Propositional Logic

1.1

Proposition

A **proposition** is a declarative sentence that is either true or false, but not both

- *Remember that a declarative sentence is a sentence that declares a fact!*

Determine which sentences are propositions:

1. 7 - 4 = 3

Yes

5. x + y = 3

No. B/c x & y are unknown, cant determine True/False

1. Odin is on our side.

Yes (T or F)

~~6. x = 7~~

1. What is a Viking's favorite food?

No (Questions are never proposition)

7. Obey the Viking Code!

No (declaration)

1. Every Saxon is under attack.

Yes

8. 8/2 = 5

Yes

Propositional Variables

Propositional Variables are variables used to denote propositions.

- conventional letters for propositional variables are p, q, r, s
- sometimes other letters are used but capital letters should never be used as they will be reserved for propositional functions.

Example: "Thor's hammer is named Mjölnir" could be denoted as **p.**

- At this point, anytime you see **p**, you know it means "Thor's hammer is named Mjölnir"

Negation of p

Consider a proposition, p. The negation of p is denoted by $\neg p$.

- In English, it is read as "It is not the case that p."

"$\neg$" means "logical not"

Let's Negate!

1. Ronnie likes Taco Bell.

r: Ronnie Likes taco bell

¬r: it is not the case that ronnie likes taco bell
(aka: Ronnie doesnt like taco bell)

1. "Thor's hammer is not named Mjölnir" ⟵ Let this be ¬m

m: Thors hammer <u>is</u> named Mjonir

$\neg(\neg m) \equiv m$

The 3 lines here means logically equivalent to

Truth Values and Truth Tables

A **proposition** can be either **True (T) or False(F).**

1. Suppose we have a proposition *p*. Create a truth table for *p* and ¬p

p	$\neg p$
T	F
F	T

Conjunctions and Disjunctions

Let *p* and *q* be propositions:

- A **conjunction** of *p* and *q* is the statement "*p* and *q*"
  - it is denoted as $p \wedge q$
  - **Both** p and q must be true for $p \wedge q$ to be true.

$p \wedge q \equiv p$ and q

- A **disjunction** of *p* and *q* is the statement "*p* or *q*"
  - it is denoted as $p \vee q$
  - p ∨ q is true whenever p is true, q is true, or both p, q are true
    - inclusive or

in apcs, regular or

p	q	p ∨ q
T	T	T
T	F	T
F	T	T
F	F	F

Examples

Let p be the statement "Ronnie has 3 cats"

Let q be the statement "Ronnie does crossfit"

1. Find $p \vee q$ Ronnie has 3 cats or ronnie does crossfit
2. Find $\neg p \wedge q$ Ronnie doesn't have 3 cats and does crossfi

Translate into propositional logic

Always define propositional statement!

1. My first cat is Hazelnut and my second cat is Pistachio.

h p

1. Thor is a deceiver or Loki is a deceiver.

t l

Exclusive Or (XOR)

Let *p* and *q* be propositions:

- The **exclusive or** of p and q is the statement *p* XOR *q*
  - sometimes abbreviated as $p \oplus q$
  - Note that $p \oplus q$ is true only when exactly one of p,q is true. It is false otherwise.
  - Also note that $p \oplus q$ can be written as $(p \lor q) \land \neg(p \land q)$

1. Let p and q be the propositions that state “A student can have a salad with dinner” and “A student can have soup with dinner,” respectively. What is $p \oplus q$, the exclusive or of p and q?

Student can hae salad w/ dinner or student can hae soup w/ dinner, but not both

1. Why is the statement “I will use all my savings to travel to Europe or to buy an electric car” an example of **exclusive or?**

Infer xor b/c you can only use "all my savings" on 1 thing.