and  $3$ . **(3)**

**Question 41**

(A) If  $\alpha, \beta$  are the zeroes of  $5x^2 + x - 6$ , find  $1/\alpha^2 + 1/\beta^2$ . **(3)**

**OR**

(B) Find a quadratic polynomial whose sum of zeroes is  $1$  and product is  $-6$ . Also find its zeroes. **(3)**

# HINTS/SOLUTIONS

## Solution 1:

A pair of linear equations in two variables is written as:

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

where  $a_1, b_1, a_2, b_2$  are not zero simultaneously.

## Solution 2:

A pair of linear equations can have **three types of solutions**:

**1. One solution** (unique solution)

**2. Infinitely many solutions**

**3. No solution**

## Solution 3:

A pair of linear equations is called **consistent** if it has

- **one solution** or

- **infinitely many solutions**

That is, at least one solution exists.



#### Solution 4:

If two lines intersect at **one point**, then the pair of linear equations has **only one solution**.

#### Solution 5:

If two lines are **parallel**, they never intersect.  
So, the pair of linear equations has **no solution**.

#### Solution 6:

Consider the pair of linear equations:

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

**(a) Condition for unique solution:**

$$a_1 \div a_2 \neq b_1 \div b_2$$

(Is case me lines intersect karti hain)

**(b) Condition for no solution:**

$$a_1 \div a_2 = b_1 \div b_2 \neq c_1 \div c_2$$

(Is case me lines parallel hoti hain)

#### Solution 7:

Given equations:

$$2x + 3y - 11 = 0$$

$$4x + 6y - 22 = 0$$

### Q8 (Infinitely many solutions)

Given:

$$kx + y = 4$$

$$6x + 2y = 8$$

Standard form:

$$kx + y - 4 = 0$$

$$6x + 2y - 8 = 0$$

Condition:

$$a_1 / a_2 = b_1 / b_2 = c_1 / c_2$$

$$\begin{aligned}\frac{k}{6} &= \frac{1}{2} = \frac{-4}{-8} \\ \frac{k}{6} &= \frac{1}{2} \\ k &= 3\end{aligned}$$

✓ **Answer:  $k = 3$**

### Q9 (Inconsistent condition)

Pair of linear equations is **inconsistent** if:

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

)Parallel lines, no solution)



### Q10 (Coincident lines)

Two coincident lines overlap completely and have **infinitely many solutions**.

### Q11 (Substitution method)

$$x + y = 7$$

$$x - y = 1$$

From first equation:

$$y = 7 - x$$

Substitute in second:

$$x - (7 - x) = 1$$

$$2x = 8$$

$$x = 4$$

$$y = 3$$

✓ **Answer:** (4, 3)

### Q12 (Elimination method)

$$2x + 3y = 13$$

$$x + y = 5$$

Multiply second by 2:

$$2x + 2y = 10$$

Subtract:

$$y = 3$$

$$x = 2$$

✓ **Answer:** (2, 3)

**Q13 (Substitution method)**

$$2x - y = 4$$

$$x + y = 5$$

$$y = 5 - x$$

$$2x - (5 - x) = 4$$

$$x = 3, y = 2$$

✓ **Answer:** (3, 2)

**Q14 (No solution)**

$$kx + 2y = 5$$

$$3x + ky = 10$$

Condition for no solution:

$$\frac{k}{3} = \frac{2}{k} \neq \frac{5}{10}$$

$$k^2 = 6$$

$$k = \sqrt{6}, -\sqrt{6}$$

✓ **Answer:**  $k = \pm\sqrt{6}$

**Q15 (Graphical)**

$$x + y = 6$$

$$x - y = 2$$

Intersection point:

$$x = 4, y = 2$$



### Solution 16

Given:

$$3x - 2y = 5 \dots\dots(1)$$

$$2x + y = 4 \dots\dots(2)$$

Multiply equation (2) by 2:

$$4x + 2y = 8 \dots\dots(3)$$

Add (1) and (3):

$$7x = 13$$

$$x = \frac{13}{7}$$

Put value of x in (2):

$$2\left(\frac{13}{7}\right) + y = 4$$

$$y = \frac{2}{7}$$

**Answer:**

$$x = \frac{13}{7}, y = \frac{2}{7}$$

### Solution 17

Given:

$$x = y + 3 \dots\dots(1)$$

$$2x + y = 11 \dots\dots(2)$$

Put value of x from (1) into (2):

$$2(y+3)+y=11$$

$$3y = 5$$

$$y = \frac{5}{3}$$

Now,

$$x = y + 3 = \frac{5}{3} + \frac{9}{3} = \frac{14}{3}$$

**Answer:**

$$x = \frac{14}{3}, y = \frac{5}{3}$$

### Solution 18

(Elimination Method)

Given:

$$5x - y = 9 \dots\dots(1)$$

$$3x + 2y = 7 \dots\dots(2)$$

Multiply (1) by 2:

$$10x - 2y = 18 \dots\dots(3)$$

Add (2) and (3):

$$13x = 25$$

$$x = \frac{25}{13}$$

Put x in (1):

$$5\left(\frac{25}{13}\right) - y = 9$$



$$y=138$$

**Answer:**

$$x = \frac{25}{13}, y = \frac{8}{13}$$

### **Solution 19**

Given:

$$4x - 3y = -5 \dots\dots(1)$$

$$2x + y = 7 \dots\dots(2)$$

Multiply (2) by 3:

$$6x + 3y = 21 \dots\dots(3)$$

Add (1) and (3):

$$10x = 16$$

$$x = \frac{8}{5}$$

Put x in (2):

$$2\left(\frac{8}{5}\right) + y = 7$$
$$y = \frac{19}{5}$$

**Answer:**

$$x = \frac{8}{5}, y = \frac{19}{5}$$

### Solution 20

Given:

$$x + 2y = 10 \dots\dots(1)$$

$$2x - y = 1 \dots\dots(2)$$

Multiply (2) by 2:

$$4x - 2y = 2 \dots\dots(3)$$

Add (1) and (3):

$$5x = 12$$

$$x = \frac{12}{5}$$

Put x in (1):

$$\frac{12}{5} + 2y = 10$$

$$y = \frac{19}{5}$$

**Answer:**

$$x = \frac{12}{5}, y = \frac{19}{5}$$

### Solution 21 (PYQ)

Let numbers be x and y

$$x + y = 27 \dots\dots(1)$$

$$x - y = 3 \dots\dots(2)$$

Add (1) and (2):

$$2x = 30$$

$$x = 15$$

$$y = 12$$



### Solution 22 (PYQ)

Let smaller number =  $x$

Larger number =  $x + 5$

$$x + (x + 5) = 25$$

$$2x = 20$$

$$x = 10$$

Larger number = 15

**Answer:**

Numbers are **10 and 15**

### Solution 23 (PYQ)

Given:

$$2x + 5y = 14 \dots\dots(1)$$

$$4x + 10y = 28 \dots\dots(2)$$

Multiply (1) by 2:

$$4x + 10y = 28$$

This is same as equation (2)

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

**Infinite solutions**

### Solution 24

Given:

$$3x + y = 8 \dots\dots(1)$$

$$6x + 2y = 16 \dots\dots(2)$$

Multiply (1) by 2:

$$6x + 2y = 16$$

Equation (2) and new equation are same

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

**Answer:**

Pair has **infinitely many solutions**

### Solution 25

Given:

$$kx + y = 2 \dots\dots(1)$$

$$3x + 3y = 6 \dots\dots(2)$$

Rewrite (2):

$$x + y = 2 \dots\dots(3)$$

For **no solution:**

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

From (1) and (3):



$$\frac{k}{1} = \frac{1}{1} \Rightarrow k = 1$$

But

$$\frac{2}{2} = 1$$

So equations become coincident

👉 To make **no solution**, coefficients same but constants different

Hence **no value of k**

**Answer:**

❌ **No such value of k**

### Solution 26

(Elimination Method)

Given:

$$7x - 3y = 5 \dots\dots(1)$$

$$3x + y = 11 \dots\dots(2)$$

Multiply (2) by 3:

$$9x + 3y = 33 \dots\dots(3)$$

Add (1) and (3):

$$16x = 38$$

$$x = \frac{19}{8}$$

Put x in (2):

$$3\left(\frac{19}{8}\right) + y = 11$$
$$y = \frac{31}{8}$$

**Answer:**

$$x = \frac{19}{8}, y = \frac{31}{8}$$

### Solution 27

(Substitution Method)

Given:

$$x - 2y = 4 \dots\dots(1)$$

$$2x + y = 1 \dots\dots(2)$$

From (1):

$$x = 4 + 2y$$

Put in (2):

$$2(4 + 2y) + y = 1$$
$$5y = -7 \Rightarrow y = -\frac{7}{5}$$
$$x = \frac{6}{5}$$

**Answer:**

$$x = \frac{6}{5}, y = -\frac{7}{5}$$



### **Solution 28**

(Graphical Method)

Equations:

$$2x + y = 6$$

$$x - y = 1$$

Convert to y form:

$$y = 6 - 2x$$

$$y = x - 1$$

Intersection point:

$$x = \frac{7}{3}, y = \frac{4}{3}$$

**Answer:**

$$\text{Solution} = \left(\frac{7}{3}, \frac{4}{3}\right)$$

### **Solution 29**

Given:

$$3x + 4y = 10 \dots\dots(1)$$

$$2x - y = 1 \dots\dots(2)$$

Multiply (2) by 4:

$$8x - 4y = 4 \dots\dots(3)$$

Add (1) and (3):

$$11x = 14$$

$$x = \frac{14}{11}$$

### Solution 30

Given:

$$x + y = 8 \dots\dots(1)$$

$$2x - y = 1 \dots\dots(2)$$

Add (1) and (2):

$$3x = 9$$

$$x = 3$$

$$y = 5$$

**Answer:**

$$x = 3, y = 5$$

### Solution 31

Let the two numbers be **x** and **y**

Given:

$$x + y = 20 \dots\dots(1)$$

$$x - y = 4 \dots\dots(2)$$

Add (1) and (2):

$$2x = 24$$

$$x = 12$$

Put  $x = 12$  in (1):

$$12 + y = 20 \Rightarrow y = 8$$

**Answer:**

Numbers are **12 and 8**



### Solution 32

Let the two numbers be **x** and **y**

Given:

$$x - y = 6 \dots\dots(1)$$

$$x + y = 24 \dots\dots(2)$$

Add (1) and (2):

$$2x = 30$$

$$x = 15$$

Put **x = 15** in (2):

$$15 + y = 24 \Rightarrow y = 9$$

**Answer:**

Numbers are **15 and 9**

### Solution 33

Given equations:

$$kx + 4y = 8 \dots\dots(1)$$

$$2x + ky = 4 \dots\dots(2)$$

For **infinitely many solutions:**

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$
$$\frac{k}{2} = \frac{4}{k} = \frac{8}{4}$$
$$\frac{8}{4} = 2$$

So,

$$2k=2 \Rightarrow k=1$$

Check:

$$\frac{4}{2} = \frac{4}{4} = 1$$

✓condition satisfied

**Answer:**

$$k = 1$$

### Solution 34

Given equations:

$$kx + y = 5 \dots\dots(1)$$

$$3x + 3y = 9 \dots\dots(2)$$

Rewrite (2):

$$x + y = 3 \dots\dots(3)$$

For **no solution**:

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

From (1) and (3):

$$\frac{k}{1} = \frac{1}{1} \Rightarrow k = 1$$

But,

$$\frac{5}{3} \neq 1$$

✓condition satisfied

**Answer:**

$$k = 1$$



### Solution 35 (A)

Given polynomial:

$$3x^2 - 5x + 2$$

If  $\alpha, \beta$  are zeroes, then

$\alpha + \beta =$  coefficient of  $x$  (with sign changed) upon coefficient of  $x^2$

$$\alpha + \beta = \frac{5}{3}$$

$\alpha\beta =$  constant term upon coefficient of  $x^2$

$$\alpha\beta = \frac{2}{3}$$

We know,

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$$

Substitute values:

$$\begin{aligned}\alpha^2 + \beta^2 &= \left(\frac{5}{3}\right)^2 - 2\left(\frac{2}{3}\right) \\ &= \frac{25}{9} - \frac{4}{3} \\ &= \frac{25}{9} - \frac{12}{9} \\ &= \frac{13}{9}\end{aligned}$$

**Answer:**

$$\alpha^2 + \beta^2 = \frac{13}{9}$$

### Solution 35 (B)

Given:

Sum of zeroes = 6

Product of zeroes = 8

Let the zeroes be  $\alpha$  and  $\beta$ .

Standard quadratic polynomial:

$$x^2 - (\text{sum})x + (\text{product})$$

So required polynomial is:

$$x^2 - 6x + 8$$

Now find its zeroes:

$$x^2 - 6x + 8 = 0$$

Factorization:

$$(x - 2)(x - 4) = 0$$
$$x = 2, 4$$

**Answer:**

Quadratic polynomial:  $x^2 - 6x + 8$

Zeroes are **2 and 4**

### Question 36 (A)

Given polynomial:

$$4x^2 + 7x - 5$$

$$\alpha + \beta = -7 \text{ upon } 4$$

$$\alpha\beta = -5 \text{ upon } 4$$

Formula:



$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$$

$$\begin{aligned}
 &= \left(\frac{-7}{4}\right)^2 - 2\left(\frac{-5}{4}\right) \\
 &= \frac{49}{16} + \frac{10}{4} \\
 &= \frac{49}{16} + \frac{40}{16} \\
 &= \frac{89}{16}
 \end{aligned}$$

**Answer:** 89 upon 16

### Question 36 (B)

Sum of zeroes =  $-1$

Product of zeroes =  $-12$

Quadratic polynomial:

$$\begin{aligned}
 &x^2 - (\text{sum})x + (\text{product}) \\
 &x^2 + x - 12
 \end{aligned}$$

Factorization:

$$(x + 4)(x - 3) = 0$$

Zeroes:  **$-4$  and  $3$**

### Question 37 (A)

Given polynomial:

$$2x^2 - 9x + 4$$

$$\alpha + \beta = 9 \text{ upon } 2$$

$$\alpha\beta = 2$$

Expression given:

$$(\alpha + \beta)^2 - 2\alpha\beta$$

$$\begin{aligned} &= \left(\frac{9}{2}\right)^2 - 2(2) \\ &= \frac{81}{4} - 4 \\ &= \frac{81 - 16}{4} \\ &= \frac{65}{4} \end{aligned}$$

**Answer:** 65 upon 4

### Question 37 (B)

Zeroes are: 3 and  $-5$

$$\text{Sum} = -2$$

$$\text{Product} = -15$$

Quadratic polynomial:

$$\begin{aligned} &x^2 - (-2)x + (-15) \\ &x^2 + 2x - 15 \end{aligned}$$



### Question 38 (A)

Given polynomial:

$$5x^2 - 3x - 1$$

$$\alpha + \beta = 3 \text{ upon } 5$$

$$\alpha\beta = -1 \text{ upon } 5$$

Formula:

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$$

$$\begin{aligned} &= \left(\frac{3}{5}\right)^2 - 2\left(\frac{-1}{5}\right) \\ &= \frac{9}{25} + \frac{2}{5} \\ &= \frac{9}{25} + \frac{10}{25} \\ &= \frac{19}{25} \end{aligned}$$

**Answer:** 19 upon 25

### Question 38 (B)

Sum of zeroes = 2

Product = -15

Quadratic polynomial:

$$x^2 - 2x - 15$$

Factorization:

$$(x - 5)(x + 3) = 0$$

Zeroes: **5 and -3**

### Question 39 (A)

Given polynomial:

$$x^2 - 6x + 1$$

$$\alpha + \beta = 6$$

$$\alpha\beta = 1$$

Formula:

$$\begin{aligned}\frac{1}{\alpha} + \frac{1}{\beta} &= \frac{\alpha + \beta}{\alpha\beta} \\ &= \frac{6}{1} \\ &= 6\end{aligned}$$

**Answer: 6**

### Question 39 (B)

**Zeroes: -2 and 7**

$$\text{Sum} = -2 + 7 = 5$$

$$\text{Product} = -14$$

Quadratic polynomial:

$$\begin{aligned}x^2 - (\text{sum})x + (\text{product}) \\ x^2 - 5x - 14\end{aligned}$$

**Answer:**  $x^2 - 5x - 14$



**Question 40 (A)**

Given polynomial:

$$2x^2 - 5x - 7$$

$$\alpha + \beta = 5 \text{ upon } 2$$

$$\alpha\beta = -7 \text{ upon } 2$$

Formula:

$$\begin{aligned}(\alpha - \beta)^2 &= (\alpha + \beta)^2 - 4\alpha\beta \\&= \left(\frac{5}{2}\right)^2 - 4\left(\frac{-7}{2}\right) \\&= \frac{25}{4} + 14 \\&= \frac{25}{4} + \frac{56}{4} = \frac{81}{4}\end{aligned}$$

**Question 40 (B)**

**Zeros:** 1 upon 2 and 3

$$\text{Sum} = 1 \text{ upon } 2 + 3 = 7 \text{ upon } 2$$

$$\text{Product} = 3 \text{ upon } 2$$

Quadratic polynomial:

$$x^2 - \frac{7}{2}x + \frac{3}{2}$$

Multiply by 2 to remove fraction:

$$2x^2 - 7x + 3$$

**Answer:**  $2x^2 - 7x + 3$

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### Question 41(A)

Given polynomial:

$$5x^2 + x - 6$$

$$\alpha + \beta = -1 \text{ upon } 5$$

$$\alpha\beta = -6 \text{ upon } 5$$

Formula:

$$\begin{aligned}\frac{1}{\alpha^2} + \frac{1}{\beta^2} &= \frac{(\alpha + \beta)^2 - 2\alpha\beta}{(\alpha\beta)^2} \\&= \frac{\left(\frac{-1}{5}\right)^2 - 2\left(\frac{-6}{5}\right)}{\left(\frac{-6}{5}\right)^2} \\&= \frac{\frac{1}{25} + \frac{12}{5}}{\frac{36}{25}} \\&= \frac{61}{36} \\&= \frac{61}{36}\end{aligned}$$