Page No.:
Date:

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Tutorial 5
(Evaluation & Measurement of Hypothesis Testing)

1] $H_0 : P = 0.7$

$H_1 : P \neq 0.7$

Level of significance $= \alpha = 0.1$

test stat : binomial var. with $p = 0.7$, $n = 15$

$x = 8$ & $np_0 = 15 \times 0.7 = 10.5$

$\therefore \quad P = 2P(X \leq 8 \text{ when } p = 0.7)$

$= 2 \sum_{x=0}^{8} b(x; 15, 0.7)$

$= 2 \times 0.1311$ (from binomial prob. table)

$= 0.2622$

$\therefore P > 0.1$ i.e. $P > \alpha$

$\therefore$ do not reject H_0. Conclude that there is insufficient reason to doubt the builder's claim.

2) $H_0 : P = 0.6$

$H_1 : P > 0.6$ given $x = 70$, $n = 100$, $p = 0.6$

$\alpha = 0.05$

$$z = \frac{x - np_0}{\sqrt{np_0q_0}}$$

$$= \frac{70 - 100 \times 0.6}{\sqrt{100 \times 0.6 \times 0.4}} = 2.04$$

$P = P(z > 2.04)$

$= 0.0207$ (from table)

as $P < \alpha$, reject H_0 & conclude that new drug is superior.

Page No.:
Date:

3) Let P_1 be proportion of Mumbai voters.
P_2 be proportion of surrounding area residents.

$$\hat{P}_1 = \frac{120}{100} = 0.6$$

$$\hat{P}_P = \frac{120 \cdot 1240}{200 \cdot 1500} = 0.514$$

$$\hat{P}_2 = \frac{240}{500} = 0.48$$

$\alpha = 0.05 \; (5\%)$

Hypothesis : $H_0 : P_1 \leq P_2$
$H_1 : P_1 > P_2$

$$Z = \frac{\hat{P}_1 - \hat{P}_2}{\sqrt{\hat{P}_P(1-\hat{P}_P)\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} = \frac{0.5 - 0.48}{\sqrt{(0.514)(1-0.514)\left(\frac{1}{200}+\frac{1}{500}\right)}}$$

$$Z = 2.869 \Rightarrow P(Z > 2.869) = 0.0044$$

Now as $P < \alpha$, reject H_0 & conclude that the proportion of Mumbai voters favouring the proposal is higher than population of surrounding area voters.

4) (a) $H_0 : p = 0.2$ the critical region is in right tail
$H_1 : p > 0.2$

(b) $H_0 : \mu = 3$ the critical region is in both tails.
$H_1 : \mu \neq 3$

(c) $H_0 : p = 0.15$ the critical region is in left tail
$H_1 : p < 0.15$

(d) $H_0 : \mu = 500$ the critical region is the right tail
$H_1 : \mu > 500$

(e) $H_0 : \mu = 15$ the critical region is in both tails.
$H_1 : \mu \neq 15$

Page No.:
Date:

5) Let μ_1 = population mean "robustness" - company A
μ_2 = " " " - company B

$H_0 : \mu_1 = \mu_2$
$H_1 : \mu_1 \neq \mu_2$ $\alpha = 0.05$

$$\bar{x}_1 = \frac{1}{n_1}\sum_{i=1}^{n_1} x_{1i} = \frac{9.3+8.8+6.8+8.7+8.5+6.7+8+6.5+9.2+7}{10}$$

$\bar{x}_1 = 7.95$

$$\frac{1}{n_2}\sum_{i=1}^{n_2} x_{2i} = \frac{11+9.8+9.9+10.2+10.1+9.7+11+11.1+10.2+9.6}{10}$$

$\therefore \bar{x}_2 = 10.26$

$$s_1^2 = \frac{1}{n_1-1}\left(\sum_{i=1}^{n_1} x_{1i}^2 - n_1\bar{x}_1^2\right) = \frac{10.865}{9} = \underline{\underline{1.207}}$$

$$s_2^2 = \frac{1}{n_2-1}\left(\sum_{i=1}^{n_2} x_{2i}^2 - n_2\bar{x}_2^2\right) = \frac{2.924}{9} = \underline{\underline{0.325}}$$

as sample variances are very different, we can't assume population variances equal, so use the "unpooled t-test"

$$V = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{1}{n_1-1}\left(\frac{s_1^2}{n_1}\right)^2 + \frac{1}{n_2-1}\left(\frac{s_2^2}{n_2}\right)^2} = \frac{\left(\frac{1.207}{10} + \frac{0.325}{10}\right)^2}{\frac{1}{9}\left(\frac{1.207}{10}\right)^2 + \frac{1}{9}\left(\frac{0.325}{10}\right)^2}$$

$\therefore V = 10.3 \approx 10$

The test statistic used to test hypo. is $T = \dfrac{\bar{x}_1 - \bar{x}_2 - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$

~~when~~ which under the null hypothesis, follows approximately t-distribution with $v = 10$ degrees of freedom. Under null hypo, $(\mu_1 - \mu_2) = 0$

$$\therefore \text{value of } T = \frac{7.95 - 10.26}{\sqrt{\frac{1.207}{10} + \frac{0.325}{10}}} = \underline{\underline{-5.9}}$$

Since the test is two-sided,

the value of test is the doubled area under density curve of t-distribution with $(v=10)$, right of the absolute value of test stat.

$$|t| = |-5.9| = 5.9.$$

i.e. $$p\text{-value} = 2P(T \geq |t|)$$
$$= 2P(T \geq 5.9)$$

$t_{0.0005}(10) = 4.587$ since $|t| = 5.9$ is even greater than $P(T \geq 5.9) < 0.0005$ so,

$p\text{-value} < 0.001$

as $P_E < \alpha$, reject null hypo. conclude that the mean "robustness" of laptop isn't same for both companies.