Scenario creation for the Unit Commitment Problem

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Outline

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Context of the project

Unit Commitment Problem

- UC Problem : finding the best commitment s.t the electrical demand is matched while the cost of these plants are minimized for a defined period of time.
- Increasing uncertainty due to intermittent renewables.
- Power generation becomes uncertain => Stochastic UC Problem on which Pr. Woodruff is working.

Pr. Woodruff's Prescient-GOSM software

- SUCP solver, on Python called "Prescient" designed for electricity grid management companies such as California Independent System Operator (CAISO) or power marketing agencies such as Bonneville Power Association (BPA). (outcome by Sandia labs & University of California Davis)
- Instead of a deterministic forecast, creation of different scenarios with their probabilities to happen : Grid Operation Scenario Maker (GOSM). => Emphasis on this in the past internships (Markov Chains - Copulas).

My internship locates in the GOSM part of the broader project.

Giving Researchers insight for the future

Uncertainty has a cost for the network. For capacity expansion and prospective cost-model for the UC commitment one important question, is

What if we were to improve the forecast technology or pay for better ?

Formalization

A scenario is a sequence of actuals or forecasts of wind or solar power production with an associated probability.

Implementation of a software simulating "plausible" wind-power scenarios for a given overall-measure of errors, the MAPE (mean absolute percent error).

Plausibility criteria

Definition

A scenario set is said to be "plausible" if :

- its distribution of errors is close to the empirical distribution of errors i.e its plausibility score is close to 1 (defined afterwards)
- its auto-correlation coefficients are close the empirical values
- When the output is forecast scenarios, the second differences are close to the empirical values

Overview : from actuals to forecasts

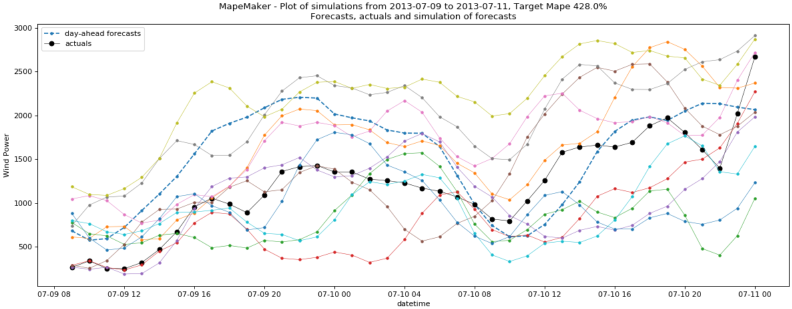


Figure – Simulation of 10 different scenarios of forecasts - CAISO Wind Power

Notations and Methodology

Measures of forecast error

Let $(x_i)_i \in \mathbb{R}^n$ and $(y_i)_i \in \mathbb{R}^n$ be two timeseries. (e.g actuals & forecasts)

$$\begin{aligned} RE &: \mathbb{R}^* \times \mathbb{R} &\to& \quad \mathbb{R} \\ & \quad x, y &\mapsto& \quad \frac{y-x}{x} \\ MARE &: \mathbb{R}^{*n} \times \mathbb{R}^n &\to& \quad \mathbb{R}_+ \\ & \quad x, y &\mapsto& \quad \sum_{i=1}^{n} \frac{|RE(x_i, y_i)|}{n} \end{aligned}$$

- Problem at 0,
- Can cause bad interpretations

Notations

$\mathcal{X}$ and $\mathcal{Y}$ is the paired input data of length n,
$\mathcal{X}_{SID}$ represent the input data from which the user wishes to simulate.
$\mathcal{E}$ denotes the random vector of errors s.t $\mathcal{E}_i = \mathbf{Y}_i - x_i$,
We want to simulate a "plausible" $\tilde{\mathbf{Y}}$ that verifies :

$$\mathbb{E}[MARE(x, \tilde{\mathbf{Y}})] = \tilde{r}$$

.

	Population	Estimation	Simulation
Error variable	$\mathcal{E}$	$\hat{\mathcal{E}}$	$\tilde{\mathcal{E}}$
Conditional distribs	$f_{\mathcal{E}\mid\mathbf{X}=x}$	$\hat{f}_{\mathcal{E}\mid\mathbf{X}=x}$	$\tilde{f}_{\mathcal{E}\mid\mathbf{X}=x}$

Methodology

1. Estimate conditional distributions of errors
2. Adjust the distributions ("simulation distributions") to meet the target mare
3. Input a Base Process to meet auto-correlation properties
4. Optimize second-differences to meet curvature properties
5. Evaluate the methods based on "scores"

Modeling the Joint Distribution

From the Relative Errors to the Errors

- Errors as IID : $\mathcal{E}/\mathbf{X}$
- Creation of Regimes, $\mathcal{E}/\mathbf{X}|\mathbf{X} \in R_k$ (2 then 3 => Problem of bounds)
- Generalization with conditional distributions $\mathcal{E}|\mathbf{X}$ => 0 ok !

Formulation of the mare :

$$m(x) = \mathbb{E}[|\mathcal{E}|\ |\mathbf{X} = x] = \int_{\varepsilon=-\infty}^{\infty} |\varepsilon| f_{\mathcal{E}|\mathbf{X}=x}(\varepsilon) d\varepsilon.$$

$$r = \mathbb{E}[\mathbb{E}[\frac{|\mathcal{E}|\ |\mathbf{X}}{\mathbf{X}}]] = \mathbb{E}[\frac{m(\mathbf{X})}{\mathbf{X}}]$$

.

Empirical joint distribution

WARNING : Can be deceptive, the actual distribution of errors for low power has smaller scale than high power.

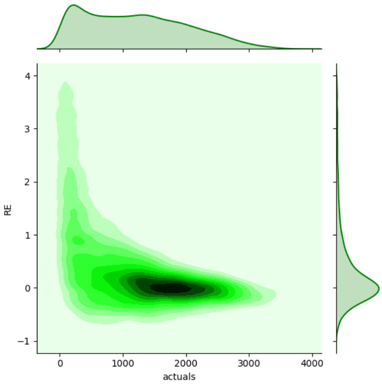


Figure – Empirical joint distribution of $(\frac{\mathcal{E}}{\mathbf{X}}, \mathbf{X})$ - CAISO Wind Power

Fitting Beta Distributions

$$\boxed{f_{\mathcal{E}|\mathbf{X}=x}(\varepsilon) = beta(\varepsilon;\ (\alpha, \beta, l, s)) = \frac{(\frac{\varepsilon - l}{s})^{\alpha - 1}(1 - \frac{\varepsilon - l}{s})^{\beta - 1}}{B(\alpha, \beta)}}$$

- 4 parameters : 2 shapes, 2 locations
- Sample of estimation
- Bound constraints
- Method of moments

Sample of estimation - insight on the input of estimation

We estimate over a sample with a% of data on the left and a% on the right. The sample of estimation is centered on $\bar{x}(x;\ a)$. For $x \in \mathcal{X}$ we take the closest the closest $\bar{x}(x';\ a)$ to estimate the parameters.

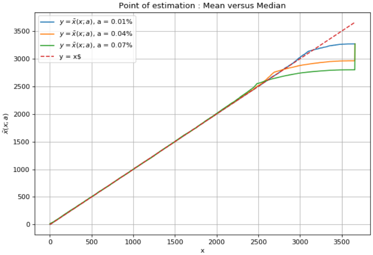


Figure – $\bar{x}(x;\ a)$- CAISO Wind Power

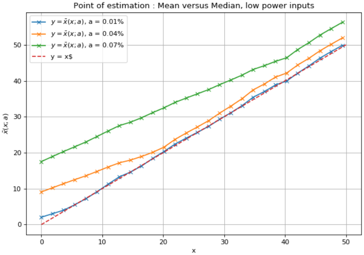


Figure – $\bar{x}(x;\ a)$- low power input - CAISO Wind Power

$\hat{S}_x = (\hat{\alpha}(x), \hat{\beta}(x), \hat{l}(x), \hat{s}(x))$

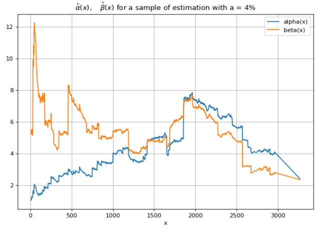


Figure – $\hat{\alpha}(x), \hat{\beta}(x)$ - CAISO Wind Power

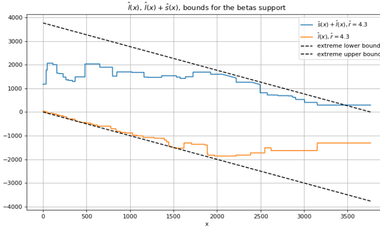
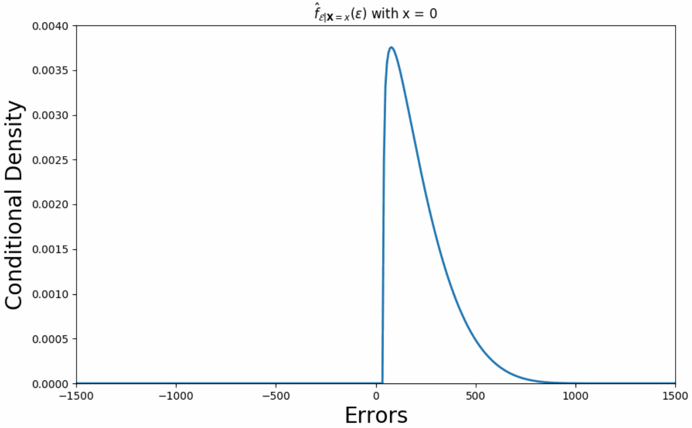
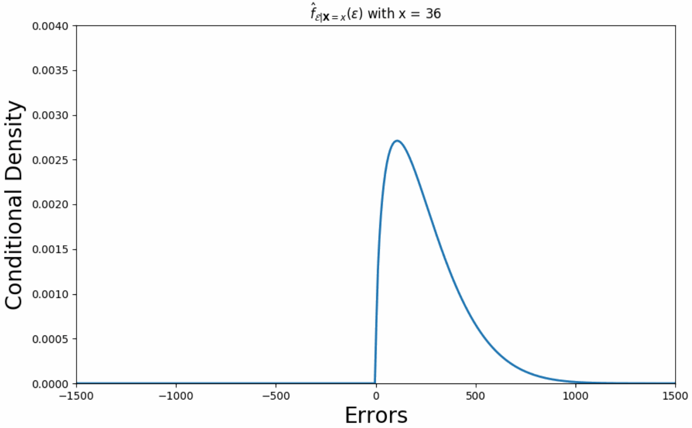


Figure – $\hat{l}(x), \hat{s}(x)$ - CAISO Wind Power

Estimated conditional distributions



Estimated conditional distributions



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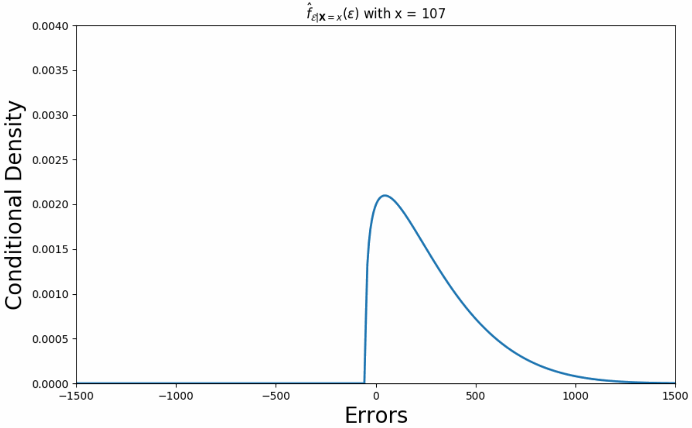
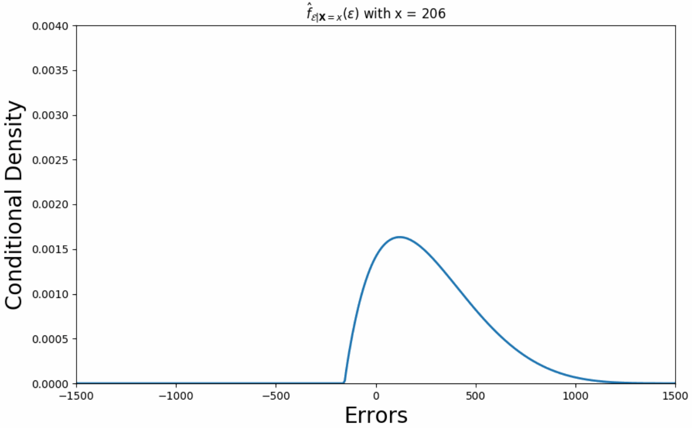
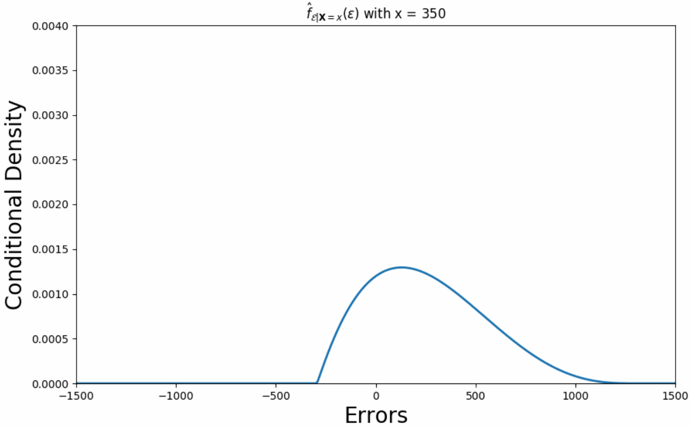


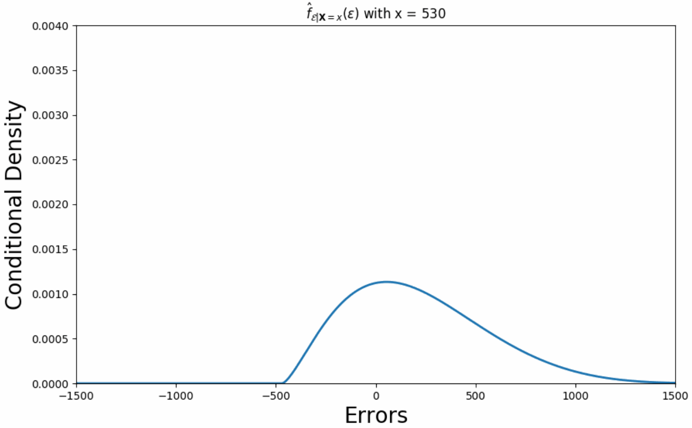
Estimated conditional distributions



Estimated conditional distributions



Estimated conditional distributions



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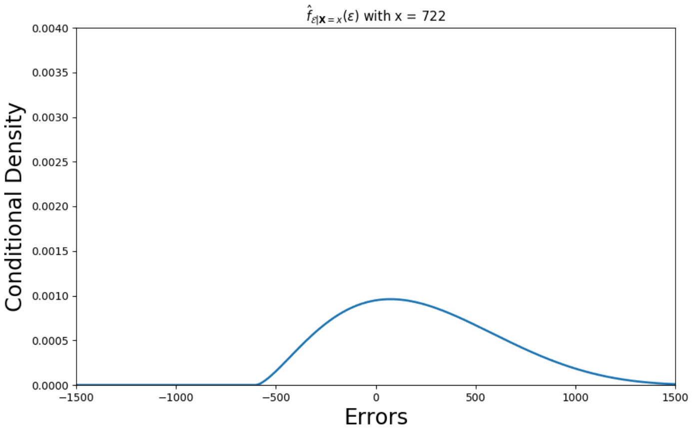
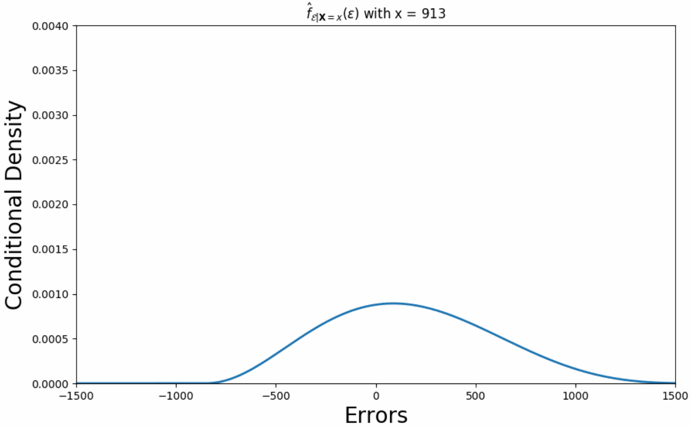


Estimated conditional distributions



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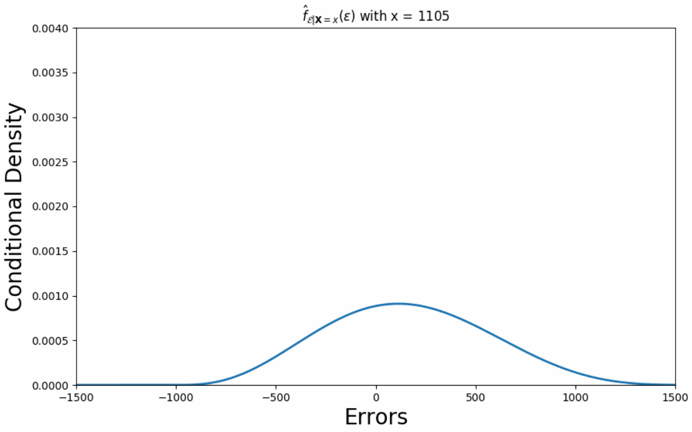

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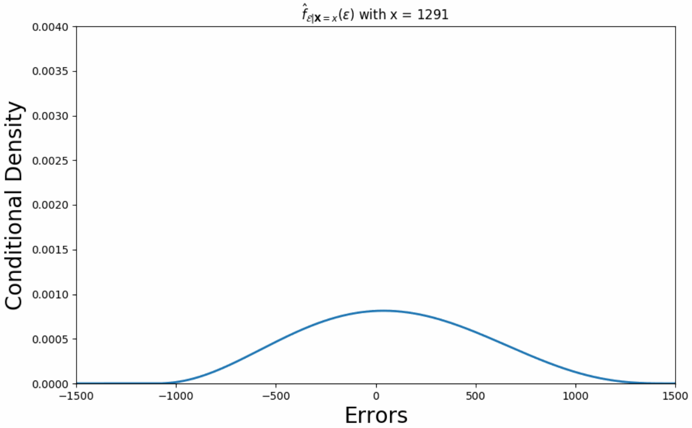

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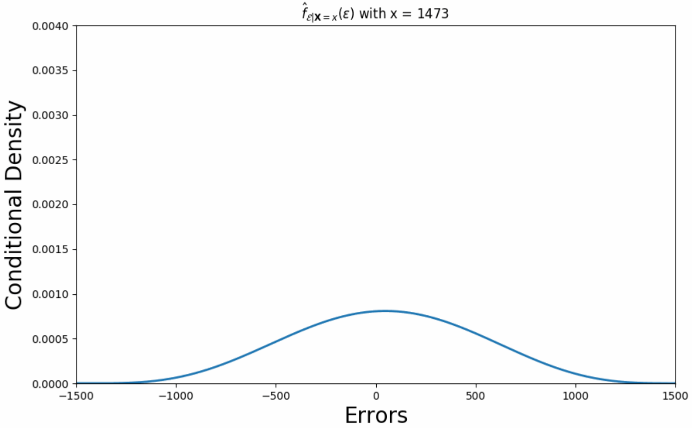

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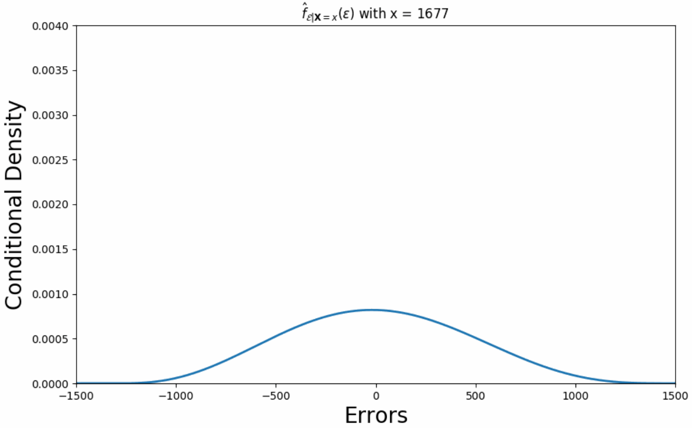

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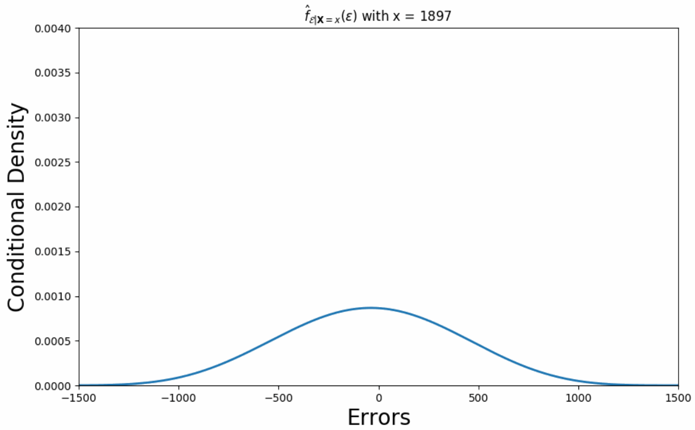

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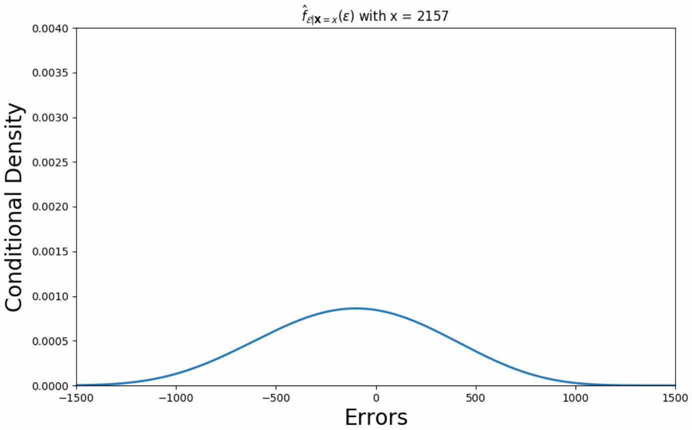

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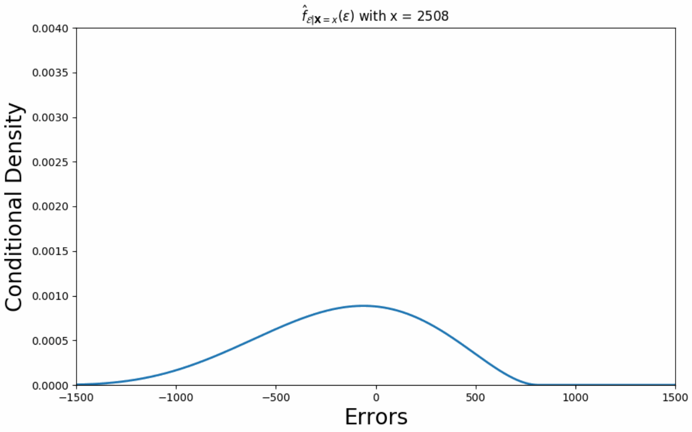

Adjusting the conditional distributions

A target mare

We want to model each conditional distribution so that we control their MAE => we control the MARE of each distribution => we control the MARE of the simulation over the $\mathcal{X}_{SID}$.
We define the following function being the mean absolute error for a beta distribution of parameters (α, β, l, s)

$$\nu(l, s; \alpha, \beta) = \int_{\varepsilon=l}^{s+l} |\varepsilon| b(\varepsilon; \alpha, \beta, l, s) d\varepsilon.$$

A program to get $\tilde{\mathcal{S}}_{x,m}$

For a given target $m(x)$, we define $\tilde{\mathcal{S}}_{x,m} = (\hat{\alpha}(x), \hat{\beta}(x), \tilde{l}(x), \tilde{s}(x))$ so that :

$$
\begin{aligned}
(\tilde{l}(x), \tilde{s}(x)) = \underset{l,s}{\operatorname{argmin}} \quad & (l - \hat{l}(x))^2 + (s - \hat{s}(x))^2 \\
\text{s.t.} \quad & l \in \mathbb{R},\ s \in \mathbb{R}_+ \\
& l \geq -x \\
& s \leq cap - x - l \\
& \nu(l, s; \hat{\alpha}(x), \hat{\beta}(x)) = m(x)
\end{aligned}
\tag{1}
$$

$\tilde{l}, \tilde{s}$ for different values of m

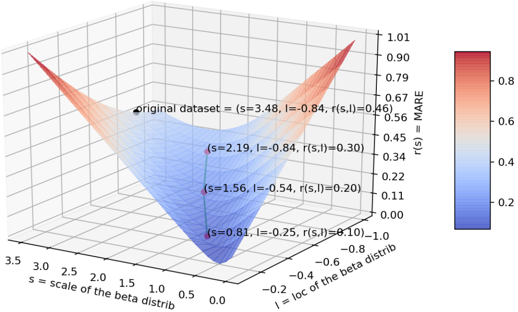

Figure – Adjusting the location parameters - CAISO Wind Power

Our Goal

Simulate conditional distributions of $\tilde{\mathcal{E}}\,|\mathbf{X}=x$ adapted to the $\mathcal{X}_{SID}$ so that :

$$\mathbb{E}_{\tilde{\mathcal{E}}}[\frac{1}{n_{SID}}\sum_{x\in\mathcal{X}_{SID}}\frac{|\tilde{\mathcal{E}}|\,|\mathbf{X}=x}{x}]=\tilde{r}$$

A target function

Let's define $\Omega_{\mathcal{X}_{SID}}$ as the set of functions $\omega_{\mathcal{X}_{SID}}$ defined on $\mathcal{X}_{SID}$ such that $\frac{1}{n_{SID}} \sum_{x \in \mathcal{X}_{SID}} \omega_{\mathcal{X}_{SID}}(x) = 1$.

$$\begin{array}{cccc} \tilde{m} & : & \mathcal{X}_{SID} \times \mathbb{R}_+ \times \Omega_{\mathcal{X}_{SID}} & \rightarrow & \mathbb{R}_+ \\ & & x, \tilde{r}, \omega & \mapsto & \tilde{r} x \omega(x), \; x > 0 \end{array}$$

Defined at zero by continuity. Allocating the MAE to each conditional distribution will define $\tilde{\mathcal{S}}_{x,\tilde{m}}$. Thus, we will have :

$$\int_{\varepsilon=-\infty}^{\infty} |\varepsilon| b(\varepsilon, \tilde{\mathcal{S}}_{x,\tilde{m}}) d\varepsilon = \tilde{m}(x; \; \tilde{r}, \omega_{\mathcal{X}_{SID}}), \quad \forall x \in \mathcal{X}_{SID}.$$

Hitting the MARE

Then, the expectancy of the MARE with the errors drawn of these distributions and with the inputs in the $\mathcal{X}_{SID}$ is :

$$\begin{aligned}
\mathbb{E}_{\tilde{\mathcal{E}}}\Big[\frac{1}{n_{SID}} \sum_{x \in \mathcal{X}_{SID}} \frac{|\tilde{\mathcal{E}}| \ |\mathbf{X} = x}{x}\Big] &= \frac{1}{n_{SID}} \sum_{x \in \mathcal{X}_{SID}} \frac{\mathbb{E}_{\tilde{\mathcal{E}}}[|\tilde{\mathcal{E}}| \ |\mathbf{X} = x]}{x} \\
&= \frac{1}{n_{SID}} \sum_{x \in \mathcal{X}_{SID}} \frac{\tilde{m}(x; \ \tilde{r}, \omega_{\mathcal{X}_{SID}})}{x} \\
&= \frac{\tilde{r}}{n_{SID}} \sum_{x \in \mathcal{X}_{SID}} \omega_{\mathcal{X}_{SID}}(x) \\
&= \tilde{r}
\end{aligned}$$

A constraint on the weight function : the plausibility assumption

This was true for all $\omega \in \Omega_{SID}$, how do we choose ω for a $\mathcal{X}_{SID}$?
If we define the following $\hat{\omega}_{\mathcal{X}}$ function,

$$\forall x \in \mathcal{X},\ \hat{\omega}_{\mathcal{X}}(x) := \frac{\hat{m}(x)}{x r_{\hat{m}}} = \frac{\int_{\varepsilon=-\infty}^{\infty} |\varepsilon| \hat{f}_{\mathcal{E}|\mathbf{X}=x}(\varepsilon) d\varepsilon}{x r_{\hat{m}}}$$

$$\mathcal{X}_{SID} = \mathcal{X}, \quad \tilde{r} = r_{\hat{m}} \implies \forall x \in \mathcal{X}, \tilde{l}(x) = \hat{l}(x) \text{ and } \tilde{s}(x) = \hat{s}(x)$$

$\hat{m}$ and $\hat{\omega}_{\mathcal{X}}$

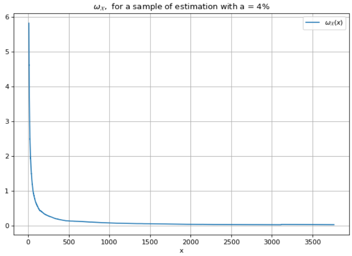

Figure – $\hat{\omega}_{\mathcal{X}}$ - CAISO Wind Power

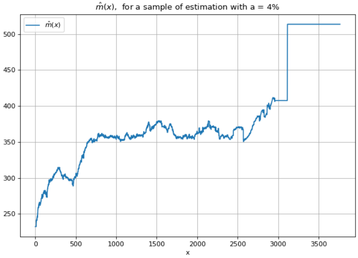

Figure – $\hat{m}$ - CAISO Wind Power

Plausiblity score

$$P_{\mathcal{X}_{SID}} = \frac{1}{n_{SID}} \sum_{x \in \mathcal{X}_{SID}} \hat{\omega}_{\mathcal{X}}(x)$$

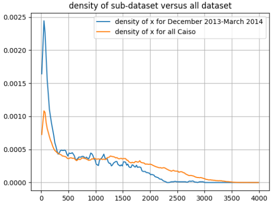

Figure – Comparision test density versus all dataset density- CAISO

NB : $P_{\mathcal{X}_{SID}}$ is the plausibility score : the closer it is from 1, the more plausible the distributions are for being close to the estimations.

Generalized weight function

$$\forall x \in \mathcal{X}_{SID},\ \tilde{\omega}_{\mathcal{X}_{SID}}(x) := \frac{\omega_{\mathcal{X}}}{P_{\mathcal{X}_{SID}}}$$

- $\frac{1}{n_{SID}} \sum_{x \in \mathcal{X}_{SID}} \tilde{\omega}_{\mathcal{X}_{SID}}(x) = 1$ and $\tilde{\omega}_{\mathcal{X}_{SID}} \in \Omega_{\mathcal{X}_{SID}}$.
- For a given $\tilde{r} \in \mathbb{R}_+$, we compute a $\tilde{\omega}_{\mathcal{X}_{SID}}$ which allocates the absolute errors across $\mathcal{X}_{SID}$ based on the allocation from $\mathcal{X}$
- With these two parameters we can compute $\tilde{m}(x;\ \tilde{r}, \tilde{\omega}_{\mathcal{X}_{SID}}),\ x \in \mathcal{X}_{SID}$

Defining $\tilde{\mathcal{E}}$ from this target function, will get us $\mathbb{E}_{\tilde{\mathcal{E}}}[\frac{1}{n_{SID}} \sum_{x \in \mathcal{X}_{SID}} \frac{|\tilde{\mathcal{E}}|\ |\mathbf{X}=x}{x}] = \tilde{r}$.

$\tilde{S}_{x,m} = (\hat{\alpha}(x), \hat{\beta}(x), \tilde{l}(x), \tilde{s}(x))$

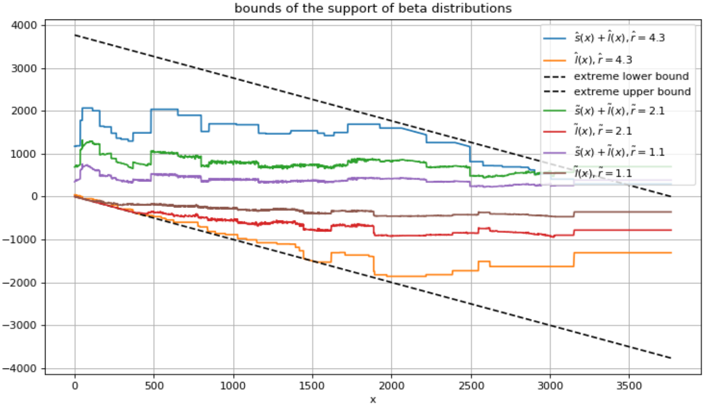

Figure – Adjusting the location parameters - CAISO Wind Power

Change in the distribution for X=750

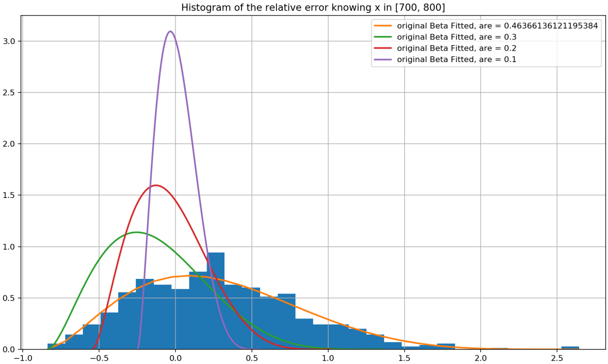

Figure – Adjusting the location parameters - CAISO Wind Power

Base Process

The idea behind Base Process - ARTAFIT

We have fitted conditional distributions but for now we have completely ignored the auto-correlation structure. The idea is to simulate a Base Process U_t of marginal Uniform in [0,1] and to simulate the errors via the transformation $\hat{F}^{-1}_{\mathcal{E}|\mathbf{X}=x_t}(U_t)$.
We are going to model $Z_t = \phi(U_t) \in (-\inf, \inf)$ as a Gaussian Process and more specifically as an ARMA process.
Heuristically, we will show that this method gets us a relatively close auto-correlation function for our simulations.

Empirical Base Process

We denote the CDF of the standard normal distribution ϕ and the CDF of the conditional distribution $\mathcal{E}|\mathbf{X} = x_t$, which is a beta distribution fit using $\mathcal{X}$, $\hat{F}_{\mathcal{E}|\mathbf{X}=x_t}$. Let us define the following timeseries $(\hat{Z}_t)_t$:

$$\forall t \leq n, \quad \hat{Z}_t = \phi^{-1}(\hat{F}_{\mathcal{E}|\mathbf{X}=x_t}(\varepsilon_t))$$

$(\hat{Z}_t)_t$ is the base process timeseries of the dataset. Its empirical distribution is close to a standard Gaussian.

Estimation of the base process

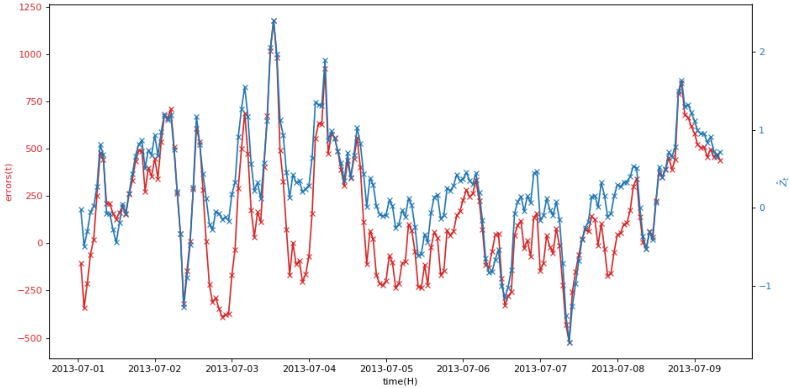

Figure – Inferring the base process from the errors

Fitting an ARMA Process

Definition

$\{\mathbf{Z}_t\}$ is a base process if

- $\{\mathbf{Z}_t\}$ follows an ARMA process of order p and q :

$$\mathbf{Z}_t = \sum_{h=1}^{p} a_h \mathbf{Z}_{t-h} + \sum_{h=1}^{q} b_h \epsilon_{t-h} + \epsilon_t$$

  Where $\{\varepsilon_t\}$ are the iid Gaussian error of mean 0 and variance σ_ϵ^2.

- $Var[\mathbf{Z}_t] = 1$, $E[\mathbf{Z}_t] = 0$, so that for all t, $\quad \mathbf{Z}_t \sim N(0,1)$.

We run a grid search over multiple (p,q) and we select the ARMA model to minimize the BIC criterion. Then, we can model directly the error by :

$$\mathcal{E}_i = F_{\mathcal{E}|\mathbf{X}=x_i}(\phi(\mathbf{Z}_i))$$

Curvature

Second difference simulated - actuals

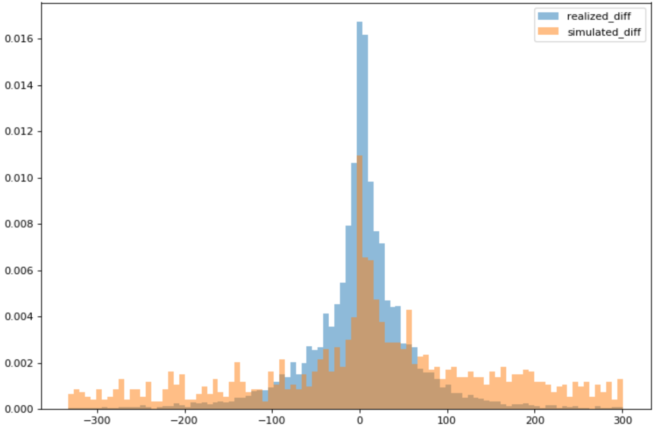

Figure – Inferring the base process from the errors

Second differences

Let $(y_i)_i \in \mathbb{R}^n$ be two time series. We define the following :

$$s_i = y_{i+1} - 2y_{i-1} + y_{i-2} \quad \forall i < n \qquad \text{(Second Differences)}$$

MIP to solve

$$
\begin{aligned}
\min \quad & \sum_{i=2}^{n} W_s \left(\lambda_i^+ + \lambda_i^- - d \right)^2 + W_\varepsilon \left(y_i - x_i - \varepsilon_i \right)^2 \\
\text{s.t.} \quad & y \in \mathbb{R}_+^n,\ \lambda^+ \in \mathbb{R}_+^n,\ \lambda^- \in \mathbb{R}_+^n,\ b \in \{0,1\}^n \\
& \lambda_i^+ - \lambda_i^- = y_i - 2y_{i-1} + y_{i-2}, \quad \forall i \leq n \\
& \lambda_i^+ \leq b_i * d_{max} \\
& \lambda_i^- \leq (1 - b_i) * d_{max}
\end{aligned}
\tag{2}
$$

Results and scores

3 types of simulation

We recall the three types of simulations :

- A) IID base process, ϕ_1
- B) ARMA base process, ϕ_2
- C) ARMA base process and curvature optimization, ϕ_3

Overview

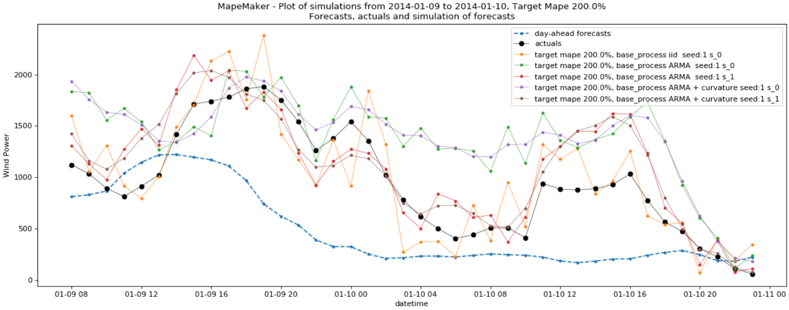

Figure – Simulation of different scenarios of forecasts - CAISO Wind Power

Validation of the mape

$$S_m(M, n_t, k) = \sqrt{\sum_{i=1}^{M} (\tilde{r} * 100\% - MAPE((x_i)_{i \leq n_t}, \phi_k((x_i)_{i \leq n_t})))^2}$$

Validation of the auto-correlation

Let us define the functionnal

$$\hat{\rho}((\varepsilon_i)_{i\leq n}, j) = \frac{1}{(n-j)\sigma^2}\sum_{i=0}^{n-j}\varepsilon_{i+j}\varepsilon_i$$

$$S_{ac}(M, n_t, k, p) = \sqrt{\sum_{i=1}^{M}\sum_{j=1}^{p}(\hat{\rho}((\varepsilon_i)_{i\leq n}, j) - \hat{\rho}((\phi_k(x_i) - x_i)_{i\leq n_t}, j))}$$

Validation of the second difference

Let us define the functionnal

$$D((y_i)_{i \leq n}) = \frac{1}{n-2} \sum_{i=0}^{n-2} y_i - 2y_{i-1} + y_{i-2}$$

$$S_{sd}(M, n_t, k) = \sqrt{\sum_{i=1}^{M} (D((y_i)_{i \leq n}) - D(\phi_k(x_i)_{i \leq n_t}))^2}$$

Total Score

The score function is the sum of those three targets weighted :

$$\begin{aligned} S(M, n_t, k, p;\ w_m, w_{ac}, w_{sd}) = w_m * S_{mare}(M, n_t, k) + \\ w_{ac} * S_{auto\_correlation}(M, n_t, k, p) + \\ w_{sd} * S_{second\_differrence}(M, n_t, k) \end{aligned}$$

Convergence of the MAPE (M)

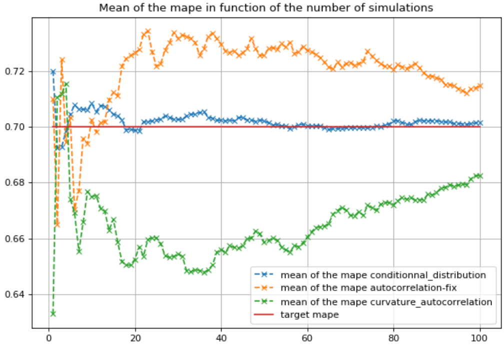

Figure – Convergence of the Mape target for the three options

Auto-correlation

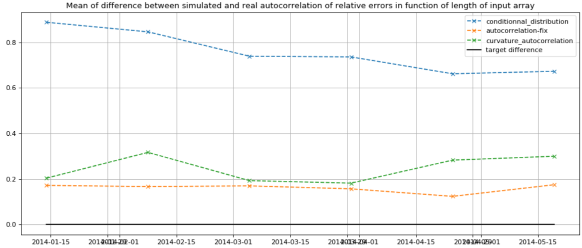

Figure – Auto-correlation ok for option 2 and 3

Second Difference

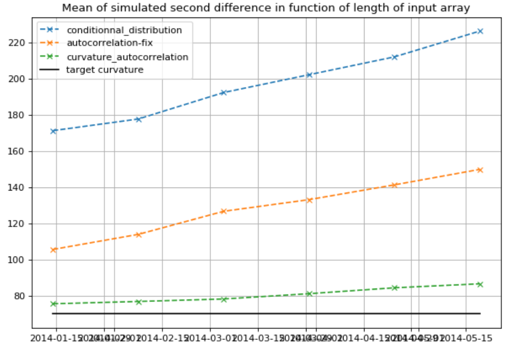

Figure – Second difference ok for option 3

Time of computation

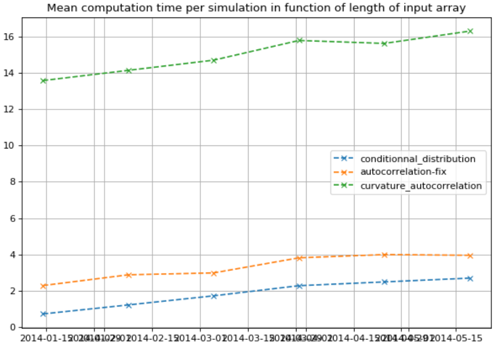

Figure – Option 3 takes a lot of time

Conclusion

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Next Year

- short term : unittest, solar power scenarios, rare events (!)
- Hope to continue working on UC Problem, more on the operational research side
- UC Berkeley, Master in Systems Engineering

Thank you !