1-1.11-4

EE24BTECH11005 - Arjun Pavanje

Question:

Find a unit vector perpendicular to both vectors $\mathbf{a}\begin{pmatrix}1\\-7\\7\end{pmatrix}$ and $\mathbf{b}\begin{pmatrix}3\\-2\\2\end{pmatrix}$

Variable	**Description**
$\mathbf{a}$	$\begin{pmatrix}1\\-7\\7\end{pmatrix}$ vector
$\mathbf{b}$	$\begin{pmatrix}3\\-2\\2\end{pmatrix}$ vector
$\mathbf{x}$	$\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}$ vector

TABLE I: Variables Used

Solution: Let $\mathbf{x}$ be a vector perpendicular to both $\mathbf{a}, \mathbf{b}$. Then,

$$\mathbf{x} = \begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}$$

$$\begin{pmatrix}\mathbf{a}^{\mathbf{T}}\\\mathbf{b}^{\mathbf{T}}\end{pmatrix}\mathbf{x} = 0 \tag{1}$$

$$\begin{pmatrix}1 & -7 & 7\\3 & -2 & 2\end{pmatrix}\mathbf{x} = 0 \tag{2}$$

Using row reduction, then back substitution,

$$\begin{pmatrix}1 & -7 & 7\\3 & -2 & 2\end{pmatrix} \xleftrightarrow{R_2=R_2+R_1} \begin{pmatrix}1 & -7 & 7\\4 & 0 & 0\end{pmatrix} \tag{3}$$

$$x_1 = 0, x_2 = 7x_3 \tag{4}$$

$$\mathbf{x} = x_2\begin{pmatrix}0\\1\\1\end{pmatrix} \tag{5}$$

Here, $x_2 \in R$, we need to pick it such that magnitude is 1

$$\|x\| = \sqrt{x^T x} = \sqrt{(x_2)^2 \begin{pmatrix} 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}} = 1 \tag{6}$$

$$2{x_2}^2 = 1 \tag{7}$$

$$x_2 = \pm \frac{1}{\sqrt{2}} \tag{8}$$

Required unit vector is $\frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$

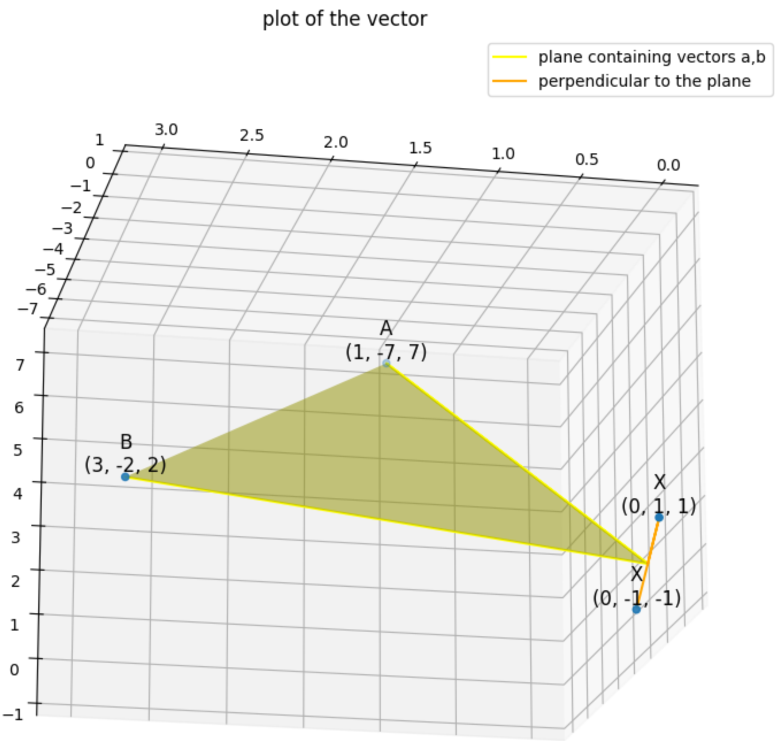

Fig. 1: Plot of the vectors