

→ Till now, we have described techniques for reasoning with a complete, consistent & unchanging model of world. In many domains, it is not possible to create such models. In this chapter, we will study techniques for solving problems with incomplete & uncertain models.

7.1

### Intro. to non-monotonic reasoning

② - B

→ Let's consider one murder story. Let Abbott, Babbitt & Cabot be suspects in a murder case.

→ Abbott was absent in the register of respectable hotel in Albany.

→ Babbitt also was absent, since Babbitt visited his brother-in-law in Brooklyn at that time.

→ Cabot also excused to be absent and he claimed to have been watching a ski meet in the catskills. But, this claim is from Cabot himself, so we should not believe "Cabot" or that he is saying truth. So, we believe :-

1. That Abbott didn't commit the crime.

2. That Babbitt " " " " .

3. n. Abbott or Babbitt or Cabot did.

→ But presently Cabot claims his absent and he had good luck that he caught in the television at the ski-meet. So, a new belief is :-

4. That Cabot did not.

→ Our beliefs ① to ④ are inconsistent, so we must choose one for rejection. Which has the weakest evidence?

① → The basis ① is good, since it is a fine old hotel.

② → The basis ② is weaker, since Babbitt's brother-in-law might be lying.

③ → The basis ③ is two-fold, means more confusing.

④ → The basis ④ is conclusive, since we can believe the evidence from television.

Thus, to resolve the inconsistency of ① to ④, we should reject ② or ③.

→ Finally, a certain arbitrariness & randomness should be noted in the organization of this analysis. The inconsistent beliefs in ① to ④ were like this:

- a belief about hotel register
- " " " prestige of the hotel
- " " " television
- Unwanted belief about brother-in-law.

→ This story illustrates some of the problems posed by uncertain and often changing knowledge. A variety of logical and computational methods have been proposed for handling such problems. Here, we discuss two approaches:-

① Non-monotonic reasoning, in which axioms & rules of inference are extended to make it possible to reason with incomplete info. These systems preserve the property that, a statement is either believed to be

Date / /

true, or believed to be false or not believed to be either.

(2) Statistical reasoning, in which "reps" is extended to allow some kind of numeric measure of certainty (rather than simply TRUE or FALSE) to be associated with each statement. E.g. statement  $A \rightarrow B$  has numerical measure assigned to it, i.e. true/false etc.

→ Conventional reasoning systems, such as "predicate logic" are designed to work with info. that has three imp. properties:

(1) It is complete with respect to domain of interest. In other words, all the facts necessary to solve prob. are present in the system or can be derived from conventional rules of first-order logic.

(2) It is consistent.

(3) The only way it can change is that new facts can be added as they become available. If these new facts are consistent with all other facts that have been asserted, then nothing will be retracted from the set of facts that are known to be true. This property is called "monotonicity".

→ Unfortunately, if any of above properties are not satisfied then conventional logic-based reasoning systems become inadequate. Non-monotonic reasoning systems are designed to be able to solve probs in which all these properties may be missing.

We must address following key issues to perform non-monotonic (N.M.) reasoning :

- 1) How knowledge Base can be extended to allow inference to be made on the basis of lack of knowledge as well as on the presence of it? for ex:, we would like to be able to say, "if you have no reason to suspect that a particular person committed a crime, then assume he didn't". or "if you have no reason to believe that someone is not getting along with her relatives, then assume that relatives will try to protect her".
- Allowing such reasoning has a significant impact on a knowledge base. Non monotonic reasoning system derive their name that means non-monotonic reasoning is called non-monotonic because of inferences that depend on lack of knowledge.

- 2) How can the knowledge base be updated properly when a new fact is added to the system or when an old one is removed ? In non-monotonic systems, since the addition of a fact can cause previously discovered proofs to become invalid.
- The usual solution to this problem is to keep track of proofs, which are called "justifications".
- 3) How can knowledge be used to help resolve conflicts when there are several inconsistent non monotonic inferences that could be drawn ?

In many techniques for reasoning non-monotoni-  
city, we are able to derive many alternatives  
that could be believed, but mostly there is not  
any way to choose one option when all of the  
options can't be believed once. for ex: as  
soon as we conclude that Abbott, Babbitt and  
Cabot all claimed that they didn't commit a  
crime, then also we conclude that one of  
them must have committed, since there is no  
one else. In this case if we choose to resolve  
the conflict by finding the person with the weakest  
absence and believing that he committed the  
crime.

~~7.2~~

## Logics for non-monotonic (N.M) reasoning

- We know that monotonicity is ~~parallel~~ to basic thing to the def<sup>n</sup> of first-order predicate logic. So, we are forced to find some alternative to support N.M. reasoning.
- Fig. shows one way of visualizing how N.M. reasoning works. The box labeled A corresponds to an

original set of wiff's.

- The large circle contains all the models of A. When we add some N.M. reasoning capabilities to A, the we get a new set of wiff's, which we have labeled 'B'. 'B' contains more info. than A does. The set of models corresponding to 'B' is shown at lower right of the large circle.
- Now suppose we add some new wiff's to A. We represent 'A' with these additions as the box C. A set of models to C is shown in smaller, interior circle, since it is disjoint with the models for B.
- To find a new set of models that satisfy C, we need to accept models that had been previously rejected. To do that, we need to eliminate the wiff's that were responsible for these models being thrown away. This is the essence of N.M. reasoning.

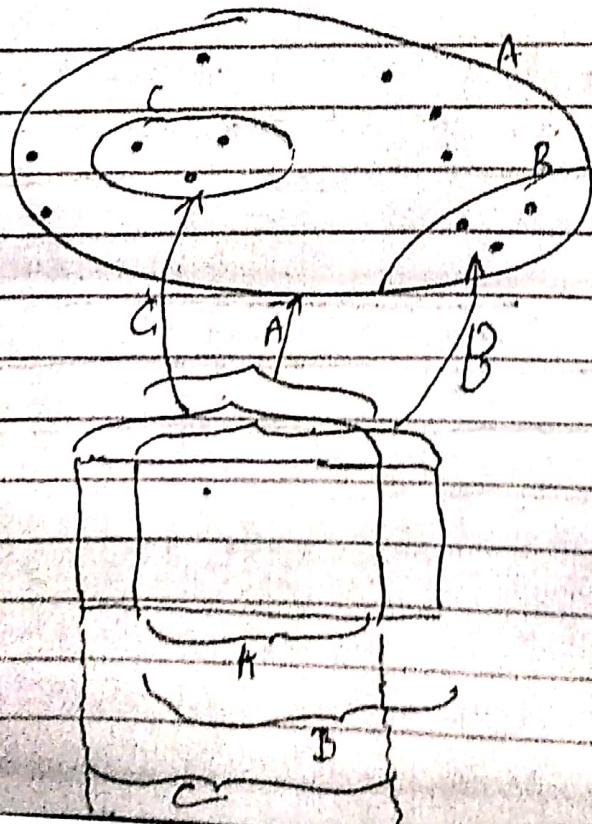
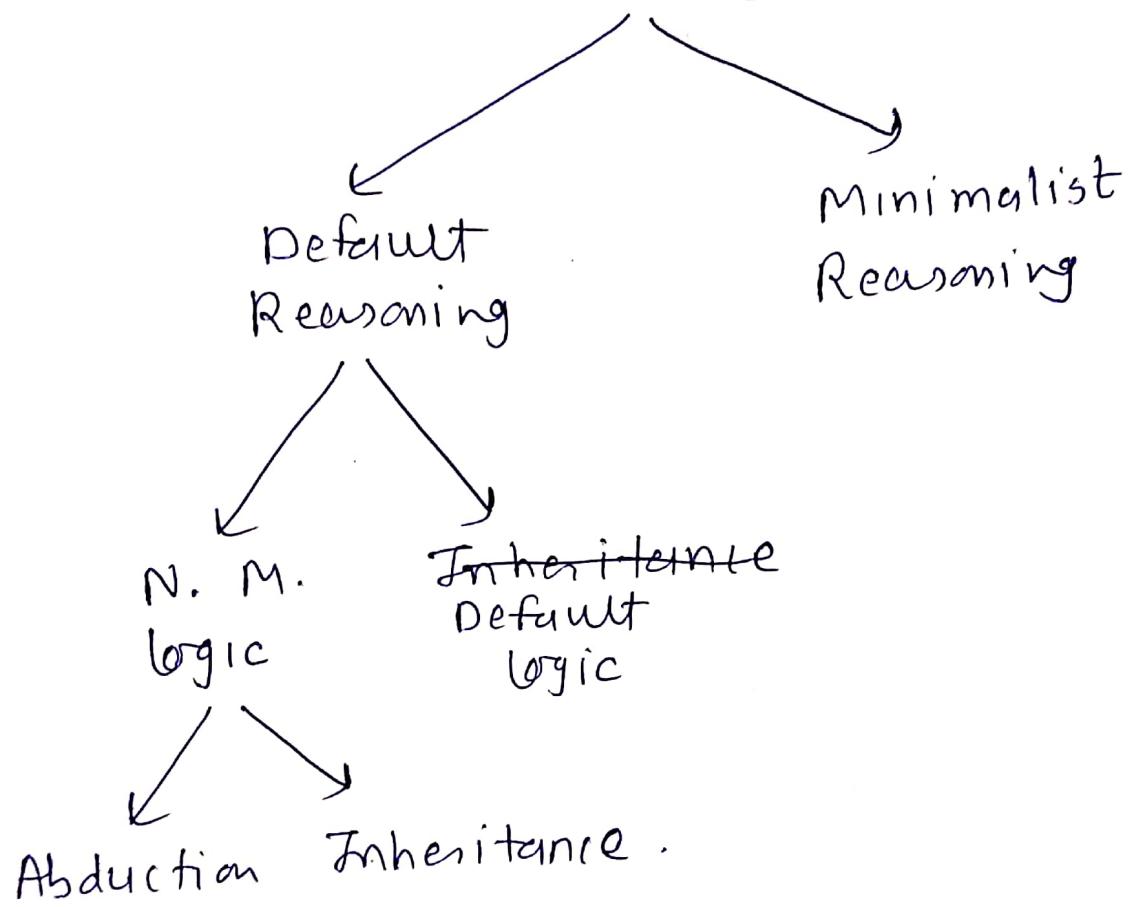


Fig.

Models, Wiff's and  
N.M. reasoning

## Subtypes of non monotonic reasoning



## Default reasoning

what is commonly called as

- We want to use N.M. reasoning to perform "default reasoning." We want to draw conclusions based on what is most likely to be true. In this section, we discuss two approaches for doing this.

~~two ways of~~ ~~N.M.~~ • N.M. logic.

~~two ways of~~ ~~N.M.~~ • Default logic

We then describe two common kinds of N.M. reasoning that can be defined on those logics:

~~two types of~~ ~~N.M.~~ { • Abduction  
• Inheritance

• N.M. logic

- One system that provides a basis for default reasoning is N.M. logic (N.M.L), in which the language of first-order predicate logic is augmented with a modal operator  $\Box$  which can be read as "is consistent".

- For ex., the formula,

$$\forall x, y : \text{Related}(x, y) \wedge \Box \text{GetAlong}(x, y) \rightarrow \neg \text{WillDefend}(x, y)$$

should be read as, "For all  $x$  and  $y$ , if  $x$  and  $y$  are related and if the fact that  $x$  gets along with  $y$  is consistent with everything else that is believed, then conclude that  $x$  will not defend  $y$ ".

- We must define what "is-consistent" means? Since consistency in this system is undecidable, so we,

(9)

(Saathi)

Date: / /

need some approximation. The one that is usually used is the PROLOG notion of negation as failure, or some variant of it. For ex., to show that 'P' is consistent, we attempt to prove  $\neg P$ . If we fail to prove  $\neg P$ , then we assume  $\neg P$  to be false and we call 'P' is consistent. Negation as failure works in pure 'PROLOG', bcoz the rest of our languages to Horn clause, then we have a decidable theory. So failure to prove something is not a proper way.

→ On the other hand, if we start with full first order predicate logic as our base lang., then we have no such guarantee. So, if we define consistency on some heuristic func', it will be more proper.

One prob.

(2) → A second problem that arises in this approach is what to do when multiple non-monotonic statements suggest ways of augmenting our knowledge that if taken together would be inconsistent. For ex., consider foll. set of assertions:

~~multiple N.M. statements~~

multiple N.M. statements	$\forall x : \text{Republican}(x) \wedge M \rightarrow \text{Pacifist}(x) \rightarrow \text{Pacifist}(x)$ (1)
	$\forall x : \text{Quaker}(x) \wedge \text{Pacifist}(x) \rightarrow \text{Pacifist}(x)$ (2)
	<del>Republican(Dick)</del> (3)
	<del>Quaker(Dick)</del> (4)

→ The def'n of NMI that we have given support to distinct ways of augmenting this knowledge base.

① In one, we first apply the first assertion, which concludes  $\neg \text{Pacifist}(\text{Dick})$ . If we have applied this first assertion, then we can't apply second one, bcoz it is contradictory; and it is not consistent to assume  $\text{Pacifist}(\text{Dick})$ .

Date \_\_\_\_\_

② The other thing we can do is, we can apply second assertion first. This results in the conclusion Pacifist (Dick), which prevents the first one from applying.

→ It is worth pointing out here that although assertions such as the ones we used to reason about Dick's pacifism look like rules, but in theory, they are just ~~not~~ ordinary wff's which can be manipulated by the standard rules for combining logical expressions.

→ So, for ex., given

$$\begin{array}{l} A \wedge B \rightarrow B \\ \neg A \wedge B \rightarrow B \end{array}$$

So, we can derive the expression as,

$$M \cdot B \rightarrow B$$

### \* Default logic

→ An alternative logic for performing default based reasoning. Some scratch is Reiter's Default logic (DL), in which a new class of inference rules is introduced.

→ In this approach, we allow inference rules of the form

$$\begin{array}{c} A : B \\ C \end{array} \quad A \wedge B \rightarrow C$$

→ Such a rule should be read as, "If A is provable

and it is consistent to assume B 'then conclude C'. As we can see, this is very similar to the N.M. expressions that we used in NML.

- There are some imp. differences b/w these two theories:
- ① The first is that in DL (Default Logic) the new inference rules are used as a basis for computing a set of logical extensions to the knowledge base. If a decision among the extensions is necessary to support prob. coloring, some other mechanism must be provided. So, if we return to the case of "Dick the Republican", then we compute two extensions, one corresponding to his being a pacifist and one corresponding to his not being a pacifist. The theory of DL does not say anything how to choose b/w these two.
- ② A second imp. diff. b/w those two theories is that, in DL, the N.M. expressions are rules of inference rather than expression in the lang. Thus, they can't be manipulated by the other rules of inference. This leads to some unexpected results. For ex., given the two rules :-

$$\frac{A : B}{B}$$

$$\frac{\neg A : B}{B}$$

No assertion about A, no conclusion about B will be drawn, since neither inference rule applies.

Date \_\_\_\_ / \_\_\_\_ / \_\_\_\_

## Abduction (अनुसूची ग्रंथि)

- For ex., suppose the axiom we have is,

$\forall x: \text{Measles}(x) \rightarrow \text{Spots}(x)$

- The axiom says that having "measles (କିରାଣୀ)" also implies having "spots (କିରାଣୀ ମୁଖୀ)".

- But suppose we notice "spots", we might like to conclude "measles". Such a conclusion is not permitted by the rules of standard logic and it may be wrong; but it may be the best guess we can make about what is going on.

- Deriving conclusions in this way is another form of 'default reasoning.' This specific form is called as 'Abductive reasoning'.

- The process of abductive reasoning can be described as, "Given two wff's ( $A \rightarrow B$ ) and ( $B$ ) for any expression ' $A$ ' and ' $B$ ', if it is consistent to assume ' $A$ ', then do so."

- In many domains, "Abductive reasoning" is particularly useful, if some measure of certainty is attached to the resulting expressions.

- The 'certainty measure' denotes the "risk" that

( $A \rightarrow B$  ना case रहे तो,  $B$  नाही) A ना होणी शक्याची,  
 परंतु तो प्रत्येक "guess" एकूण अविष्ट तो ही problem  
 नाहिं शक्य... (सोटले  $\neg A \rightarrow B$  नाही, A derive करीने,  
 तर  $\vdash$  abduction)

Date    /    /

the abductive process may be wrong. For ex., if want to produce ' $A$ ' from ' $B$ ' in ' $A \rightarrow B$ ' inference, then antecedent (~~सिवाय नहीं लितर~~ literal रूप से,  $(A)$  सिवाय नहीं लितर) literal वि ( $B'$ ) produce फूट दें तो... (तभी direct ' $B$ ' रिहा) ' $A'$  assume नहीं फूट देंगे...)

- Abductive reasoning is not a kind of logic like  
X DL and NML. But it is a very useful kind of  
reasoning.

$A \rightarrow B$

## Inheritance

5

- One way to use N.M. reasoning is as a basis for inheriting attr's values from a prototype description of a class to the individual entities that belong to the class. (Generalized class to specialized class)

## Example

- We derived "baseball knowledge base" in chap-4. In this chap, we also described algo. for implementing inheritance.

- We can say like:- "An object inherits attr.s values from all the classes of which it is a member, if it doesn't lead to a contradiction", that means a value from a more restricted class has precedence over a value from a broader class: (Specialized class = Restricted class & generalized class = Broader class)"

Date: / /

14

- Is this logical idea able to provide basis more formally? To see how, let's return to the baseball example:-

- We can write a rule ~~for~~ inheritance of a default value for the height of a Baseball player as:-

Baseball-player(x) : height(x, 6-1)  
height(x, 6-1)

- Now suppose we assert Pitcher(Three-Finger-Brown). Since, "Three-Finger-Brown" is a baseball player, our rule allows us to conclude that his height is 6-1.

- TF, on the other hand, we had asserted a conflicting value for "Three-finger Brown" has an axiom like:-

$$\forall x, y, z : \text{height}(x, y) \wedge \text{height}(x, z) \rightarrow (y = z)$$

जोहर 'x' ने 'y' height दिया, और 'x' ने 'z' height दिया,  
 तो y = z ओही, becoz जोहर या उसके बाहर 'x' ने  
 अद्वितीय height दिया नहीं दिया....

- Above rule ~~prohibits~~ prohibits someone from having more than one height

- But now, let's encode the default rule for the height of adult male in general. If we pattern after one "baseball players", then we get

Adult - Male (x) : height(x, 5-10)  
height(x, 5-10)

T-2.2

## Minimalist reasoning

- Till now, we have talked about general methods that provide things that are generally true. These methods are based on some variant of the idea of "minimal model."
- The idea behind using minimal models as a basis for N.M. reasoning about the world is the following :-
  - There are many fewer true statements than false ones.
  - If something is true & relevant, then it assumes that it has entered into our knowledge base.
- Therefore, assume that the only true statements are those that necessarily must be true in order to ~~fix~~ maintain consistency of the knowledge base.
- The closed world assumption
- The CWA says that the objs that satisfy any predicate P. The CWA is particularly powerful as a basis for reasoning with DB's, which are assumed to be complete with respect to the properties they describe.
- For ex., a personnel DB can safely be assumed to list all company's employees.

Date \_\_\_\_\_

- Similarly, an airline DB can be assumed to contain a complete list of all the routes flown by that airline. So, if I ask if there is a direct flight from Ahmedabad to Dubai, then the answer should be "no" if none can be found in that DB.
- Although the CWA is simple & powerful, it can fail to produce an appropriate answer for two reasons.
  - (1) Its assumptions are not always true in the world
  - (2) Some parts of the world are not really "flocable".
- The second kind of problem that CWA arises from the fact that it is a purely syntactic reasoning process. Let's look at two specific examples of this problem.
- Consider a knowledge base that consists of just a single statement:
 

$A(\text{Joe}) \vee B(\text{Joe})$
- In this, suppose we allow to conclude both  $A(\text{Joe})$  &  $B(\text{Joe})$  to be true, but neither of them always be true for Joe. So, unfortunately the extended knowledge base is:

$$\left. \begin{array}{l} A(\text{Joe}) \vee B(\text{Joe}) \\ \neg A(\text{Joe}) \\ \neg B(\text{Joe}) \end{array} \right\} \text{which is totally inconsistent.}$$