Matgeo Presentation

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November 6, 2024

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Problem

Problem Statement

Find the area of the region bounded by the curves,

$$y^2 = 4ax \tag{2.1}$$

$$x^2 = 4ay \tag{2.2}$$

Solution

General Equation of Conic

General equation of conic with directrix $\mathbf{n}^\top \mathbf{x} = c$ is,

$$g(\mathbf{x}) = \mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{3.1}$$

$$\mathbf{V} = \|\mathbf{n}\|^2 \mathbf{I} - e^2 \mathbf{n}\mathbf{n}^\top \tag{3.2}$$

$$\mathbf{u} = ce^2\mathbf{n} - \|\mathbf{n}\|^2 \mathbf{F} \tag{3.3}$$

$$f = \|\mathbf{n}\|^2 \|\mathbf{F}\|^2 - c^2 e^2 \tag{3.4}$$

Where,

$\mathbf{V}$ = A symmteric matrix obtained by eigen value decomposition

$\mathbf{F}$ = Focus of conic

$\mathbf{e}$ = eccentricity of conic

$\mathbf{n}$ = normal vector of directrix

Equation of given conics in Matrix form

$y^2 = 4ax$ can be represented in Matrix form as,

$$\begin{pmatrix} x & y \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + 2 \begin{pmatrix} -2a & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + 0 = 0 \tag{3.5}$$

On comparing it with the general equation of a conic given in the previous slide we get,

$$\mathbf{V_1} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \mathbf{u_1} = \begin{pmatrix} -2a \\ 0 \end{pmatrix}, f_1 = 0; \tag{3.6}$$

$x^2 = 4ay$ can be represented in Matrix form as,

$$\begin{pmatrix} x & y \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + 2 \begin{pmatrix} 0 & -2a \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + 0 = 0 \tag{3.7}$$

Equation of given conics in Matrix form

On comparing it with the general equation of a conic given in the previous slide we get,

$$\mathbf{V_2} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \mathbf{u_2} = \begin{pmatrix} 0 \\ -2a \end{pmatrix}, f_2 = 0; \tag{3.8}$$

Points of Intersection

The intersection of two conics with parameters $\mathbf{V}_i, \mathbf{u}_i, f_i, i = 1, 2$ is defined as,

$$\mathbf{x}^\top (\mathbf{V_1} - \mathbf{V}_2)\, \mathbf{x} + 2\,(\mathbf{u_1} - \mathbf{u_2})^\top \mathbf{x} + (f_1 - f_2) = 0 \tag{3.9}$$

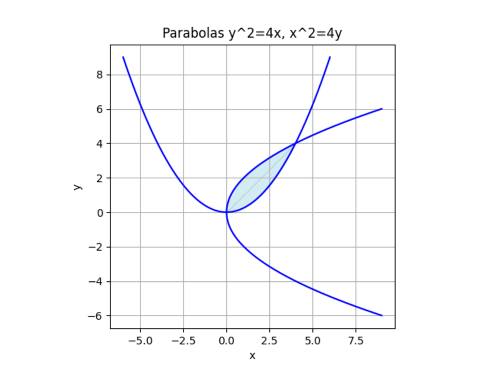
On solving we get the points of intersection to be $\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 4 \\ 4 \end{pmatrix}$

Area Required

Area between the 2 parabolas is,

$$\int_0^4 2\sqrt{x}dx - \int_0^4 \frac{x^2}{4}dx = \frac{16}{3} \tag{3.10}$$

The area between the curves $y^2 = 4x, x^2 = 4y$ is, $\frac{16}{3}$ units



Codes

C Code:

```
void yparabola_gen(FILE *fptr, double a, double num_points, double **vertex){
        for(int i=num_points; i>=0; i--){
                double t = 3*i/num_points;
                double **output=createMat(2,1);
                output[1][0]=vertex[0][0]+a*t*t;
                output[0][0]=vertex[1][0]+2*a*t;
                fprintf(fptr,"%lf,%lf\n",output[0][0],output[1][0]);
                freeMat(output,2);
        }
        for(int i=0; i<=num_points; i++){
                double t = -3*i/num_points;
                double **output=createMat(2,1);
                output[1][0]=a*t*t;
                output[0][0]=2*a*t;
                fprintf(fptr,"%lf,%lf\n",output[0][0],output[1][0]);
                freeMat(output,2);
        }
}
```

Codes

```
void xparabola_gen(FILE *fptr, double a, double num_points, double **vertex){
        for(int i=num_points;i>=0;i--){
                double t = 3*i/num_points;
                double **output=createMat(2,1);
                output[0][0]=vertex[0][0]+a*t*t;
                output[1][0]=vertex[1][0]+2*a*t;
                fprintf(fptr,"%lf,%lf\n",output[0][0],output[1][0]);
                freeMat(output,2);
        }
        for(int i=0;i<=num_points;i++){
                double t = -3*i/num_points;
                double **output=createMat(2,1);
                output[0][0]=a*t*t;
                output[1][0]=2*a*t;
                fprintf(fptr,"%lf,%lf\n",output[0][0],output[1][0]);
                freeMat(output,2);
        }
}
```

Codes

```
int main() {
        double x1, y1;
        x1 = 0; y1 = 0;
        int m = 2, n = 1;
        double **vertex = createMat(m, n);
        vertex[0][0] = x1;
        vertex[1][0] = y1;
        double radius = 4;
        FILE *fptr;
        fptr = fopen("line_points.txt", "w");
        if (fptr == NULL) {
                printf("Error opening file!\n");
                return 1;
        }
        double a = 1;
        yparabola_gen(fptr, a, 1000,vertex);
        xparabola_gen(fptr, a, 1000,vertex);
        fclose(fptr);
        return 0;
        }
```

Codes

Python Code:

```
import numpy as np
import matplotlib.pyplot as plt
points = np.loadtxt("line_points.txt", delimiter=',', max_rows=len(list(open("./
    line_points.txt"))))
centre=np.array([points[0][0],points[0][1]])
x1 = points[:2002, 0]
y1 = points[:2002, 1]
x2 = points[-2002:,0]
y2 = points[-2002:,1]
plt.figure()
plt.plot(x1, y1, label='y^2=4x', color='blue')
plt.plot(x2, y2, label='x^2=4y', color='blue')
plt.fill_between(x2, y1, y2, where=(y2 >= y1), color='lightblue', alpha=0.5)
plt.fill_between(x1, y1, y2, where=(y2 >= y1), color='lightblue', alpha=0.5)
plt.gca().set_aspect('equal', adjustable='box')
plt.xlabel("x")
plt.ylabel("y")
plt.grid(True)
plt.show()
```