

**B.E. / B.Tech. Mechanical Engineering (Model Curriculum) Semester-III**  
**BS202 - Applied Mathematics-III (PDE, Probability & Statistics)**

P. Pages : 2

Time : Three Hours



**GUG/S/25/14056**

Max. Marks : 80

- Notes : 1. All questions carry equal marks.  
 2. All questions are compulsory.  
 3. Non programmable calculate is permitted.

1. a) Solve  $x(y^2 - z^2)p + y(z^2 - x^2)q = z(x^2 - y^2)$ . 8

b) Solve  $(D^3 - 3D' - 2D^3)z = \cos(x + 2y) - e^y(3 + 2x)$  8

**OR**

2. a) Show that  $u = e^x(x \cos y - y \sin y)$  is harmonic and find  $v$  such that  $f(z) = u + iv$  is an analytic function. 8

b) Show that  $u = 2x - x^3 + 3xy^2$  is a harmonic function. 8

Find the harmonic conjugate and the corresponding analytic function  $f(z) = u + iv$

3. a) An urn holds 5 white and 3 black marbles. If two marbles are drawn at random without replacement and  $X$  denotes the number of white marbles. 8

- i) Find the probability function and  
 ii) The distribution function.

b) Let  $X$  be random variable with density function 8

$$f(x) = \begin{cases} 3x^2, & 0 \leq x \leq 1 \\ 0, & \text{otherwise} \end{cases} \quad \text{find } E(X) \text{ and } \text{Var}(X)$$

**OR**

4. a) A random variable  $X$  has the density function given by 8

$$f(x) = \begin{cases} e^{-x}, & x \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

Find the coefficient of skewness and kurtosis.

b) Find the distribution function for r.v.  $X$  whose density 8

$$\text{Function is } f(x) = \begin{cases} \frac{x}{2}, & 0 \leq x \leq 2 \\ 0, & \text{otherwise} \end{cases}$$

Hence find  $P\left(\frac{1}{2} < X < 3/2\right)$  and  $P(X > 1)$ .

5. a) Solve the differential equation  $\frac{d^2x}{dt^2} - 3\frac{dx}{dt} + 2x = 0$  given  $X(0)=1, x'(0)=0$  by matrix method. 8

- b) Use Sylvester's theorem to verify that  $\sin^2 A + \cos^2 A = 1$ , Where  $A = \begin{bmatrix} 2 & 4 \\ 3 & 1 \end{bmatrix}$ . 8

**OR**

6. a) Verify Caley Hamiltonian theorem for the matrix  $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$  And find its inverse. Also express  $A^5 - 4A^4 - 7A^3 + 11A^2 - A - 10I$  As a linear polynomial in A. 8

- b) Find the model matrix B corresponding to the matrix  $A = \begin{bmatrix} 3 & 4 \\ 4 & -3 \end{bmatrix}$  and verify that  $B^{-1}AB$  is a diagonal form. 8

7. a) Obtained the Fourier series for  $f(x) = x - x^2$  in the interval  $-1 < X < 1$  8
- b) Give that  $f(x) = x + x^2$  for  $-\pi < x < \pi$  find the Fourier Expression of f(x). 8

**OR**

8. a) Find L. T. of  $\frac{e^{-at} - e^{-bt}}{t}$ , hence evaluate  $\int_0^\infty \frac{e^{-at} - e^{-bt}}{t}$  8

- b) Find  $L^{-1} \left\{ \frac{s}{s^4 + 4a^4} \right\}$  8

9. a) Find by Newton's Rapson method, the real root of  $3x - \cos x - 1 = 0$  8
- b) Using Regula False method, find the roots of the following Equations correct to third decimal place  $x \log_{10} x - 1.2 = 0$ . 8

**OR**

10. a) Solve the following system of equations by Crout's method  
 $x + y + z = 1, 3x + y - 3z = 5, x - 2y - 5z = 10$  8
- b) Solve by Gauss-Seidel method,  
 $x + 7y - 3z = -22, 5x - 2y + 3z = 18, 2x - y + 6z = 22$  8

\*\*\*\*\*