

# Black-Scholes Model

## Description

The Black-Scholes Model or Black-Scholes-Merton formula is a pricing model to price European style options. The model works by calculating the expected value the buyer receives and the expected cost the buyer pays. The difference between them can be used to calculate the theoretical premium.

European options : Derivative contracts giving the holder the right, but not the obligation, to buy (call) or sell (put) an asset at a fixed strike price, but only on the contract's specific expiration date, unlike American options that allow early exercise.

## Formula

$S$  : stock price

$K$  : strike price

$r$  : risk-free interest rate

$T$  : time to expiration

$\sigma$  : standard deviation of log returns (volatility)

To get the price we derive two components  $d_1$  and  $d_2$

$$d_1 = \frac{\ln(\frac{S}{K}) + (r + \frac{\sigma^2}{2})T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

Price of a Call option (buying)

$$C = S\mathcal{N}(d_1) - Ke^{-rT}\mathcal{N}(d_2)$$

Price of a Put option (selling)

$$P = Ke^{-rT}\mathcal{N}(-d_2) - S\mathcal{N}(-d_1)$$

## Call Option

**Expected benefit - Value of stock the call buyer might receive**

$$S\mathcal{N}(d_1)$$

$\mathcal{N}(d_1)S = \text{number of shares (adjusted for probability)} \times \text{Current stock price}$

- This is what the option buyer stands to GET
- If they exercise they receive  $S$
- $\mathcal{N}(d_1)$  is the probability adjusted amount of that stock

### Expected cost - Cost the buyer might pay

$$Ke^{-rT}\mathcal{N}(d_2)$$

$$\mathcal{N}(d_2)Ke^{-rT} = (\text{Probability of Paying}) \times (\text{Strike Price}) \times (\text{Discount factor})$$

- This is what the option buyer stands to PAY
- If they exercise they pay  $K$
- $\mathcal{N}(d_2)$  is the probability they'll actually have to pay

Call option price = Value of stock being bought - Cost of buying the stock at strike price

$$C = S\mathcal{N}(d_1) - Ke^{-rT}\mathcal{N}(d_2)$$

### Put Option

#### Expected Benefit: Cash the seller might receive

$$Ke^{-rT}\mathcal{N}(-d_2)$$

$$Ke^{-rT}\mathcal{N}(-d_2) = (\text{Strike price}) \times (\text{Discount factor}) \times (\text{Probability of receiving})$$

- This is what the seller stands to GET
- If they exercise they receive  $K$
- $\mathcal{N}(-d_2)$  is the probability they'll actually receive this cash

#### Expected Cost : Value of the stock the seller might give up

$$S\mathcal{N}(-d_1)$$

$$S\mathcal{N}(-d_1) = (\text{Current Stock price}) \times (\text{Number of shares adjusted for probability})$$

- This is what the seller stands to give up/pay
- If they exercise they give stock worth  $S$
- $\mathcal{N}(-d_1)$  is the probability they have to give the stock

Put option = Cash received from selling the stock at strike price - Value of stock being sold

$$P = Ke^{-rT}\mathcal{N}(-d_2) - S\mathcal{N}(-d_1)$$