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ASSIGNMENT 1

EE24BTECH11005 - Arjun Pavanje*

1) Without using tables prove that

$$\left(\sin\left(12^{\circ}\right)\right)\left(\sin\left(48^{\circ}\right)\right)\left(\sin\left(54^{\circ}\right)\right) = \frac{1}{8}$$

$$(1982 - 2Marks)$$

2) Show that

$$16\left(\cos\left(\frac{2\pi}{15}\right)\right)\left(\cos\left(\frac{4\pi}{15}\right)\right)\left(\cos\left(\frac{8\pi}{15}\right)\right)$$
$$\left(\cos\left(\frac{16\pi}{15}\right)\right) = 1$$
$$(1983 - 2Marks)$$

3) Find all the solution of

$$4\cos^2(x)\sin(x) - 2\sin^2(x) = 3\sin(x)$$

$$(1983 - 2Marks)$$

4) Find the values of $x \in (-\pi, +\pi)$ which satisfy the equation

$$8^{(1+|\cos(x)|+|\cos^2(x)|+|\cos^3(x)|+...)} = 4^3$$

(1984 - 2Marks)

5) Prove that

$$\tan(\alpha) + 2\tan(2\alpha) + 4\tan(4\alpha) + 8\cot(8\alpha)$$
$$= \cot(\alpha)$$

$$(1988 - 2Marks)$$

6) ABC is a triangle such that

$$\sin(2A + B) = \sin(C - A) = -\sin(B + 2C)$$
$$= \frac{1}{2}$$

If A, B and C are in arithmetic progression, determine the values of A, B and C. (1990 - 5Marks)

- 7) If $\exp\left\{\left(\sin^2(x) + \sin^4(x) + \sin^6(x) + \dots \infty\right) \ln 2\right\}$ satisfies the equation $x^2 9x + 8$, find the value of $\frac{\cos(x)}{\cos(x) + \sin(x)}$, $0 < x < \frac{\pi}{2}$ (1991 4*Marks*)
- 8) Show that the value of $\frac{\tan(x)}{\tan(3x)}$, wherever defined never lies between $\frac{1}{3}$ and 3 (1992 4*Marks*)

9) Determine the smallest positive value of x (in degrees) for which

$$\tan\left(x + 100^{\circ}\right) = \tan\left(x + 50^{\circ}\right)\tan\left(x\right)\tan\left(x - 50^{\circ}\right)$$

(1993 - 5Marks)

10) Find the smallest positive number *p* for which the equation

$$\cos(p\sin(x)) = \sin(p\cos(x))$$

has a solution $x \in [0, \pi]$ (1995 – 5*Marks*)

11) Find all values of θ in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ satisfying the equation

$$(1 - \tan(\theta)) (1 + \tan(\theta)) \sec^2(\theta) + 2^{\tan^2(\theta)} = 0$$

$$(1996 - 2Marks)$$

12) Prove that the values of the function

$$\frac{\sin(x)\cos(3x)}{\sin(3x)\cos(x)}$$

does not lie between $\frac{1}{3}$ and 3 for any real x (1997 – 5Marks)

13) Prove that

$$\sum_{k=1}^{n-1} (n-k) \cos\left(\frac{2k\pi}{n}\right) = -\frac{n}{2}$$

, where $n \ge 3$ (1)

(1997 - 5Marks)

14) In any triangle *ABC*, prove that

$$\cot\left(\frac{A}{2}\right) + \cot\left(\frac{B}{2}\right) + \cot\left(\frac{C}{2}\right) =$$

$$\cot\left(\frac{A}{2}\right)\cot\left(\frac{B}{2}\right)\cot\left(\frac{C}{2}\right)$$

$$(2000 - 3Marks)$$

15) Find the range of values of for which

$$2\sin(t) = \frac{1 - 2x + 5x^2}{3x^2 - 2x - 1}$$

$$t \in \left[-\frac{\pi}{2}, \frac{\pi}{2} \right]$$
 (2005 – 2*Marks*)