APPENDIX A

In the following three sections of this appendix, we elaborate on the reformulations of objective function (2), power balance constraints (4e), and chance constraints (5) and (7), respectively. Recall we use linear decision rules in this paper, and thereby, we substitute power production of each unit i, i.e., $p_{isrt}(\gamma)$ with $p_{isrt} + \alpha_{isrt}1^\top\gamma$.

A. Objective Function

By implementing linear decisions rules, objective function (2) is written as

$$\min_{\mathbf{y}} k^T\mathbf{y} + \sum_{srt}\pi_s \max_{D\in\mathcal{P}_{srt}} w_r \min_{\mathbf{p},\alpha,\mathbf{x},\mathbf{u}} \mathbb{E}^D[c^T(\mathbf{p}_{srt} + \alpha_{srt}1^\top\gamma) + h^T\mathbf{u}_{srt}]. \tag{12a}$$

Recall we assume that the mean of the forecast error uncertainty vector γ is zero, i.e. $\mathbb{E}^D(\gamma) = \mu = 0$. Therefore, the second row of (12a) reduces to $c^T\mathbf{p}_{srt} + h^T\mathbf{u}_{srt}$. Now, the forecast error γ does no longer exist in the objective function. Similarly, constraints (4e), (5) and (7) will be reformulated in the next two sections of this appendix in a way that they will not include γ. As a result, maximization operator $\max_{D\in\mathcal{P}_{srt}}$ in min-max-min objective function (12a) can be removed [20], yielding a min-min objective function, which can be combined to a single minimization function as

$$\min_{\mathbf{y},\mathbf{p},\alpha,\mathbf{x},\mathbf{u}} l^T\mathbf{y} + \sum_{s,r,t}\pi_s w_r(c^T\mathbf{p}_{srt} + h^T\mathbf{u}_{srt}). \tag{12b}$$

B. Power Balance Constraints

The linear decisions rules reformulate the power balance constraints (4e) to

$$\mathbf{1}^\top(\mathbf{p} + \alpha_{srt}1^\top\gamma) + \mathbf{1}^\top(\mathbf{m}_{srt} + \gamma) = \mathbf{1}^\top\mathbf{d}_{srt}, \ \forall s,r,t. \tag{13a}$$

In order to satisfy (13a) for all realizations of the forecast error γ, we set the first-order coefficient of γ equal to zero [18]. Thus, by matching the zero- and first-order coefficients of γ on both sides of (13a), we obtain

$$\mathbf{1}^\top\alpha_{srt} = -\mathbf{1}, \quad \forall s,r,t \tag{13b}$$

$$\mathbf{1}^\top\mathbf{p}_{srt} + \mathbf{1}^\top\mathbf{m}_{srt} = \mathbf{1}^\top\mathbf{d}_{srt}, \quad \forall s,r,t. \tag{13c}$$

C. Chance Constraints

The mathematical process to recast the generic chance constraint (8) as (9) is explained in details in [17]. For an example, we provide here the reformulation of chance constraint (5a). Other chance constraints can be analytically reformulated in the same manner. By implementing the linear decision rules, chance constraint (5a) is written as

$$\min_{\mathbb{P}\in\mathcal{P}_{srt}} D\left(p_{isrt} + \alpha_{isrt}1^\top\gamma \leqslant \overline{p}_i x_{isrt}\right) \geqslant 1-\epsilon, \ \forall i,s,r,t. \tag{14a}$$

According to (9), chance constraint (14a) is equivalent to

$$p_{isrt} \leqslant \overline{p}_i x_{isrt} - \sqrt{\frac{1-\epsilon_i}{\epsilon_i}}\sqrt{\alpha_{isrt}1^\top\Sigma_{srt}1\alpha_{isrt}}, \quad \forall i,s,r,t, \tag{14b}$$

which can be rewritten as

$$p_{isrt} \leqslant \overline{p}_i x_{isrt} - \sqrt{\frac{1-\epsilon_i}{\epsilon_i}}\|\alpha_{isrt}1^\top\Sigma_{srt}^{1/2}\|_2, \ \forall i,s,r,t. \tag{14c}$$

APPENDIX B

According to all reformulations explained in Appendix A, the resulting mixed-integer second-order cone problem for the proposed DRCC model is

$$\min_{\mathbf{y},\mathbf{p},\alpha,\mathbf{x},\mathbf{u}} l^T\mathbf{y} + \sum_{s,r,t}\pi_s w_r(c^T\mathbf{p}_{srt} + h^T\mathbf{u}_{srt}) \tag{15a}$$

subject to:

(4a)-(4d),(4f),(6),(13b)-(13c),(14c) (15b)

$$p_{isrt} \geqslant \underline{p}_i x_{isrt} + \sqrt{\frac{1-\epsilon_i}{\epsilon_i}}\|\alpha_{isrt}1^\top\Sigma_{srt}^{1/2}\|_2, \ \forall i,s,r,t \tag{15c}$$

$$p_{isrt} - p_{isr(t-1)} \leqslant \overline{r}_i x_{isr(t-1)} + rs_i(1-x_{isr(t-1)}) - \sqrt{\frac{1-\epsilon_i}{\epsilon_i}}\|(v_{srt})^\top\hat{\Sigma}_{srt}^{1/2}\|_2, \ \forall i,s,r,t \tag{15d}$$

$$p_{isrt} - p_{isr(t-1)} \geqslant -\underline{r}_i x_{isrt} - rs_i(1-x_{isrt}) + \sqrt{\frac{1-\epsilon_i}{\epsilon_i}}\|(v^{srt})^\top\hat{\Sigma}_{srt}^{1/2}\|_2, \ \forall i,s,r,t \tag{15e}$$

$$H_l^{\mathrm{G}}\mathbf{p}_{srt} + H_l^{\mathrm{W}}\mathbf{m}_{srt} - H_l^{\mathrm{D}}\mathbf{d}_{srt} \leqslant \overline{f}_l - \sqrt{\frac{1-\epsilon_l}{\epsilon_l}}\|(H_l^{\mathrm{G}}\alpha_{srt}1^\top + H_l^{\mathrm{W}})\Sigma_{srt}^{1/2}\|_2, \ \forall l,s,r,t \tag{15f}$$

$$H_l^{\mathrm{G}}\mathbf{p}_{srt} + H_l^{\mathrm{W}}\mathbf{m}_{srt} - H_l^{\mathrm{D}}\mathbf{d}_{srt} \geqslant -\overline{f}_l + \sqrt{\frac{1-\epsilon_l}{\epsilon_l}}\|(H_l^{\mathrm{G}}\alpha_{srt}1^\top + H_l^{\mathrm{W}})\Sigma_{srt}^{1/2})\|_2, \ \forall l,s,r,t \tag{15g}$$

$$p_{isr(t-1)} \leqslant rs_i x_{isr(t-1)} + (\overline{p}_i - rs_i)(x_{isrt} - u_{isrt}) - \sqrt{\frac{1-\epsilon_i}{\epsilon_i}}\|\alpha_{isr(t-1)}1^\top\Sigma_{sr(t-1)}^{1/2}\|_2, \ \forall i,s,r,t \tag{15h}$$

$$p_{isrt} \leqslant \overline{p}_i x_{isrt} - (\overline{p}_i - rs_i)u_{isrt} - \sqrt{\frac{1-\epsilon_i}{\epsilon_i}}\|\alpha_{isrt}1^\top\Sigma_{srt}^{1/2}\|_2, \ \forall i,s,r,t \tag{15i}$$

$$p_{isrt} - p_{isr(t-1)} \leqslant (\underline{p}_i + \overline{r}_i)x_{isrt} - (\underline{p}_i + \overline{r}_i - rs_i)u_{isrt} - \underline{p}_i x_{isr(t-1)} - \sqrt{\frac{1-\epsilon_i}{\epsilon_i}}\|(v_{srt})^\top\hat{\Sigma}_{srt}^{1/2}\|_2, \ \forall i,s,r,t \tag{15j}$$

$$p_{isrt} - p_{isr(t-1)} \geqslant -rs_i x_{isr(t-1)} - (\underline{p_i} + \underline{r}_i - rs_i)u_{isrt} - (rs_i - \underline{r}_i)x_{isrt} + \sqrt{\frac{1-\epsilon_i}{\epsilon_i}}\|(v_{srt})^\top\hat{\Sigma}_{srt}^{1/2}\|_2, \ \forall i,s,r,t. \tag{15k}$$

APPENDIX C

Assuming a Gaussian probability distribution with mean μ and covariance Σ, the generic chance constraint (8) can be rewritten as [23]

$$v^\top\mu + \Phi^{-1}(1-\epsilon)\sqrt{v^\top\Sigma v} \leqslant b, \tag{16}$$

where $\Phi^{-1}(1-\epsilon)$ is the inverse cumulative distribution function. The key difference between (9) and (16) is the coefficient of $\sqrt{v^\top\Sigma v}$. Fig. 7 illustrates the values of coefficients $\sqrt{\frac{1-\epsilon}{\epsilon}}$ and $\Phi^{-1}(1-\epsilon)$ as a function of confidence level, and thereby, their effects on tightening the chance constraints. For the same value of confidence level, it can be concluded that

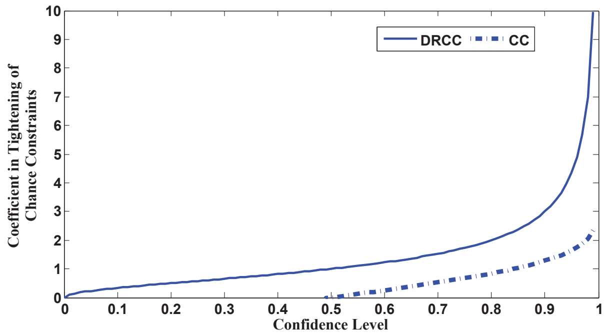

Fig. 7. Tightening coefficient of chance constraints in the DRCC and CC models

the DRCC model provides comparatively tighter chance constraints, which results in more robust solutions. This tightening effect is more remarkable for higher values of confidence level.

Appendix D

To assess the *ex-post* and *ex-ante* violation probabilities, we use the test dataset containing 5,000 trajectories. The ex-post violation probability is calculated through the out-of-sample analysis based on the simulation of the unit commitment problem with considering the recourse actions of load shedding and wind spillage. To do so, for each sample trajectory j, the tight relaxed unit commitment problem is carried out to calculate the violation indicator as follows:

$$\widehat{I}_j = \begin{cases} 1 & \text{if } ls_j > 0 \text{ or } ws_j > 0, \\ 0 & \text{otherwise}, \end{cases} \tag{17}$$

where ls_j and ws_j indicate the value of load shedding and wind spillage for sample trajectory j. We use (17) to assess the number of violations, and subsequently, the ex-post violation probability is calculated by

$$\widehat{V} = \frac{1}{N} \sum_{j=1}^{N} \widehat{I}_j, \tag{18}$$

where N is the number of sample trajectories. In contrast, for calculating the ex-ante violation probability, we do not run the tight relaxed unit commitment problem. According to the optimal in-sample values, we calculate the recourse power production of generating units for each sample trajectory by using linear decision rules. Then, we obtain the violation indicator for each trajectory j by

$$I_j = \begin{cases} 1 & \text{if } v_k^\top \gamma_j > b_k, \\ 0 & \text{otherwise}, \end{cases} \tag{19}$$

where k indicates each chance constraint from (5) and (7). We investigate the satisfaction of all chance constraints by I_j. Then, we assess the ex-ante violation probability with

$$V = \frac{1}{N} \sum_{j=1}^{N} I_j. \tag{20}$$