

ASSIGNMENT 1

EE24BTECH11005 - Arjun Pavanje*

- 1) Without using tables prove that

$$\left(\sin(12^\circ)\right)\left(\sin(48^\circ)\right)\left(\sin(54^\circ)\right) = \frac{1}{8}$$

(1982 – 2Marks)

- 2) Show that

$$16 \left(\cos\left(\frac{2\pi}{15}\right)\right) \left(\cos\left(\frac{4\pi}{15}\right)\right) \left(\cos\left(\frac{8\pi}{15}\right)\right) \left(\cos\left(\frac{16\pi}{15}\right)\right) = 1$$

(1983 – 2Marks)

- 3) Find all the solution of

$$4 \cos^2(x) \sin(x) - 2 \sin^2(x) = 3 \sin(x)$$

(1983 – 2Marks)

- 4) Find the values of $x \in (-\pi, +\pi)$ which satisfy the equation

$$8(1 + |\cos(x)| + |\cos^2(x)| + |\cos^3(x)| + \dots) = 4^3$$

(1984 – 2Marks)

- 5) Prove that

$$\tan(\alpha) + 2 \tan(2\alpha) + 4 \tan(4\alpha) + 8 \cot(8\alpha) = \cot(\alpha)$$

(1988 – 2Marks)

- 6) ABC is a triangle such that

$$\sin(2A + B) = \sin(C - A) \\ = -\sin(B + 2C) = \frac{1}{2}$$

If A , B and C are in arithmetic progression, determine the values of A , B and C .
(1990 – 5Marks)

- 7) If

$$\exp\left\{\left(\sin^2(x) + \sin^4(x) + \sin^6(x) + \dots \infty\right)\right. \\ \left. (\ln 2)\right\}$$

satisfies the equation $x^2 - 9x + 8$, find the value of $\frac{\cos(x)}{\cos(x) + \sin(x)}$, $0 < x < \frac{\pi}{2}$ (1991 – 4Marks)

- 8) Show that the value of $\frac{\tan(x)}{\tan(3x)}$, wherever defined never lies between $\frac{1}{3}$ and 3 (1992 – 4Marks)
- 9) Determine the smallest positive value of x (in degrees) for which

$$\tan(x + 100^\circ) = \tan(x + 50^\circ) \tan(x) \tan(x - 50^\circ)$$

(1993 – 5Marks)

- 10) Find the smallest positive number p for which the equation

$$\cos(p \sin(x)) = \sin(p \cos(x))$$

has a solution $x \in [0, \pi]$ (1995 – 5Marks)

- 11) Find all values of θ in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ satisfying the equation

$$(1 - \tan(\theta))(1 + \tan(\theta)) \sec^2(\theta) + 2^{\tan^2(\theta)} = 0$$

(1996 – 2Marks)

- 12) Prove that the values of the function

$$\frac{\sin(x) \cos(3x)}{\sin(3x) \cos(x)}$$

does not lie between $\frac{1}{3}$ and 3 for any real x (1997 – 5Marks)

- 13) Prove that

$$\sum_{k=1}^{n-1} (n-k) \cos\left(\frac{2k\pi}{n}\right) = -\frac{n}{2}$$

, where $n \geq 3$ (1997 – 5Marks)

- 14) In any triangle ABC , prove that

$$\cot\left(\frac{A}{2}\right) + \cot\left(\frac{B}{2}\right) + \cot\left(\frac{C}{2}\right) = \cot\left(\frac{A}{2}\right) \cot\left(\frac{B}{2}\right) \cot\left(\frac{C}{2}\right)$$

(2000 – 3Marks)

- 15) Find the range of values of t for which

$$2 \sin(t) = \frac{1 - 2x + 5x^2}{3x^2 - 2x - 1}, \quad t \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \quad (2005 – 2Marks)$$