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Tutorial 6

ARKA HALDI
20181300014

1. (a) $P(H) = \lambda$

$P(T) = 1-\lambda$ $\quad$ P(first head at k+1)

$= (1-\lambda)^{k} \lambda$

(b) Let M be no: of tosses required to get first head

Let $S = E[M]$

As tosses are ~~indepen~~ independent & equation is additive,

$S = \lambda \times 1 + (1-\lambda)(S+1)$

$= \cancel{\lambda} + S + 1 - \lambda S - \cancel{\lambda}$

$\therefore \lambda S = 1 \qquad \therefore S = \frac{1}{\lambda}$

2. X: random variable

(a) $var(X) = E[X - E(X)^2]$

to prove: $var(X) = E[X^2] - (E[X])^2$

Now, we have

$var(X) = E[(X - E(X))^2]$

$= E[X^2 - 2X E(X) + E^2(X)]$

$= E[X^2] - 2E[X E(X)] + E^2(X)$

$= E[X^2] - 2E^2[X] + E^2[X]$

$= E[X^2] - E^2[X]$ $\quad$ thus proved

(b) $E(X) = 0 \qquad E[X^2] = 1$

(i) $var(X) = E[X^2] - E^2[X]$

$= 1$

(ii) $Y = a + bX$

* $E[Y^2] = E[(a+bX)^2] = E[a^2 + 2abX + b^2X^2]$

$= a^2 + 2ab \cancel{E[X]}^{\,0} + b^2 \cancel{E[X^2]}^{\,1}$

$= a^2 + b^2$

* $E[Y] = E[a+bX] = a + bE[X] = a + b(0) = a$

* $var(Y) = E[Y^2] - E^2(Y)$

$= (a^2+b^2) - (a^2)$

$= b^2$

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3) A => Aku predicts given house is winning house.
~A => ——— " ——— is not ——— " ———

B => event that given horse wins.
~B => ——— " ——— doesn't win.

ⓐ: given a horse, prob it wins

$$P(B) = P(B,A) + P(\cancel{\ } B, \sim A)$$

$$= P(B|A) \cdot P(A) + P(B|\sim A) \cdot P(\sim A)$$

$$= (0.99)(\cancel{\ } \times 10^{-5}) + (1-0.99)(1-10^{-5})$$

$$P(B) = 1.99 \times 10^{-5}$$

ⓑ Prob that Aku predicts black beauty is winning.

$$P(A|B) = \frac{P(A,B)}{P(B)} = \frac{P(A|B) \cdot P(B)}{P(B)} = \frac{0.99 \times 10^{-5}}{1.99 \times 10^{-5}}$$

$$P(A|B) = \underline{\underline{0.499}}$$

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