NCERT Presentation

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Problem Statement

Of all the closed right circular cylindrical cans of given volume $100cm^3$, find the dimensions of the can which has minimum surface area

Solution

Solution

Surface Area of cylinder is given by,

$$2\pi rh + 2\pi r^2 \tag{3.1}$$

Volume of a cylinder is given by,

$$\pi r^2 h \tag{3.2}$$

where r is the radius of the cylinder, h is the height of the cylinder. Given that volume is $100cm^3$,

$$\pi r^2 h = 100 \tag{3.3}$$

$$h = \frac{100}{\pi r^2} \tag{3.4}$$

Equation (3.1) becomes,

$$\left(\frac{200}{r} + 2\pi r^2\right) \tag{3.5}$$

Theoretical Solution

To minimize surface area, differentiate equation (1) and set it to zero,

$$\frac{4\pi r^3 - 200}{r^2} = 0 \tag{3.6}$$

value of r at which satisfies is, $\left(\frac{50}{\pi}\right)^{\frac{1}{3}}$. We can verify that this is a minima by differentiating equation (6),

$$\frac{400}{r^3} + 4\pi \tag{3.7}$$

Thus we see that at the above value of r, it is a minima.
Can of given volume will have maximum surface area when radius is $\left(\frac{50}{\pi}\right)^{\frac{1}{3}}$ cm, height is $2\left(\frac{\pi}{50}\right)^{\frac{1}{3}}$

Computational solution 1

We need to minimize,

$$\left(\frac{200}{r} + 2\pi r^2\right) \tag{3.8}$$

Applying gradient descent theorem,

$$r_{n+1} = r_n - \mu f'(r_n) \tag{3.9}$$

where μ is the step size,

$$f'(r_n) = -\frac{200}{r_n^2} + 4\pi r_n \tag{3.10}$$

Final Difference Equation comes out to be,

$$r_{n+1} = r_n (1 - 4\pi) + \frac{200}{r_n^2} \tag{3.11}$$

Taking inital guess as 2, step size 0.01, tolerence as 0.0001.
We get minimum value of r to be $2.515397787094116cm \approx \left(\frac{50}{\pi}\right)^{\frac{1}{3}} cm$

Computational Solution 2

We can also solve it using *cvxpy* module in python. On running the code we get,
Minimum value of r is, 2.515299390016942cm,
Minimum surface area is, 119.26542080485049cm^2

Computational Solution