

## Setting Optimal Target Service Level: An Optimization Approach

Choosing an inventory service level is often treated as a policy decision rather than an economic one. Teams debate whether the target should be 95 percent, 98 percent, or 99 percent, but rarely ask the more fundamental question: what does it cost to choose one value over another? Inventory exists to absorb uncertainty: uncertainty in demand, uncertainty in replenishment timing, and uncertainty in growth. That uncertainty creates unavoidable tradeoffs. Holding more inventories reduces the chance of stockouts, but it increases carrying cost. Holding less inventory saves costs, but exposes the business to lost sales and, in many cases, permanent loss of customer value.

The correct way to choose a service level is to treat it as an optimization problem, not a convention. Specifically, the service level should be selected to minimize total expected economic cost, where cost includes not only the visible cost of holding excess inventory, but also the full impact of stock outs, including lost margin today and reduced customer lifetime value tomorrow. This approach follows classic inventory logic: choose a protection level that balances overage and underage costs but extend the shortage side to include long-run economic damage.

The optimal service level is not a promise. It is the outcome of a measurable trade-off.

### What does service level really mean in an inventory system

Service level in this framework is defined as Cycle Service Level (CSL): the probability that demand during the replenishment lead time can be met entirely from inventory. This is a planning-time definition. It describes how much tail risk the system is designed to absorb over the lead time window.

If demand during the lead time has mean  $\mu_L$  and standard deviation  $\sigma_L$ , a common planning expression is:

$$Q = \mu_L + z \times \sigma_L$$

Here,  $z$  is the safety factor. Increasing  $z$  means holding more buffer inventory above expected demand, reducing the probability that demand exceeds supply during lead time.

The corresponding service level is linked to  $z$  via the standard normal cumulative distribution function:  $SL = \Phi(z)$

Operationally, teams often measure performance using Fill Rate (FR), the fraction of demand served immediately from inventory. The framework defines lost service as:  $LS = 1 - FR$

CSL is the intended probability-based protection target. Fill rate is an observed outcome. The goal of the framework is to choose CSL economically, then validate operational outcomes using fill rate and lost service.

### **The three economic forces behind service level decisions**

Selecting a service level requires balancing three distinct economic forces. Ignoring any one of them biases the decision.

First, holding costs reflects the cost of being prepared. Inventory ties up capital, incurs storage and handling cost, and creates exposure to aging and obsolescence. Let  $h$  denote the unit holding cost and let  $S$  denote excess inventory. Holding cost is:

$$\text{Holding cost} = h \times S$$

As service level increases, expected excess inventory increases. This cost is smooth and highly visible, which is why organizations often exceed it when setting service targets.

Second, immediate opportunity cost captures the margin lost when inventory is insufficient. Let  $m$  denote unit contribution margin and let  $B$  denote unmet demand. Immediate opportunity cost is:

$$\text{Immediate opportunity cost} = m \times B$$

In competitive environments, not all unmet demand behaves as a short-term deferral. Some portions trigger permanent customer displacement. Let  $p$  denote the probability that unmet demand causes a customer to move away. Then the purely immediate component can be written as:

$$\text{Immediate opportunity cost} = m \times B \times (1 - p)$$

Third—and most important—future opportunity cost captures long-run value destruction. Stockouts can reduce trust, suppress future growth, and permanently reduce customer value. A compact representation of this effect uses customer lifetime value, growth, and lost service:

$$\text{Future opportunity cost} = \text{LTV} \times \text{LS} \times p \times D \times \mu$$

Here,  $D$  is a demand scale term and  $\mu$  is a growth-rate factor. Structurally, this term says that the cost of stockouts is not just today's lost sale, but a discounted stream of future losses amplified by growth and competition.

### **The Optimization Problem: Formal Definition**

Calling this a full optimization problem requires making explicit what is being optimized, what can be chosen, and what assumptions are in place. In this framework, the service level is not fixed; it is derived as the solution to a cost-minimization problem under uncertainty.

### Decision variables

The primary decision variable is the target service level SL. Equivalently, the decision can be represented by the safety factor  $z$ , linked by:

$$SL = \Phi(z)$$

Choosing SL implicitly determines how much inventory buffer is held relative to variability.

### Objective function

The objective is to minimize total expected economic cost as a function of SL:

Expected Total Cost (SL)

= Expected Holding Cost + Expected Immediate Opportunity Cost + Expected Future Opportunity Cost

A compact representation is: Expected Total Cost (SL)

$$= h \times E[S(SL)] + m \times E[B(SL)] \times (1 - p) + LTV \times E[LS(SL)] \times p \times D \times \mu$$

Each expectation depends on SL, because changing the service level shifts the balance between excess inventory and shortages.

### Constraints

The optimization is subject to the following constraints:  $0 \leq SL \leq 1$

Planned inventory must be non-negative:  $Q = \mu_L + z \times \sigma_L \geq 0$

Service level must be consistent with the safety-factor definition:  $SL = \Phi(z)$

Implicit constraints related to replenishment granularity and lead times are treated as parameters in this stage and are reflected through the lot-size term introduced later.

### Assumptions

Several simplifying assumptions underpin the formulation:

- Demand over the replenishment lead time is stochastic with estimable mean and variability.
- Excess inventory and unmet demand are monotonic functions of the service level.
- Holding cost is approximately linear in excess inventory.
- Immediate opportunity cost is approximately linear in unmet demand.
- Long-run customer value loss scales with lost service, customer value, growth, and competition intensity.
- Replenishment lot size and lead times are fixed over the optimization horizon.

### **Optimality condition and interpretation**

At the optimal service level  $SL^*$ , the marginal cost of increasing SL equals the marginal benefit of reducing shortages. Increasing SL raises expected holding cost but reduces immediate margin loss and long-run customer value loss. The optimum is where these marginal effects balance.

This yields the familiar critical-fractile structure:

$$SL^* = \text{effective shortage cost} \div (\text{holding cost} + \text{effective shortage cost})$$

In this framework, effective shortage cost is:

$$\text{Effective shortage cost} = m + p \times \mu \times LTV$$

Substituting gives the canonical result:

$$SL^* = (m + p \times \mu \times LTV) \div (h + m + p \times \mu \times LTV)$$

This expression makes the economics explicit: service level rises when shortages are more damaging and falls when inventory is more expensive to hold.

Practical approximation with replenishment granularity

When discrete replenishment or irregular demand prevents a clean closed-form solution, a practical approximation is:

$$SL^* = 1 - h \div [ h + m \times (1 - p) + (LTV \times p \times D \times \mu) \div Q ]$$

Here,  $Q$  is the replenishment lot size. This approximation shows how granularity influences service levels: coarse replenishment increments increase exposure per decision and can justify higher service targets even when underlying demand statistics remain unchanged.

### **Why the solution follows a newsvendor structure**

The structure of the optimal service-level equation mirrors the classic newsvendor result: the optimal service level equals the ratio of shortage cost to total cost. The difference is that shortage cost here includes long-run customer value loss, not just immediate margin loss. When customer lifetime value or competitive pressure is high, the economic cost of stockouts grows substantially, pushing the optimal service level upward.

Holding costs provides the opposing force. When holding cost is high, additional protection becomes expensive, and the model pushes service levels lower. The framework's value lies in making this balancing act explicit and quantitative.

### **The role of uncertainty and variability**

Uncertainty determines how expensive it is to maintain protection. Higher demand variability increases the inventory required to achieve any given service level. As variability rises, the marginal cost of increasing the safety factor increases significantly.

This explains why highly volatile items often rationally have lower optimal service levels than stable items. It is not neglected; it is an economically sound response to the rising cost of tail protection under uncertainty.

### **Model Limitations**

No model is universal, and this framework has limitations that should be stated explicitly. First, the quality of the result depends on the quality of the input. Estimates of margin, holding cost, customer lifetime value, growth, and competition sensitivity are inherently uncertain. Misestimation can lead to under- or over-protection.

Second, the linear cost assumptions simplify reality. In practice, stockouts may trigger nonlinear effects such as contractual penalties, reputational cascades, or threshold-based churn.

Third, the relationship between cycle service level and operational metrics such as fill rate may be complex in multi-period systems with backorders, substitutions, or demand censoring.

Finally, replenishment constraints beyond simple lot sizes such as shared upstream constraints or correlated lead-time disruptions—can materially change the optimization and may require a richer model.

These limitations do not undermine the framework; they define where calibration and judgment are required.

### **Why customer segmentation is mathematically unavoidable**

Once service level depends on margin, customer lifetime value, growth, competition, replenishment granularity, and uncertainty, uniform service targets become indefensible. Different customer segments impose very different economic costs when stockouts occur. High-value, fast-growing, competitive segments have much larger effective shortage costs, which push their optimal service levels upward. Lower-value or less sensitive segments have lower penalties and therefore lower optimal service levels.

Segmentation is not a policy choice layered on top of the model. It is the direct consequence of mathematics. Even modest differences in lifetime value or substitution probability can justify materially different service targets.

The practical challenge is to choose segmentation that reflects real economic differences without creating operational complexity. The best segmentation separates segments with meaningfully different shortage penalties, not arbitrary distinctions.

**Conclusion:** service level is a capital allocation decision

Inventory service level is not an operational KPI. It is a capital allocation decision under uncertainty.

Every increase in service level commits additional capital to safety stock. Every stockout exposes margin and long-term customer value to loss. The optimal service level lies at the balance point between these forces.

This framework replaces arbitrary targets with economic reasoning. It explains why service levels vary across customers and products. It connects business strategy directly to inventory policy.

When organizations stop asking “what service level feels safe?” and start asking “what service level minimizes total economic cost for this segment?”, inventory planning moves from habit to strategy—and service level becomes a true lever of long-term value.