Rejection-based EEG Signal Classification

Artyom Matveev[1]
matveev.as@phystech.edu
[1]Moscow Institute of Physics and Technology, Institutskiy lane, 9, 141700, Dolgoprudny city, Russia

This paper presents a new generative classification model in the context of Brain-Computer Interface (BCI). This model involves the concept of Riemannian geometry in the manifold of covariance matrices. The main idea is to solve a classification problem by finding the closest class for a given object in probabilistic terms. If the given object is out of a confidence interval of a specific class, the object will be rejected. The probabilistic space takes Riemannian structure of the data into account. The computational experiments are carried out using publicly available EEG datasets. We show that the rejection strategy leads to near-optimal classification accuracy in most test scenarios.

Keywords: *Brain-Computer Interface, EEG Classification, Generative Modeling, Riemannian Gaussian Distribution, Riemannian Geometry.*

1 Introduction

A Brain-Computer Interface (BCI) is a technology that measures and analyzes signals of a human brain. On the one hand, such technology is useful for people with various disabilities in order to provide them with effective rehabilitation [1]. On the other hand, it could also be a great tool for healthy people, e.g. in the video game industry [2]. There are three main types of BCI: invasive (implant-based), partially invasive (electrocorticography-based), and non-invasive (electroencephalography-based). Electroencephalography (EEG)-based BCIs are the most prominent ones due to their portability, subject-friendliness, and cost-effectiveness. EEG-BCIs have been widely used to solve various tasks, such as motor imagery (MI), event-related potential (ERP), steady state visually evoked potential (SSVEP), etc. This paper focuses on MI-EEG, although the method we propose here might be applied to other EEG-BCI tasks. In the MI-EEG task, the individual mentally stimulates physical movement, which activates the cortical sensorimotor systems [3], and a BCI device records EEG signals. Various machine-learning approaches have been utilized to decode an individual's intentions [4].

Since the 1990s, researchers have been actively developing various methods for MI-EEG classification [5]. Conventional methods are usually divided into several stages in which the first stage is often devoted to building subject-dependent frequency and spatial filters discriminating between EEG datasets corresponding to two different classes of MI [6]. Such spatial filters perform a linear combination of the EEG signals to create new signals with maximal variance in one condition and minimal variance in the other condition. Once these spatial filters have been designed, the (log-)variance of the spatially filtered signals is used as features by a supervised classification algorithm. Linear Discriminant Analysis (LDA) is often used to perform this processing [7]. In addition to that, there have been invented much more sophisticated algorithms that can take the Riemannian structure of the data into account and improve the classification results of previous studies [8].

Main contributions. We describe a new approach to the classification of EEG signals. First, we reconstruct the probability density function of each class, taking the Riemannian Gaussian distribution of data into account. Second, we define a specific confidence interval for each class so that we can use it in our rejection strategy. Third, we solve the classification problem by evaluating the statistical significance of data concerning the classes' distributions.

2 Problem statement

Let $\mathfrak{D} = \{(\mathbf{X}_i, y_i)_{i=1}^L\}$ be the given dataset, where a segment of EEG signals

$$\mathbf{X}_i = [\boldsymbol{\chi}_1, \dots, \boldsymbol{\chi}_j, \dots, \boldsymbol{\chi}_{n_C}]^\intercal, \mathbf{X}_i \in \mathbb{R}^{n_C \times n_T} \text{ and } j \in \mathcal{J} = \{1, \dots, n_C\}.$$

The vector $\boldsymbol{\chi}_j$ is called the j-th EEG channel and $y_i \in \mathbb{Y} = \{1, \dots, M\}$, where M is the number of classes, is called the class label. Here each EEG channel $\boldsymbol{\chi}_j$ has n_T sampled points on each epoch duration. Here and hereafter we suppose that each EEG channel measurement has been previously bandpass filtered, centered, and scaled. For the i-th segment, the spatial covariance matrix is estimated using the *Sample Covariance Matrix* (SCM) $\mathbf{P} \in \mathbb{S}_{++}$, where $\mathbb{S}_{++} \coloneqq \{\mathbf{P} \in \mathbb{R}^{n_C \times n_C} : \mathbf{P} = \mathbf{P}^\intercal \text{ and } \mathbf{x}^\intercal \mathbf{P} \mathbf{x} > 0, \forall \mathbf{x} \in \mathbb{R}^{n_C} \setminus \{0\}\}$, such as:

$$\mathbf{P}_i = \frac{1}{n_T - 1} \mathbf{X}_i \mathbf{X}_i^\intercal \tag{1}$$

2.1 Probabilistic nature of the data

In this study, we consider that $\mathbf{P}_i$ has a Riemannian structure and therefore can be represented in terms of probability distributions on Riemannian manifolds. The main idea is that a spatial SCM belongs to a particular manifold and all the work can be done in the Riemannian space. Note that the space of symmetric positive-definite matrices $\mathbb{S}_{++}$ is a differentiable Riemannian manifold $\mathcal{M}$. In this case, each matrix $\mathbf{P}_i$ is a point on the manifold $\mathcal{M}$ and the distance between these points can be measured using a specific metric

$$d^2(\mathbf{P}_1, \mathbf{P}_2) = (\log^2(\mathbf{P}_1^{-\frac{1}{2}} \mathbf{P}_2 \mathbf{P}_1^{-\frac{1}{2}})) \qquad \mathbf{P}_1, \mathbf{P}_2 \in \mathcal{M} \tag{2}$$

where matrix logarithms and powers are understood as self-adjoint matrix functions, obtained by taking logarithms and powers of eigenvalues. Using the distance (2), we define Riemannian Gaussian distributions on the manifold $\mathcal{M}$ to be given by parameterized probability density function, as in [9],

$$p(\mathbf{P}|\bar{\mathbf{P}}, \sigma) = (Z(\sigma))^{-1} \exp\left[\frac{d^2(\mathbf{P}, \bar{\mathbf{P}})}{2\sigma^2}\right], \tag{3}$$

where $\bar{\mathbf{P}} \in \mathcal{M}$ is the centre-of-mass parameter, and $\sigma > 0$ is the standard deviation. The normalizing factor $Z(\sigma)$ is given by the integral

$$Z(\sigma) = \int_{\mathcal{M}} \exp\left[-\frac{d^2(\mathbf{P}, \bar{\mathbf{P}})}{2\sigma^2}\right] dv(\mathbf{P}), \tag{4}$$

where $dv(\mathbf{P}) = \det(\mathbf{P})^{-n_C}\{d\mathbf{P}\}$ is the Riemannian volume. Here,

$$\{d\mathbf{P}\} = \prod_{i \leqslant j} d\mathbf{P}_{ij}$$

We can simplify the notation by diagonalizing the matrix $\mathbf{P} \in \mathcal{M}$ as $\mathbf{P} = \mathbf{U}\mathbf{E}\mathbf{U}^\dagger$ where $\mathbf{E}$ is a diagonal matrix with diagonal elements $(r_1, \dots, r_{n_C}) \in \mathbb{R}^{n_C}$, and $\mathbf{U}\mathbf{U}^\dagger = \mathbf{I}_{n_C}$. As has already been mentioned, the matrix $\mathbf{P} \in \mathcal{M}$ follows the Riemannian structure, and therefore we consider that it also follows the Riemannian Gaussian density (3). The normalizing factor $Z(\sigma)$ in (4) reduces to a specific multiple integral $z(\sigma)$. Specifically,

$$Z(\sigma) = \Omega_{n_C} z(\sigma), \tag{5}$$

where Ω_{n_C} is a numerical constant that appears after the integration of a uniformly distributed matrix $\mathbf{U}$ out of (4), and

$$z(\sigma) = \frac{1}{n_C!} \int_{\mathbb{R}^{n_C}} \prod_{i<j} \sinh \left| \frac{r_i - r_j}{2} \right| \prod_i \exp\left[-\frac{r_i^2}{2\sigma^2} \right] dr_i \tag{6}$$

2.2 Parameter estimation

The main features that we need to get from the distribution are the center-of-mass (mean) $\bar{\mathbf{P}}$ and the standard deviation σ. We can get the mean easily by using the maximum likelihood estimation as follows,

$$\hat{\mathbf{P}} = \underset{\mathbf{P}\in\mathcal{M}}{\arg\min} \sum_{i=1}^{\ell} d^2(\mathbf{P}_i, \mathbf{P}), \tag{7}$$

where $\hat{\mathbf{P}}$ is the estimated mean of the data $\mathbf{P}_i$ with respect to the distance (2).

A much harder problem is to find the maximum likelihood estimate of σ. It is necessary to compute the partition function (5) of the distribution (3) to solve such a problem. The issue is that we have to be able to compute the integral (6) effectively given that the dimension n_C might be in the tens or hundreds; it is a regular case in practical tasks like EEG signal classification. In particular, we have to compute the logarithm of the multiple integral (6). We will be using the approximation of (6) mentioned in [**?**] to get optimal results.

Based on the preposition 2 of the paper [**?**], we can substitute the computation of $\log z(\sigma)/n_C^2$ with the following expression up to an error of the order of $1/n_C^2$,

$$\frac{1}{2}\Phi\left(\frac{1}{2}t\right)$$

where $t = n_C\sigma^2$ remains constant in the limit where $n_C \to \infty$, $\sigma \to 0$, and

$$\Phi(\xi) = \frac{\xi}{6} - \frac{\mathrm{Li}_3(e^{-\xi}) - \mathrm{Li}_3(1)}{\xi^2}$$

Here, Li_3 is the trilogarithm function, $\mathrm{Li}_3(\mu) = \sum_{k=1}^{\infty} \mu^k/k^3$. In other words, we have just got the approximation of $\log z(\sigma)/n_C^2$,

$$z_a(\sigma) = \frac{1}{2}\Phi\left(\frac{1}{2}t\right) \tag{8}$$

Now we can compute the standard deviation with the following nonlinear equation,

$$\varphi(\hat{\sigma}) = \frac{1}{\ell}\sum_{i=1}^{\ell} d^2(\mathbf{P}_i, \hat{\mathbf{P}}); \varphi(\sigma) = \sigma^3 \frac{d}{d\sigma} z_a(\sigma) \tag{9}$$

2.3 Density reconstruction

As soon as we know how to compute the mean and standard deviation, we can get these parameters for each matrix in each class. Our aim here is to reconstruct the distribution density of each class. Each class can be characterized with two matrices, the mean matrix $\mathbf{M}$ and the covariance matrix $\mathbf{\Sigma}$, such that each row in these matrices represents the corresponding parameters of a matrix $\mathbf{P}_i$. We have just got all the necessary information to complete the reconstruction task for a class, and our next step is to do the same thing for an unknown matrix $\mathbf{P}_j$ from the test set $\mathfrak{D}_{\mathcal{C}} \subset \mathfrak{D}$.

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