Testing of hypothesis (Large samples)

Remarks : The **p-value** defines the smallest value of alpha for which the null hypothesis can be rejected.

For example, if the p-value of a test is 0.038, the null hypothesis cannot be rejected at alpha= 0.01 because 0.038 is the smallest value of alpha for which the null hypothesis can be rejected. However, the null hypothesis can be rejected for alpha= 0.05.

For two-tailed test, we must double the p-value

p-value < alpha, reject null

p-value > = alpha, accept null

Z test for one sample mean and two sample mean

Example

```
install.packages("distributions3")
library(distributions3)
x=160
mu=165
n=100
s=10
z_stat=(x-mu)/(s/sqrt(n))
z_stat

Z=Normal(0,1)
p_value=2*cdf(Z,-5)
p_value

2*pnorm(5,lower.tail = FALSE)

alpha=0.05
if(p_value<alpha){
  print("Reject Null Hypothesis")
} else{
  print("Accept Null Hypothesis")
}
```

Z test for one sample proportion

```
prop.test(x, n, p = NULL, alternative = "two.sided", correct = TRUE)
```

- **x**: a vector of counts of successes
- **n**: a vector of count trials
- **alternative**: a character string specifying the alternative hypothesis
- **correct**: a logical indicating whether Yates' continuity correction should be applied where possible

Note that, by default, the function prop.test() used the Yates continuity correction, which is really important if either the expected successes or failures is < 5. If you don't want the correction, use the additional argument correct = FALSE in prop.test() function. The default value is TRUE. (This option must be set to FALSE to make the test mathematically equivalent to the uncorrected z-test of a proportion.)

```
prop.test(x = 95, n = 160, p = 0.5, correct = FALSE, alternative =
"less")
```

Z test for two sample proportion

```
prop.test(x = c(490, 400), n = c(500, 500), alternative = "less")
```

Exercise

1. A machine is set to produce metal plates of thickness 1.5 cms with standard deviation of 0.2 cm. A sample of 100 plates produced by the machine gave an average thickness of 1.52 cms. Is the machine fulfilling the purpose?

Write a R program for above problem.

2. The average marks scored by 32 boys are 72 with SD of 8, while that for 36 girls is 70 with SD of 6. Test at 1% LOS whether the boys perform equal as girls.

Write a R program for above problem.

3. The fatality rate of typhoid patients is believed to be 17.26%. In a certain year 640 patients suffering from typhoid were treated in a metropolitan hospital and only 63 patients died. Can you consider the hospital efficient? [1% LOS]

Write a R program for above problem.

4. 15.5% of a random sample of 1600 undergraduates were smokers, where as 20% of a random sample of 900 postgraduates were smokers in a states. Can we conclude that less number of undergraduates are smokers than the postgraduates?

Write a R program for above problem.

5. A random sample of 400 members is found to have a mean of 4.45 cm. Can it be reasonably regarded as a sample from a large population whose mean is 5 cm and whose variance is 4 cms?

Write a R program for above problem.

Application/Case Study

1. To Plan the Marketing Strategies

Many businesses often use hypothesis testing to determine the impact of the newly implemented marketing techniques, campaigns or other tactics on the sales of the product. For example, the marketing department of the company assumed that if they spend more the digital advertisements it would lead to a rise in sales. To verify this assumption, the marketing department may raise the digital advertisement budget for a particular period, and then analyse the collected data at the end of that period. They have to perform hypothesis testing to verify their assumption.

Case Study: Experience Marketing Services reported that the typical American spends a mean of 144 minutes (2.4 hours) per day accessing the Internet via a mobile device. (Source: The 2014 Digital Marketer, available at ex.pn/1kXJifX.) To test the validity of this statement, you select a sample of 30 friends and family. The result for the time spent per day accessing the Internet via a mobile device (in minutes) are stored in Internet_Mobile_Time.csv file.

Is there evidence that the populations mean time spent per day accessing the Internet via a mobile device is different from 144 minutes? Use the p-value approach and a level of significance of 0.05.

Dataset (in minutes): 72,144,48,72,36,360,44,30,432,24, 288,144,144,240,432,144,144,144,576,216,72,72,144,288,144,36,288,48, 288,144.

2. Process capability Analysis

Every manufacturing process has variation associated with it. Since process variation can never be totally eliminated, the variability in a process should be minimized to improve product quality. Process capability analysis deals with the techniques used to understand the

variability of a process and its effect on the product performance. Process capability analysis is an important engineering decision-making tool and has found application in a number of areas: as a criterion for vendor selection, reducing variability in a manufacturing process, specifying process requirements for new equipment, predicting how well the process will hold tolerances, assisting product designers in selecting or modifying a process and formulating quality improvement programs. Process capability analysis techniques have helped manufacturers control the quality of goods produced.

Reading material:

https://onlinelibrary.wiley.com/doi/abs/10.1002/qre.2290

https://onlinelibrary.wiley.com/doi/abs/10.1002/qre.1713

3. Quality control

Quality is demanded by every customer in the products they purchase in this era of science and technology, claiming for better products and services alike. This demand produces pressure on the manufacturers to conform to customers' wishes by offering products and/or services incorporating increased quality levels, applying quality control methods, practicing statistical quality control, etc. Manufacturers intensely control and improve the quality of their products in order to make them better while also aiming at establishing a competitive edge. Hypothesis testing is one of the useful tools of statistical methodology in quality control and improvement.

Case Study: In quality-control applications of hypothesis testing, the null and alternative hypotheses are frequently specified as H0: The production process is performing satisfactorily. Ha: The production process is performing unsatisfactorily. Accordingly, α is sometimes referred to as the producer's risk while β is called the consumer's risk. An injection molder produces plastic golf tees. The process is designed to produce tees with a mean weight of 0.256 ounces. To investigate whether the injection molder is operating satisfactorily, 40 tees were randomly sampled from the last hour's production.
a) Write null and alternative hypotheses in terms of the true mean weight of golf tees.

b) What is the test score?

c) What is the rejection region?

Data: 0.247 0.251 0.254 0.253 0.253 0.248 0.253 0.255 0.256 0.252

0.253 0.252 0.253 0.256 0.254 0.256 0.252 0.251 0.253 0.251

0.253 0.253 0.248 0.251 0.253 0.256 0.254 0.250 0.254 0.255

0.249 0.250 0.254 0.251 0.251 0.255 0.251 0.253 0.252 0.253