

9-9.3-1

EE24BTECH11005 - Arjun Pavanje

Question:

Find the area of the region in the first quadrant enclosed by the X axis, the line $y = x$ and the circle $x^2 + y^2 = 32$

Variable	Description
h	Point lying on the line
m	Slope of line
e	Eccentricity of conic
F	Focus of conic
f	$\ \mathbf{u}\ ^2 - r^2$
V	A symmetric matrix given by eigenvalue decomposition
u	Centre of circle
r	Radius of circle

TABLE I: Variables Used

Solution: Line equation of form $\mathbf{x} = \mathbf{h} + k\mathbf{m}$

$$\mathbf{x} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} + k \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad (1)$$

Equation of circle is of form $\mathbf{x}^\top \mathbf{V} \mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0$ with

$$\mathbf{u} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, f = \|\mathbf{u}\|^2 - r^2, \mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad (2)$$

If a line intersects the conic, k value of intersecting point is given by,

$$k_i = \frac{-\mathbf{m}^\top (\mathbf{V}\mathbf{h} + \mathbf{u}) \pm \sqrt{[\mathbf{m}^\top (\mathbf{V}\mathbf{h} + \mathbf{u})]^2 - g(h) (\mathbf{m}^\top \mathbf{V} \mathbf{m})}}{\mathbf{m}^\top \mathbf{V} \mathbf{m}} \quad (3)$$

On substituting values of $\mathbf{u}, \mathbf{m}, \mathbf{h}, \mathbf{V}$ we get,

$$k = \pm 4 \quad (4)$$

Points of intersection with circle are, $\begin{pmatrix} 4 \\ 4 \end{pmatrix}, \begin{pmatrix} -4 \\ -4 \end{pmatrix}$

Angle between given line $y = x$ and x axis is 45°

Area bound between the circle, line, X axis, in the first quadrant is,

$$\frac{45}{360} \pi r^2 = 4\pi \quad (5)$$

Required Area = 4π sq. units

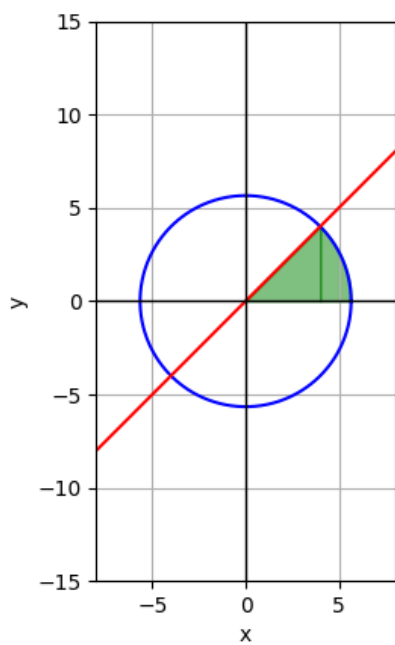


Fig. 1: Circle $y^2 + x^2 = 32$, Line $x = y$