Adaptive Control of Inverted Pendulum on Cart Using Physics-Informed Neural Networks

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Abstract

This study extends the work of Seo [1] by employing a Physics-Informed Neural Network (PINN) approach to address the control problem of an inverted pendulum on a cart, a quintessential challenge in control theory and engineering. Diverging from traditional control methods, our strategy leverages PINNs to capture the system's intricate dynamics and accurately predict control actions for transitioning the pendulum to an inverted position within a predetermined timeframe. Utilizing the DeepXDE library [2], a tool for solving differential equations via PINNs, we generated synthetic data that adheres to the physical laws governing the pendulum-cart system. Our findings validate the PINNs' capability in achieving precise control, marking a significant step forward in devising advanced control mechanisms for nonlinear and dynamically intricate systems.

1 Background

The inverted pendulum on a cart system is a fundamental problem in control theory. The goal is to apply forces to the cart such that the pendulum is in it's inverted state after a given fixed time. Previous works have explored various control methods, including PID controlers and linear quadratic regulators [3]. Building on the advancements in PINNs, this study is inspired by Seo's exploration of PINNs in real-world optimization tasks, including pendulum control, path optimization, and spacecraft trajectory planning [1]. Our work extends PINNs' application to the inverted pendulum problem, seeking innovative solutions in the realm of dynamic system control.

2 Methods

Physics-Informed Neural Networks (PINNs) are a class of deep learning models that incorporate physical laws into their learning process, enabling them to predict outcomes that adhere to the underlying physics of the problem. This characteristic makes PINNs particularly suitable for solving differential equations and control problems where traditional models may struggle.

In his innovative study, Seo leverages Physics-Informed Neural Networks (PINNs) to address a variety of real-world optimization problems, demonstrating a marked improvement in efficiency over reinforcement learning techniques. By integrating physical laws directly into the neural network's training process, PINNs act as a powerful tool for navigating complex systems and constraints to find optimal solutions across diverse tasks. Seo's work showcases this methodology in three distinct applications: swinging up a pendulum to a desired state, identifying the shortest-time path between two points, and optimizing a spacecraft's swingby trajectory using minimal thrust. This approach, which blends the precision of physics with the adaptability of machine learning, offers a powerful tool for solving optimization tasks that are characterized by their governing laws, constraints, and specific goals, proving particularly effective since the PINN model converges in significantly less iterations than other methods.

In our investigation of the inverted pendulum on a cart control problem, we define the following initial and boundary conditions, and the governing equations of the system's dynamics.

2.1 Initial Conditions

At the initial time $t = 0$, the system is at rest with the conditions:

- $x(0) = 0$ – The cart is positioned at the origin.
- $F(0) = 0$ – The initial force on the cart is zero.
- $\theta(0) = 0$ – The pendulum is in the downward vertical position.

2.2 Boundary Conditions

We apply the following boundary conditions at the temporal boundaries:

- Neumann conditions at $t = 0$:
  1. $\left.\frac{\partial \theta}{\partial t}\right|_{t=0} = 0$ – The angular velocity of the pendulum is zero.
  2. $\left.\frac{\partial x}{\partial t}\right|_{t=0} = 0$ – The velocity of the cart is zero.
- Custom conditions at $t = T$:
  1. $\theta(T) = \pi$ – The pendulum reaches an inverted position.

2.3 Equations of Dynamics

The dynamics of the system are described by the following differential equations:

$$\begin{bmatrix} \ddot{x} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} + \begin{bmatrix} \frac{-1}{(T(M+m)+Mm\ell^2)} & \frac{m^2 g\ell^2}{T(M+m)+Mm\ell^2} \\ \frac{m\ell b}{T(M+m)+Mm\ell^2} & \frac{-mg\ell(M+m)}{T(M+m)+Mm\ell^2} \end{bmatrix} \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{T(M+m)+Mm\ell^2} \end{bmatrix} u \tag{1}$$

From (1), we get

$$\dot{x} = -\frac{(I+m\ell^2)b\dot{x}}{I(M+m)+Mm\ell^2} + \frac{m^2 g\ell^2\phi}{I(M+m)+Mm\ell^2} + \frac{(I+m\ell^2)u}{I(M+m)+Mm\ell^2},$$
$$\ddot{\phi} = -\frac{mb\ell\dot{\phi}}{I(M+m)+Mm\ell^2} + \frac{mg(N+m)\ell\phi}{I(M+m)+Mm\ell^2} + \frac{m\ell u}{I(M+m)+Mm\ell^2}.$$

Where:

- x and θ represent the cart's position and the pendulum's angle, respectively.
- $\dot{x}$ and $\dot{\theta}$ are their respective velocities.
- $\ddot{x}$ and $\ddot{\theta}$ are their respective accelerations.
- u is the control input force.
- m and M are the masses of the pendulum and cart, respectively.
- ℓ is the length of the pendulum.
- g is the acceleration due to gravity.
- b is the damping coefficient.
- T is time.

3 Results and Analysis

In the conducted study, PINN model successfully addressed the optimization problem, adhering to a diverse array of initial and boundary conditions, as well as the predefined goal state. This multifaceted problem resulted in a composite loss function within the PINN framework, which effectively guided the learning process.

The simulation details and the corresponding results are available for review in the associated GitHub repository. Through the simulation, it is evident that the PINN model strategically applies minimal forces incrementally. This approach ensures that the system smoothly transitions towards the target state at $t = T$, all the while conforming to the constraints of maximal force application. Such a methodical force application showcases the PINN's capability to not only learn from the physical dynamics but also to optimize control input in a restrained environment.

4 Discussion and Conclusions

In our research, we have adopted the PINN methodology to tackle real-world optimization challenges, demonstrating its robustness across various test cases, including the control of an inverted pendulum. PINN excels in solving inverse problems, often ill-posed in nature, by integrating governing equations directly into the learning process, rather than relying on sequential, explorative steps characteristic of other machine learning techniques like Reinforcement Learning (RL) and Genetic Algorithms (GA). This direct integration allows for a more efficient learning pathway with fewer iterations needed for convergence, particularly advantageous for designing trajectories in non-real-time scenarios. While RL may surpass PINN in tasks necessitating interactive, real-time decision-making, PINN is superior in single-pass optimizations, especially when a precise model is known.

5 References

References

[1] Seo, J. Solving real-world optimization tasks using physics-informed neural computing. *Sci Rep*, 14:202, 2024. DOI: 10.1038/s41598-023-49977-3.

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