ASSIGNMENT 1

EE24BTECH11005 - Arjun Pavanje*

1) Without using tables prove that

$$(\sin(12^\circ))(\sin(48^\circ))(\sin(54^\circ)) = \frac{1}{8}$$

$(1982 - 2Marks)$

2) Show that

$$16\left(\cos\left(\frac{2\pi}{15}\right)\right)\left(\cos\left(\frac{4\pi}{15}\right)\right)\left(\cos\left(\frac{8\pi}{15}\right)\right)\left(\cos\left(\frac{16\pi}{15}\right)\right) = 1$$

$(1983 - 2Marks)$

3) Find all the solution of

$$4\cos^2(x)\sin(x) - 2\sin^2(x) = 3\sin(x)$$

$(1983 - 2Marks)$

4) Find the values of $x \in (-\pi, +\pi)$ which satisfy the equation

$$8^{\left(1+|\cos(x)|+|\cos^2(x)|+|\cos^3(x)|+\ldots\right)} = 4^3$$

$(1984 - 2Marks)$

5) Prove that

$$\tan(\alpha) + 2\tan(2\alpha) + 4\tan(4\alpha) + 8\cot(8\alpha) = \cot(\alpha)$$

$(1988 - 2Marks)$

6) ABC is a triangle such that

$$\sin(2A + B) = \sin(C - A) = -\sin(B + 2C) = \frac{1}{2}$$

If A, B and C are in arithmetic progression, determine the values of A, B and C. $(1990 - 5Marks)$

7) If $\exp\left\{\left(\sin^2(x) + \sin^4(x) + \sin^6(x) + \ldots\infty\right)\ln 2\right\}$ satisfies the equation $x^2 - 9x + 8$, find the value of $\frac{\cos(x)}{\cos(x)+\sin(x)}$, $0<x<\frac{\pi}{2}$ $(1991 - 4Marks)$

8) Show that the value of $\frac{\tan(x)}{\tan(3x)}$, wherever defined never lies between $\frac{1}{3}$ and 3 $(1992 - 4Marks)$

9) Determine the smallest positive value of x (in degrees) for which

$$\tan(x + 100^\circ) = \tan(x + 50^\circ)\tan(x)\tan(x - 50^\circ)$$

$(1993 - 5Marks)$

10) Find the smallest positive number p for which the equation

$$\cos(p\sin(x)) = \sin(p\cos(x))$$

has a solution $x \in [0, \pi]$ $(1995 - 5Marks)$

11) Find all values of θ in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ satisfying the equation

$$(1 - \tan(\theta))(1 + \tan(\theta))\sec^2(\theta) + 2^{\tan^2(\theta)} = 0$$

$(1996 - 2Marks)$

12) Prove that the values of the function

$$\frac{\sin(x)\cos(3x)}{\sin(3x)\cos(x)}$$

does not lie between $\frac{1}{3}$ and 3 for any real x $(1997 - 5Marks)$

13) Prove that

$$\sum_{k=1}^{n-1}(n-k)\cos\left(\frac{2k\pi}{n}\right) = -\frac{n}{2}$$

, where $n \geq 3$ $(1997 - 5Marks)$

14) In any triangle ABC, prove that

$$\cot\left(\frac{A}{2}\right) + \cot\left(\frac{B}{2}\right) + \cot\left(\frac{C}{2}\right) = \cot\left(\frac{A}{2}\right)\cot\left(\frac{B}{2}\right)\cot\left(\frac{C}{2}\right)$$

$(2000 - 3Marks)$

15) Find the range of values of for which

$$2\sin(t) = \frac{1 - 2x + 5x^2}{3x^2 - 2x - 1}$$

, $t \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ $(2005 - 2Marks)$