

Assignment 2

EE24BTECH11005 - Arjun Pavanje

A. FILL IN THE BLANKS

- 1) Let a, b, c be positive real numbers. Let

$$\theta = \tan^{-1} \left(\sqrt{\frac{a(a+b+c)}{bc}} \right) + \tan^{-1} \left(\sqrt{\frac{b(a+b+c)}{ca}} \right) + \tan^{-1} \left(\sqrt{\frac{c(a+b+c)}{ab}} \right)$$

Then $\tan(\theta) =$ _____ (1981 – 2Marks)

- 2) The numerical value of $\tan \left\{ 2 \tan^{-1} \left(\frac{1}{5} \right) - \frac{\pi}{4} \right\}$ is equal to _____ (1984 – 2Marks)

- 3) The greater of the two angles

$$A = 2 \tan^{-1} (2\sqrt{2} - 1) \text{ and}$$

$$B = 3 \sin^{-1} \left(\frac{1}{3} \right) + \sin^{-1} \left(\frac{3}{5} \right)$$

is _____ (1989 – 2Marks)

C. MCQs WITH ONE CORRECT ANSWER

- 1) The value of $\tan \left[\cos^{-1} \left(\frac{4}{5} \right) + \tan^{-1} \left(\frac{2}{3} \right) \right]$ is (1983 – 1Mark)

- a) $\frac{6}{17}$ b) $\frac{7}{16}$
c) $\frac{16}{7}$ d) None

- 2) If we consider only the principle values of the inverse trigonometric functions then the value of

$$\tan \left(\cos^{-1} \left(\frac{1}{5\sqrt{2}} \right) - \sin^{-1} \left(\frac{4}{\sqrt{17}} \right) \right)$$

is _____ (1994)

- a) $\frac{\sqrt{29}}{3}$ b) $\frac{29}{3}$
c) $\frac{\sqrt{3}}{29}$ d) $\frac{3}{29}$

- 3) The number of real solutions of

$$\tan^{-1} \left(\sqrt{x(x-1)} \right) + \sin^{-1} \left(\sqrt{x^2 + x + 1} \right) = \frac{\pi}{2}$$

is _____ (1999 – 2Marks)

- a) zero b) one
c) two d) infinite

- 4) If

$$\sin^{-1} \left(x - \frac{x^2}{2} + \frac{x^3}{4} - \dots \right) + \cos^{-1} \left(x^2 - \frac{x^4}{2} + \frac{x^6}{4} - \dots \right) = \frac{\pi}{2}$$

for $0 < |x| < \sqrt{2}$, then x equals (2001S)

- a) $\frac{1}{2}$ b) 1
c) $-\frac{1}{2}$ d) -1

- 5) The value of x for which

$$\sin \left(\cot^{-1} (1+x) \right) = \cos \left(\tan^{-1} (x) \right)$$

is _____ (2004S)

- a) $\frac{1}{2}$ b) 1
c) 0 d) $-\frac{1}{2}$

- 6) If $0 < x < 1$, then

$$\sqrt{1+x^2} \left[\left\{ x \cos \left(\cot^{-1} (x) \right) + \sin \left(\cot^{-1} (x) \right) \right\}^2 - 1 \right]^{\frac{1}{2}} =$$

is _____ (2008)

- a) $\frac{x}{\sqrt{1+x^2}}$ b) x
c) $x\sqrt{1+x^2}$ d) $\sqrt{1+x^2}$

- 7) The value of

$$\cot \left(\sum_{n=1}^{23} \cot^{-1} \left(1 + \sum_{k=1}^n 2k \right) \right)$$

is _____ (JEEAdv.2013)

- a) $\frac{23}{25}$ b) $\frac{25}{23}$
 c) $\frac{23}{24}$ d) $\frac{24}{23}$

D. MCQs WITH ONE OR MORE THAN ONE CORRECT

1) The principal value of $\sin^{-1}\left(\sin\left(\frac{2\pi}{3}\right)\right)$ is
 (1986 – 2Marks)

- a) $-\frac{2\pi}{3}$ b) $\frac{2\pi}{3}$
 c) $\frac{4\pi}{3}$ d) none

2) If $\alpha = 3 \sin^{-1}\left(\frac{6}{11}\right)$ and $\beta = 3 \cos^{-1}\left(\frac{4}{9}\right)$, where the inverse trigonometric functions take only the principal values, then the correct option(s) is(are)
 (JEEAdv.2015)

- a) $\cos(\beta) > 0$ b) $\sin(\beta) < 0$
 c) $\cos(\alpha + \beta) > 0$ d) $\cos(\alpha) < 0$

3) For non-negative integers n , let

$$f(n) = \frac{\sum_{k=0}^n \sin\left(\frac{k+1}{n+2}\pi\right) \sin\left(\frac{k+2}{n+2}\pi\right)}{\sum_{k=0}^n \sin^2\left(\frac{k+1}{n+2}\pi\right)}$$

Assuming $\cos^{-1}(x)$ takes values in $[0, \pi]$, which of the following options is/are correct
 (JEEAdv.2019)

- a) $\lim_{n \rightarrow \infty} f(n) = \frac{1}{2}$
 b) $f(4) = \frac{\sqrt{3}}{2}$
 c) If $\alpha = \tan\left(\cos^{-1}(f(6))\right)$, then $\alpha^2 + 2\alpha - 1 = 0$
 d) $\sin\left(7 \cos^{-1}(f(5))\right) = 0$

E. SUBJECTIVE PROBLEMS

1) Find the value of:

$$\cos\left(2 \cos^{-1}(x) + \sin^{-1}(x)\right)$$

where $0 \leq \cos^{-1}(x) \leq \pi$ and $-\frac{\pi}{2} \leq \sin^{-1}(x) \leq \frac{\pi}{2}$
 (1981 – 2Marks)

2) Find all the solution of

$$4 \cos^2(x) \sin(x) - 2 \sin^2(x) = 3 \sin(x)$$

(1983 – 2Marks)