

B.Tech. (Model Curriculum) Semester-I
BSC103 - Engineering Mathematics-I

P. Pages : 2

Time : Three Hours



GUG/S/25/13166

Max. Marks : 80

- Notes : 1. All questions carry equal marks.
 2. All questions are compulsory.
 3. Non programmable calculator is permitted.

1. a) If $y = \tan^{-1} x$, then prove that $(1+x^2)y_{n+1} + 2nxy_n + n(n-1)y_{n+1} = 0$ and find $y_n(0)$. 8

b) Evaluate from the following limits 8

$$\lim_{x \rightarrow 0} \log_{\tan^2 x} (\tan^2 2x)$$

OR

2. a) Prove that 8

$$e^x \cdot \sin x = x + x^2 + \frac{1}{3}x^3 + \dots + \frac{2^n}{n!} \cdot \sin\left(\frac{n\pi}{4}\right)x^n + \dots$$

b) If $y = \cos(m \log x)$, then show that 8

$$x^2 y_{n+2} + (2n+1)xy_{n+1} + (m^2 + n^2)y_n = 0$$

3. a) If $z(x+y) = x^2 + y^2$, then prove that $\left(\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)^2 = 4\left(1 - \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)$ 8

b) If $u = \sin^{-1}\left(\frac{x^2 + y^2}{\sqrt{x} + \sqrt{y}}\right)$, then find the value of $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2}$. 8

OR

4. a) If $z = f(x, y)$, $x = u + v$, $y = uv$, then show that 8

$$\frac{\partial^2 z}{\partial x^2} - y \frac{\partial^2 z}{\partial y^2} = \frac{1}{u-v} \left(u \frac{\partial^2 z}{\partial u^2} - v \frac{\partial^2 z}{\partial v^2} \right)$$

b) If $u = x^2 - y^2$, $v = 2xy$, $f(x, y) = q(u, v)$ then show that 8

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 4(x^2 + y^2) \left(\frac{\partial^2 \theta}{\partial u^2} + \frac{\partial^2 \theta}{\partial v^2} \right)$$

5. a) If $u = \frac{x+y}{x-y}$, $v = \frac{xy}{(x-y)^2}$. Are u and v functionally related? If so, find the relationship. 8

- b) If $u = x + y + z, u^2 v = y + z, u^3 w = z$, then show that $\frac{\partial(u, v, w)}{\partial(x, y, z)} = u^{-5}$. 8

OR

6. a) If $u = xy + yz + zx, v = x + y + z, w = x^2 + y^2 + z^2$ then find $\frac{\partial(u, v, w)}{\partial(x, y, z)}$. Are u, v, w are functionally related? If so find the relationship. 8

- b) Find the minimum value of $x^2 + y^2 + z^2$, given that $ax + by + cz = p$. 8

7. a) Show that $\int_0^\infty x^n \cdot e^{-k^2 x^2} \cdot dx = \frac{1}{2 \cdot k^{n+1}} \cdot \Gamma\left(\frac{n+1}{2}\right)$. 8

- b) Evaluate $\int_0^1 (1-x^3)^{-1/2} \cdot dx$. 8

OR

8. a) Show that $\int_0^\infty \frac{e^{-x} - e^{-ax}}{x \cdot \sec x} dx = \frac{1}{2} \cdot \log\left(\frac{a^2 + 1}{2}\right), a < 0$ 8

- b) Find the RMs value of $f(x) = e^x + 1$ over the range $x = 0$ to $x = 2$. 8

Substitute the value of e^2 and e^4 in the result and evaluate it further.

9. a) Find the equation of regression lines and coefficient of correlation to the following data 8

x	3	5	6	8	9	11
y	2	3	4	6	5	8

- b) Fit a straight -line $y = a + bx$ to the following data by the method of least squares 8

x	0	1	3	6	8
y	1	3	2	5	4

OR

10. a) Find the coefficient of correlation and equation of lines of regression from the following data. 8

x	1	2	3	4	5
y	2	5	3	8	7

- b) Fit the parabola $y = ax + bx^2$ for the following data by least square method. 8

x	0	1	2	3
y	2	4	10	15
