

9.1.8

EE24BTECH11005 - Arjun Pavanje

Question: Solve the differential equation $y' + y = e^x$ with initial conditions $y(0) = \frac{1}{2}$
Solution:

Theoretical Solution:

This is a linear differential equation of the first order.

$$\frac{dy}{dx} + y = e^x \quad (1)$$

$$(2)$$

Finding integrating factor

$$e^{\int 1 dx} \quad (3)$$

$$= e^x \quad (4)$$

Multiplying both sides of (2) with integrating factor,

$$\frac{dy}{dx} (e^x) + ye^x = e^{2x} \quad (5)$$

$$\frac{d(ye^x)}{dx} = e^{2x} \quad (6)$$

$$ye^x = \frac{e^{2x}}{2} + c \quad (7)$$

$$y = \frac{e^x}{2} + ce^{-x} \quad (8)$$

On substituting initial conditions we get,

$$y = \frac{e^x}{2} \quad (9)$$

Computational Solution:

Aim is to find a computational solution to the differential equation using trapezoidal law.

$$y' + y = e^x \quad (10)$$

Integrating by setting limits after discretizing,

$$\int_{y_n}^{y_{n+1}} dy = \int_{x_n}^{x_{n+1}} -y dx + \int_{x_n}^{x_{n+1}} e^x dx \quad (11)$$

We can solve the two integrals on the *R.H.S* by using trapezoid method

$$\int_{x_n}^{x_{n+1}} y dx = h \left[\frac{y(x_{n+1}) + y(x_n)}{2} \right] \quad (12)$$

$$\int_{x_n}^{x_{n+1}} e^x dx = h \left[\frac{e^{x_{n+1}} + e^{x_n}}{2} \right] \quad (13)$$

Substituting into equation (11)

$$y_{n+1} - y_n = -h \left(\frac{y_{n+1} + y_n}{2} \right) + h \left(\frac{e^{x_{n+1}} + e^{x_n}}{2} \right) \quad (14)$$

$$y_{n+1} (2 + h) = y_n (2 - h) + h e^{x_n} \left(\frac{1 + e^h}{2} \right) \quad (15)$$

The general difference equation comes out to be,

$$y_{n+1} = y_n \left(\frac{2 - h}{2 + h} \right) + e^{x_n} h \left(\frac{1 + e^h}{2(2 + h)} \right) \quad (16)$$

Below is a comparison between Simulated Plot and Theoretical Plot.

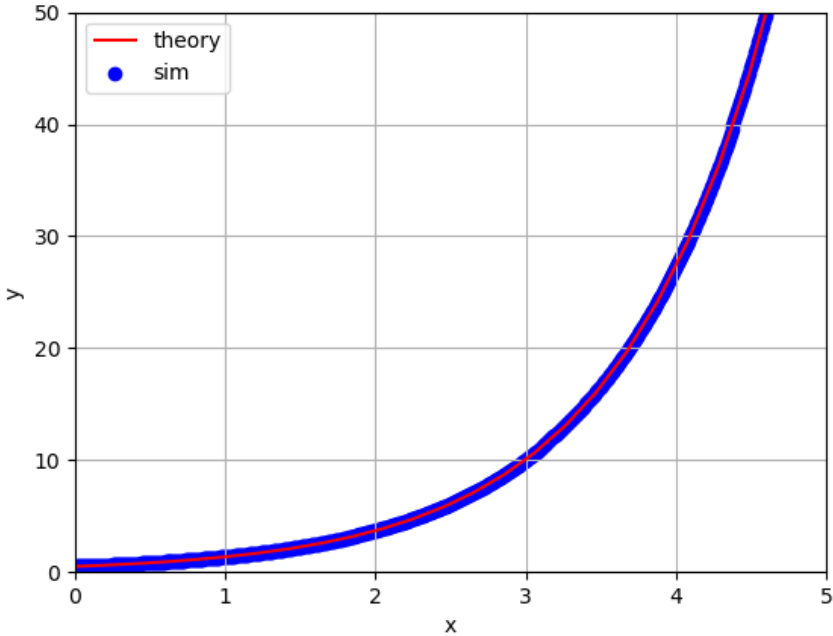


Fig. 1: Computational vs Theoretical solution of $\frac{dy}{dx} = -\frac{y^2+y+1}{x^2+x+1}$