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Tutorial 4

Ex1] Independent Component Analysis

Mixing statistically indep. sources.

now, variance of mixture

$$= \text{var}(x) = \langle (x - \langle x \rangle)^2 \rangle$$

$$= \langle x^2 \rangle - (\langle x \rangle)^2$$

$$= \left\langle \left(\sum_i w_i s_i \right)^2 \right\rangle - \left(\left\langle \sum_i w_i s_i \right\rangle \right)^2$$

$$= \left\langle \left(\sum_i w_i s_i \right) \left(\sum_j w_j s_j \right) \right\rangle - \left(\sum_i w_i \langle s_i \rangle \right) \left(\sum_j w_j s_j \right)$$

$$= \left\langle \sum_{ij} w_i w_j s_i s_j \right\rangle - \sum_{ij} w_i w_j \langle s_i \times s_j \rangle$$

$$= \sum_i w_i w_j \left(\langle s_i s_j \rangle - \langle s_i \times s_j \rangle \right) + \sum_{ij} w_i w_j \left(\langle s_i s_j \rangle - \langle s_i \rangle \langle s_j \rangle \right)$$

$$= \sum_i w_i^2 \left(\langle s_i s_j \rangle - \langle s_i \rangle^2 \right) + \sum_{i,j} w_i w_j \left(\langle s_i \rangle \langle s_j \rangle - \langle s_i \times s_j \rangle \right)$$

s_i & s_j are statistically independent for $i \neq j$

$\Rightarrow \langle s_i \rangle \langle s_j \rangle - \langle s_i \times s_j \rangle = 0$

& $\text{var}(s_i) = 1$

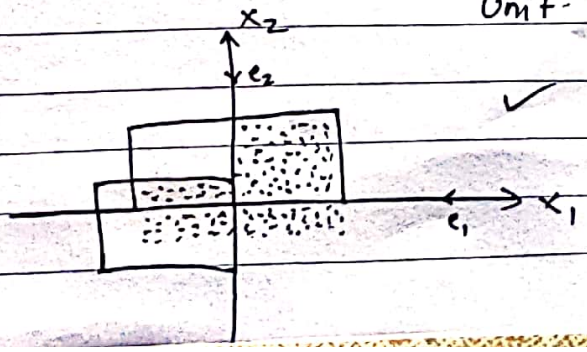
$\therefore \text{var}(x) = \sum_i w_i^2$

to guarantee that mixture has unit variance,

$\text{var}(x) = 1$

$\therefore \boxed{\sum_i w_i^2 = 1}$ This constraint has to be imposed on the weights w_i for the mixture to have unit variance

Ex2] (a)



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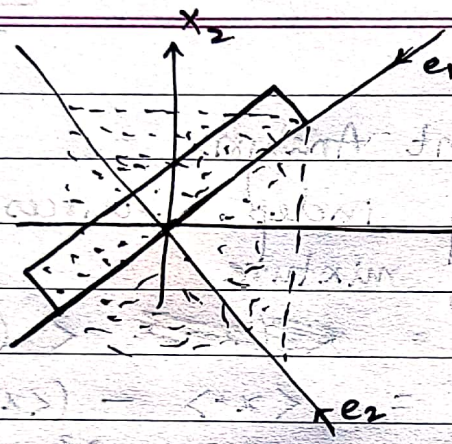
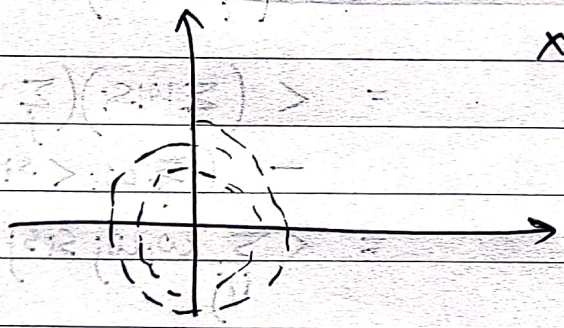
(b)



(c)



x — can't be seperated into independent components