

# 2023 April 6 Shift 1

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EE24BTECH11005 - Arjun Pavanje

16) If  $2x^y + 3y^x = 20$  then  $\frac{dy}{dx}$  at  $\left(\frac{2}{2}\right)$  is equal to,

a)  $-\left(\frac{3+\log_e 8}{2+\log_e 4}\right)$

b)  $-\left(\frac{2+\log_e 8}{3+\log_e 4}\right)$

c)  $-\left(\frac{3+\log_e 4}{2+\log_e 8}\right)$

d)  $-\left(\frac{3+\log_e 16}{4+\log_e 8}\right)$

17) If the system of equations

$$x + y + az = b$$

$$2x + 5y + 2z = 6$$

$$x + 2y + 3z = 3$$

has infinitely many solutions, then  $2a + 3b$  is equal to,

a) 28

b) 20

c) 25

d) 23

18) Let  $\left(1 + x + 2x^2\right)^{20} = a_0 + a_1x + a_2x^2 + \cdots + a_{40}x^{40}$ . Then,  $a_1 + a_3 + a_5 + \cdots + a_{37}$  is equal to,

a)  $2^{20}(2^{20} + 21)$

b)  $2^{19}(2^{20} + 21)$

c)  $2^{20}(2^{20} - 21)$

d)  $2^{19}(2^{20} - 21)$

19) Let  $5f(x) + 4f\left(\frac{1}{x}\right) = \frac{1}{x} + 3, x > 0$ , then  $\int_1^2 f(x) dx$  is equal to,

a)  $10 \log_e 2 - 6$

b)  $10 \log_e 2 + 6$

c)  $5 \log_e -3$

d)  $5 \log_e 2 + 3$

20) The mean and variance of a set of 15 numbers are 12 and 14 respectively. The mean and variance of another set of 15 numbers are 14 and  $\sigma^2$  respectively. If the variance of all the 30 numbers in the two sets is 13, then  $\sigma^2$  is equal to,

a) 12

b) 10

c) 11

d) 9

- 21) Let the tangents to the curve  $x^2 + 2x - 4y + 9 = 0$  at the point  $\mathbf{P}\left(\frac{1}{3}\right)$  on it meet the y-axis at  $\mathbf{A}$ . Let the line passing through  $\mathbf{P}$  and parallel to the line  $x - 3y = 6$  meet the parabola  $y^2 = 4x$  at  $\mathbf{B}$ . If  $\mathbf{B}$  lies on the line  $2x - 3y = 8$ , then  $(AB)^2$  is equal to

- 22) Let the point  $\left(\frac{p}{p+1}\right)$  lie inside the region

$$E = \{(x, y) : 3 - x \leq y \leq \sqrt{9 - x^2}, 0 \leq x \leq 3\}$$

If the set of all values of  $p$  in the interval  $\left(\frac{a}{b}\right)$  then  $b^2 + b - a^2$  is equal to \_\_\_\_\_

- 23) Let  $y = y(x)$  be a solution of the differential equation

$$(x \cos x) dy + (xy \sin x + y \cos x - 1) dx = 0, 0 < x < \frac{\pi}{2}$$

If  $\frac{\pi}{3}y\left(\frac{\pi}{3}\right) = \sqrt{3}$ , then  $\left|\frac{\pi}{6}y''\left(\frac{\pi}{6}\right) + 2y'\left(\frac{\pi}{6}\right)\right|$  \_\_\_\_\_

- 24) Let  $a \in \mathbb{Z}$  and  $[t]$  be the greatest integer  $\leq t$ . Then the number of points, where the function  $f(x) = [a + 13 \sin x]$ ,  $x \in (0, \pi)$  is not differentiable is \_\_\_\_\_

- 25) If the area of the region

$$S = \{(x, y) : 2y - y^2 \leq x^2 \leq 2y, x \geq y\}$$

is equal to  $\left(\frac{n+2}{n+1} - \frac{\pi}{n-1}\right)$  then the natural number  $n$  is equal to \_\_\_\_\_

- 26) The number of ways of giving 20 distinct oranges to 3 children such that each child gets atleast one orange is \_\_\_\_\_

- 27) Let the image of the point  $\mathbf{P}\left(\frac{1}{2}, \frac{3}{2}\right)$  in the plane  $2x - y + z = 9$  be  $\mathbf{Q}$ . If the coordinates of

the point  $\mathbf{R}$  are  $\left(\frac{6}{10}, \frac{7}{10}\right)$ . Then the square of the area of triangle  $PQR$  is \_\_\_\_\_

- 28) Let A circle passing through the point  $\mathbf{P}\left(\frac{\alpha}{\beta}\right)$  in the first quadrant touches the two coordinate axes at the points  $\mathbf{A}, \mathbf{B}$ . The point  $\mathbf{P}$  is above the line  $\mathbf{AB}$ . The point  $\mathbf{Q}$  on the line segment  $\mathbf{AB}$  is the foot of perpendicular from  $\mathbf{P}$  on  $\mathbf{AB}$ . If  $\mathbf{PQ}$  is equal to 11 units, then value of  $\alpha\beta$  is \_\_\_\_\_

- 29) The coefficient of  $x^{18}$  in the expansion of  $\left(x^4 - \frac{1}{x^3}\right)^{15}$  is \_\_\_\_\_

- 30) Let  $A = \{1, 2, 3, 4, \dots, 10\}$ ,  $B = \{0, 1, 2, 3, 4\}$ . The number of elements in the relation  $R = \{(a, b) \in A \times A : 2(a - b)^2 + 3(a - b) \in B\}$  is \_\_\_\_\_