

# Assignment 2

EE24BTECH11005 - Arjun Pavanje

## A. FILL IN THE BLANKS

- 1) Let  $a, b, c$  be positive real numbers. Let

$$\theta = \tan^{-1} \left( \sqrt{\frac{a(a+b+c)}{bc}} \right) + \tan^{-1} \left( \sqrt{\frac{b(a+b+c)}{ca}} \right) + \tan^{-1} \left( \sqrt{\frac{c(a+b+c)}{ab}} \right)$$

Then  $\tan(\theta) =$  \_\_\_\_\_ (1981 – 2Marks)

- 2) The numerical value of  $\tan \left\{ 2 \tan^{-1} \left( \frac{1}{5} \right) - \frac{\pi}{4} \right\}$  is equal to \_\_\_\_\_ (1984 – 2Marks)

- 3) The greater of the two angles

$$A = 2 \tan^{-1} (2\sqrt{2} - 1) \text{ and}$$

$$B = 3 \sin^{-1} \left( \frac{1}{3} \right) + \sin^{-1} \left( \frac{3}{5} \right)$$

is \_\_\_\_\_ (1989 – 2Marks)

## C. MCQs WITH ONE CORRECT ANSWER

- 1) The value of  $\tan \left[ \cos^{-1} \left( \frac{4}{5} \right) + \tan^{-1} \left( \frac{2}{3} \right) \right]$  is (1983 – 1Mark)

- a)  $\frac{6}{17}$                       b)  $\frac{7}{16}$   
c)  $\frac{16}{7}$                       d) None

- 2) If we consider only the principle values of the inverse trigonometric functions then the value of

$$\tan \left( \cos^{-1} \left( \frac{1}{5\sqrt{2}} \right) - \sin^{-1} \left( \frac{4}{\sqrt{17}} \right) \right)$$

is \_\_\_\_\_ (1994)

- a)  $\frac{\sqrt{29}}{3}$                       b)  $\frac{29}{3}$   
c)  $\frac{\sqrt{3}}{29}$                       d)  $\frac{3}{29}$

- 3) The number of real solutions of

$$\tan^{-1} \left( \sqrt{x(x-1)} \right) + \sin^{-1} \left( \sqrt{x^2 + x + 1} \right) = \frac{\pi}{2}$$

is \_\_\_\_\_ (1999 – 2Marks)

- a) zero                      b) one  
c) two                      d) infinite

- 4) If

$$\sin^{-1} \left( x - \frac{x^2}{2} + \frac{x^3}{4} - \dots \right) + \cos^{-1} \left( x^2 - \frac{x^4}{2} + \frac{x^6}{4} - \dots \right) = \frac{\pi}{2}$$

for  $0 < |x| < \sqrt{2}$ , then  $x$  equals (2001S)

- a)  $\frac{1}{2}$                       b) 1  
c)  $-\frac{1}{2}$                       d) -1

- 5) The value of  $x$  for which

$$\sin \left( \cot^{-1} (1+x) \right) = \cos \left( \tan^{-1} (x) \right)$$

is \_\_\_\_\_ (2004S)

- a)  $\frac{1}{2}$                       b) 1  
c) 0                      d)  $-\frac{1}{2}$

- 6) If  $0 < x < 1$ , then

$$\sqrt{1+x^2} \left[ \left\{ x \cos \left( \cot^{-1} (x) \right) + \sin \left( \cot^{-1} (x) \right) \right\}^2 - 1 \right]^{\frac{1}{2}} =$$

is \_\_\_\_\_ (2008)

- a)  $\frac{x}{\sqrt{1+x^2}}$                       b)  $x$   
c)  $x\sqrt{1+x^2}$                       d)  $\sqrt{1+x^2}$

- 7) The value of

$$\cot \left( \sum_{n=1}^{23} \cot^{-1} \left( 1 + \sum_{k=1}^n 2k \right) \right)$$

is \_\_\_\_\_ (JEEAdv.2013)

- a)  $\frac{23}{25}$                       b)  $\frac{25}{23}$   
 c)  $\frac{23}{24}$                       d)  $\frac{24}{23}$

#### D. MCQs WITH ONE OR MORE THAN ONE CORRECT

1) The principal value of  $\sin^{-1}\left(\sin\left(\frac{2\pi}{3}\right)\right)$  is  
 (1986 – 2Marks)

- a)  $-\frac{2\pi}{3}$                       b)  $\frac{2\pi}{3}$   
 c)  $\frac{4\pi}{3}$                       d) none

2) If  $\alpha = 3 \sin^{-1}\left(\frac{6}{11}\right)$  and  $\beta = 3 \cos^{-1}\left(\frac{4}{9}\right)$ , where the inverse trigonometric functions take only the principal values, then the correct option(s) is(are)  
 (JEEAdv.2015)

- a)  $\cos(\beta) > 0$                       b)  $\sin(\beta) < 0$   
 c)  $\cos(\alpha + \beta) > 0$                       d)  $\cos(\alpha) < 0$

3) For non-negative integers  $n$ , let

$$f(n) = \frac{\sum_{k=0}^n \sin\left(\frac{k+1}{n+2}\pi\right) \sin\left(\frac{k+2}{n+2}\pi\right)}{\sum_{k=0}^n \sin^2\left(\frac{k+1}{n+2}\pi\right)}$$

Assuming  $\cos^{-1}(x)$  takes values in  $[0, \pi]$ , which of the following options is/are correct  
 (JEEAdv.2019)

- a)  $\lim_{n \rightarrow \infty} f(n) = \frac{1}{2}$   
 b)  $f(4) = \frac{\sqrt{3}}{2}$   
 c) If  $\alpha = \tan\left(\cos^{-1}(f(6))\right)$ , then  $\alpha^2 + 2\alpha - 1 = 0$   
 d)  $\sin\left(7 \cos^{-1}(f(5))\right) = 0$

#### E. SUBJECTIVE PROBLEMS

1) Find the value of:

$$\cos\left(2 \cos^{-1}(x) + \sin^{-1}(x)\right)$$

where  $0 \leq \cos^{-1}(x) \leq \pi$  and  $-\frac{\pi}{2} \leq \sin^{-1}(x) \leq \frac{\pi}{2}$   
 (1981 – 2Marks)

2) Find all the solution of

$$4 \cos^2(x) \sin(x) - 2 \sin^2(x) = 3 \sin(x)$$

(1983 – 2Marks)