9.1.8

EE24BTECH11005 - Arjun Pavanje

Question: Solve the differential equation $y' + y = e^x$ with initial conditions $y(0) = \frac{1}{2}$
Solution:
Theoretical Solution:
This is a linear differential equation of the first order.

$$\frac{dy}{dx} + y = e^x \tag{1}$$
$$\tag{2}$$

Finding integrating factor

$$e^{\int 1 dx} \tag{3}$$
$$= e^x \tag{4}$$

Multiplying both sides of (2) with integrating factor,

$$\frac{dy}{dx}(e^x) + ye^x = e^{2x} \tag{5}$$
$$\frac{d(ye^x)}{dx} = e^{2x} \tag{6}$$
$$ye^x = \frac{e^{2x}}{2} + c \tag{7}$$
$$y = \frac{e^x}{2} + ce^{-x} \tag{8}$$

On substituting initial conditions we get,

$$y = \frac{e^x}{2} \tag{9}$$

Computational Solution:
Aim is to find a computational solution to the differential equation using trapezoidal law.

$$y' + y = e^x \tag{10}$$

Integrating by setting limits after discretizing,

$$\int_{y_n}^{y_{n+1}} dy = \int_{x_n}^{x_{n+1}} -y dx + \int_{x_n}^{x_{n+1}} e^x dx \tag{11}$$

We can solve the two integrals on the $R.H.S$ by using trapezoid method

$$\int_{x_n}^{x_{n+1}} y dx = h\left[\frac{y(x_{n+1}) + y(x_n)}{2}\right] \tag{12}$$

$$\int_{x_n}^{x_{n+1}} e^x dx = h\left[\frac{e^{x_{n+1}} + e^{x_n}}{2}\right] \tag{13}$$

Here, h is the step-size. Using smaller values of h will give a more accurate plot. Substituting into equation (11) we get,

$$y_{n+1} - y_n = -h\left(\frac{y_{n+1} + y_n}{2}\right) + h\left(\frac{e^{x_{n+1}} + e^{x_n}}{2}\right) \tag{14}$$

$$y_{n+1}(2+h) = y_n(2-h) + he^{x_n}\left(1+e^h\right) \tag{15}$$

The general difference equation comes out to be,

$$y_{n+1} = y_n\left(\frac{2-h}{2+h}\right) + e^{x_n}h\left(\frac{1+e^h}{2+h}\right) \tag{16}$$

$$x_{n+1} = x_n + h \tag{17}$$

Below is a comparission between Simulated Plot and Theoretical Plot.

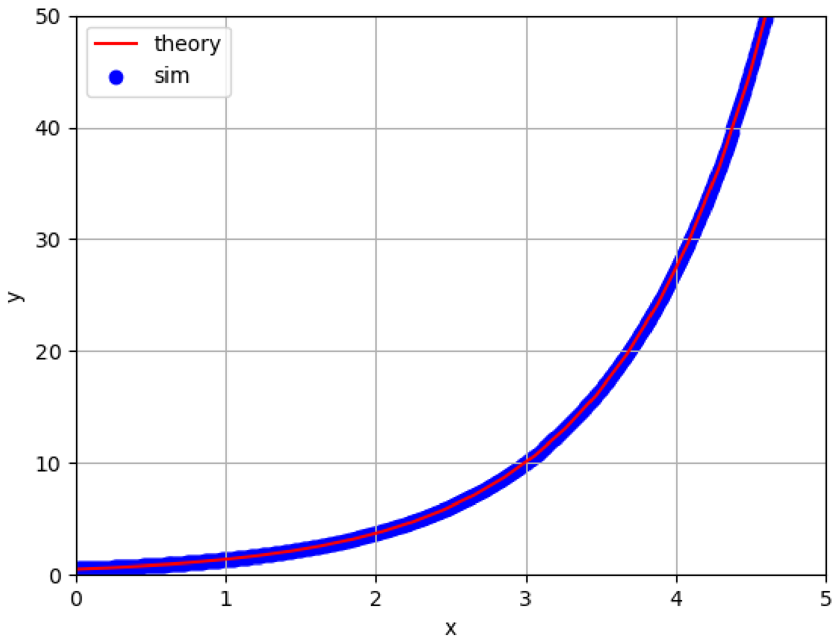

Fig. 1: Computational vs Theoretical solution of $\frac{dy}{dx} = -\frac{y^2+y+1}{x^2+x+1}$