Assignment 1

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1.5 Determine the base of the numbers in each case for the following operations to be correct:

(a) $\frac{67}{5} = 11$

(b) $15 \times 3 = 51$

(c) $123 + 120 = 303$

Solution:

(a) We can rewrite the equation as $67 = 11 \times 5$. Say the base of the numbers is b. Converting everything to base-10 we can write,

$$67 = 6 \times b^1 + 7 \times b^0$$
$$11 = 1 \times b^1 + 1 \times b^0$$
$$5 = 5 \times b^0$$

Above equation becomes,

$$6b + 7 = (b + 1)5$$

We get $b = -2$. Which is not possible. However, if we consider that the number 11 is in base-10, we get,

$$6b + 7 = (11 \times 5) = 55$$

We get $b = 8$, i.e. Octal
Base of numbers is 8

(b) Say the base of the numbers is b. Converting everything to base-10 we can write,

$$\begin{aligned} 15 &= 1 \times b^1 + 5 \times b^0 \\ 51 &= 5 \times b^1 + 1 \times b^0 \\ 3 &= 3 \times b^0 \end{aligned}$$

Above equation becomes,

$$(b+5)3 = 5b + 1$$

We get $b = 7$
Base of numbers is 7

(c) Say the base of the numbers is b. Converting everything to base-10 can write,

$$\begin{aligned} 123 &= 1 \times b^2 + 2 \times b^1 + 3 \times b^0 \\ 120 &= 1 \times b^2 + 2 \times b^1 + 0 \times b^0 \\ 303 &= 3 \times b^2 + 0 \times b^1 + 3 \times b^0 \end{aligned}$$

Above equation becomes,

$$\begin{aligned} (b^2 + 2b + 3) + (b^2 + 2b) &= 3b^2 + 3 \\ b^2 - 4b = (b-4)(b) &= 0 \end{aligned}$$

We get $b = 4, 0$, but since $b = 0$ has no meaning, $b = 4$
Base of numbers is 4

1.6 The solutions to the quadratic equation $x^2 - 13x + 22 = 0$ are $x = 7$ and $x = 2$. What is the base of the numbers?

Solution

Let the numbers be in base b then, the equation in decimal system becomes,

$$\begin{aligned} &13 = 1 \times b^1 + 3 \times b^0 \\ &22 = 2 \times b^1 + 2 \times b^0 \\ &x^2 - (b+3)x + (2b+2) = 0 \end{aligned}$$

As 7_{10} and 2_{10} are the solutions

$$\begin{aligned} 7^2 - 7(b+3) + (2b+2) = 0 \\ 2^2 - 2(b+3) + (2b+2) = 0 \end{aligned}$$

From the two equations we get,

$$49 - 21 + 2 = 5b$$
$$4 - 6 + 2 = (2 - 2)b$$

The value of k can be anything for $x = 2$ to be a solution and $k = 6$ for $x = 7$ to be the solution.
Base of numbers is 6 for $x = 7$ to be a solution, and can be anything for $x = 2$ to be a solution.

1.31 How many printing characters are there in ASCII? How many of them are special characters (not letters or numerals)?
Solution:
There are 95 printing characters in ASCII out of which 33 are special characters (not letters or numerals).

1.32 What bit must be complemented to change an ASCII letter from capital to lowercase and vice versa?
Solution:
The ASCII values for uppercase and lowercase English letters are,

- Uppercase letters have ASCII values from 65 to 90.
- Lowercase letters have ASCII values from 97 to 122.

→ In binary form, the ASCII values are represented using 7 bits. We notice that the difference in ASCII values of an alphabet in its Lowercase and Uppercase form is 32.

→ So if we subtract the ASCII form of any Lowercase alphabet by 32 (or $(100000)_2$ in binary) we get its Uppercase form. To convert from Uppercase to Lowercase, just add 32(or $(100000)_2$ in binary) to the ASCII value of the Uppercase alphabet.

→ We notice that this turns out to be just flipping (complementing) the 6^{th} bit.

We shall verify this with an example,

- 'A' is represented as 1000001 in ASCII binary.
- 'a' is represented as 1100001 in ASCII binary.

We can notice that they only differ in their 6^{th} bit.
To convert an ASCII alphabet from upper to lowercase or vice-versa, just complement (flip) the 6^{th} bit.

1.14 Obtain the 1's and 2's complements of the following binary numbers:

(a) 11100010

(b) 00011000

(c) 10111101

(d) 10100101

(e) 11000011

(f) 01011000

Solution:

→ x's complement of a number N in base r having n digits is defined as $(r^n - x) - N$

→ To take 1's complement of a binary number just subtract the binary number of n digits from the binary equivalent of 2^n. This comes out to be, just replacing 1's with 0's, and the 0's with 1's in the original number.

→ 2's complement of a binary number is just 1 added to its 1's complement

(a) 11100010

→ 1's complement: 00011101

❇ Adding 1:

$$\begin{array}{r} 1 \\ 00011101 \\ +\quad 1 \\ \hline 00011110 \end{array}$$

→ 2's complement: 00011110

(b) 00011000

→ 1's complement: 11100111

❇ Adding 1:

$$\begin{array}{r} 111 \\ 11100111 \\ +\quad 1 \\ \hline 11101000 \end{array}$$

→ 2's complement: 11101000

(c) 10111101

→ 1's complement: 01000010

❇ Adding 1:

$$\begin{array}{r} 01000010 \\ +\quad 1 \\ \hline 01000011 \end{array}$$

→ 2's complement: 01000011

(d) 10100101

→ 1's complement: 01011010

❇ Adding 1:

$$\begin{array}{r} 01011010 \\ +\quad 1 \\ \hline 01011011 \end{array}$$

→ 2's complement: 01011011

(e) 11000011

→ 1's complement: 00111100

❇ Adding 1:

$$\begin{array}{r} 00111100 \\ +\quad 1 \\ \hline 00111101 \end{array}$$

→ 2's complement: 00111101

(f) 01011000

→ 1's complement: 10100111

❇ Adding 1:

$$\begin{array}{r} {\scriptstyle 111} \\ 10100111 \\ +\quad 1 \\ \hline 10101000 \end{array}$$

→ 2's complement: 10101000