Solid s, liquid ℓ. Container walls at uniform temperature T_w, melt initially at uniform temperature T_m at which liquid and solid would coexist. Latent heat of fusion H_s^ℓ energy needed to melt unit mass of solid. Container of characteristic dimension d.
In each phase

$$\frac{\partial T}{\partial t} = \kappa \nabla^2 T \qquad (a)$$

At the solid–liquid interface, mass flux $J_i\ell^s$ from liquid to solid, unit normal n_ℓ^s directed from liquid into solid. Interface propagates normal to itself with speed c.
Balance of total energy:

$$-H_s^\ell J_\ell^s = \boldsymbol{n}_\ell^s \cdot [k\nabla T]_\ell^s, \qquad (b)$$

Balance of mass:

$$\rho_s(\boldsymbol{v}_s \cdot \boldsymbol{n}_\ell^s - c) = J_\ell^s = \rho_\ell(\boldsymbol{v}_\ell \cdot \boldsymbol{n}_\ell^s - c) \qquad (c)$$

Rigid solid $\boldsymbol{v}_s = 0$.
Assumption: liquid and solid of identical properties

$$\rho_\ell = \rho_s = \rho \;\; \Rightarrow \;\; \boldsymbol{v}_\ell = 0$$

Kinetics of solidification:

$$J_\ell^s = \lambda(T_m - T); \qquad (d)$$

if temperature T at the interface $< T_m$, then mass is converted from liquid into solid.
Kinematics: $\boldsymbol{r}$, position vector of a point on the interface

$$\frac{\mathrm{d}\boldsymbol{r}}{\mathrm{d}t} \cdot \boldsymbol{n}_\ell^s = c \qquad (e)$$

End of formulation.
Eliminate J_ℓ^s between (d) and (b):

$$\lambda H_s^\ell(T_m - T) + \boldsymbol{n}_\ell^2 \cdot [k\nabla T]_\ell^s = 0. \qquad (f)$$

Then there are time derivatives only in (a) and in (e).

There are three time scales. Conduction (from (a)) $t_c = d^2/\kappa$, thermal diffusivity κ. Kinetic (from (d) and (c)) $t_k = \rho d/(\lambda \Delta T)$, time taken for the interface to propagate across container if temperature at interface is T_w. Third time scale (from (b) and (c)) $t_H = \rho H_s^\ell d^2/(k\Delta T), = t_c H_s^\ell/(c_p \Delta T)$ (specific heat c_p).
Osher has $t_c \approx t_H \gg t_k$ so that $T = T_m$ at the interface. For $t_H \approx t_k \gg t_c$, heat conduction will be quasi-steady. This conclusion, and the time scales, can be obtained by introducing dimensionless variables, which you should do if you decide to use this model problem.