2021 March 18 Shift 1

EE24BTECH11005 - Arjun Pavanje

1) If $\sum_{k=1}^{31}\binom{31}{k}\binom{31}{k-1} - \sum_{k=1}^{30}\binom{30}{k}\binom{30}{k-1} = \frac{\alpha(60!)}{(30!)(31!)}$, where $\alpha \in R$, then the value of 16α is equal to,

a) 1411
b) 1320
c) 1615
d) 1855

2) Let a function $fN \to N$ be defined by,

$$f(n) = \begin{cases} 2n, & n = 2, 4, 6, 8, \dots \\ n-1, & n = 3, 7, 11, 15, \dots \\ \frac{n+1}{2}, & n = 1, 5, 9, 13, \dots \end{cases}$$

then f is,

a) one-one but not onto
b) onto but not one-one
c) neither one-one nor onto
d) one-one and onto

3) If the system of linear equations,

$$2x + 3y - z = -2$$
$$x + y + z = 4$$
$$x - y + |\lambda| z = 4\lambda - 4$$

where $\lambda \in R$, has no solution then,

a) $\lambda = 7$
b) $\lambda = -7$
c) $\lambda = 8$
d) $\lambda^2 = 1$

4) Let A be a matrix of order $3 \times 3, |A| = 2$. Then

$$\left| |A|\, adj\left(5 adj\left(A^3\right)\right)\right|$$

is equal to,

a) 512×10^6
b) 256×10^6

c) 1024×10^6

d) 256×10^{11}

5) The total number of 5 digit numbers, formed by using the digits $1, 2, 3, 5, 6, 7$ without repitition which are a multiple of 6 is,

a) 36

b) 48

c) 50

d) 72

6) Let $A_1, A_2, A_3, \ldots$ be an increasing geometric progression of positive real numbers. If $A_1A_3A_5A_7 = \frac{1}{1296}$, and $A_2 + A_4 = \frac{7}{36}$, then the value of $A_6 + A_8 + A_{10}$ is,

a) 33

b) 37

c) 43

d) 47

7) Let $[t]$ denote the greatest integer less than or equal to t. Then, the value of the integral $\int_0^1 \left[-8x^2 + 6x - 1\right] dx$ is equal to,

a) -1

b) $-\frac{5}{4}$

c) $\frac{\sqrt{17}-13}{8}$

d) $\frac{\sqrt{17}-16}{8}$

8) Let $f : R \to R$ be defined as,

$$f(x) = \begin{cases} [e^x], & x < 0 \\ ae^x + [x-1], & 0 \le x < 1 \\ b + [\sin(\pi x)], & 1 \le x < 2 \\ [e^{-x}] - c, & x \ge 2 \end{cases}$$

where $a, b, c \in R$ and $[t]$ denotes greatest integer less than or equal to t. Then, which of the following statements is true?

a) There exists $a, b, c \in R$ such that f is continous of R

b) If f is discontinous at exactly one point, $a + b + c = 1$

c) If f is discontinous at exactly one point, $a + b + c \neq 1$

d) f is discontinous at atleast two points for any values of a, b, c

9) The area of the region

$$S = \left\{(x, y) : y^2 \le 8x, y \ge \sqrt{2}x, x \ge 1\right\}$$

is,

a) $\frac{13\sqrt{2}}{6}$ b) $\frac{11\sqrt{2}}{6}$

c) $\frac{5\sqrt{2}}{6}$ d) $\frac{19\sqrt{2}}{6}$

10) Let the solution curve $y = f(x)$ of the differential equation,

$$\left[\frac{x}{\sqrt{x^2 - y^2}} + e^{\frac{y}{x}}\right] x\frac{dy}{dx} = x + \left[\frac{x}{\sqrt{x^2 - y^2}} + e^{\frac{y}{x}}\right] y$$

pass through the points $\begin{pmatrix}1\\0\end{pmatrix}, \begin{pmatrix}2\alpha\\\alpha\end{pmatrix}, \alpha > 0$. Then α is equal to,

a) $\frac{1}{2}\exp\left(\frac{\pi}{6} + \sqrt{e} - 1\right)$ b) $\frac{1}{2}\exp\left(\frac{\pi}{3} + \sqrt{e} - 1\right)$

c) $\exp\left(\frac{\pi}{6} + \sqrt{e} - 1\right)$ d) $2\exp\left(\frac{\pi}{6} + \sqrt{e} - 1\right)$

11) Let $y = y(x)$ be the solution to the differential equation $x\left(1 - x^2\right)\frac{dy}{dx} + \left(3x^2y - y - 4x^3\right) = 0, x > 1$ with $y(2) = -2$. Then $y(3)$ is equal to

a) -18 b) -12

c) -6 d) -3

12) The number of real solutions of $x^7 + 5x^3 + 3x + 1 = 0$ is equal to,

a) 0 b) 1

c) 3 d) 5

13) Let the eccentricity of the hyperbola $H : \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ be $\sqrt{\frac{5}{2}}$ and the length of the latus rectum be $6\sqrt{2}$. If $y = 2x + c$ is a tangent to hyperbola H, then the value of c^2 is equal to,

a) 18 b) 20

c) 24 d) 32

14) If the tangents drawn at the point $O\begin{pmatrix}0\\0\end{pmatrix}, P\begin{pmatrix}1 + \sqrt{5}\\2\end{pmatrix}$ on the circle $x^2 + y^2 - 2x - 4y = 0$ intersect at point Q, then the area of triangle OPQ is equal to

a) $\frac{3+\sqrt{5}}{2}$ b) $\frac{4+2\sqrt{5}}{2}$

c) $\frac{5+3\sqrt{5}}{2}$

d) $\frac{7+3\sqrt{5}}{2}$

15) If two distinct points Q, R lie on the line of intersection of the planes $-x+2y-z=0$ and $3x-5y+2z=0$ and $PQ = PR = \sqrt{18}$ where the point P is $\begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$, then the area of the triangle PQR is equal to

a) $\frac{2}{3}\sqrt{38}$

b) $\frac{4}{3}\sqrt{38}$

c) $\frac{8}{3}\sqrt{38}$

d) $\sqrt{\frac{152}{3}}$