

# 9-9.3-1

EE24BTECH11005 - Arjun Pavanje

**Question:**

Find the area of the region in the first quadrant enclosed by the  $X$  axis, the line  $y = x$  and the circle  $x^2 + y^2 = 32$

Variable	Description
<b>h</b>	Point lying on the line
<b>m</b>	Slope of line
<b>e</b>	Eccentricity of conic
<b>F</b>	Focus of conic
$f$	$\ \mathbf{u}\ ^2 - r^2$
<b>V</b>	A symmetric matrix given by eigenvalue decomposition
<b>u</b>	Centre of circle

TABLE I: Variables Used

**Solution:** Line equation of form  $\mathbf{x} = \mathbf{h} + k\mathbf{m}$

$$\mathbf{x} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} + k \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad (1)$$

Equation of parabola of form  $\mathbf{x}^\top \mathbf{V} \mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0$  is

$$\mathbf{u} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, f = \|\mathbf{u}\|^2 - r^2, \mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad (2)$$

If a line intersects the conic,  $k$  value of intersecting point is given by,

$$k_i = \frac{-\mathbf{m}^\top (\mathbf{V}\mathbf{h} + \mathbf{u}) \pm \sqrt{[\mathbf{m}^\top (\mathbf{V}\mathbf{h} + \mathbf{u})]^2 - g(h)(\mathbf{m}^\top \mathbf{V}\mathbf{m})}}{\mathbf{m}^\top \mathbf{V}\mathbf{m}} \quad (3)$$

On substituting values of  $\mathbf{u}, \mathbf{m}, \mathbf{h}, \mathbf{V}$  we get,

$$k = \pm 4 \quad (4)$$

Points of intersection with circle are,  $\begin{pmatrix} 4 \\ 4 \end{pmatrix}, \begin{pmatrix} -4 \\ -4 \end{pmatrix}$

Angle between given line  $y = x$  and  $x$  axis is  $45^\circ$

Area bound between the circle, line,  $X$  axis, in the first quadrant is,

$$\frac{45}{360} \pi r^2 = 4 \quad (5)$$

Required Area = 4 sq. units

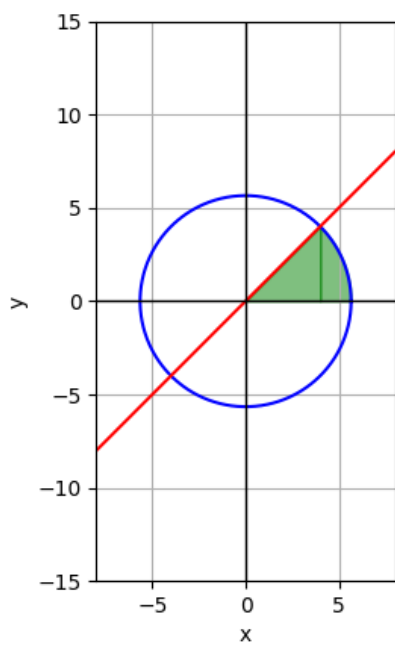


Fig. 1: Circle  $y^2 + x^2 = 32$ , Line  $x = y$