

1-1.11-4

EE24BTECH11005 - Arjun Pavanje

Question:

Find a unit vector perpendicular to both vectors $\mathbf{a} \begin{pmatrix} 1 \\ -7 \\ 7 \end{pmatrix}$ and $\mathbf{b} \begin{pmatrix} 3 \\ -2 \\ 2 \end{pmatrix}$

Variable	Description
\mathbf{a}	$\begin{pmatrix} 1 \\ -7 \\ 7 \end{pmatrix}$ vector
\mathbf{b}	$\begin{pmatrix} 3 \\ -2 \\ 2 \end{pmatrix}$ vector
\mathbf{x}	$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$ vector

TABLE I: Variables Used

Solution: Let \mathbf{x} be a vector perpendicular to both \mathbf{a}, \mathbf{b} . Then,

$$\begin{pmatrix} a^T \\ b^T \end{pmatrix} \mathbf{x} = 0 \quad (1)$$

$$\begin{pmatrix} 1 & -7 & 7 \\ 3 & -2 & 2 \end{pmatrix} \mathbf{x} = 0 \quad (2)$$

Using row reduction, then back substitution,

$$\begin{pmatrix} 1 & -7 & 7 \\ 3 & -2 & 2 \end{pmatrix} \xleftrightarrow{R_2=R_2+R_1} \begin{pmatrix} 1 & -7 & 7 \\ 4 & 0 & 0 \end{pmatrix} \quad (3)$$

$$x_1 = 0, x_2 = x_3 \quad (4)$$

$$\mathbf{x} = x_2 \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \quad (5)$$

Here, $x_2 \in R$, we need to pick it such that magnitude is 1

$$\|x\| = \sqrt{x^T x} = \sqrt{(x_2)^2 \begin{pmatrix} 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}} = 1 \quad (6)$$

$$\sqrt{2x_2^2} = 1 \quad (7)$$

$$x_2 = \pm \frac{1}{\sqrt{2}} \quad (8)$$

Required unit vector is $\frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$ or $-\frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$

plot of the vector

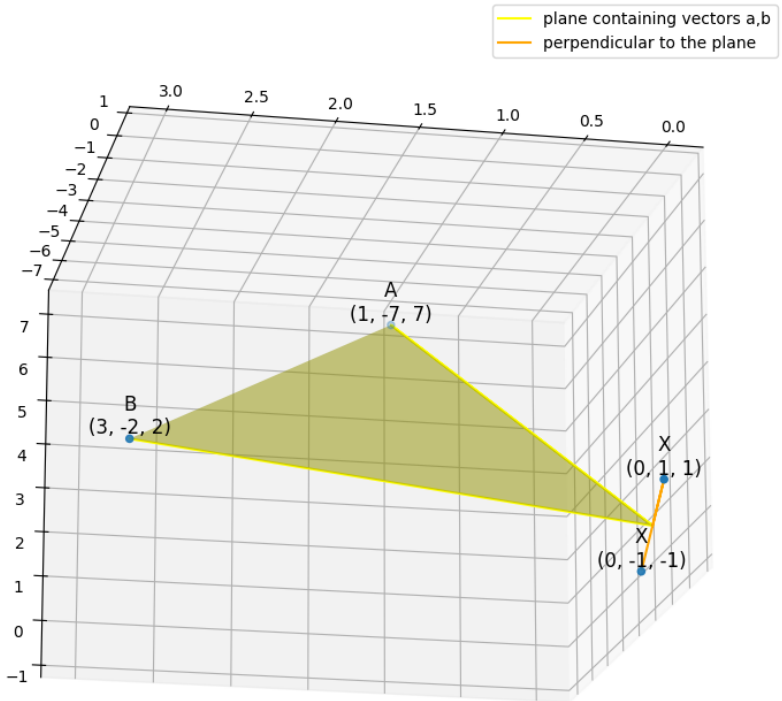


Fig. 1: Plot of the vectors