Assignment 2

EE24BTECH11005 - Arjun Pavanje

A. Fill in the Blanks

1) Let a, b, c be positive real numbers. Let $\theta = \tan^{-1}\left(\sqrt{\frac{a(a+b+c)}{bc}}\right) + \tan^{-1}\left(\sqrt{\frac{b(a+b+c)}{ca}}\right) + \tan^{-1}\left(\sqrt{\frac{c(a+b+c)}{ab}}\right)$ Then $\tan(\theta) =$ _____ $(1981 - 2Marks)$

2) The numerical value of $\tan\left\{2\tan^{-1}\left(\frac{1}{5}\right) - \frac{\pi}{4}\right\}$ is equal to _____ $(1984 - 2Marks)$

3) The greater of the two angles $A = 2\tan^{-1}\left(2\sqrt{2} - 1\right)$ and $B = 3\sin^{-1}\left(\frac{1}{3}\right) + \sin^{-1}\left(\frac{3}{5}\right)$ is _____ $(1989 - 2Marks)$

C. MCQs with One Correct Answer

1) The value of $\tan\left[\cos^{-1}\left(\frac{4}{5}\right) + \tan^{-1}\left(\frac{2}{3}\right)\right]$ is $(1983 - 1Mark)$

a) $\frac{6}{17}$
c) $\frac{16}{7}$
b) $\frac{7}{16}$
d) None

2) If we consider only the principle values of the inverse trigonometric functions then the value of

$$\tan\left(\cos^{-1}\left(\frac{1}{5\sqrt{2}}\right) - \sin^{-1}\left(\frac{4}{\sqrt{17}}\right)\right)$$

is (1994)

a) $\frac{\sqrt{29}}{3}$
c) $\frac{\sqrt{3}}{29}$
b) $\frac{29}{3}$
d) $\frac{3}{29}$

3) The number of real solutions of

$$\tan^{-1}\left(\sqrt{x(x-1)}\right) + \sin^{-1}\left(\sqrt{x^2 + x + 1}\right) = \frac{\pi}{2}$$

is $(1999 - 2Marks)$

a) zero
c) two
b) one
d) infinite

4) If $\sin^{-1}\left(x - \frac{x^2}{2} + \frac{x^3}{4} - \ldots\right) + \cos^{-1}\left(x^2 - \frac{x^4}{2} + \frac{x^6}{4} - \ldots\right) = \frac{\pi}{2}$ for $0 < |x| < \sqrt{2}$, then x equals $(2001S)$

a) $\frac{1}{2}$
c) $-\frac{1}{2}$
b) 1
d) -1

5) The value of x for which $\sin\left(\cot^{-1}(1 + x)\right) = \cos\left(\tan^{-1}(x)\right)$ is $(2004S)$

a) $\frac{1}{2}$
c) 0
b) 1
d) $-\frac{1}{2}$

6) If $0 < x < 1$, then $\sqrt{1 + x^2}\left[\left\{x\cos\left(\cot^{-1}(x)\right) + \sin\left(\cot^{-1}(x)\right)\right\}^2 - 1\right]^{\frac{1}{2}} =$ (2008)

a) $\frac{x}{\sqrt{1+x^2}}$
c) $x\sqrt{1 + x^2}$
b) x
d) $\sqrt{1 + x^2}$

7) The value of $\cot\left(\sum_{n=1}^{23}\cot^{-1}\left(1 + \sum_{k=1}^{n} 2k\right)\right)$ is $(JEEAdv.2013)$

a) $\frac{23}{25}$
c) $\frac{23}{24}$
b) $\frac{25}{23}$
d) $\frac{24}{23}$

D. MCQs with One or More than One Correct

1) The principal value of $\sin^{-1}\left(\sin\left(\frac{2\pi}{3}\right)\right)$ is $(1986 - 2Marks)$

a) $-\frac{2\pi}{3}$
c) $\frac{4\pi}{3}$
b) $\frac{2\pi}{3}$
d) none

2) If $\alpha = 3\sin^{-1}\left(\frac{6}{11}\right)$ and $\beta = 3\cos^{-1}\left(\frac{4}{9}\right)$, where the inverse trigonometric functions take only the principal values, then the correct option(s) is(are) $(JEEAdv.2015)$

a) $\cos(\beta) > 0$
b) $\sin(\beta) < 0$
c) $\cos(\alpha+\beta) > 0$
d) $\cos(\alpha) < 0$

3) For non-negative integers n, let

$$f(n) = \frac{\sum_{k=0}^{n} \sin\left(\frac{k+1}{n+2}\pi\right) \sin\left(\frac{k+2}{n+2}\pi\right)}{\sum_{k=0}^{n} \sin^2\left(\frac{k+1}{n+2}\pi\right)}$$

Assuming $\cos^{-1}(x)$ takes values in $[0, \pi]$, which of the following options is/are correct ($JEEAdv.2019$)

a) $\lim_{n\to\infty)} f(n) = \frac{1}{2}$
b) $f(4) = \frac{\sqrt{3}}{2}$
c) If $\alpha = \tan\left(\cos^{-1}(f(6))\right)$, then $\alpha^2 + 2\alpha - 1 = 0$
d) $\sin\left(7\cos^{-1}(f(5))\right) = 0$

E. Subjective Problems

1) Find the value of: $\cos\left(2\cos^{-1}(x) + \sin^{-1}(x)\right)$ at $x = \frac{1}{5}$ where $0 \leq \cos^{-1}(x) \leq \pi$ and $-\frac{\pi}{2} \leq \sin^{-1}(x) \leq \frac{\pi}{2}$ $(1981 - 2Marks)$

2) Find all the solution of $4\cos^2(x)\sin(x) - 2\sin^2(x) = 3\sin(x)$ $(1983 - 2Marks)$