# NCERT Presentation

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# Problem

# Problem Statement

Solve the following system of equations,

$$2x + y - 6 = 0 \tag{2.1}$$

$$4x - 2y - 4 = 0 \tag{2.2}$$

# Solution

## Solution

Representing using matrices,

$$\begin{pmatrix} 2 & 1 \\ 2 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 6 \\ 2 \end{pmatrix} \tag{3.1}$$

We shall solve this system of equations by LU Decomposition. Any non-sigular matrix can be represented as a product of a lower triangular matrix $L$ and an upper triangular matrix $U$

$$A\mathbf{x} = LU\mathbf{x} = \mathbf{b} \tag{3.2}$$

# Row Reduction

Applying row reduction on $A$,

$$\begin{pmatrix} 2 & 1 \\ 2 & -1 \end{pmatrix} \xrightarrow{R_2 = R_2 - R_1} \begin{pmatrix} 2 & 1 \\ 0 & -2 \end{pmatrix} \tag{3.3}$$

Let

$$L = \begin{pmatrix} 1 & 0 \\ l_{21} & 1 \end{pmatrix} \tag{3.4}$$

$l_{21}$ is the multiplier used to zero $a_{21}$, so $l_{21} = 1$.
Now,

$$A = \begin{pmatrix} 2 & 1 \\ 2 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ 0 & -2 \end{pmatrix} \tag{3.5}$$

We have thus obtained LU Decomposition of the matrix $A$.

# Doolittle's Algorithm

The LU Decomposition of matrix $A$ can also be obtained by Doolittle's Algorithm. This gives us update equations to construct the $L$ and $U$ matrix.

Elements of $U$ matrix:

For each column $j$,

$$U_{ij} = \begin{cases} A_{ij} & i = 0 \\ A_{ij} - \sum_{k=0}^{i-1} L_{ik} U_{kj} & i > 0 \end{cases} \tag{3.6}$$

Elements of $L$ matrix:

For each row $i$,

$$L_{ij} = \begin{cases} \frac{A_{ij}}{U_{ij}} & j = 0 \\ \frac{A_{ij} - \sum_{k=0}^{j-1} L_{ik} U_{kj}}{U_{ij}} & j > 0 \end{cases} \tag{3.7}$$

# Backsubstitution

The above proccess decomposes any non-sigular matrix $A$ into an upper-triangular matrix $U$ and a lower-triangular matrix $L$. Now, let

$$U\mathbf{x} = \mathbf{y} \tag{3.8}$$

$$L\mathbf{y} = \mathbf{b} \tag{3.9}$$

Substituting $L$ in equation (9),

$$\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 6 \\ 2 \end{pmatrix} \tag{3.10}$$

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 6 \\ -4 \end{pmatrix} \tag{3.11}$$

# Forward substitution

Backsubstituting into equation (8),

$$\begin{pmatrix} 2 & 1 \\ 0 & -2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 6 \\ -4 \end{pmatrix} \tag{3.12}$$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \end{pmatrix} \tag{3.13}$$

# Graph