

# 9.7.7

EE24BTECH11005 - Arjun Pavanje

**Question:** Solve the differential equation  $\frac{dy}{dx} + \frac{y^2+y+1}{x^2+x+1} = 0$ , with initial conditions  $y(0) = 0$

**Solution:**

**Theoretical Solution:**

This is a linear differential equation of the first order.

$$\frac{dy}{dx} = -\frac{y^2 + y + 1}{x^2 + x + 1} \quad (1)$$

$$\frac{dy}{y^2 + y + 1} = -\frac{dx}{x^2 + x + 1} \quad (2)$$

Integrating on both sides,

$$\int \frac{dy}{y^2 + y + 1} = - \int \frac{dx}{x^2 + x + 1} \quad (3)$$

Completing the square,

$$\int \frac{dy}{\left(y + \frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = - \int \frac{dx}{\left(x + \frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} \quad (4)$$

We know that  $\int \frac{dx}{x^2+a^2} = \tan^{-1}\left(\frac{x}{a}\right)$

$$\tan^{-1}\left(\frac{2y+1}{\sqrt{3}}\right) = -\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) + c \quad (5)$$

$$\tan^{-1}\left(\frac{2y+1}{\sqrt{3}}\right) + \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) = c \quad (6)$$

We know  $\tan^{-1} x + \tan^{-1} y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$

$$\tan^{-1}\left(\frac{2x+1+2y+1}{2-4xy-2x-2y}\right) = c \quad (7)$$

On simplifying we get,

$$(x+y+1) = A(1-x-y-2xy) \quad (8)$$

Where A is a constant. On substituting initial conditions we get,

$$(x+y+1) = (1-x-y-2xy) \quad (9)$$

$$y = \frac{-x}{1+x} \quad (10)$$

### Computational Solution:

By first principle of derivatives,

$$y'(t) = \lim_{h \rightarrow 0} \frac{y(t+h) - y(t)}{h} \quad (11)$$

$$y(t+h) = y(t) + hy'(t) \quad (12)$$

If we repeat the above process iteratively, we obtain the points to plot. Taking smaller step-size  $h$  will give more accurate plots. On discretizing the process we get,

$$y(x_{n+1}) = y(x_n) + hy'(x_n) \quad (13)$$

$$x_{n+1} = x_n + h \quad (14)$$

If we denote  $y(x_n)$  as  $y_n$ , the equation (14) becomes,

$$y_{n+1} = y_n + hy'_n \quad (15)$$

The above equation is the general difference equation.

In the given question,

$$y' = -\frac{y^2 + y + 1}{x^2 + x + 1} \quad (16)$$

Difference Equation can be written as,

$$y_{n+1} = y_n - h \left( \frac{y_n^2 + y_n + 1}{x_n^2 + x_n + 1} \right) \quad (17)$$

Below is a comparison between Simulated Plot and Theoretical Plot.

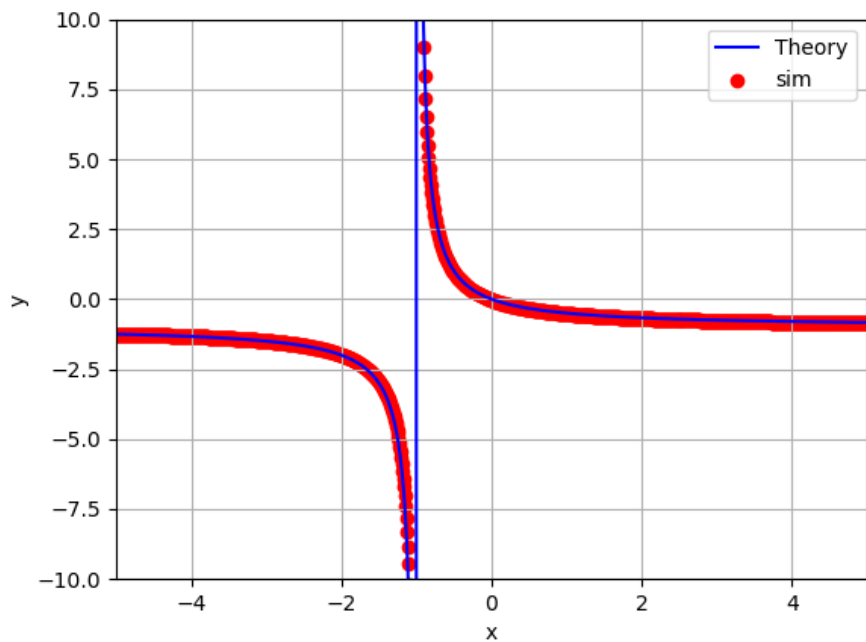


Fig. 1: Computational vs Theoretical solution of  $\frac{dy}{dx} = -\frac{y^2+y+1}{x^2+x+1}$