

2021 March 18 Shift 1

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EE24BTECH11005 - Arjun Pavanje

16) If $2x^y + 3y^x = 20$ then $\frac{dy}{dx}$ at $\left(\frac{2}{2}\right)$ is equal to,

a) $-\left(\frac{3+\log_e 8}{2+\log_e 4}\right)$

b) $-\left(\frac{2+\log_e 8}{3+\log_e 4}\right)$

c) $-\left(\frac{3+\log_e 4}{2+\log_e 8}\right)$

d) $-\left(\frac{3+\log_e 16}{4+\log_e 8}\right)$

17) If the system of equations

$$x + y + az = b$$

$$2x + 5y + 2z = 6$$

$$x + 2y + 3z = 3$$

has infinitely many solutions, then $2a + 3b$ is equal to,

a) 28

b) 20

c) 25

d) 23

18) Let $\left(1 + x + 2x^2\right)^{20} = a_0 + a_1x + a_2x^2 + \cdots + a_{40}x^{40}$. Then, $a_1 + a_3 + a_5 + \cdots + a_{37}$ is equal to,

a) $2^{20}(2^{20} + 21)$

b) $2^{19}(2^{20} + 21)$

c) $2^{20}(2^{20} - 21)$

d) $2^{19}(2^{20} - 21)$

19) Let $5f(x) + 4f\left(\frac{1}{x}\right) = \frac{1}{x} + 3, x > 0$, then $\int_1^2 f(x) dx$ is equal to,

a) $10 \log_e 2 - 6$

b) $10 \log_e 2 + 6$

c) $5 \log_e -3$

d) $5 \log_e 2 + 3$

20) The mean and variance of a set of 15 numbers are 12 and 14 respectively. The mean and variance of another set of 15 numbers are 14 and σ^2 respectively. If the variance of all the 30 numbers in the two sets is 13, then σ^2 is equal to,

a) 12

b) 10

c) 11

d) 9

- 21) Let the tangents to the curve $x^2 + 2x - 4y + 9 = 0$ at the point $\mathbf{P}\left(\frac{1}{3}\right)$ on it meet the y-axis at \mathbf{A} . Let the line passing through \mathbf{P} and parallel to the line $x - 3y = 6$ meet the parabola $y^2 = 4x$ at \mathbf{B} . If \mathbf{B} lies on the line $2x - 3y = 8$, then $(AB)^2$ is equal to

- 22) Let the point $\left(\frac{p}{p+1}\right)$ lie inside the region

$$E = \{(x, y) : 3 - x \leq y \leq \sqrt{9 - x^2}, 0 \leq x \leq 3\}$$

If the set of all values of p in the interval $\left(\frac{a}{b}\right)$ then $b^2 + b - a^2$ is equal to _____

- 23) Let $y = y(x)$ be a solution of the differential equation

$$(x \cos x) dy + (xy \sin x + y \cos x - 1) dx = 0, 0 < x < \frac{\pi}{2}$$

If $\frac{\pi}{3}y\left(\frac{\pi}{3}\right) = \sqrt{3}$, then $\left|\frac{\pi}{6}y''\left(\frac{\pi}{6}\right) + 2y'\left(\frac{\pi}{6}\right)\right|$ _____

- 24) Let $a \in \mathbb{Z}$ and $[t]$ be the greatest integer $\leq t$. Then the number of points, where the function $f(x) = [a + 13 \sin x]$, $x \in (0, \pi)$ is not differentiable is _____

- 25) If the area of the region

$$S = \{(x, y) : 2y - y^2 \leq x^2 \leq 2y, x \geq y\}$$

is equal to $\left(\frac{n+2}{n+1} - \frac{\pi}{n-1}\right)$ then the natural number n is equal to _____

- 26) The number of ways of giving 20 distinct oranges to 3 children such that each child gets atleast one orange is _____

- 27) Let the image of the point $\mathbf{P}\left(\frac{1}{2}, \frac{3}{2}\right)$ in the plane $2x - y + z = 9$ be \mathbf{Q} . If the coordinates of

the point \mathbf{R} are $\left(\frac{6}{10}, \frac{7}{10}\right)$. Then the square of the area of triangle PQR is _____

- 28) Let A circle passing through the point $\mathbf{P}\left(\frac{\alpha}{\beta}\right)$ in the first quadrant touches the two coordinate axes at the points \mathbf{A}, \mathbf{B} . The point \mathbf{P} is above the line \mathbf{AB} . The point \mathbf{Q} on the line segment \mathbf{AB} is the foot of perpendicular from \mathbf{P} on \mathbf{AB} . If \mathbf{PQ} is equal to 11 units, then value of $\alpha\beta$ is _____

- 29) The coefficient of x^{18} in the expansion of $\left(x^4 - \frac{1}{x^3}\right)^{15}$ is _____

- 30) Let $A = \{1, 2, 3, 4, \dots, 10\}$, $B = \{0, 1, 2, 3, 4\}$. The number of elements in the relation $R = \{(a, b) \in A \times A : 2(a - b)^2 + 3(a - b) \in B\}$ is _____