

# Quantum Gate Operators: Essentials

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## **1 Gate Overview**

To define them, gates are operators that allows us to manipulate bits. They use logic and math to do so, and are extremely important for computation.

This edition will be more mathematically based to effectively learn all the operators.

## **2 Classical Gate Operators**

Before we dive into quantum gates, it is first important to explore classical gates and how they can modify regular bits. The following are some important classical gates:

## 2.1 NOT Gate

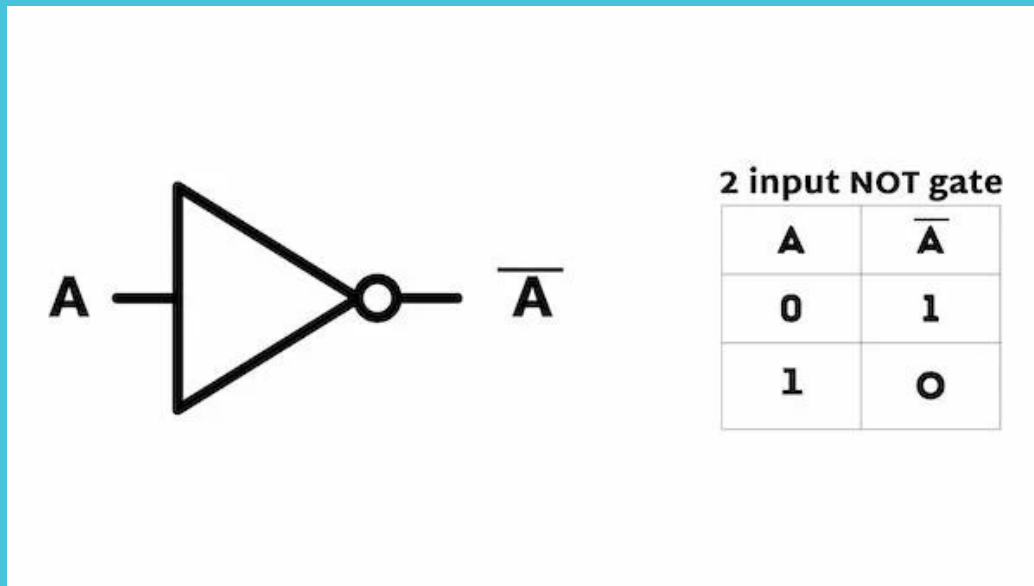


Figure 1: Enter Caption

The NOT Gate is very simple; whatever bit is entered, it simply outputs the opposite. So if you enter a 1, it exits a 0, and vice versa. The truth table can be seen on the right ( all the possible combinations ). The gate only takes one input, and has one output.

## 2.2 AND Gate

The AND gate is another common and important gate in classical computing. It can take as many inputs as needed, but it needs to have at least two.

If all of the inputs are equal to 1, the gate outputs 1; but if even one of the inputs is equal to 0, the gate would output 0. There are four possible entries that can be seen in the truth table

### **2.3 OR Gate**

This is somewhat similar to the AND Gate but with an important purpose; only one of the inputs need to be 1 to output 1, not both. Just like before, it can take as many inputs as needed, but needs to have at least two

Paying attention to the names of the gates, this makes sense; AND means that many things must all be true, while OR refers to at least one thing of many being true; this corresponds to the actual operations that the gates perform.

### **2.4 XOR Gate**

While there are many more classical gates, the last one that will be looked at today will be the XOR Gate

The XOR Gate works in a way where it will output 1 if the two inputs are different, and output 0 if the two inputs are the same.

The XOR Gate is extremely important for various encryption methods; it is often used to encrypt a message using a special key, such that it can be sent without being intercepted/

### **3 Definition Of Quantum Gates**

Quantum Gates act differently from classical gates, but that is because qubits behave differently from regular bits

Qubits can also be in superposition, not only discrete states, so gates must operate in that fashion as well.

#### **3.0.1 Mathematical Representations of Gates**

As stated before, quantum states can be represented by two-dimensional row vectors; in the same way, gate operators can be represented by matrices.

Essentially, then, applying matrices to quantum states can be represented through vector matrix multiplication.

### 3.1 Unitary Matrices

One property about quantum gates is that they are unitary matrices. Unitary matrices are matrices such that when they are multiplied by their conjugate transpose, they result in the identity matrix. They are also square matrices, meaning that they have the same number of rows and columns.

To take the conjugate transpose of a matrix is very simple; to start, the rows of the matrix need to be swapped with the columns ( meaning that the first row becomes the first column and the second row becomes the second column ). Refer to the example below.

Secondly, any imaginary components will have their signs replaced ( for example, positive to negative ); mathematically, we are replacing  $i$  with  $-i$  for any of the elements in the matrix. Again, refer to the example below

The Identity matrix is a very important matrix; its image is represented below. It should thus be a fact that the original matrix times its conjugate transpose results in the identity matrix.

### **3.2 Reversibility and Probability**

Another property is that all quantum gates are reversible, meaning that any operator done on a state can also be undone. Furthermore, even after a gate operation, the sum of all probabilities must still add to 1; this results in normalization being conserved.