# Entanglement: A Brief Introduction

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# 1 What Is Quantum Entanglement?

Quantum entanglement refers to a scenario where the particles behave in such a way that their quantum states become linked

To clarify, this means that measuring the state of one particle would allow some information to be gained regarding the state of the other particle

# 2 Analogies for Entanglement

In case the concept of entanglement is still hard to understand, here are some analogies to better comprehend what this type of relationship means.

Figure 1: Refer to this image for the analogy

Think of entangled particles as two shoes inside a shoe box; if the two shoes were moved galaxies away from each other, and you identified one shoe as being the shoe for the right foot, it would be common sense that the other shoe would have to be for the left foot; this reflects entanglement as it shows that knowing information about one state allows for knowning something else about another state.

Entanglement can also be represented through the example of dice; pretend that such a pair of dice existed, such that they always added to seven when they were rolled. if one die rolled 6 ( measuring the state of the die ), it could be noted that the other die needed to be 1.

Thus, by just knowing the value of one of the two dice, the value of the other could be found.

## 3 Causes of Entanglement

Entanglement can be caused naturally through a variety of ways at the particulate level. The following give some insight into these processes

### 3.1 Interaction of Particles

Interactions between particles can be defined as collisions, the exchange of energy, or even being impacted by the same force. This results in their quantum states becoming mixed with each other.

This has to do with the nature of evolution of quantum states; quantum states evolve based on the Schrodinger Equation, which describes how their wave functions are able to change over time.

If two particles are to interact in the way described above, it is possible that their Schrodinger equations become mixed in such a way that they come together to form a complete system; this would ensure that their states depended on each other, resulting in entanglement.

### 3.2 Laws of Conservation of Matter

Conservation laws also heavily influence entanglement between particles, especially when a particle splits into two smaller particles which need to maintain the net original state

For example, if a single particle has a net 0 spin, and it is split into two smaller particles, the net angular momentum of both particles needs to be 0; angular momentum refers to the momentum of an object due to spinning, and so an object which has 0 spin would have 0 angular momentum

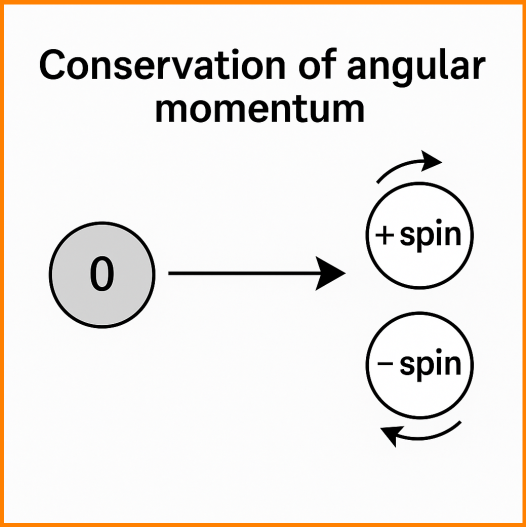


Figure 2: Angular momentum is just momentum due to the spinning of an object; momentum is conserved in the absence of external forces, so if a mass with no spin split up, the two spins of the seperated masses would need to cancel each other out. This would ultimately cause their entangled nature.

What this would mean, continuing the example, would be that if one particle had a right spin ( spun to the right ), it would have to be true that the other particle had a left spin such that both the spins cancelled out to equate to 0, thus conserving angular momentum

Of course, any conservation laws assume there is no external disturbance; if there were, these values couldn't be assumed to be conserved.

# 4 BONUS: Entangleement in Computers

The following passage gives a few means to which qubits can be purposely entangles for the sake of computation; the previous causes were natural, and therefore random.

## 4.1 Gates

Quanum gates can be applied to qubits in order to change their state; if a qubit can be represented through a vector, a gate can be represented through a matrice; thus, applying a gate to a state is like doing matrice vector multiplication

The way to do this is by using the Hadamard gate and the CNOT gate, both of which are extremely important.

Below is an image of what the circuitry would look like

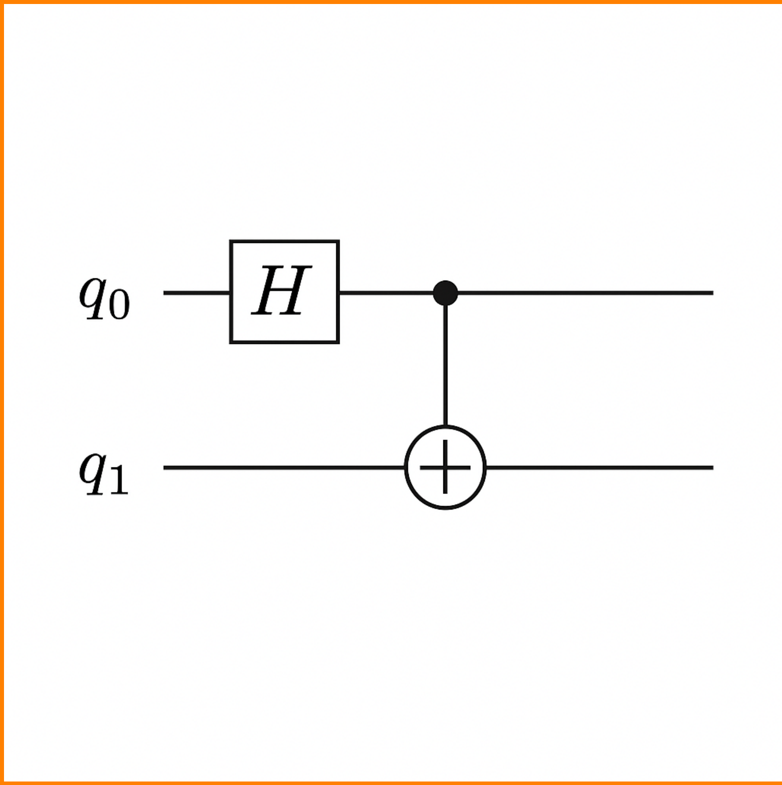


Figure 3: This quantum circuit can be coded by applying a hadamard gate to the first qubit, and then connecting the first and second qubits with the CNOT gate

The gates need to be understood in order to visualize their impact; the Hadamard gate has the ability to put the qubit in superposition; from the image it can be seen that q0 has the Hadamard applied and thus is in a probabilistic mixing of multiple states.

The second gate seen is the CNOT gate; this is the gate which has the plus symbol, and it is what causes entanglement

The CNOT gate works in such a way that the state of the second qubit, q1, depends on the state of the first qubit, q0; if the first qubit collapses to the $|0\rangle$ state, the second qubit stays the same, but if it collapses to the $|1\rangle$ state the state of the second qubit flips

Typically in quantum computing, it is assumed that every qubit is initialized in the $|0\rangle$ state, so if the first qubit is $|0\rangle$ then the second qubit will also be $|0\rangle$ and vice versa

This is why the first qubit is put into superposition; it essentially shows that the state of the second qubit depends on whatever the first eventually becomes, showing the entangled relationship that exists

## 5 Quantum Noise

A very important obstacle to consider when dealing with entanglement is noise; noise refers to any unintended interaction that occurs within a quantum system between a qubit and it's environment; basically, it causes unpredictability within a quantum system due to the fact that it cannot be controlled.

### 5.1 Analogy

If the definition of noise didn't make sense, this may be a good analogy to better comprehend it

Figure 4: Refer to this image for the analogy!

Pretend that you and a friend are having a conversation; you are in sync and can both hear each other properly, resulting in a good flow of communication. This represents the ideal state for two qubits in entanglement. Their states are perfectly connected.

But now imagine that suddenly the room gets very loud, and so you cannot hear each other properly; or one of you start stuttering or mumbling such that the conversation becomes unclear. This is a result of the system around you, and now you and your friend are not in sync anymore

This essentially represents noise; due to these disturbances, the entangled state becomes at risk; especially for quantum computers and information transportation, this becomes a very large problem .

It is important to note that noise doesn't cause errors just with entanglement; noise affects the output or connection within/between states, so it could also affect superposition, gate operations, measurement, and more.

## 6 Examples of Noise

The following are examples of noise that occur in the real world. These are just two examples, but there are many other examples which I encourage you to research yourself. .

### 6.1 Bit-Flip Noise

This is a type of noise which causes a qubit to flip from the state to the state or vice versa; for obvious reasons, this could cause computation errors due to the states being crucial for representing data in computers

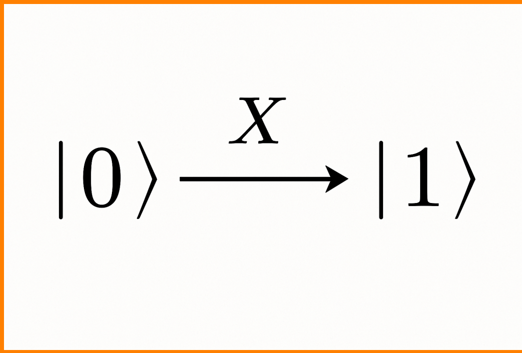


Figure 5: The bit flip error can be modeled through the Pauli-X; this gate operation causes a bit to flip its state. Obviously, the error isn't induced by the gate itself, but due to noise disturbance.

### 6.2 Amplitude Dampening

Amplitude dampening is another common type of noise; typically in quantum mechanics, the state is regarded as having higher energy than the state. In amplitude dampening, the qubit looses energy such that over time it goes from the state to the state

$$\alpha |0\rangle + \beta |1\rangle \xrightarrow{\text{amplitude damping}} \alpha |0\rangle$$

Figure 6: This represents amplitude encoding; at first, both the 1 and 0 have probability amplitudes, but after some time, only the 0 state has an amplitude, implying that the amplitude for the 1 state had deteriorated over time.

if you can imagine the mathematical representation of superposition in a qubit, it would look as if the amplitude for the state would be decreasing while the amplitude for the would b increasing until eventually the state would compltely be ( hence the name amplitude dampening )

# 7 Ways to Decrease Noise

The following are ways to decrease the influence of noise; noise can never truly be removed, so quantum computers need to figure out whats to decrease/manage it in order to still obtain accurate computations.

## 7.1 Cooling Down System

Heat is often a main cause of noise in the system; this is because is is extenral to the quantum system and causes distrurbances. Obviously then, cooling down the system to the extent where heat can be mitigated would reduce noise. This is why you see many quantum computers stored in cool chambers to ensure coherence

### 7.2 Circuit Reduction

Larger or longer circuits are more at risk of being affected by noise; to do this, quantum scientists often try to design/utilize smaller and more effecient circuits for tasks; this ensures that they are less at risk of being subject to noise

## 8 Quantum Bell States

Bell states represent quantum states involving two qubits which can be regarded as entangled to the maximal potential limit' thus, the serve as the most ideal form of entanglement. The following are the four bell states:

$$|\Phi^+\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

$$|\Phi^-\rangle = \frac{|00\rangle - |11\rangle}{\sqrt{2}}$$

$$|\Psi^+\rangle = \frac{|01\rangle + |10\rangle}{\sqrt{2}}$$

$$|\Psi^-\rangle = \frac{|01\rangle - |10\rangle}{\sqrt{2}}$$

Figure 7: These are the four quantum bell states; these states represent pure entanglement betwen two qubits and are extremely fundamental for quantum mechanics.

As you can see, all the states are in superposition; this again reinforced the entangled nature, as the state of the second qubit depends on the first

Notice that each ket has two values/numbers; this represents the state itself. This means that the ket with the values 1 and 0 would be a quantum state where one qubit is 1 and the other is 0, for example.

The first two bell states, , represent entanglement such that the second qubit will always collapse into the state that the first qubit measures; notice that the second one has a minus while the first one has a plus; this represents a phase change

For simplicity, phase represents how quantum states interact; they don't change the probabilities of measuring a state

Phase affect how two states may interact based on interfence; if two states have the same phase ( both positive or both negative ) they constructively interfere, and if they have different phases, it is destructive. So the first two bell states only differ by these means.

The second two bell states operate in such a way that the second qubit will always be the opposite state of the first; again they differ due to a phase change which affects the interference.

# 9 Excercises

## 9.1 Multiple Choice Practice

Question 1

**2. In an entangled Bell state $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$, measuring the first qubit as $|0\rangle$ guarantees that the second qubit will be:**

A) $|0\rangle$

B) $|1\rangle$

C) A superposition of 0 and 1

D) Unknown

Figure 8: Answer: A; as seen from the quantum state, each ket with two numbers represents a potential two qubit system. It relates entanglement between the two qubits by showing that if the first qubit measures 0, the second will be 0, and if the first is 1, the second will also be 1. This is also one of the bell states.

## Question 2

**3. Which quantum gate is typically used to create entanglement between two qubits?**

A) Hadamard gate

B) Pauli-X gate

C) CNOT gate

D) Z gate

Figure 9: Answer: C; while the Hadamard gate is used in the circuit which involves entanglement, the CNOT gate is what actually connects the two qubits and makes the state of the second one dependent on the first.

## Question 3

**7. Amplitude damping models what real-world process?**

A) Qubit losing energy and falling to $|0\rangle$

B) A photon splitting into two

C) Interference of qubits

D) A qubit increasing its energy level

Figure 10: Answer: A; none of the other options besides A make sense. Regardless, amplitude dampening refers to when the energy level of a state drops from 1 to 0 due to the amplitude for the 1 state decreasing over time, resulting in the amplitude for state 0 increasing over time

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