

controlled recursion + recursion Ke base case se return maarte time kuch work karna!!

## BACKTRACKING:

y print 1 n times :  $n=5$  O/P  $\Rightarrow 1 1 1 1 1$

```
void print (int n)
if (n==0) return;
print (n-1); 0
cout << "1" << endl;
```

}

```
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0  
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if (n==0) return;  
print (n-1);  
cout << "1" << endl;

}

1  
void print (int n)  
if (n==0) return;  
print (n-1); 0  
cout << "1" << endl;

}

2  
void print (int n)  
if (n==0) return;  
print (n-1); 0  
cout << "1" << endl;

}

⑥  $f(S, sum=0)$

3  
 $f(n, sum=0)$

$\uparrow$  (n=-0) ret sum;

$sum = sum + n$  3

$f(n-1, sum);$

2  $\uparrow$  80  
 $f(n, sum=0)$

$\uparrow$  (n=-0) ret sum;

$sum = sum + n$  5

$f(n-1, sum);$

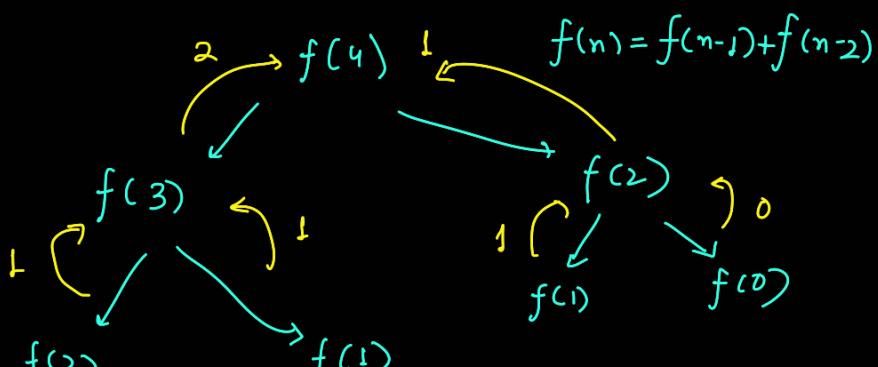
1  $\uparrow$  80  
 $f(n, sum=0)$

$\uparrow$  (n=-0) ret sum;

$sum = sum + n$  6

$f(n-1, sum);$

[Recursion tree for fibonacci sequence]:



6  
 $f(0, sum=0)$

$\uparrow$  (n=-0) ret sum;

$sum = sum + n$

$f(n-1, sum);$



$$f(4) = 3 \quad \text{and}$$

$$2^0 + 2^1 + 2^2 + \dots \underbrace{\quad \quad \quad}_{n \text{ terms}}$$

$$\frac{2^0 (2^n - 1)}{2-1} = 2^n - 1$$

$$\boxed{\text{Time complexity} = O(2^n)}$$

Recursion on Subsequences : a contiguous/non-contiguous sequence, which follows the order.

$$\text{arr} = [3, 1, 2], n=3$$

3      1      2

$$\text{Total} = 2^n = 2^3 = \underline{\underline{8}}$$

all subsequences  $\Rightarrow$

$\{\}$	$\times$	$\times$	$\times$
$\{3\}$	$\checkmark$	$\times$	$\times$
$\{1\}$	$\times$	$\checkmark$	$\times$
$\{2\}$	$\times$	$\times$	$\checkmark$
$\{3, 1\}$	$\checkmark$	$\checkmark$	$\times$
$\{1, 2\}$	$\times$	$\checkmark$	$\checkmark$
$\{3, 2\}$	$\checkmark$	$\times$	$\checkmark$
$\{3, 1, 2\}$	$\checkmark$	$\checkmark$	$\checkmark$

(

using pick not pick strategy:

$\Rightarrow \underline{\underline{V V Imp}}$

Piece of  
code

```
void printsbs( int idx, vector<int> ds, vector<int> arr, int n ) {
```

```
    if (idx == n) {
        for (auto x: ds) cout << x << " ";
        cout << endl;
        return;
    }
}
```

```
ds.push_back( arr[idx] ); // pick → arr
```

```
printsbs( idx+1, ds, arr, n );
```

this func<sup>n</sup> prints all  
subsets from idx to n.

```
ds.pop_back(); // not pick
```

```
printsbs( idx+1, ds, arr, n );
```

```
}
```

$$arr = [3, 2, 1]$$

0

```
void printsbs( int idx, vector<int> ds, vector<int> arr, int n ) {
    if (idx == n) {
        for (auto x: ds) cout << x << " ";
        cout << endl;
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    ds.push_back( arr[idx] ); // pick
    printsbs( idx+1, ds, arr, n ); 0
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```

1 [3]

```
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    if (idx == n) {
        for (auto x: ds) cout << x << " ";
        cout << endl;
        return;
    }
    ds.push_back( arr[idx] ); // pick
    printsbs( idx+1, ds, arr, n ); 0
    ds.pop_back(); // not pick
    printsbs( idx+1, ds, arr, n );
}
```

0 1 P → 3 2 1

2 [3, 2]

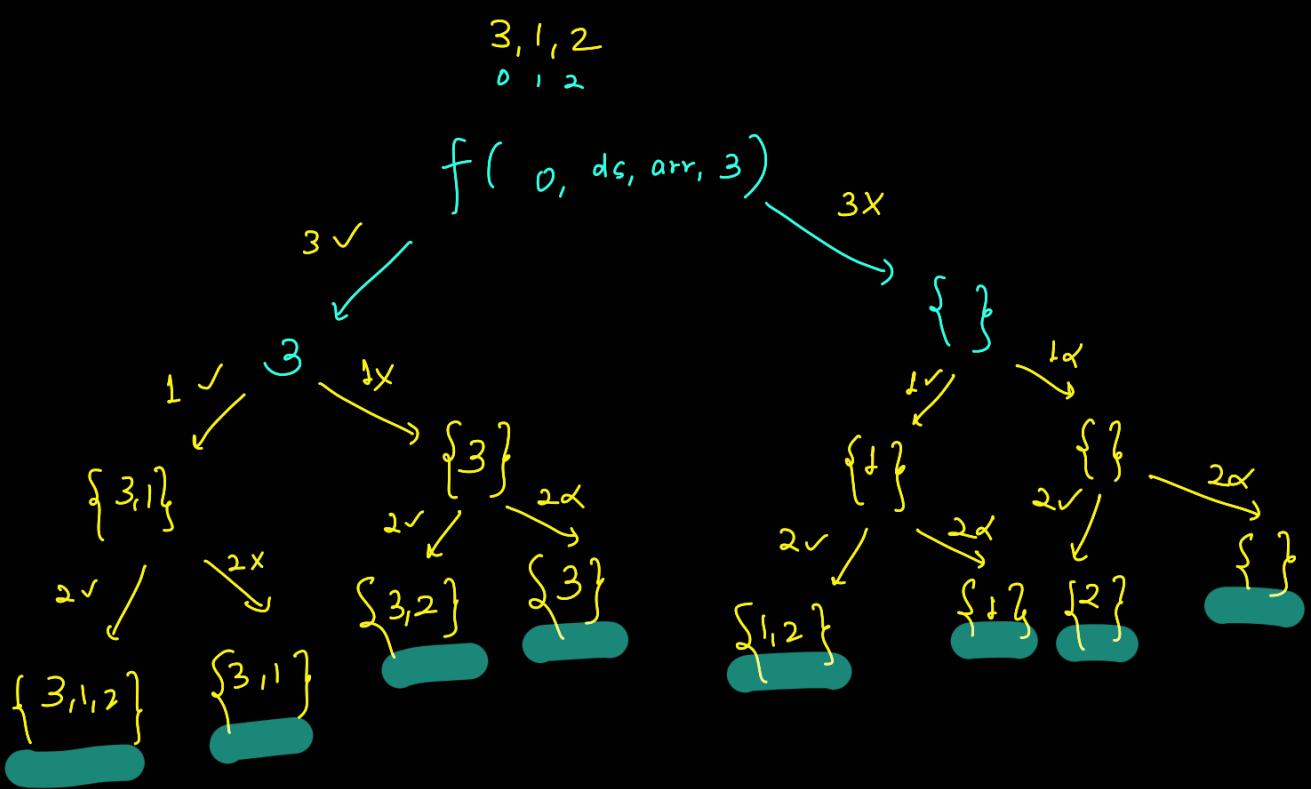
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        cout << endl;
        return;
    }
    ds.push_back( arr[idx] ); // pick
    printsbs( idx+1, ds, arr, n ); 0
    ds.pop_back(); // not pick
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}
```

3, 2

3 [3, 2]

```
void printsbs( int idx, vector<int> ds, vector<int> arr, int n ) {
    if (idx == n) {
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        cout << endl;
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    if (idx == n) {
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    }
    ds.push_back( arr[idx] ); // pick
    printsbs( idx+1, ds, arr, n );
    ds.pop_back(); // not pick
    printsbs( idx+1, ds, arr, n );
}
```

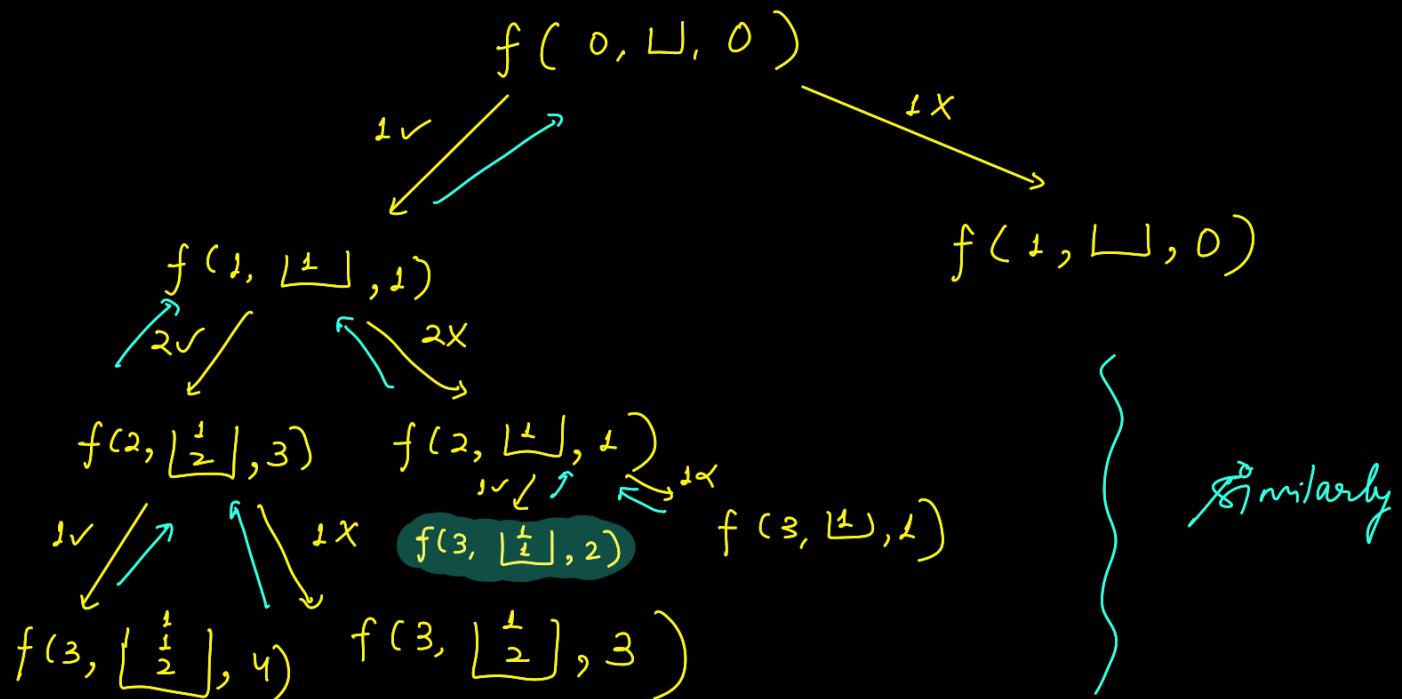


Subseq. sum  $k$ .

$$k = 2$$

$[1, 2, 1]$

$f(i, \text{L}, \text{sum})$



Pattern 2

Ques  $\rightarrow$  Point only one subsequence whose sum is  $k$ .

★ ★ ★ technique to print only one ans  $\rightarrow$  functional recursion  
 $\hookrightarrow$  base case:

cond  $\rightarrow$  satisfied  
 return true;

else return false;

}

$f() == \text{true}) \text{return};$

else return false;

Pattern 3  $\Rightarrow$  Give the count of subsequences with sum k.

$[1, 2, 1] \quad k = 2 \quad \text{OIP: count} = 2$

$f()$  {  
base case

return 1  $\rightarrow$  cond<sup>n</sup> satisfies;

return 0  $\rightarrow$  cond<sup>n</sup> do not satisfies.

$\left. \begin{array}{l} \text{left} = f(); \\ \text{right} = f(); \\ \text{return left + right;} \end{array} \right\}$  agar n recursion calls ho toh:  
 $s = 0$   
 $\text{for } (i = 1 \rightarrow n) \{$   
 $\quad s += f();$   
 $\text{return } s;$

$\text{arr} = [1, 2, 1] \quad \text{sum} = 2$ .

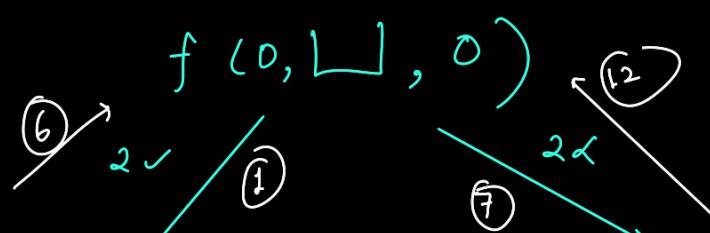


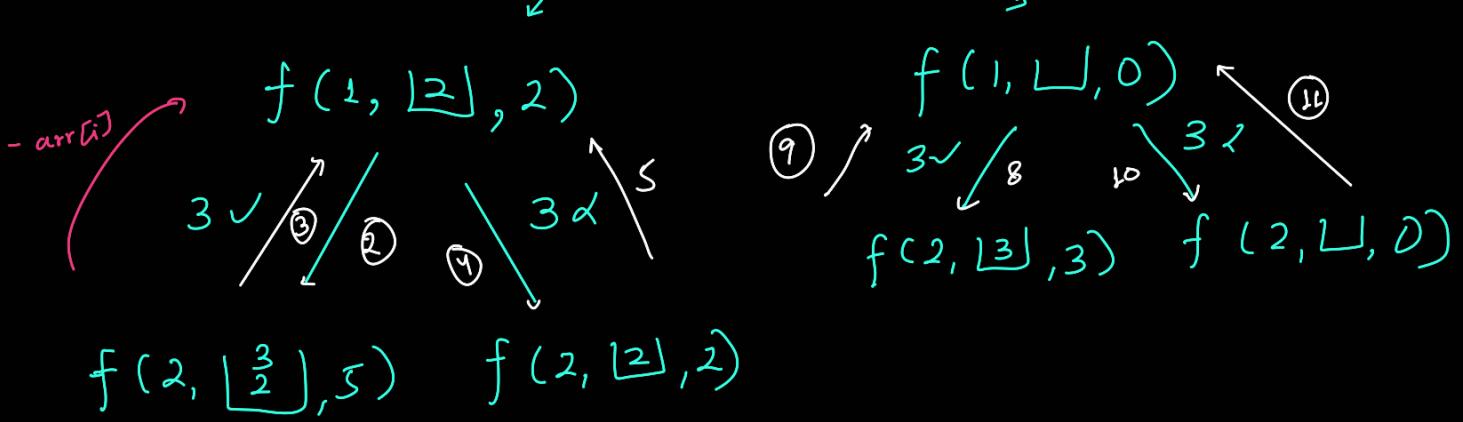
Ques  $\Rightarrow$  Subset Sum  $\rightarrow \{2, 3\}$  sum

Subsets	$\subseteq$	$\subseteq$	$\{2, 3\}$	5
	$\alpha$	$\alpha$	$\{\}$	0 OIP = $\{0, 2, 3, 5\}$
	$\subseteq$	$\alpha$	$\{2\}$	2
	$\alpha$	$\subseteq$	$\{3\}$	3

$f(i, ds, sum)$

$\sqcup = \{2, 3\}$





base  $f(i == n)$  {  
 $\text{ans.push\_back}(\text{sum});$

while backtracking,

- subtract the sum.

## # Generate Permutations of an array/string:

(Recursion tree):  $f(\text{index}, \text{nums}, \text{ans})$

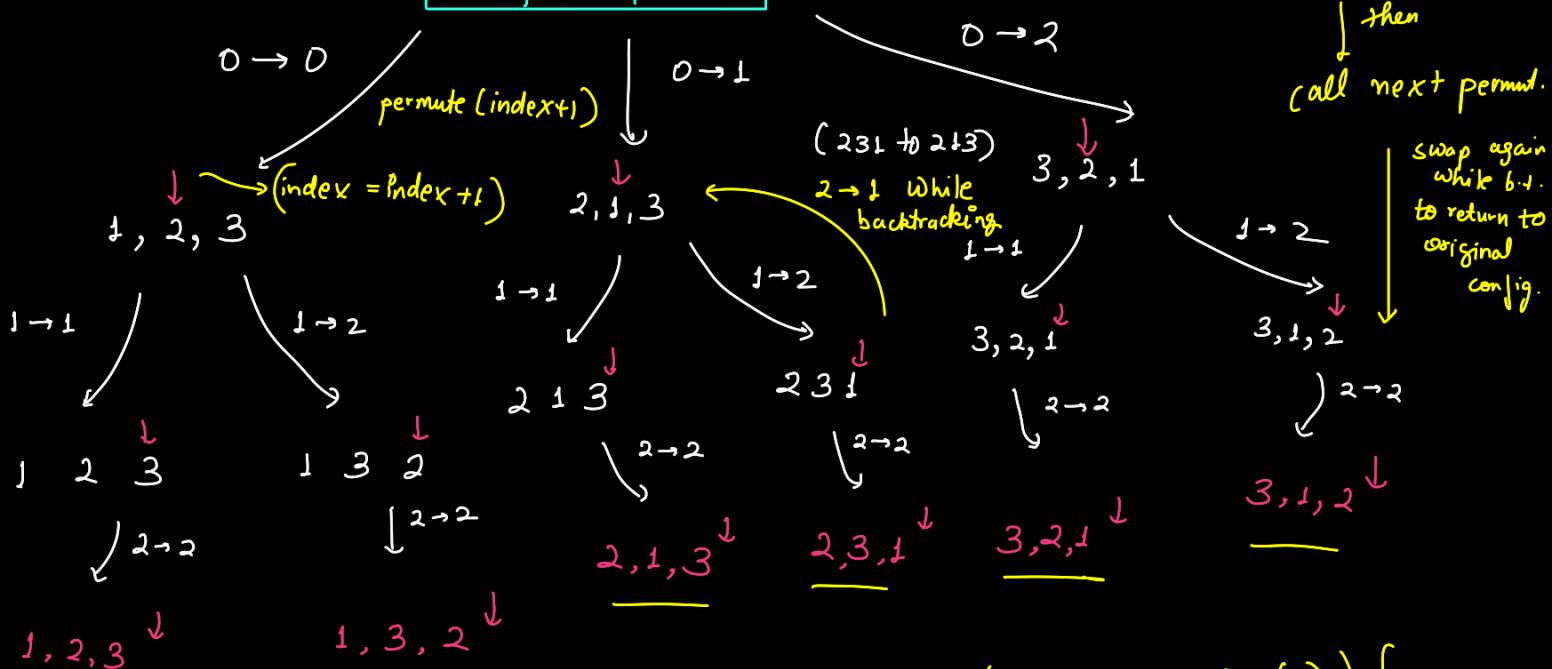


$i = \text{index} \rightarrow n-1$

swap

then  
call next perm.

swap again  
while b.t.  
to return to  
original config.



base case: if ( $\text{index} == \text{arr.size}()$ ) {  
 $\text{ans.push\_back}(\text{arr});$







