# 9.1.8

EE24BTECH11005 - Arjun Pavanje

**Question:** Of all the closed right circular cylindrical cans of given volume $100cm^3$, find the dimensions of the can which has minimum surface area
**Solution:**
Surface Area of cylinder is given by,

$$2\pi rh + 2\pi r^2 \tag{1}$$

where $r$ is the radius of the cylinder, $h$ is the height of the cylinder.
Volume of a cylinder is given by,

$$\pi r^2 h \tag{2}$$

where $r$ is the radius of the cylinder, $h$ is the height of the cylinder.
Given that volume is $100cm^3$,

$$\pi r^2 h = 100 \tag{3}$$

$$h = \frac{100}{\pi r^2} \tag{4}$$

Equation (1) becomes,

$$\left(\frac{200}{r} + 2\pi r^2\right) \tag{5}$$

**Theoretical Solution**
To minimize surface area, differentiate equation (1) and set it to zero,

$$\frac{4\pi r^3 - 200}{r^2} = 0 \tag{6}$$

value of $r$ at which satisfies is, $\left(\frac{50}{\pi}\right)^{\frac{1}{3}}$. We can verify that this is a minima by differentiating equation (6),

$$\frac{400}{r^3} + 4\pi \tag{7}$$

Thus we see that at the above value of $r$, it is a minima.
Can of given volume will have maximum surface area when radius is $\left(\frac{50}{\pi}\right)^{\frac{1}{3}} cm$, height is $2\left(\frac{\pi}{50}\right)^{\frac{1}{3}}$
**Computational Solution:**
We need to minimize,

$$\left(\frac{200}{r} + 2\pi r^2\right) \tag{8}$$

Applying gradient descent theorem,

$$r_{n+1} = r_n - \mu f'(r_n) \tag{9}$$
$$\tag{10}$$

where $\mu$ is the step size,

$$f'(r_n) = -\frac{200}{r_n^2} + 4\pi r_n \tag{11}$$

Final Difference Equation comes out to be,

$$r_{n+1} = r_n(1 - 4\pi) + \frac{200}{r_n^2} \tag{12}$$

Taking inital guess as 2, step size 0.01, tolerence as 0.0001. We get minimum value of $r$ to be $2.515397787094116cm \approx \left(\frac{50}{\pi}\right)^{\frac{1}{3}} cm$

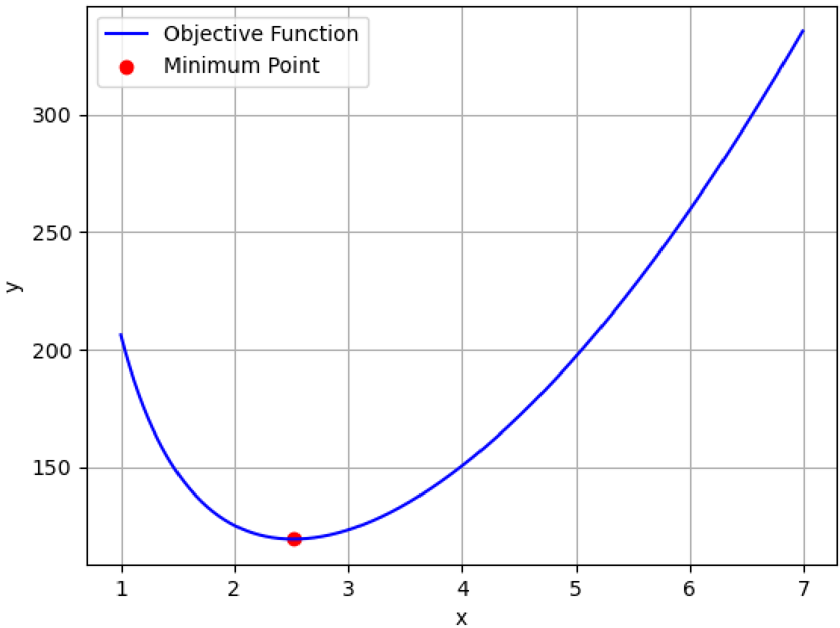


Fig. 1: Minimizing $\left(\frac{200}{r} + 2\pi r^2\right)$. Surface Area function with point of minima