MATH 239 Spring 2024: Tutorial 7 problems

Friday July 12, Tuesday July 16

Some of these problems will be discussed during the tutorials with a TA. No solutions will be provided outside of the tutorials.

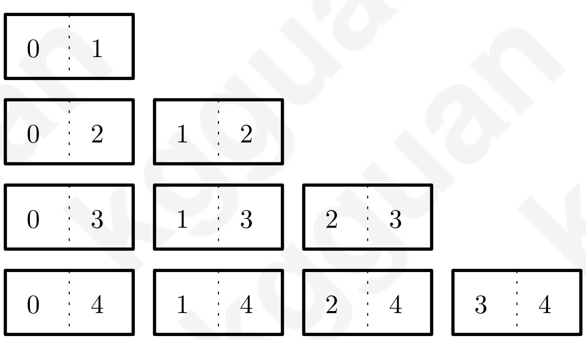
T7-1. Let G be a graph where every vertex has even degree. Prove that every cut of G has even number of edges, and G does not have a bridge.

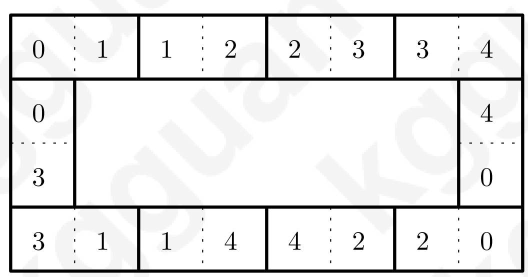
T7-2. Let T be a tree on n vertices where every vertex has degree 1 or 5. Prove that $n \equiv 2 \pmod 4$. State and prove a generalization of this result.

T7-3. Let T be a tree with $n \geq 2$ vertices, and let x be a fixed vertex in T. For any vertex v in T, define $d(v)$ to be the length of the unique x, v-path in T. Prove that

$$\sum_{v \in V(T)} d(v) \leq \binom{n}{2}.$$

Extra problem (time permitting): Let $n \geq 3$ be an integer. Consider a set of 1×2 domino tiles where each tile contains two distinct numbers from the set $\{0, 1, \ldots, n\}$ (one number in each 1×1 square), and all $\binom{n+1}{2}$ unordered pairs of distinct numbers are represented. For example, when $n = 4$, we have the set represented in the diagram on the left. The *main rule* in placing dominoes is that when two dominoes share an edge, the adjacent numbers are the same. We wish to place all the dominoes in a rectangular pattern following the main rule. An example is represented in the diagram on the right.





Prove that there exists a way to place all $\binom{n+1}{2}$ dominoes in a rectangular pattern following the main rule if and only if n is even.