Group quiz

EE24BTECH11038 - M.B.S Aravind

Forward Euler method:- Given an ODE and an initial value $y(x_0) = y_0$, Euler's method yield's approximate solution values at equidistant x-values $x_0, x_1 = x_0 + h, x_2 = x_0 + 2h, \ldots$ namely

$$y_1 = y_0 + hf(x_0, y_0) \tag{0.1}$$

$$y_2 = y_1 + hf(x_1, y_1) \tag{0.2}$$

In general

$$y_n = y_{n-1} + hf(x_{n-1}, y_{n-1}) \tag{0.3}$$

where the step h equals, e.g., 0.1 or 0.2 or a smaller value for greater accuracy

Analysis of parameter variations

Given differential equation is

$$\frac{di}{dt} = \frac{v(t) - Ri}{L} \tag{0.4}$$

$$i_{n+1} = i_n + h\frac{v(t) - Ri}{L} \tag{0.5}$$

- **Varying R**
  - Let the values of L,T,α be 1H, 1 sec, 0.5 respectively.
  - The plot of i vs. time (for different values of R is given by)

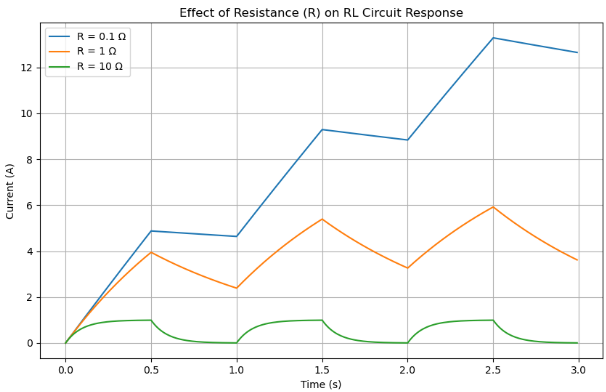

Fig. 0.1: Caption

- **R=0.1Ω**
    1) The current reaches a high peak value as a result of minimal resistance restricting the flow.
    2) The decay phase is slow because the inductor dominates, storing energy and releasing it gradually.
- **R=1Ω**
    1) Slower current buildup than in the low resistance case.
    2) The peak current is lower, as more energy is dissipated in the resistor.
- **R=10Ω**
    1) The peak current is significantly lower because the resistor limits current flow.
    2) The response stabilizes much faster, as energy is quickly dissipated.

Varying L

- Let the values of R,T,α be 1Ω, 1 sec, 0.5 respectively.
- **L=0.1H**
    1) Faster current response.
    2) Smaller time constant (τ = L/R), leading to a quicker rise and fall.
- **L=1H**
    1) Balanced response between resistive and inductive behavior.
    2) Current change is neither too abrupt nor too slow.
- **L=10H**
    1) Smoother transitions as the inductor resists rapid current changes.
    2) Larger time constant, leading to slow response and gradual decay.
    3) The inductor stores more energy, resulting in a lagged response.
- The plot of i vs. time (for different values of L is given by)

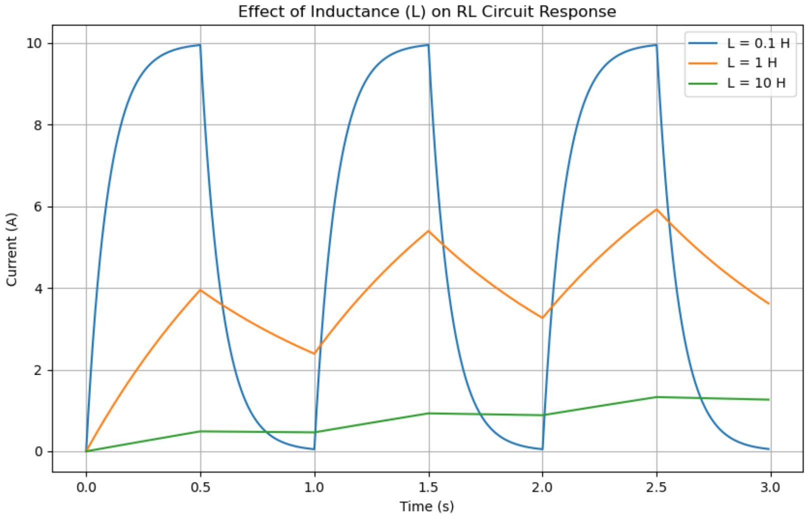

Fig. 3.1: Caption

Error analysis

The error in the forward Euler method can be analyzed using a Taylor series expansion. Given a general differential equation:

$$\frac{dy}{dt} = f(t, y) \tag{3.1}$$

with an exact solution y(t), we expand it around t_n using the Taylor series:

$$y(t_{n+1}) = y(t_n) + h\frac{dy}{dt} + h^2\frac{dy}{dt} + O(h^3) \tag{3.2}$$

Substituting the differential equation $\frac{dy}{dt} = f(t, y)$ and using the Euler approximation:

$$y_{n+1} = y_n + hf(t_n, y_n) \tag{3.3}$$

The local truncation error is given by the omitted higher-order terms:

$$E_n = \frac{h^2}{2}\frac{d^2y}{dt^2} + O(h^3) \tag{3.4}$$

Plotting the response using the Euler method for different values of h where R,L,T,α are 1Ω,1H,1 sec, 0.5 respectively

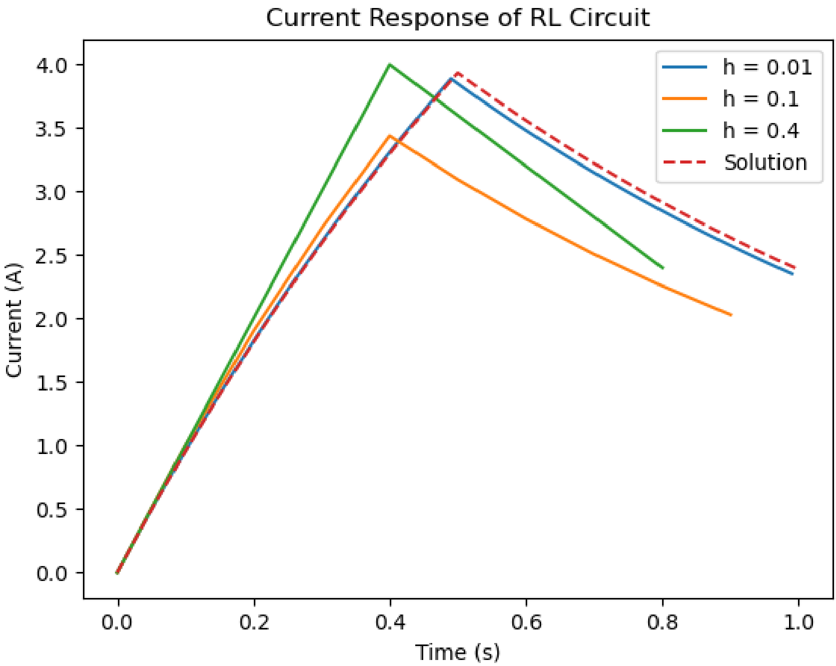

Fig. 3.2: Caption

h value	Deviation in 1st step	Deviation in 2nd step
0.01	0.000498	0.000986
0.1	0.048	0.087
0.4	0.7032	-0.5149

TABLE 3: Caption

There is a noticeable deviation for every value of h, and as h increases, the deviation also increases. This indicates that the Euler method is not stable for larger values of h