2023 April 6 Shift 1

EE24BTECH11005 - Arjun Pavanje

16) If $2x^y + 3y^x = 20$ then $\frac{dy}{dx}$ at $\begin{pmatrix}2\\2\end{pmatrix}$ is equal to,

a) $-\left(\frac{3+\log_e 8}{2+\log_e 4}\right)$

b) $-\left(\frac{2+\log_e 8}{3+\log_e 4}\right)$

c) $-\left(\frac{3+\log_e 4}{2+\log_e 8}\right)$

d) $-\left(\frac{3+\log_e 16}{4+\log_e 8}\right)$

17) If the system of equations

$$x + y + az = b$$
$$2x + 5y + 2z = 6$$
$$x + 2y + 3z = 3$$

has infinitely many solutions, then $2a + 3b$ is equal to,

a) 28

b) 20

c) 25

d) 23

18) Let $\left(1 + x + 2x^2\right)^{20} = a_0 + a_1x + a_2x^2 + \cdots + a_{40}x^{40}$. Then, $a_1 + a_3 + a_5 + \cdots + a_{37}$is equal to,

a) $2^{20}\left(2^{20} + 21\right)$

b) $2^{19}\left(2^{20} + 21\right)$

c) $2^{20}\left(2^{20} - 21\right)$

d) $2^{19}\left(2^{20} - 21\right)$

19) Let $5f(x) + 4f\left(\frac{1}{x}\right) = \frac{1}{x} + 3, x > 0$, then $\int_1^2 f(x)\,dx$ is equal to,

a) $10\log_e 2 - 6$

b) $10\log_e 2 + 6$

c) $5\log_e -3$

d) $5\log_e 2 + 3$

20) The mean and variance of a set of 15 numbers are 12 and 14 respectively. The mean and variance of another set of 15 numbers are 14 and σ^2 respectively. If the variance of all the 30 numbers in the two sets is 13, then σ^2 is equal to,

a) 12

b) 10

c) 11 d) 9

21) Let the tangents to the curve $x^2 + 2x - 4y + 9 = 0$ at the point $\mathbf{P}\begin{pmatrix}1\\3\end{pmatrix}$ on it meet the y-axis at **A**. Let the line passing through **P** and parallel to the line $x - 3y = 6$ meet the parabola $y^2 = 4x$ at **B**. If **B** lies on the line $2x - 3y = 8$, then $(AB)^2$ us equal to

22) Let the point $\begin{pmatrix}p\\p+1\end{pmatrix}$ lie inside the region

$$E = \left\{(x, y) : 3 - x \le y \le \sqrt{9 - x^2}, 0 \le x \le 3\right\}$$

If the set of all values of p in the interval $\begin{pmatrix}a\\b\end{pmatrix}$ then $b^2 + b - a^2$ is equal to _____

23) Let $y = y(x)$ be a solution of the differential equation

$$(x\cos x)\,dy + (xy\sin x + y\cos x - 1)\,dx = 0, 0 < x < \frac{\pi}{2}$$

If $\frac{\pi}{3}y\left(\frac{\pi}{3}\right) = \sqrt{3}$, then $\left|\frac{\pi}{6}y''\left(\frac{\pi}{6}\right) + 2y'\left(\frac{\pi}{6}\right)\right|$_____

24) The Let $a \in Z$ and $[t]$ be the greatest integer $\le t$. Then the number of points, where the function $f(x) = [a + 13\sin x], x \in (0, \pi)$ is not differentiable is_____

25) If the area of the region

$$S = \left\{(x, y) : 2y - y^2 \le x^2 \le 2y, x \ge y\right\}$$

is equal to $\left(\frac{n+2}{n+1} - \frac{\pi}{n-1}\right)$ then the natural number n is equal to _____

26) The number of ways of giving 20 distinct oranges to 3 children such that each child gets atleast one orange is _____

27) Let the image of the point $\mathbf{P}\begin{pmatrix}1\\2\\3\end{pmatrix}$ in the plane $2x - y + z = 9$ be **Q**. If the coordinates of the point **R** are $\begin{pmatrix}6\\10\\7\end{pmatrix}$. Then the square of the area of triangle PQR is _____

28) Let A circle passing through the point $\mathbf{P}\begin{pmatrix}\alpha\\\beta\end{pmatrix}$ in the first quadrant touches the two coordinate axes at the points **A**, **B**. The point **P** is above the line **AB**. The point **Q** on the line segment **AB** is the foot of perpendicular from **P** on **AB**. If **PQ** is equal to 11 units, then value of $\alpha\beta$ is _____

29) The coeffecient of x^{18} in the expansion of $\left(x^4 - \frac{1}{x^3}\right)^{15}$is _____

30) Let $A = \{1, 2, 3, 4, \ldots, 10\}, B = \{0, 1, 2, 3, 4\}$. The number of elements in the relation $R = \left\{(a, b) \in A \times A : 2(a - b)^2 + 3(a - b) \in B\right\}$ is _____