9-9.3-1

EE24BTECH11005 - Arjun Pavanje

1) If $lim_{x \to 0} \frac{\left[\sin^{-1} x - \tan^{-1} x\right]}{3}$ is equal to L, then the value of $(6L + 1)$ is,

   a) $\frac{1}{2}$ b) 2

   c) $\frac{1}{6}$ d) 6

2) For all four circles M, N, O, P, the following four equations are given,

$$\text{Circle M: } x^2 + y^2 = 1$$
$$\text{Circle N: } x^2 + y^2 - 2x = 0$$
$$\text{Circle O: } x^2 + y^2 - 2x - 2y + 1 = 0$$
$$\text{Circle P: } x^2 + y^2 - 2y = 0$$

   If the centre of circle M is joined with the centre of circle N, furthur centre of circle N is joined with centre of circle O, centre of circle O is joined with centre of circle P and lastly, the centre of circle P is joined with the centre of circle M, then these lines form the sides of a,

   a) Rectangle   b) Square

   c) Parallelogram   d) Rhombus

3) Let $\left(1 + x + 2x^2\right)^{20} = a_0 + a_1 x + a_2 x^2 + \cdots + a_{40} x^{40}$. Then, $a_1 + a_3 + a_5 + \cdots + a_{37}$ is equal to,

   a) $2^{20} \left(2^{20} + 21\right)$ b) $2^{19} \left(2^{20} + 21\right)$

   c) $2^{20} \left(2^{20} - 21\right)$ d) $2^{19} \left(2^{20} - 21\right)$

4) Let ,

$$A + 2B = \begin{pmatrix} 1 & 2 & 0 \\ 6 & -3 & 3 \\ -5 & 3 & 1 \end{pmatrix}$$

$$2A - B = \begin{pmatrix} 2 & -1 & 5 \\ 2 & -1 & 6 \\ 0 & 1 & 2 \end{pmatrix}$$

   If $tr(A)$ denotes the sum of all diagonal entries of the matrix A, then $tr(A) - tr(B)$ is,

a) 0   b) 1

c) 2   d) 3

5) The equation of one of the straight lines which pass through the point $\begin{pmatrix} 1 \\ 3 \end{pmatrix}$ and make an angle $\tan^{-1} \sqrt{2}$ with the straight line $y + 1 = 3\sqrt{2}x$ is,

a) $5\sqrt{2}x + 4y - 15 + 4\sqrt{2} = 0$ b) $4\sqrt{2}x - 5y - 5 + 4\sqrt{2} = 0$

c) $4\sqrt{2}x + 5y - 4\sqrt{2} = 0$ d) $4\sqrt{2}x + 5y - \left(15 + 4\sqrt{2}\right) = 0$

6) The number of times digit 3 will be written when listing the integers from 1 to 1000 is _____

7) The equation of the planes parallel to the plane $x - 2y + 2z - 3 = 0$ which are at unit distace from the point $(1, 2, 3)$ is $ax + by + cz + d = 0$. If $(b - d) = k(c - a)$, then the positive value of k is_____

8) Let $f(x), g(x)$ be two functions satisfying $f\left(x^2\right) + g(4 - x) = 4x^3$ and $g(4 - x) + g(x) = 0$, then the value of $\int_{-4}^{4} f\left(x^2\right)\,dx$ is,_____

9) The mean age of 25 teachers in a school is 40 years. A teacher retires at the age of 60 years and a new teacher is appointed in his place. If the mean age of the teachers in this school now is 39 years, then the age of the newly appointed teacher is_____

10) A square $ABCD$ has all its vertices on the curve $x^2y^2 = 1$. The midpoints of its sides also lie on the same curve. Then, the square of the area of $ABCD$ is_____

11) The missing value in the following figure is,_____
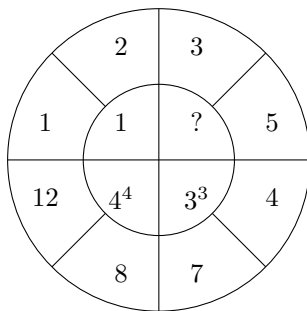
12) The number of solutions of the equation $|\cot x| = \cot x + \left(\frac{1}{\sin x}\right)$ in the interval $[0, 2\pi]$ is_____

13) Let z_1, z_2 be the roots of the equations $z_2 + a_z + 12 = 0$ and z_1, z_2 form an equilateral triangle with origin. Then, the value of $|a|$ is _____

14) Let the plane $ax + by + cz + d = 0$ bisect the line joining the points $\begin{pmatrix} 4 \\ -3 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 3 \\ -5 \end{pmatrix}^{3}$ at right angles. If a, b, c, d are integers, then the minimum value of $\left(a^2 + b^2 + c^2 + d^2\right)$ is,_____

15) If $f(x) = \int \frac{[5x^8 + x^6]}{\left[x^2 + 1 + 2x^7\right]^2} dx, (x \geq 0)$, $f(0) = 0$ and $f(1) = \frac{1}{k}$, then the value of k is,_____