Group quiz

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Fourier series:- The Fourier Series is a way to represent any periodic function as a sum of sines and cosines.
For a function f(x) that is periodic with period T, its Fourier Series is given by:

$$f(x) = a_0 + \sum_{n=1}^{n=\infty} \left[a_n cos\left(\frac{2\pi nx}{T}\right) + b_n sin\left(\frac{2\pi nx}{T}\right) \right] \tag{0.1}$$

$$\text{or } f(x) = \sum_{n=-\infty}^{n=\infty} C_n e^{j\omega_0 t} \tag{0.2}$$

a_0 is the DC component
a_n,b_n are the fourier coefficients given by

$$a_0 = \frac{1}{T} \int_T f(x)dx \tag{0.3}$$

$$a_n = \frac{2}{T} \int_T f(x)cos\left(\frac{2\pi nx}{T}\right) dx \tag{0.4}$$

$$b_n = \frac{2}{T} \int_T f(x)sin\left(\frac{2\pi nx}{T}\right) dx \tag{0.5}$$

Calculating the coefficients
Given voltage function is

$$V(t) = \begin{cases} 10, & 0 < t < \alpha T \\ 0, & \alpha T < t < T \end{cases}$$

1.DC coefficient a_0

$$a_0 = \frac{1}{T}\left[\int_0^{\alpha T} 10\, dt + \int_{\alpha T}^{T} 0\, dt\right] \tag{0.6}$$

$$a_0 = 10\alpha \tag{0.7}$$

2.Cosine coefficient a_n

$$a_n = \frac{2}{T}\int_0^{\alpha T} 10\cos\left(\frac{2\pi nt}{T}\right) dt \tag{0.8}$$

$$a_n = \frac{20}{T} \cdot \frac{T}{2\pi n} \left[\sin(2\pi n\alpha) - \sin(0)\right] \tag{0.9}$$

$$a_n = \frac{10}{\pi n} \sin(2\pi n\alpha) \tag{0.10}$$

3.sine function b_n

$$b_n = \frac{2}{T}\int_0^{\alpha T} 10\sin\left(\frac{2\pi nt}{T}\right)dt \tag{0.11}$$

$$b_n = \frac{2\cdot 10}{T}\left[\frac{-T}{2\pi n}\cos\left(\frac{2\pi nt}{T}\right)\right]_0^{\alpha T} \tag{0.12}$$

$$b_n = \frac{-20}{2\pi n}\left[\cos(2\pi n\alpha) - \cos(0)\right] \tag{0.13}$$

$$b_n = \frac{-10}{\pi n}\left[\cos(2\pi n\alpha) - 1\right] \tag{0.14}$$

$$b_n = \frac{10}{\pi n}\left[1 - \cos(2\pi n\alpha)\right] \tag{0.15}$$

Thus the fourier series for V(t) is given by

$$V(t) = 10\alpha + \sum_{n=1}^{\infty}\left(\frac{10}{\pi n}\sin(2\pi n\alpha)\cos\left(\frac{2\pi nt}{T}\right) + \frac{10}{\pi n}(1-\cos(2\pi n\alpha))\sin\left(\frac{2\pi nt}{T}\right)\right) \tag{0.16}$$

Calculating I(t)
Given differential equation is

$$L\frac{di}{dt} + iR = 10\alpha + \sum_{n=1}^{\infty}\left(\frac{10}{\pi n}\sin(2\pi n\alpha)\cos\left(\frac{2\pi nt}{T}\right) + \frac{10}{\pi n}(1-\cos(2\pi n\alpha))\sin\left(\frac{2\pi nt}{T}\right)\right) \tag{0.17}$$

Since the system is linear, we solve for each harmonic component separately:
Assume a solution of the form:

$$I_n = A_n\cos(n\omega_0 t) + B_n\sin(n\omega_0 t). \tag{0.18}$$

Substituting into the differential equation and solving for and , we obtain:

$$A_n = \frac{b_n R}{R^2 + (n\omega_0 L)^2}, \quad B_n = \frac{b_n n\omega_0 L}{R^2 + (n\omega_0 L)^2}. \tag{0.19}$$

Thus, the total current response is given by:

$$I(t) = \frac{10\alpha}{R}\left(1 - e^{-\frac{R}{L}t}\right) + \sum_{n=1}^{\infty}\frac{10}{n\pi}\sin(2\pi\alpha n)\left[\frac{R\cos(n\omega_0 t) + n\omega_0 L\sin(n\omega_0 t)}{R^2 + L^2(n\omega_0)^2} - \frac{R}{R^2 + L^2(n\omega_0)^2}e^{-\frac{R}{L}t}\right] \tag{0.20}$$

$$+ \sum_{n=1}^{\infty}\frac{10}{n\pi}(1 - \cos(2\pi\alpha n))\left[\frac{R\sin(n\omega_0 t) - Ln\omega_0\cos(n\omega_0 t)}{R^2 + L^2(n\omega_0)^2} + \frac{n\omega_0 L}{R^2 + L^2(n\omega_0)^2}e^{-\frac{R}{L}t}\right] \tag{0.21}$$

Effect of time constant

1) τ << **T**
   a) since $e^{-\frac{R}{L}t}$ approaches zero quickly, the transient terms in the Fourier series expression disappear.

b) After simplifying the terms A_n and B_n by assuming $\frac{L}{R} \to 0$, the final i(t) will be

$$\approx \frac{10\alpha}{R} + \frac{10}{\alpha R} \sum_{n=1}^{\infty} \left(\frac{\sin\left((2\pi\alpha - \omega_0 t)n\right)}{n} \right) \tag{1.1}$$

2) τ **= T**

a) For τ=T the transient and steady-state components coexist with significant contributions. The full Fourier expression is used without approximation

3) τ >>**T**

a) Assuming $\frac{R}{L} \to 0$

b) I(t) simplifies to

$$\approx \frac{10}{\pi\omega_0 L} \sum_{n=1}^{\infty} \frac{\cos((2\pi\alpha - \omega_0 t)n)}{n^2} \tag{3.1}$$