ASSIGNMENT 1

ARJUN DOSAJH
2021113016

Q1. (a) P: It rains

Q: Raju carries an umbrella

$$((P \rightarrow Q) \land Q) \longrightarrow P$$

P	Q	$P \rightarrow Q$	$(P \rightarrow Q) \land Q$	$((P \rightarrow Q) \land Q) \rightarrow P$
F	F	T	F	T
F	T	T	T	F
T	F	F	F	T
T	T	T	T	T

Since it's not a tautology, the statement is invalid

(b) A : weather is warm

B : sky is clear

C : We go swimming

D : We go boating

$$\{[(A \land B) \longrightarrow (C \lor D)] \land [\sim(\sim C \longrightarrow \sim B)]\} \longrightarrow (A \lor D)$$

Since the truth table is huge, I made it on excel and I'm attaching a screenshot here

A	B	C	D	A . B	C + D	(A.B) → (C+D)	~C	~B	~C →~B	~(~C →~B)	[(A.B) → (C+D)] . [~(~C → ~B)]	A + D	{[(A.B) → (C+D)] . [~(~C → ~B)]} → (A + D)
F	F	F	F	F	F	T	T	T	T	F	F	F	T
F	F	F	T	F	T	T	T	T	T	F	F	T	T
F	F	T	F	F	T	T	F	T	T	F	F	F	T
F	F	T	T	F	T	T	F	T	T	F	F	T	T
F	T	F	F	F	F	T	T	F	F	T	T	F	F
F	T	F	T	F	T	T	T	F	F	T	T	T	T
F	T	T	F	F	T	T	F	F	T	F	F	F	T
F	T	T	T	F	T	T	F	F	T	F	F	T	T
T	F	F	F	F	F	T	T	T	T	F	F	T	T
T	F	F	T	F	T	T	T	T	T	F	F	T	T
T	F	T	F	F	T	T	F	T	T	F	F	T	T
T	F	T	T	F	T	T	F	T	T	F	F	T	T
T	T	F	F	T	F	F	T	F	F	T	F	T	T
T	T	F	T	T	T	T	T	F	F	T	T	T	T
T	T	T	F	T	T	T	F	F	T	F	F	T	T
T	T	T	T	T	T	T	F	F	T	F	F	T	T

this is not a tautology
∴ the statement is not valid

Q2.

A = true

B = true

X = false

Y = false

(a) $\neg(A \vee X) = \neg(\text{true OR false}) = \neg(\text{true}) = \text{false}$

(b) $A \vee (X \wedge Y) = \text{true OR (false and false)} = \text{true}$

(c) $A \wedge (X \vee (B \wedge Y)) = T \wedge (F \vee (T \wedge F)) = T \wedge (F \vee F)$

$= T \wedge F = \text{False}$

(d) $[(A \wedge X) \vee \neg B] \wedge \neg[(A \wedge X) \vee \neg B] = [(T \wedge F) \vee \neg T]$

$\wedge$

$\neg[(T \wedge F) \vee \neg T]$

$= [F \vee F] \wedge \neg(F \vee F) = F \wedge T = \text{False}$

(e) $(P \wedge Q) \wedge (\neg A \vee X) = (P \wedge Q) \wedge (\neg T \vee F)$

$= (P \wedge Q) \wedge F = \text{False}$ ($\because$ anything $\wedge F$ = false)

(f) $[(X \wedge Y) \rightarrow A] \rightarrow [X \rightarrow (Y \rightarrow A)]$

$= [(F \wedge F) \rightarrow T] \rightarrow [F \rightarrow (F \rightarrow T)]$

$= [F \rightarrow T] \rightarrow [F \rightarrow T] = T \rightarrow T = \text{True}$

Q3. (a) $P \rightarrow \neg Q, \ \neg Q \rightarrow R \Rightarrow P \rightarrow R$

Formal proof

let $\neg Q = S$

$\therefore P \rightarrow S$ and $S \rightarrow R$

from hypothetical syllogism,

$P \rightarrow S, \ S \rightarrow R \Rightarrow P \rightarrow R$

Resolution method :

Conversion to CNF :

$P \rightarrow \neg Q \qquad \neg Q \rightarrow R$

$= \neg P \vee \neg Q \qquad = Q \vee R$

$\therefore$ premises : $\neg P \vee \neg Q$ and $Q \vee R$

$P \rightarrow R$

$\neg P \vee R$

We need to prove that $\neg P \vee R$

$\Rightarrow$ Resolution rule of inference

$\underbrace{\neg P \vee \neg Q \qquad Q \vee R}$

$\neg P \vee R$

<u>hence proved</u>

(b) $\Rightarrow$ $((P \vee Q) \wedge \neg P) \rightarrow Q$

Resolution :-

Convert to CNF

$((P \vee Q) \wedge \neg P) \rightarrow Q$

$\neg((P \vee Q) \wedge \neg P) \vee Q$

$(\neg(P \vee Q) \vee P) \vee Q$

$((\neg P \vee P) \wedge (\neg Q \vee P)) \vee Q$

$\underbrace{(\neg P \vee P)}_{\text{TRUE}}$

$(\text{TRUE} \wedge (\neg Q \vee P)) \vee Q$

$(\neg Q \vee P) \vee Q = \underbrace{(\neg Q \vee Q)}_{\text{TRUE}} \vee P$

$= (TRUE) \vee P = TRUE$

Negation of our conclusion = FALSE

The empty clause is unsatisfiable (always false)

Original premise is empty clause

Since conclusion is tautology, we have proved by resolution

Formal Proof

$$((P \vee Q) \wedge \neg P) \rightarrow Q$$

$$= \neg((P \vee Q) \wedge \neg P) \vee Q$$

$$= (\neg(P \vee Q) \vee P) \vee Q$$

$$= ((\neg P \wedge \neg Q) \vee P) \vee Q$$

$$= ((\neg P \vee P) \wedge (\neg Q \vee P)) \vee Q$$

$$= (TRUE \wedge (\neg Q \vee P)) \vee Q$$

$$= (\neg Q \vee P) \vee Q$$

$$= P \vee (\neg Q \vee Q)$$

$$= P \vee (TRUE) = TRUE$$

Conclusion is true

$\therefore$ entailment = TRUE

hence proved