9-9.3-1

EE24BTECH11005 - Arjun Pavanje

Question:
Find the area of the region in the first quadrant enclosed by the X axis, the line $y = x$ and the circle $x^2 + y^2 = 32$

Variable	Description
$\mathbf{h}$	Point lying on the line
$\mathbf{m}$	Slope of line
$\mathbf{e}$	Eccentricity of conic
$\mathbf{F}$	Focus of conic
f	$\|\mathbf{u}\|^2 - r^2$
$\mathbf{V}$	A symmetric matrix given by eigenvalue decomposition
$\mathbf{u}$	Centre of circle

TABLE I: Variables Used

Solution: Line equation of form $\mathbf{x} = \mathbf{h} + k\mathbf{m}$

$$\mathbf{x} = \begin{pmatrix}0\\0\end{pmatrix} + k\begin{pmatrix}1\\1\end{pmatrix} \tag{1}$$

Equation of parabola of form $\mathbf{x}^\top\mathbf{V}\mathbf{x} + 2\mathbf{u}^\top\mathbf{x} + f = 0$ is

$$\mathbf{u} = \begin{pmatrix}0\\0\end{pmatrix}, f = \|\mathbf{u}\|^2 - r^2, \mathbf{V} = \begin{pmatrix}1 & 0\\0 & 1\end{pmatrix} \tag{2}$$

If a line intersects the conic, k value of intersecting point is given by,

$$k_i = \frac{-\mathbf{m}^\top(\mathbf{V}\mathbf{h} + \mathbf{u}) \pm \sqrt{[\mathbf{m}^\top(\mathbf{V}\mathbf{h} + \mathbf{u})]^2 - g(h)(\mathbf{m}^\top\mathbf{V}\mathbf{m})}}{\mathbf{m}^\top\mathbf{V}\mathbf{m}} \tag{3}$$

On substituting values of $\mathbf{u}, \mathbf{m}, \mathbf{h}, \mathbf{V}$ we get,

$$k = \pm 4 \tag{4}$$

Points of intersection with circle are, $\begin{pmatrix}4\\4\end{pmatrix}, \begin{pmatrix}-4\\-4\end{pmatrix}$
Area bound between the circle, line, X axis, in the first quadrant is,

$$\int_0^4 x dx + \int_4^{4\sqrt{2}} \sqrt{32 - x^2} dx \tag{5}$$

$$= 4 \tag{6}$$

Required Area = 4 sq. units

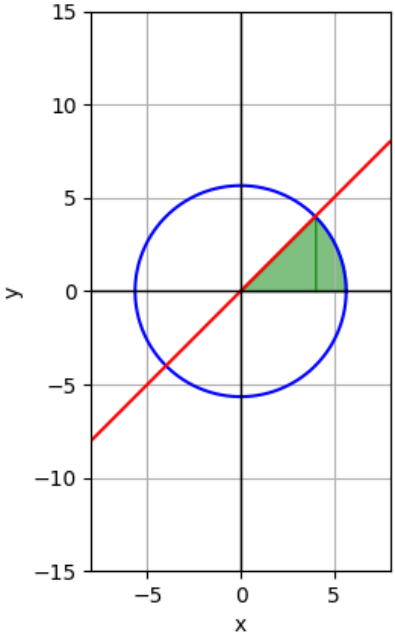

Fig. 1: Circle $y^2 + x^2 = 32$, Line $x = y$