

SublonK

Sublinear Prover *PlonK*



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#ePrint $\mathcal{S} \times \mathcal{K}$: Sublinear Prover
 $\mathcal{P} \times \mathcal{K}$



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Muhammed Esgin

@mfesgin

What a catchy paper title!

#ePrint $\mathcal{S} \times \mathcal{K}$: Sublinear Prover
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Kostas Kryptos

@kostascrypto

Only cryptographers can read this title



Jack

@j_mcph

lol



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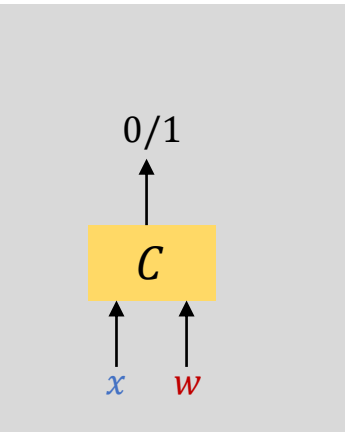


Rohit Sinha

Swirlds Labs

Succinct Non-Interactive Arguments of Knowledge (SNARKs)

Common Reference String (CRS)



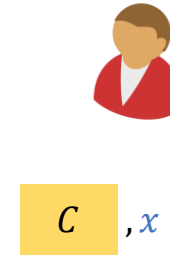
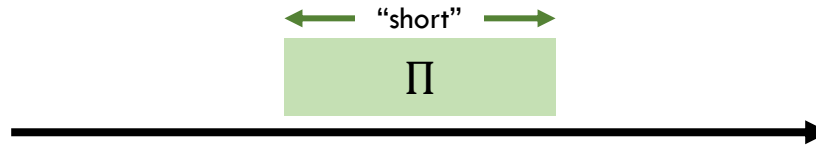
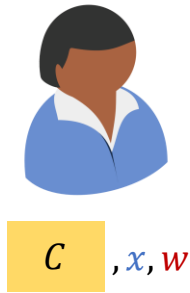
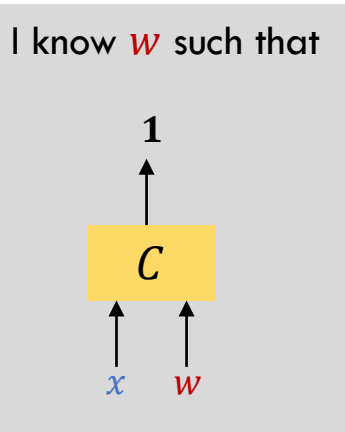
C, x, w



C, x

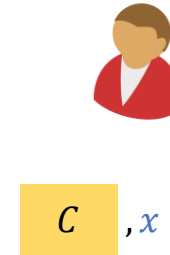
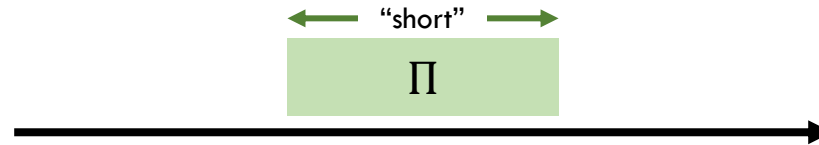
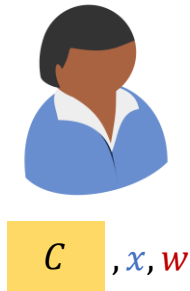
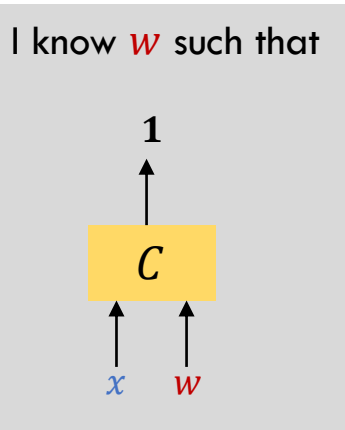
Succinct Non-Interactive Arguments of Knowledge (SNARKs)

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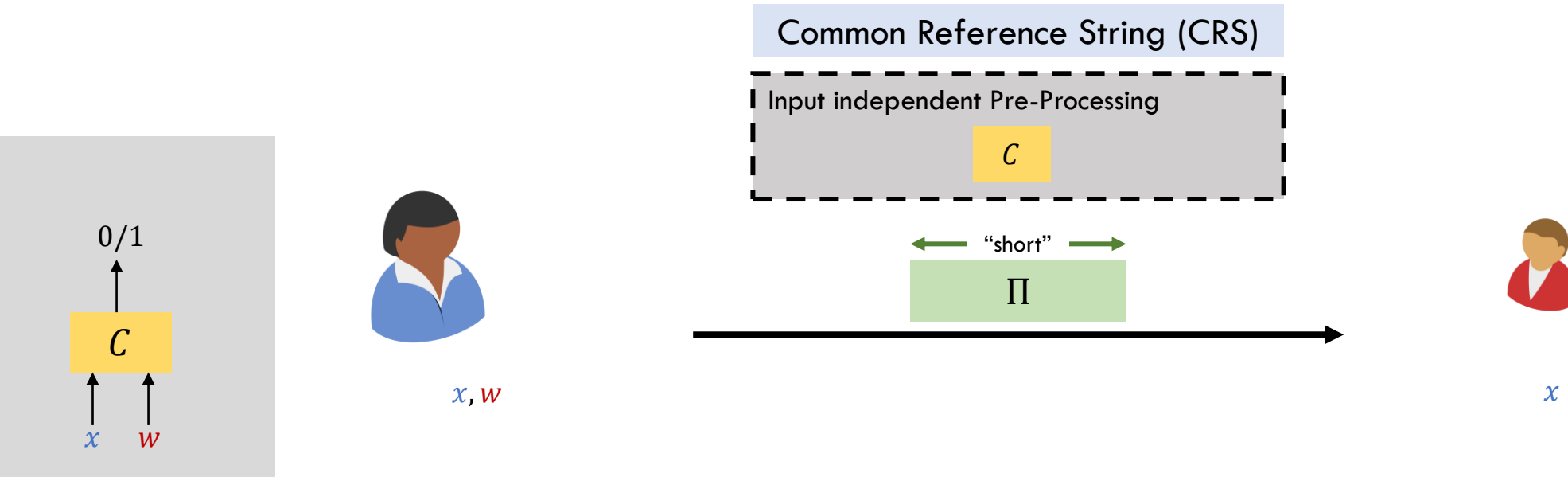
Common Reference String (CRS)



Short proof: $|\Pi| < |w|$

Prover Time: Grows with $|C|$

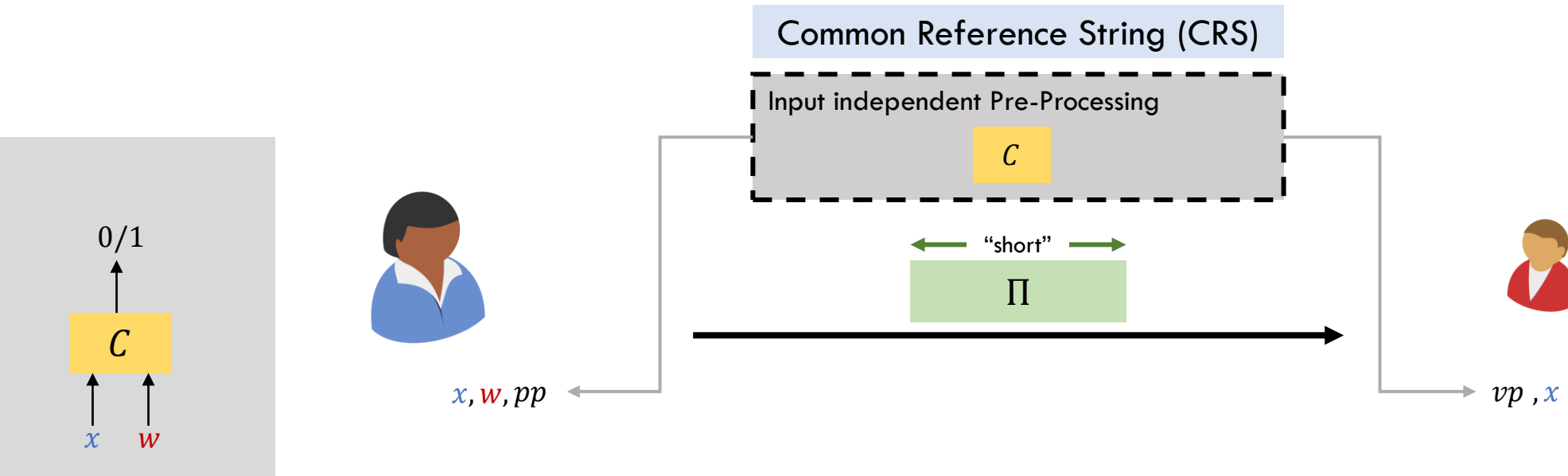
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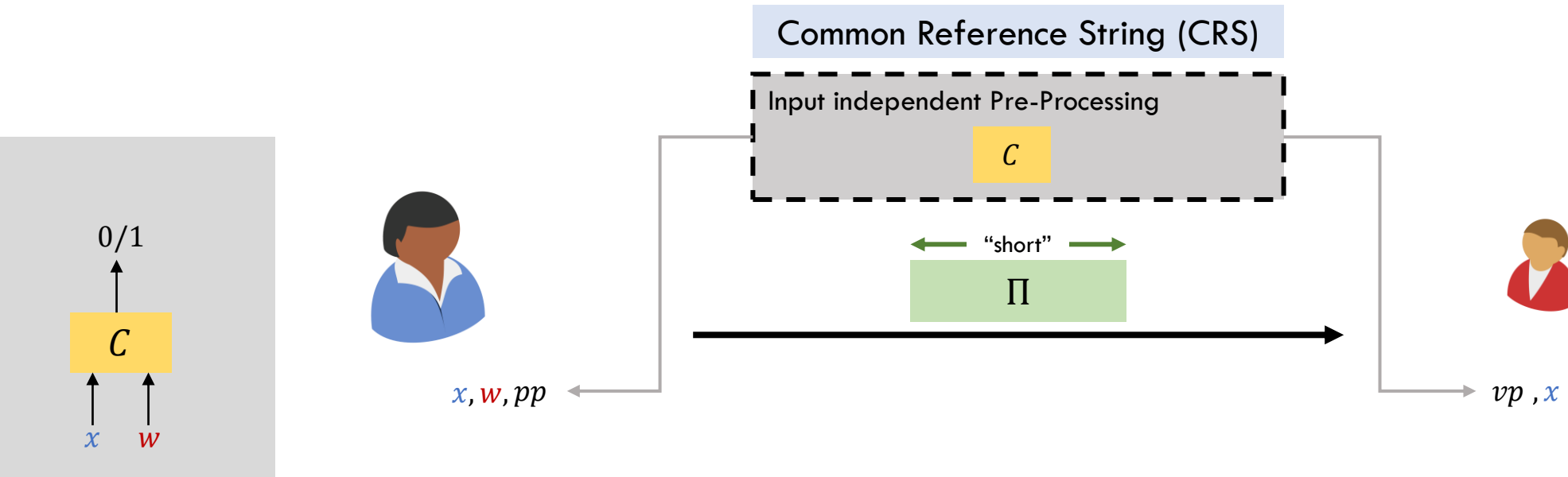
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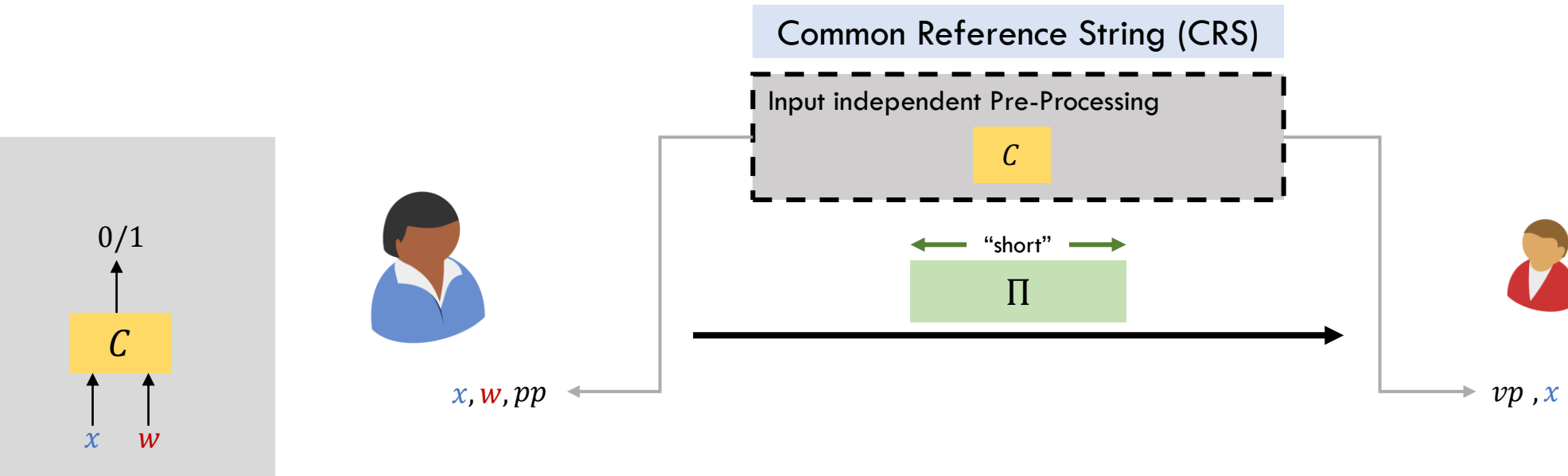


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Verification Time $< |x| + |C|$

Succinct Non-Interactive Arguments of Knowledge (SNARKs)



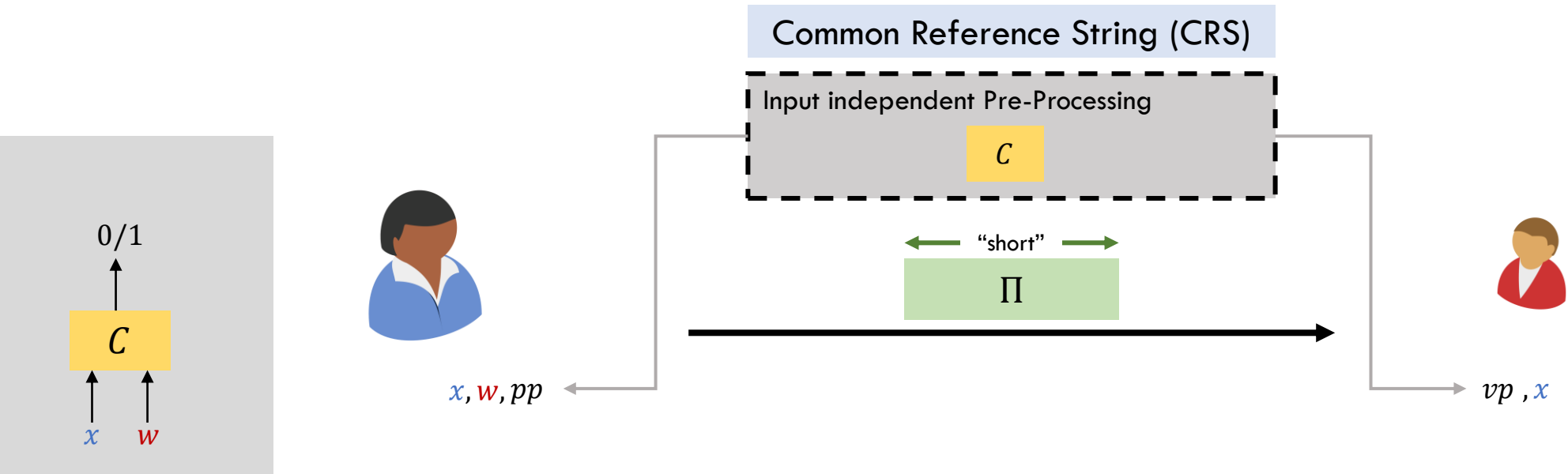
PlonK [Gabizon-Williamson-Ciobotaru'19]

Short proof: $|\Pi| = O(1)$

Prover Time: Grows with $|C|$

Verification Time = $|x| + O(1)$

Succinct Non-Interactive Arguments of Knowledge (SNARKs)



PlonK [Gabizon-Williamson-Ciobotaru'19]

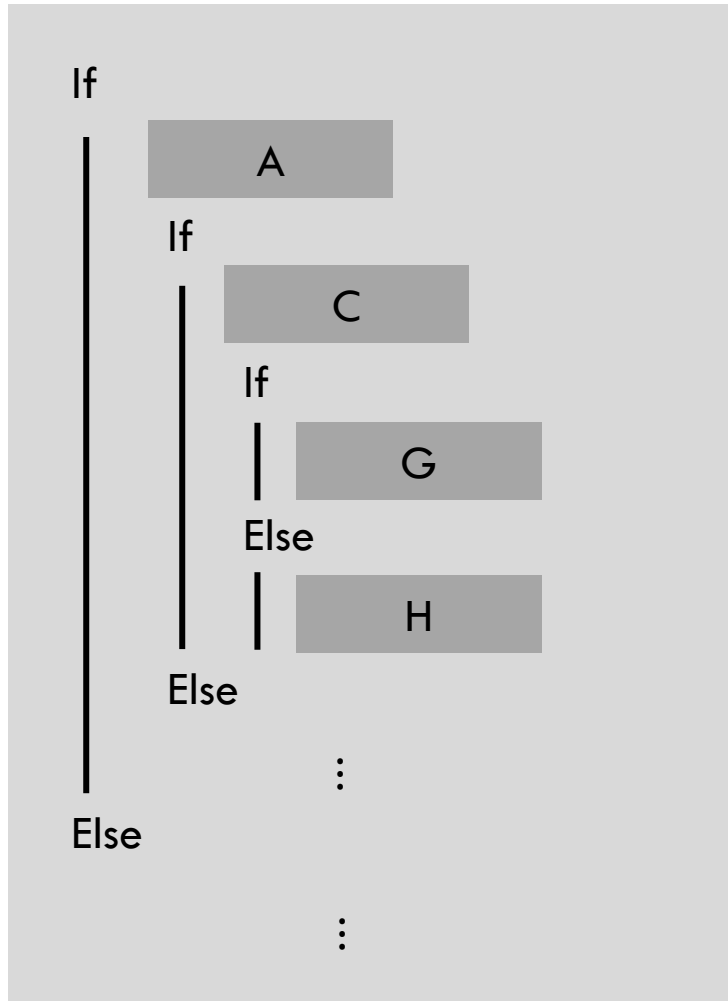
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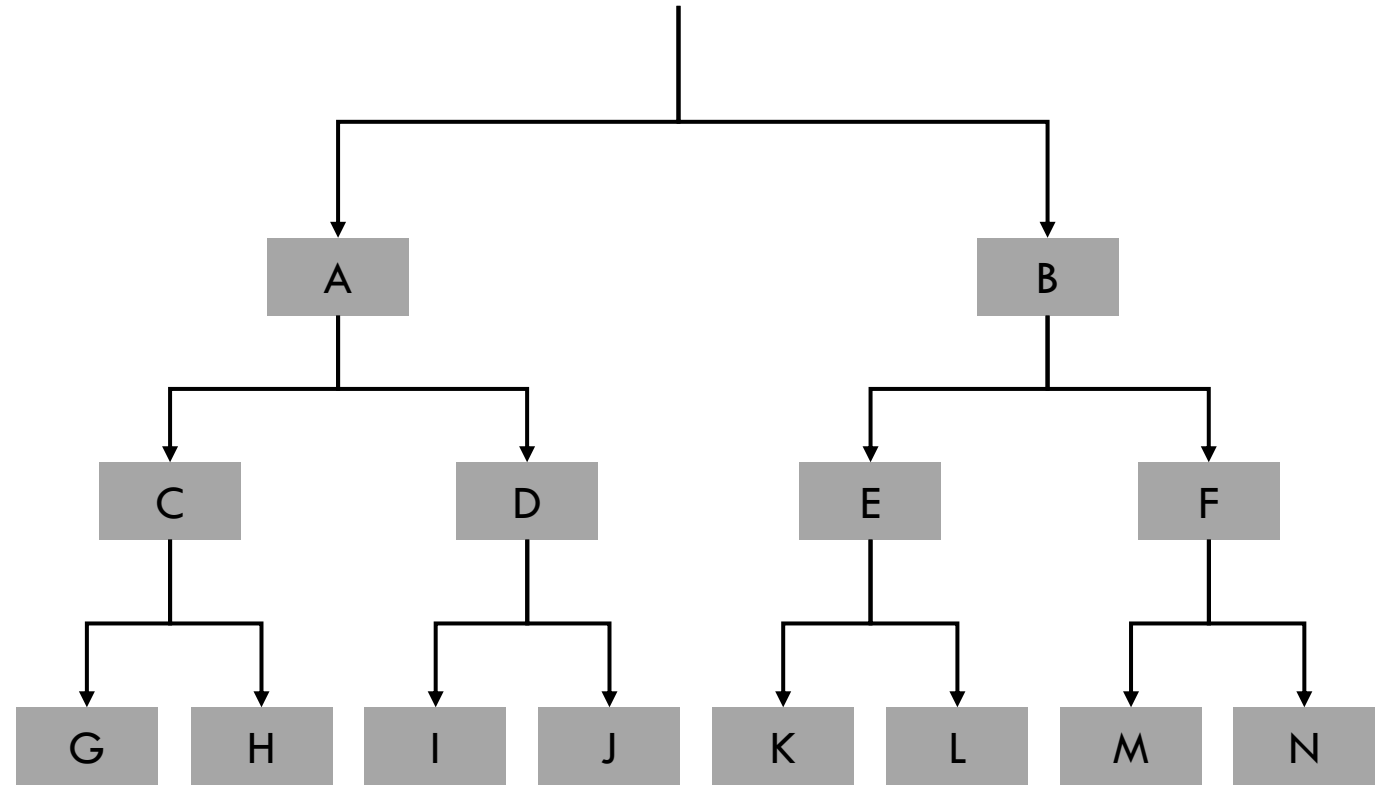
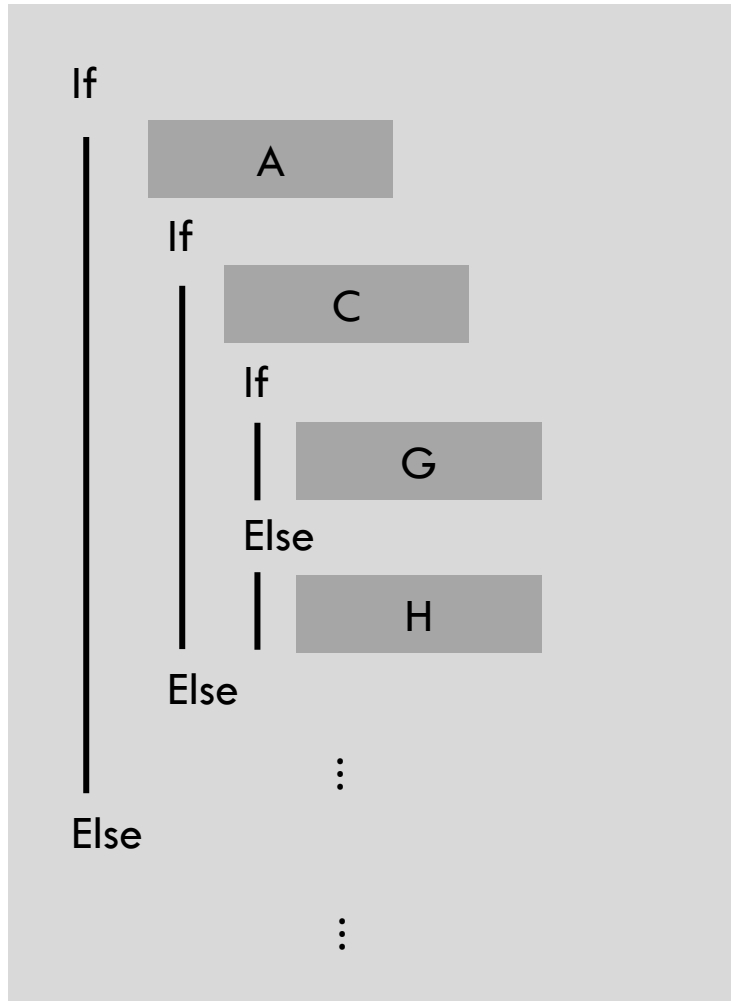
Verification Time = $|x| + O(1)$

- widely used in practice
- proof size of 600 bytes
- support for custom and lookup gates.

SNARKs for Nested-Ifs



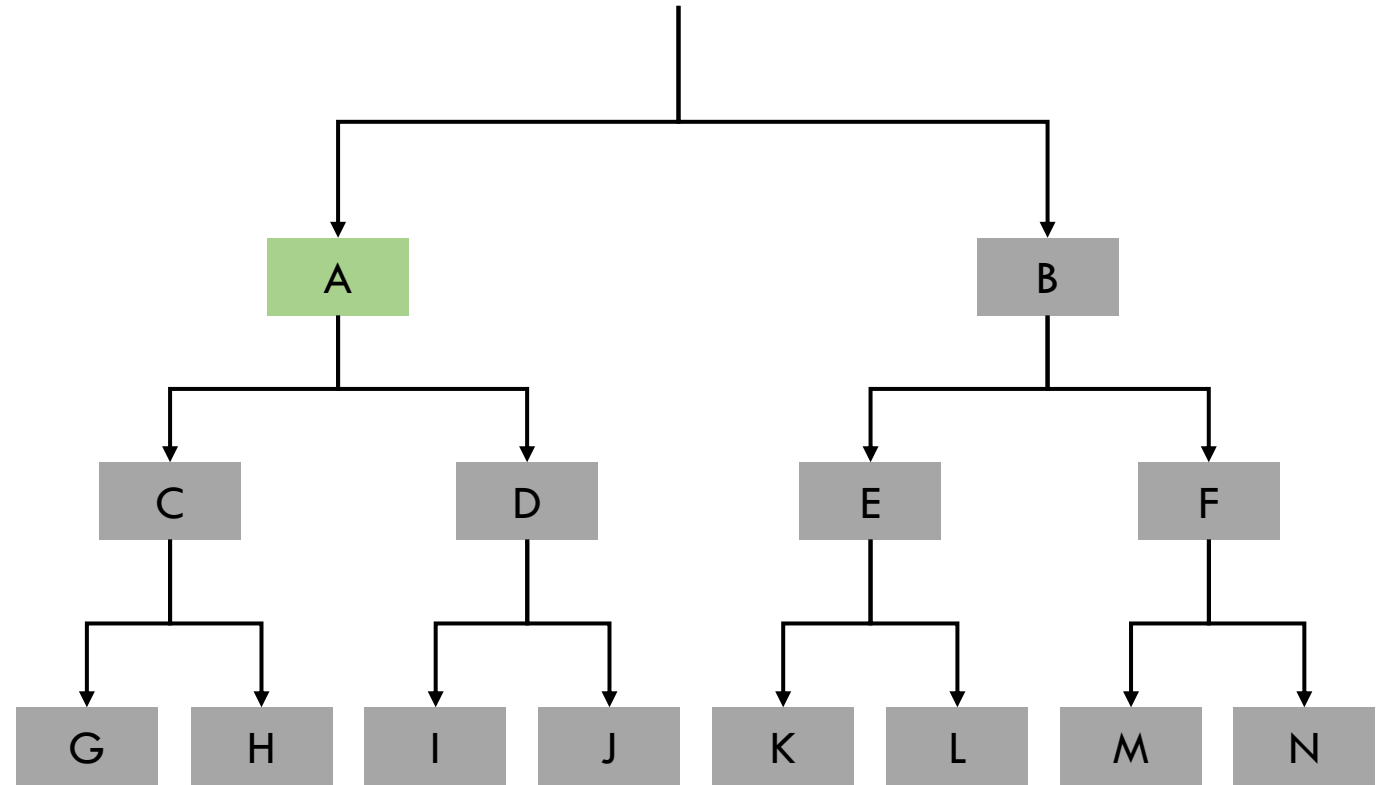
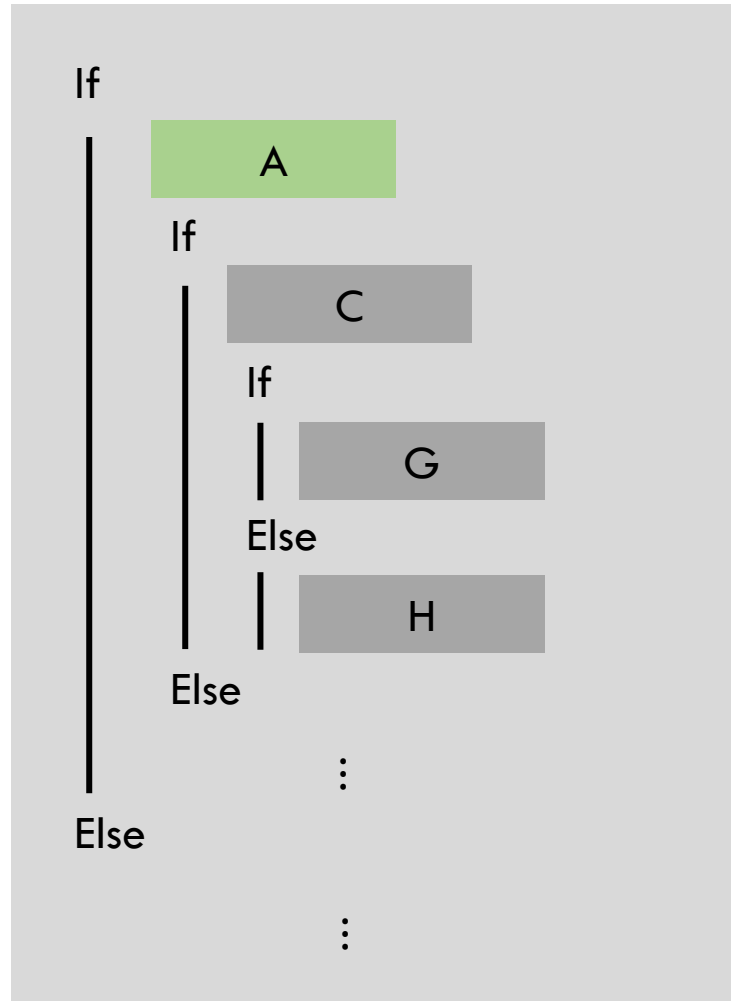
SNARKs for Nested-Ifs



Circuit representation of the program

SNARKs for Nested-Ifs

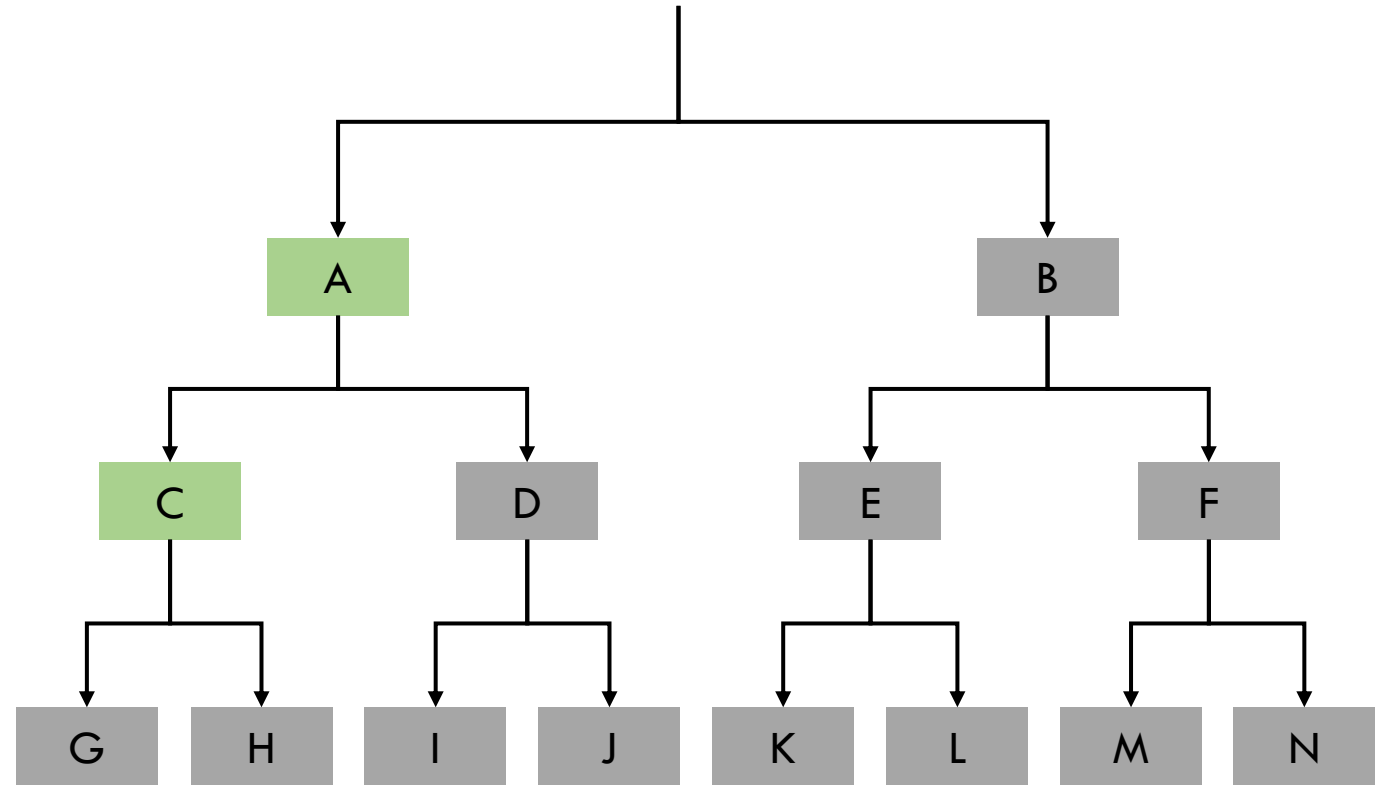
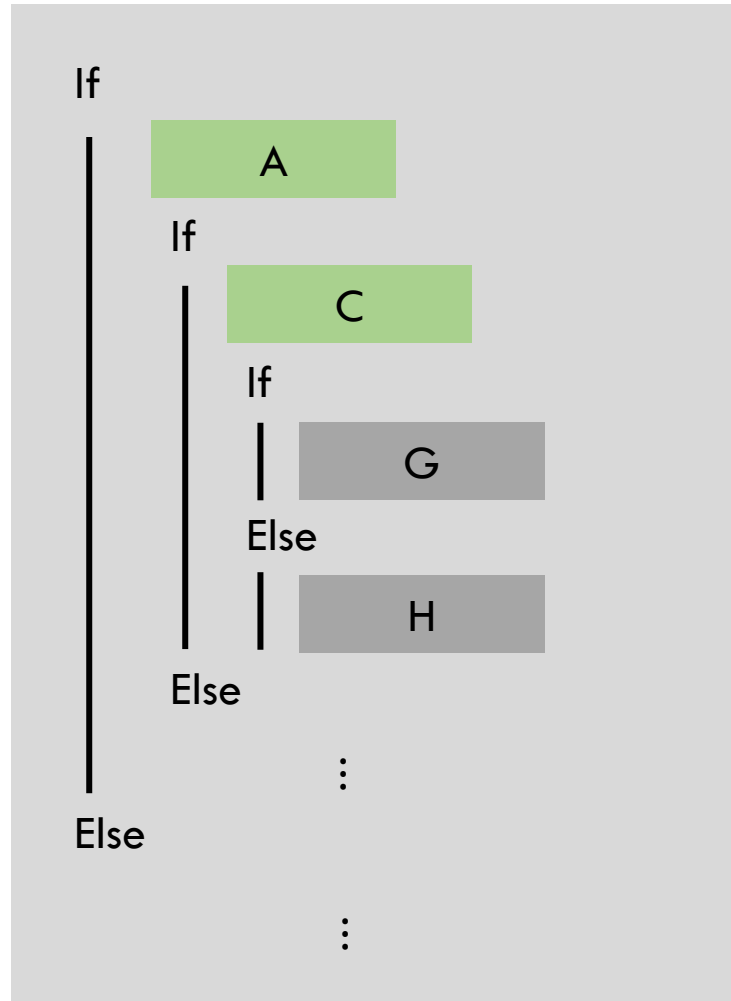
Example execution on some input



Circuit representation of the program

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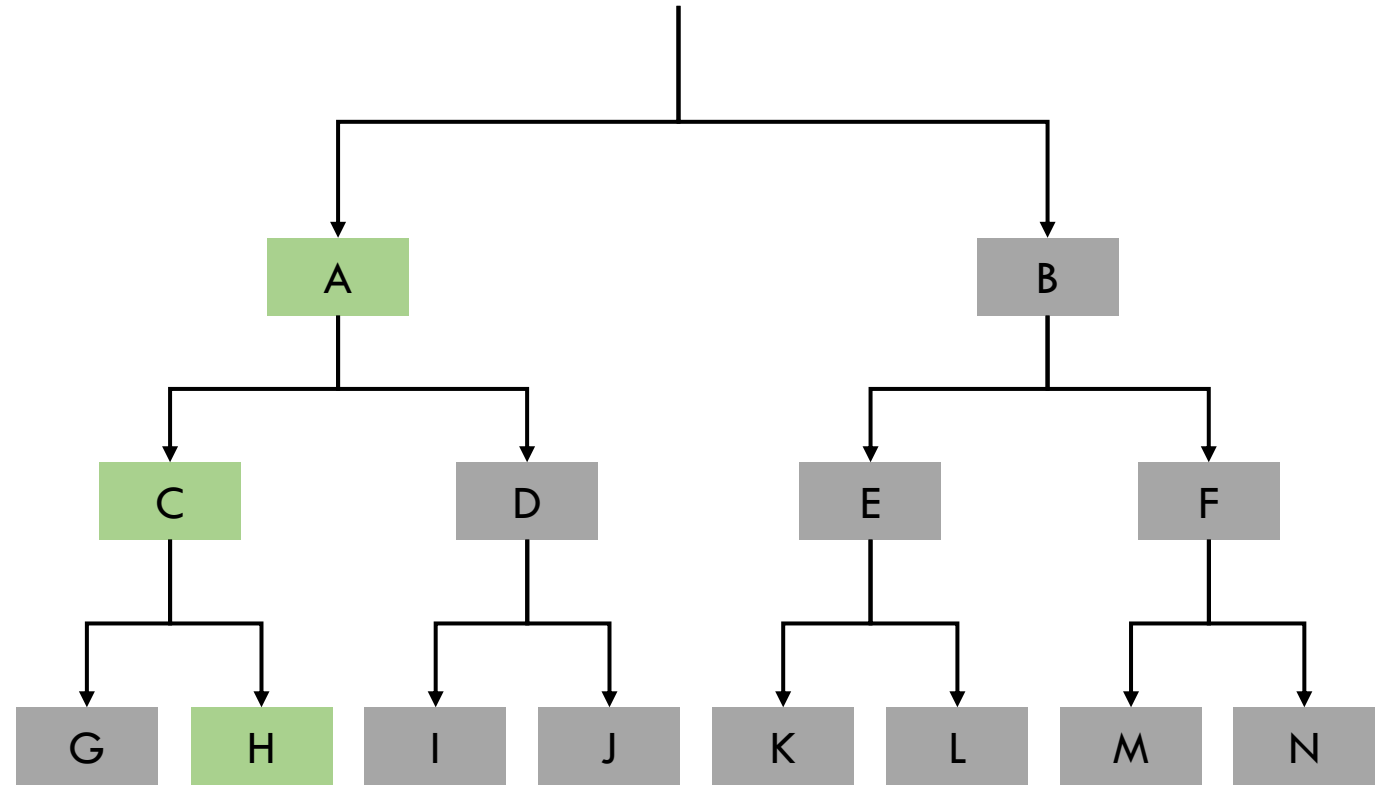
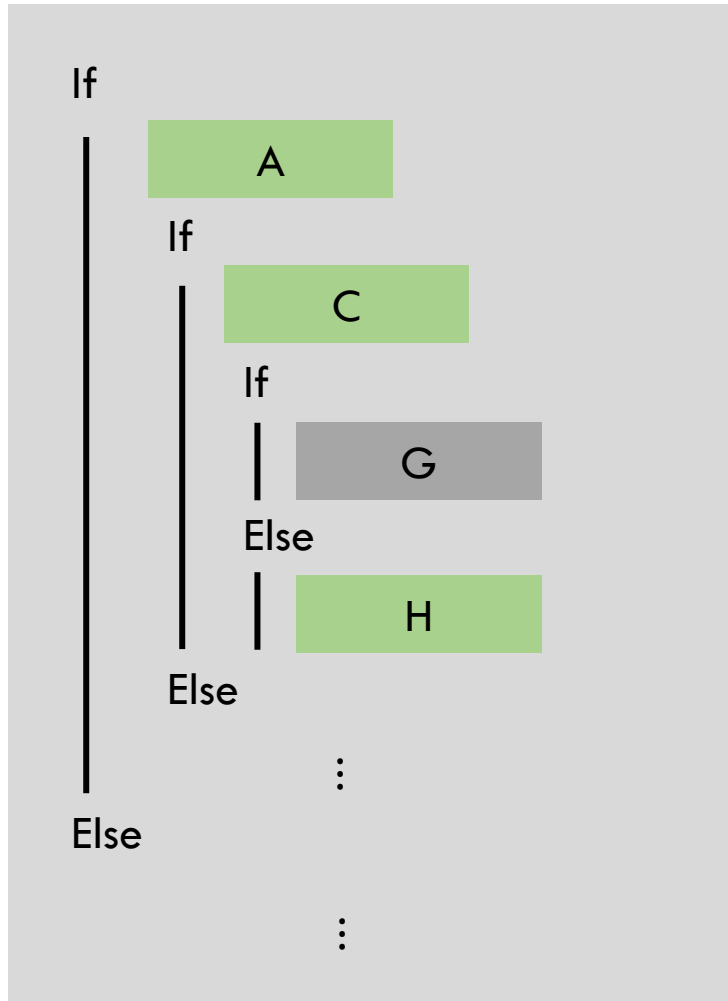
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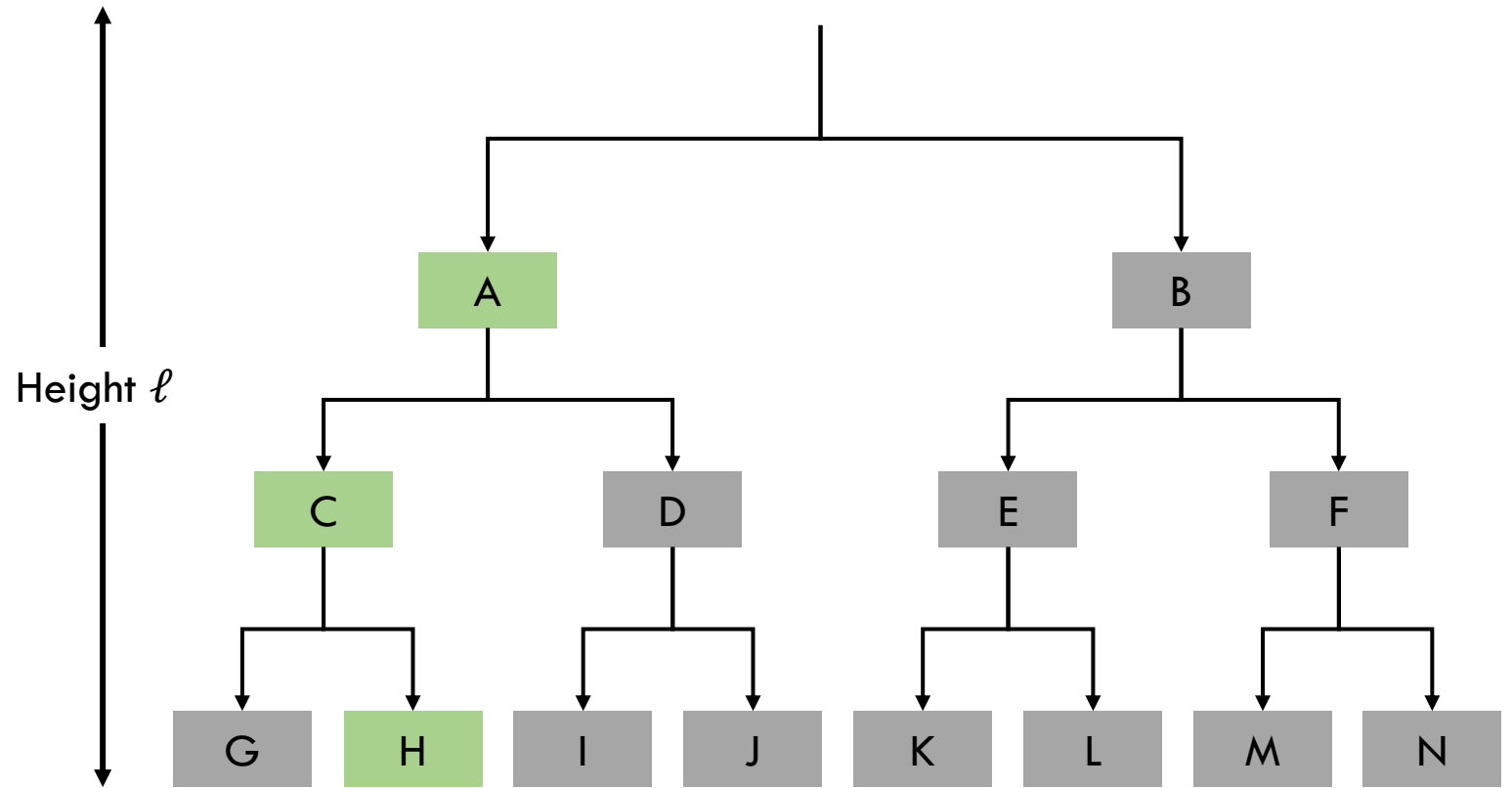
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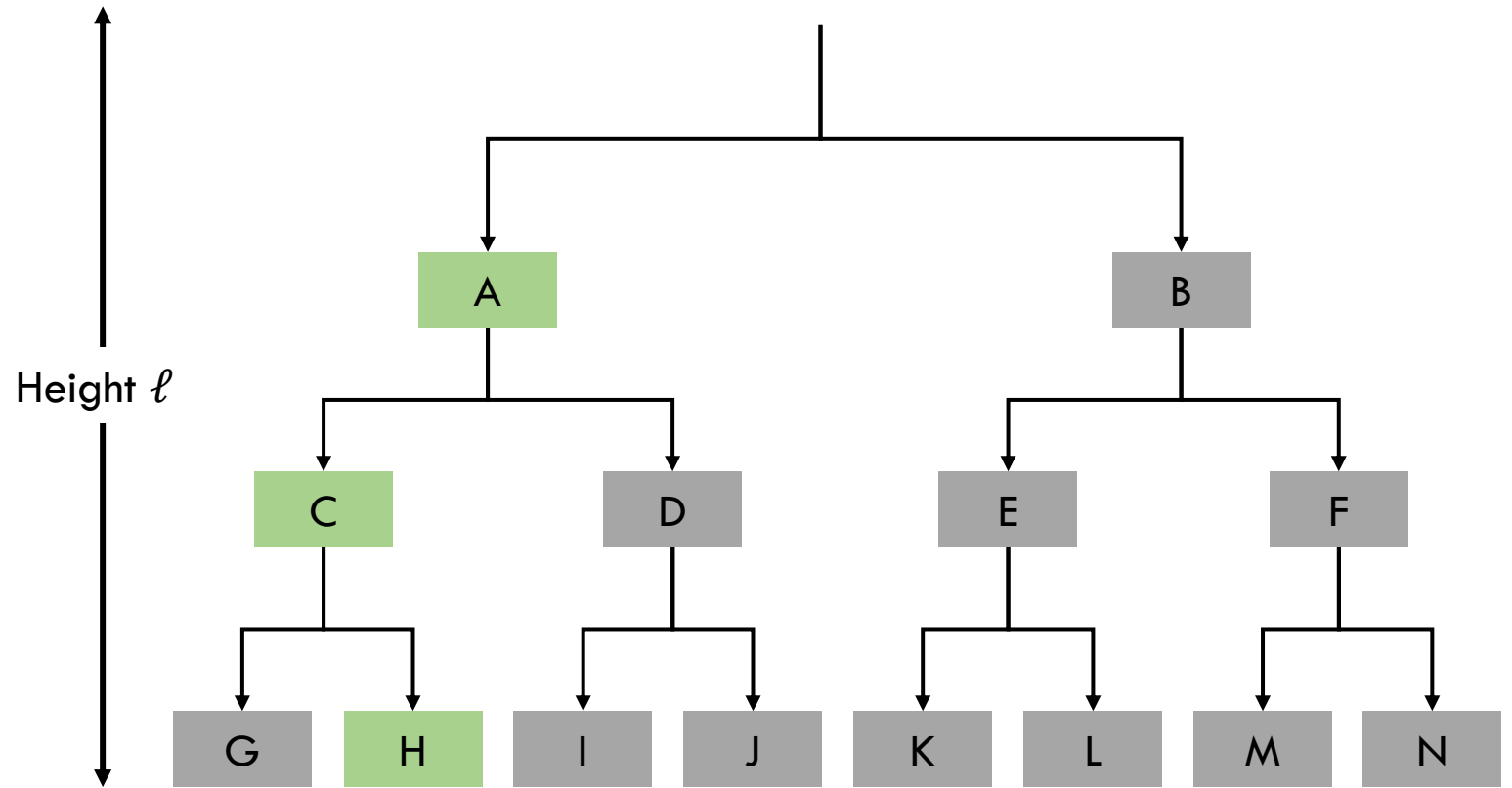


Circuit representation of the program

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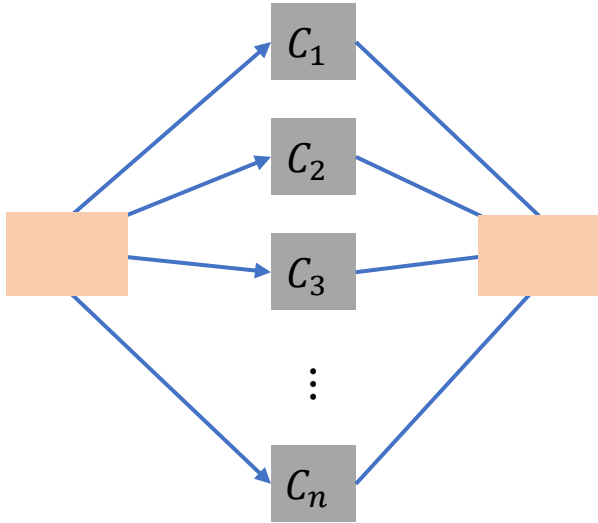
Time to compute SNARK: grows with $O(2^\ell)$

Fraction of **active circuit**: $\ell/2^\ell$



Circuit representation of the program

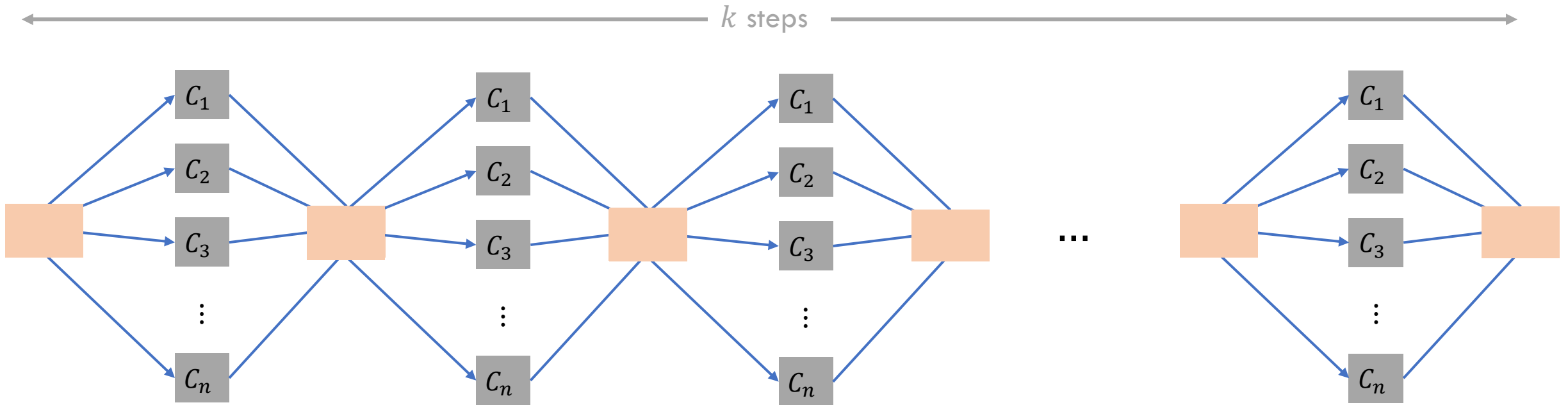
Layered Branching Circuit



Choice on n code segments, each of size s .

Only **one** code segment **active** for any input.

Layered Branching Circuit

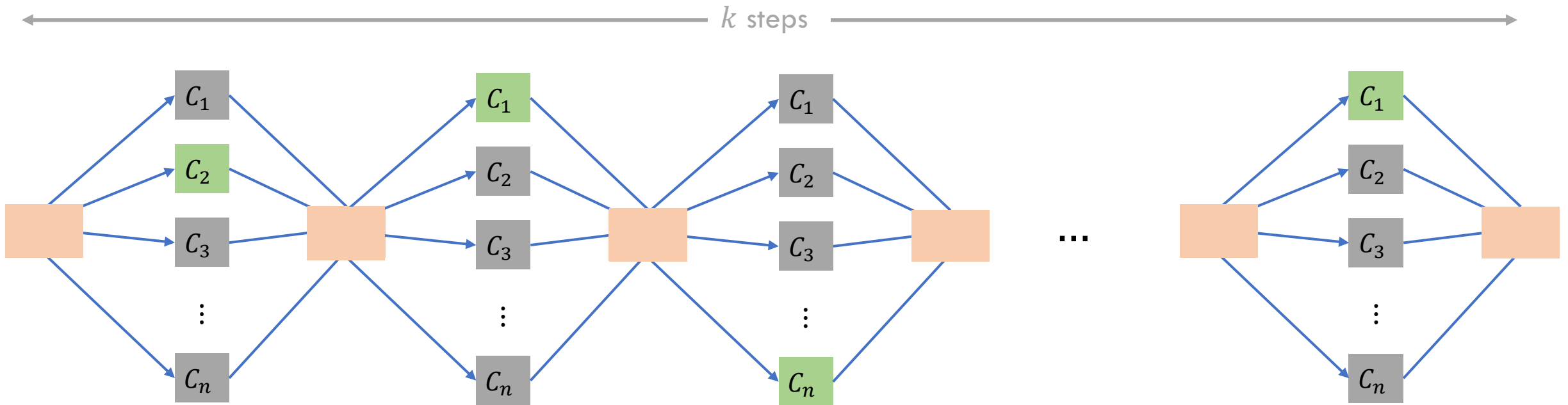


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$$|C| = n \cdot k \cdot s$$

Layered Branching Circuit

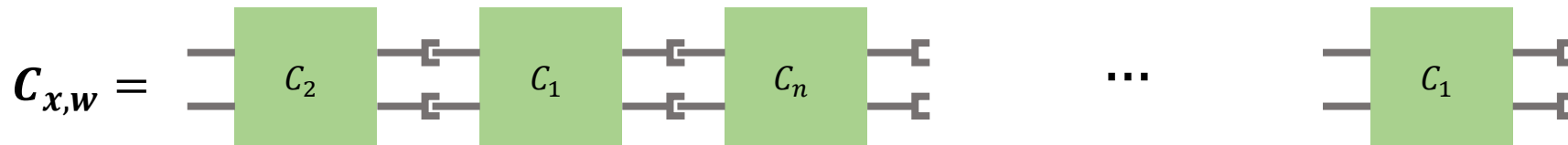
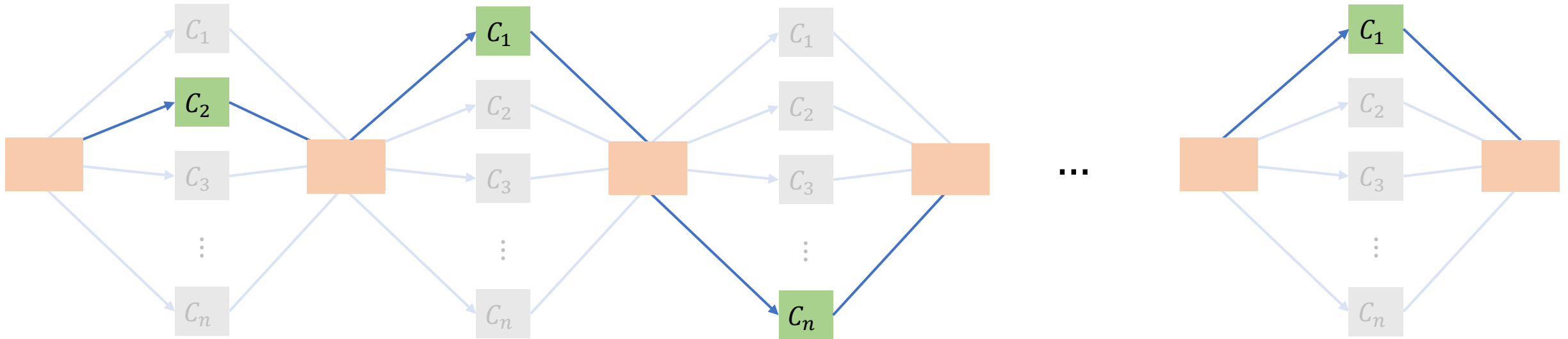


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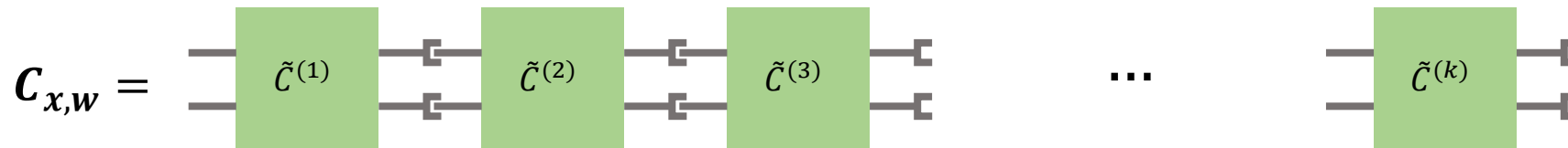
Layered Branching Circuit





Size of active circuit
 $k \cdot s$

Layered Branching Circuit

Active Circuit for an input x, w



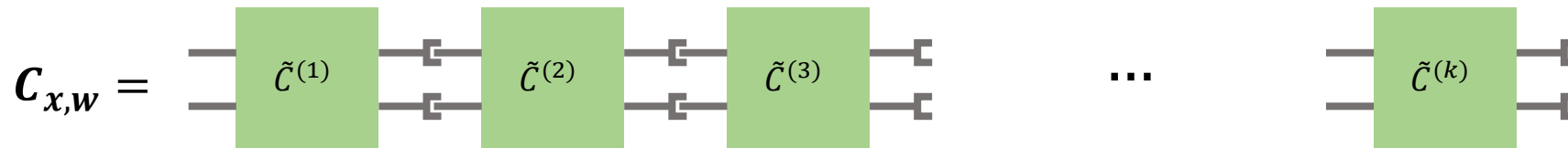
$j \in [k]$  Each $\tilde{C}^{(j)}$ is a circuit C_i 
 $i \in [n]$


$$|C_{x,w}| = k \cdot s$$

Layered Branching Circuit


Also captures the notion of [Rollups](#) in Blockchains.

Active Circuit for an input x, w



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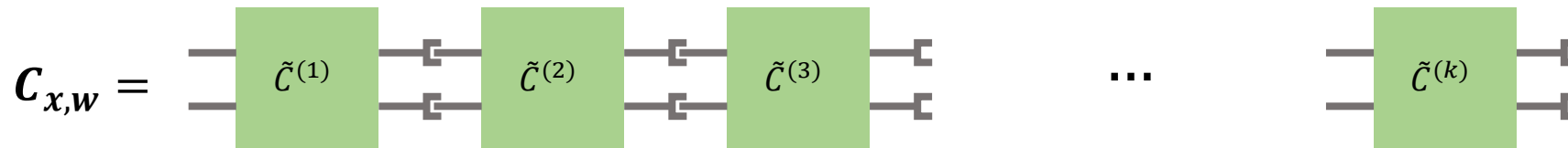
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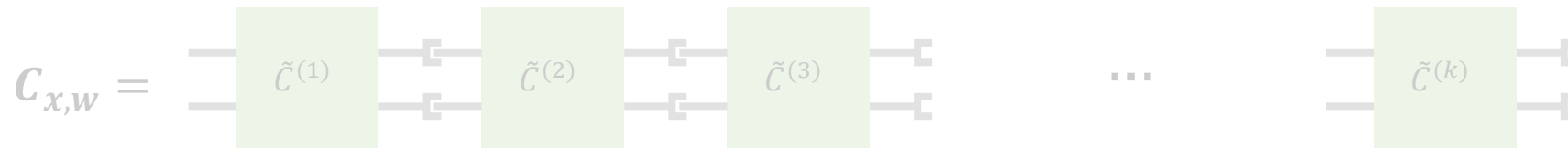


Can we construct SNARKs for Layered Branching Circuits where the **online prover time** grows only with the **size of the active circuit**?

$$|C_{x,w}| = k \cdot s$$

Layered Branching Circuit

Active Circuit for an input x, w



Allow the prover to perform a one-time input independent pre-processing of the entire circuit C

Can we construct SNARKs for Layered Branching Circuits where the **online prover time** grows only with the size of the active circuit?

$$|C_{x,w}| = k \cdot s$$

Prior Works

Prior Works

A la carte proof cost

Buffet [Wahby-Setty-Ren-Blumberg-Walfish'15], **vRAM** [Zhang-Genkin-Katz-Papadopoulos-Papamanthou'18], **Mirage** [Kosba-Papadopoulos-Papamanthou-Song'20]

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(Non-Uniform) Incrementally Verifiable Proof

Sangria, SuperNova [Kothapalli-Setty'22], ProtoStar [Bünz-Chen'23]

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Not black-box in
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Input dependent prover pre-processing.

STARKs

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Our Result

Theorem: Sublinear Prover \mathcal{P}_{InK}

Online Prover Time - $O(ks (\log ks + \log n))$

Proof Size - $O(1)$

Black-box in Cryptography with input independent pre-processing for prover and verifier.

Universal setup, support for custom and lookup gates.

Our Result

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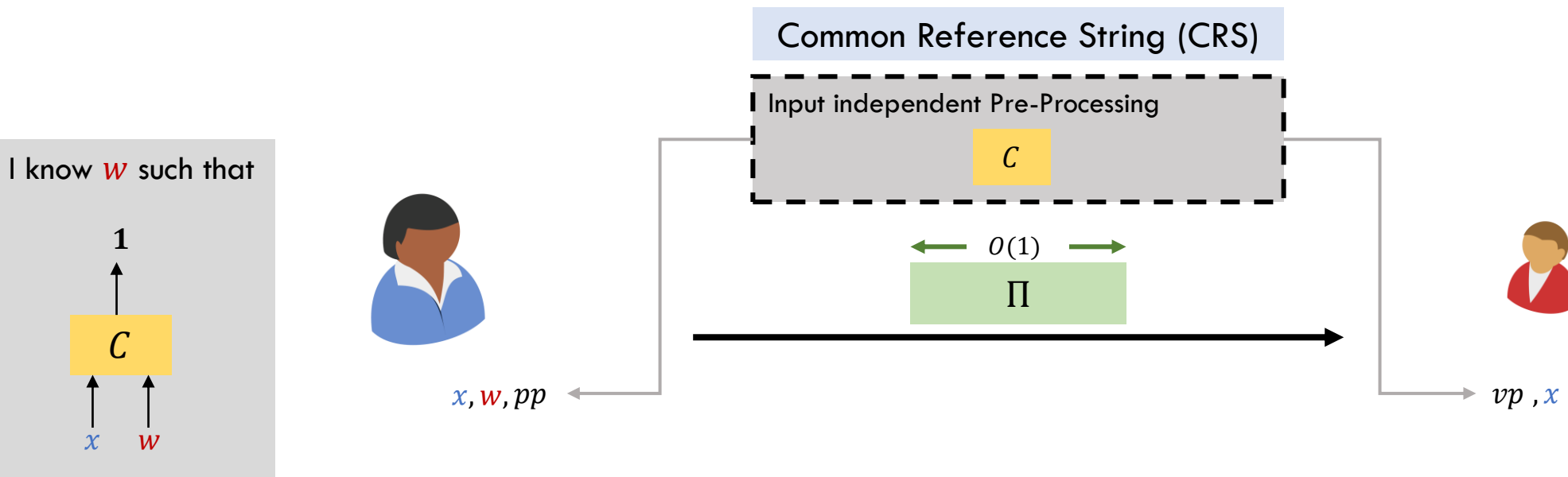
Proof Size - $O(1)$

Black-box in Cryptography with input independent pre-processing for prover and verifier.

Universal setup, support for custom and lookup gates.

A concurrent work [Di-Xia-Nguyen-Tyagi'23] uses similar ideas to achieve an almost identical result where the online prover time is $\tilde{O}((k + n)s)$.

\mathcal{PlonK} [Gabizon-Williamson-Ciobotaru'19]

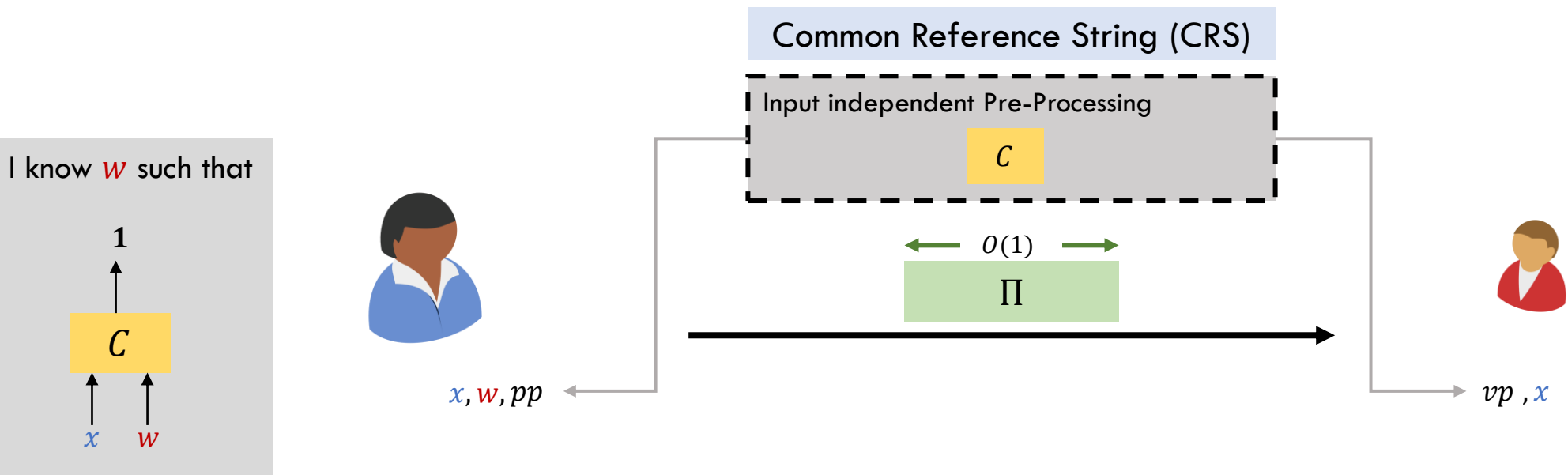


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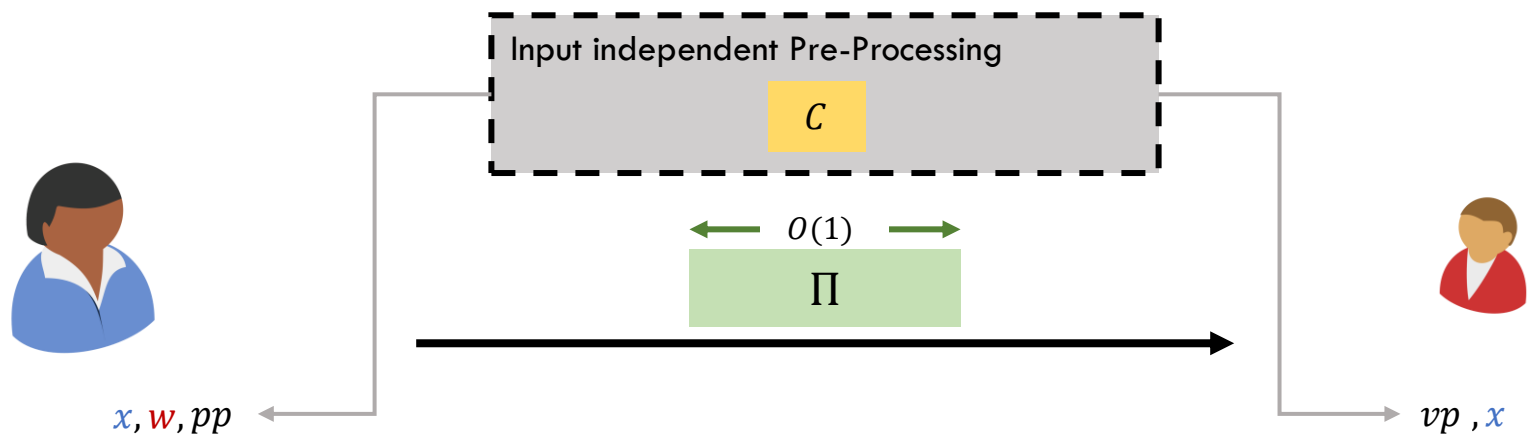
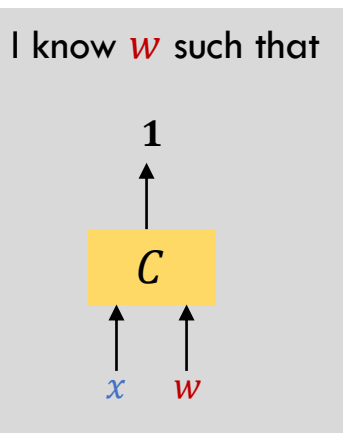
Verification Time $\approx |x| + O(1)$

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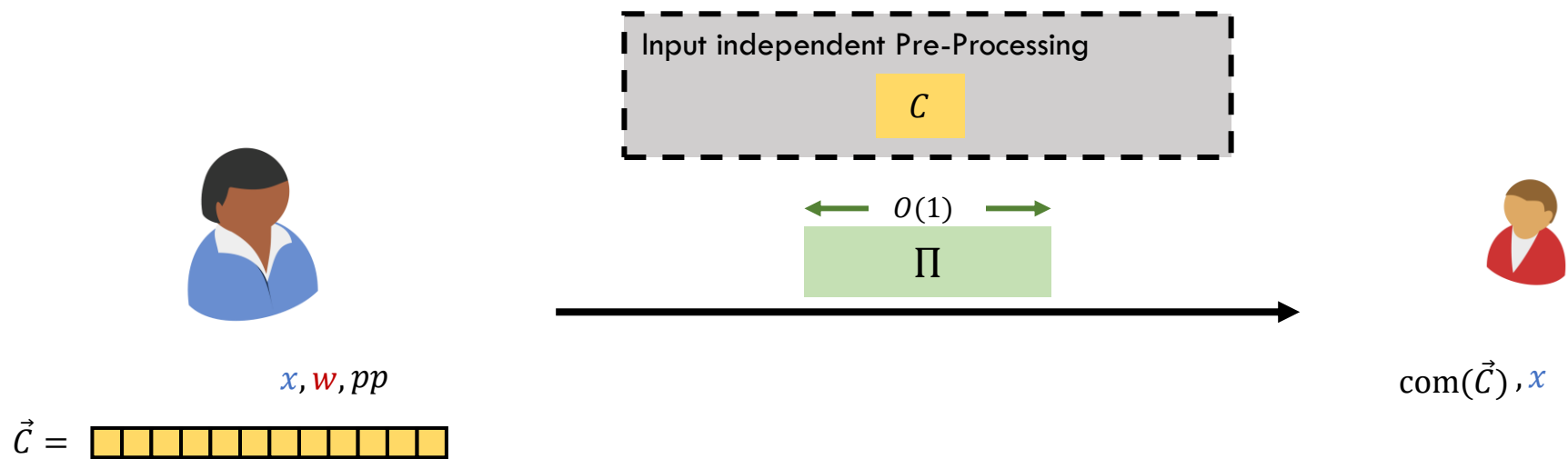


Ignore CRS in this talk for simplicity.

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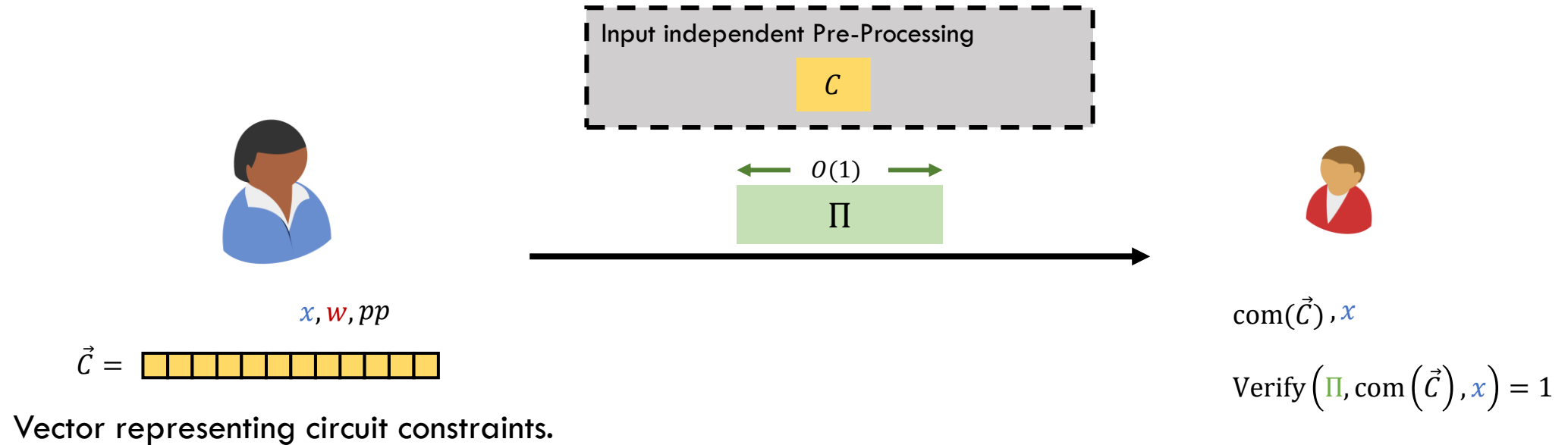


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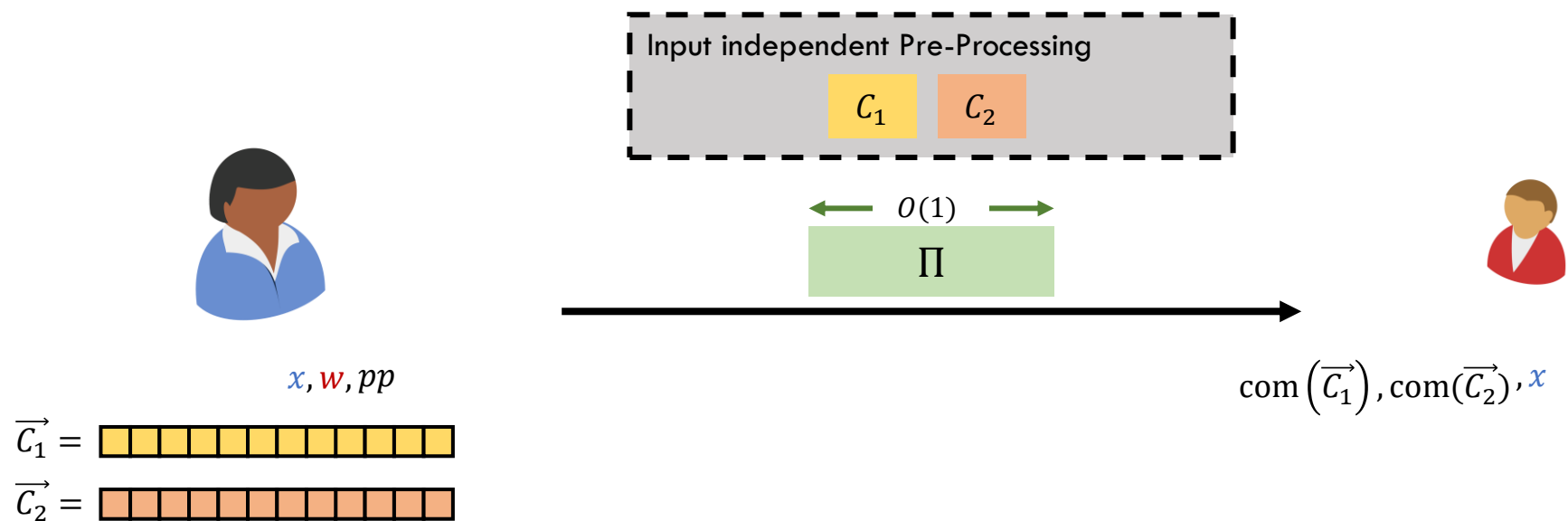


Vector representing circuit constraints.

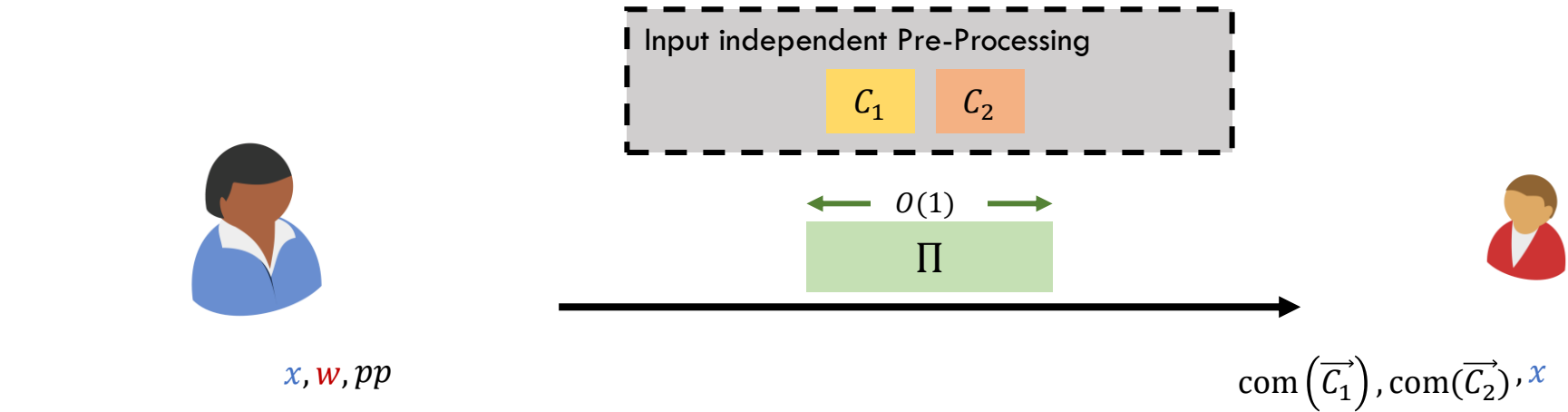
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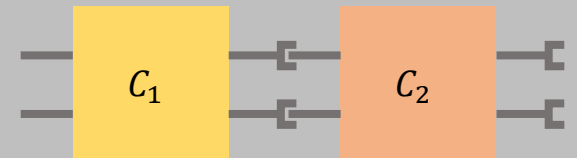


$\vec{C}_1 =$

$\vec{C}_2 =$

Observation

$\text{com}(\vec{C}_1) \circ \text{com}(\vec{C}_2) = \text{com}(\vec{C}_1 \parallel \vec{C}_2)$ is pre-processed commitment* for



Sublinear Prover \mathcal{P} on \mathcal{K}



x, w, pp



$\text{com}(C_1), \text{com}(C_2), \dots, \text{com}(C_n)$

x

Sublinear Prover \mathcal{P} on \mathcal{K}



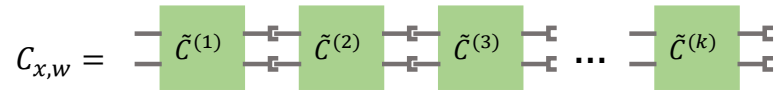
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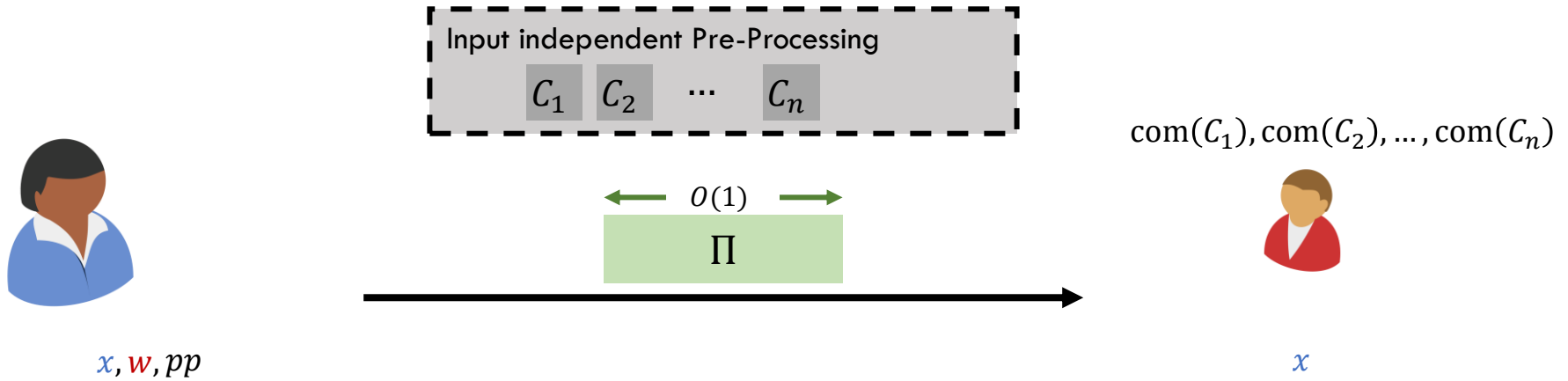
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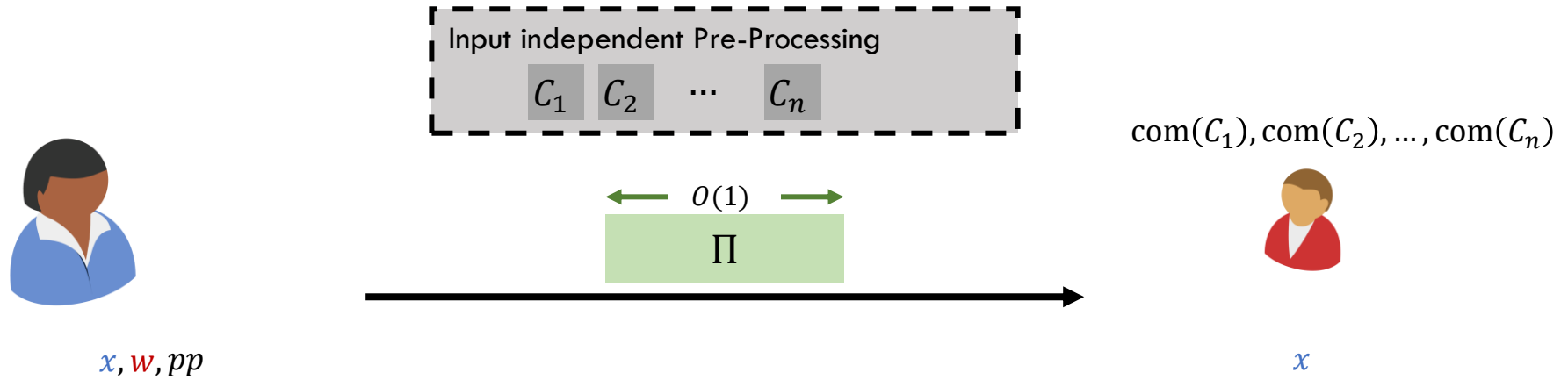
Sublinear Prover \mathcal{PlonK}



$$C_{x,w} = \tilde{C}^{(1)} \tilde{C}^{(2)} \tilde{C}^{(3)} \dots \tilde{C}^{(k)}$$

$$\text{Verify}(\Pi, \text{com}(C_{x,w}), x) = 1$$

Sublinear Prover \mathcal{PlonK}

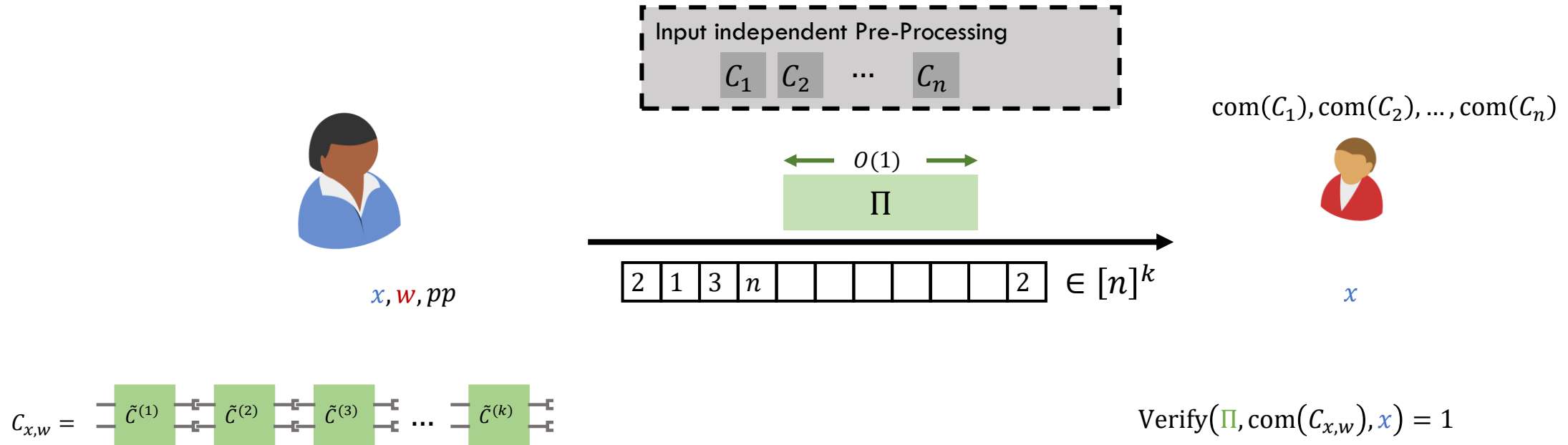


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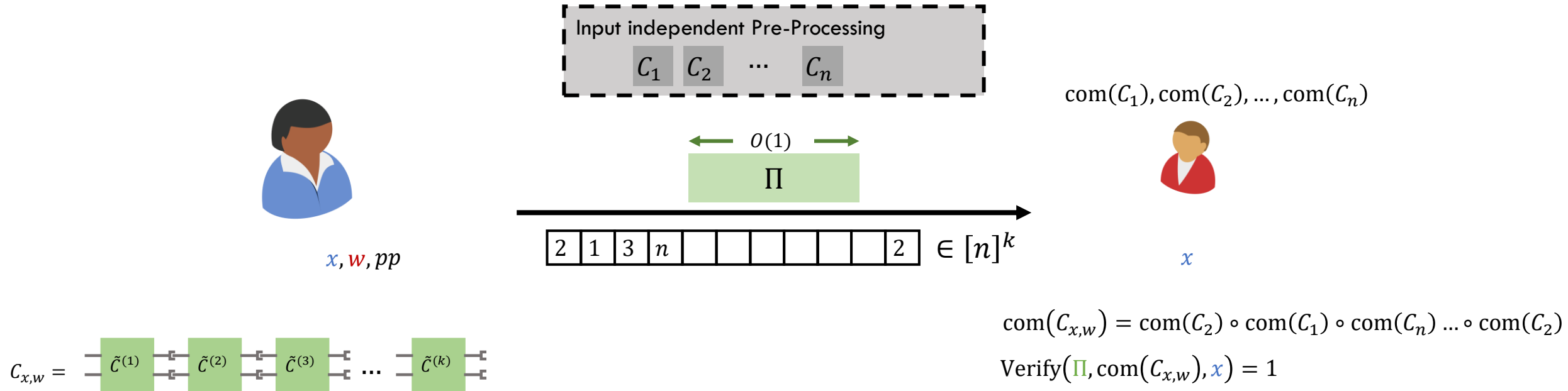
Key Insight: Generate $\text{com}(C_{x,w})$ on the fly

Sublinear Prover \mathcal{P}_{onK}



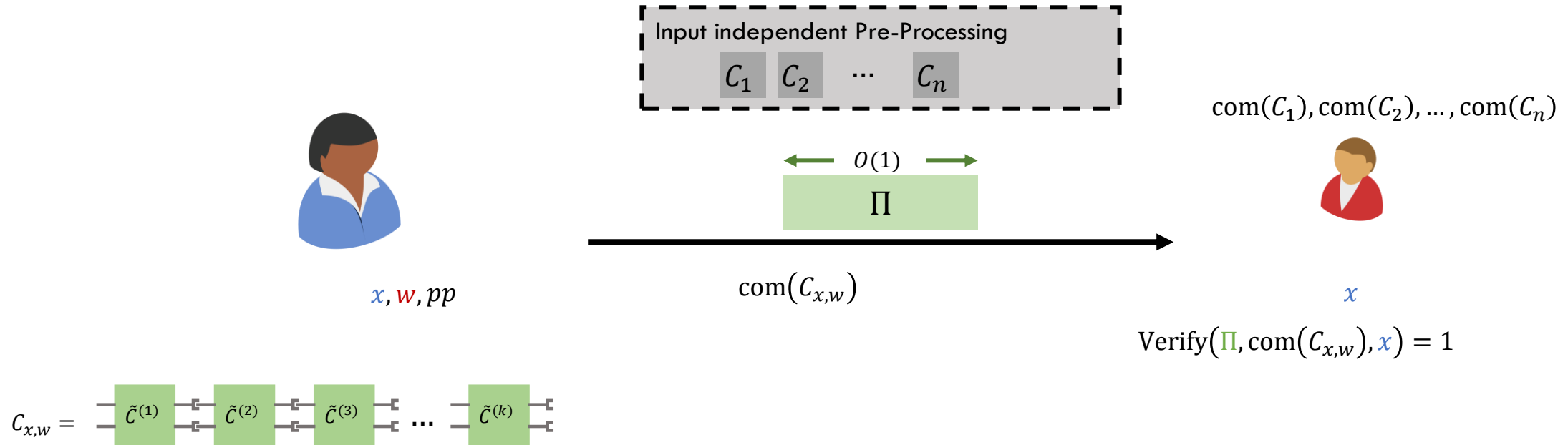
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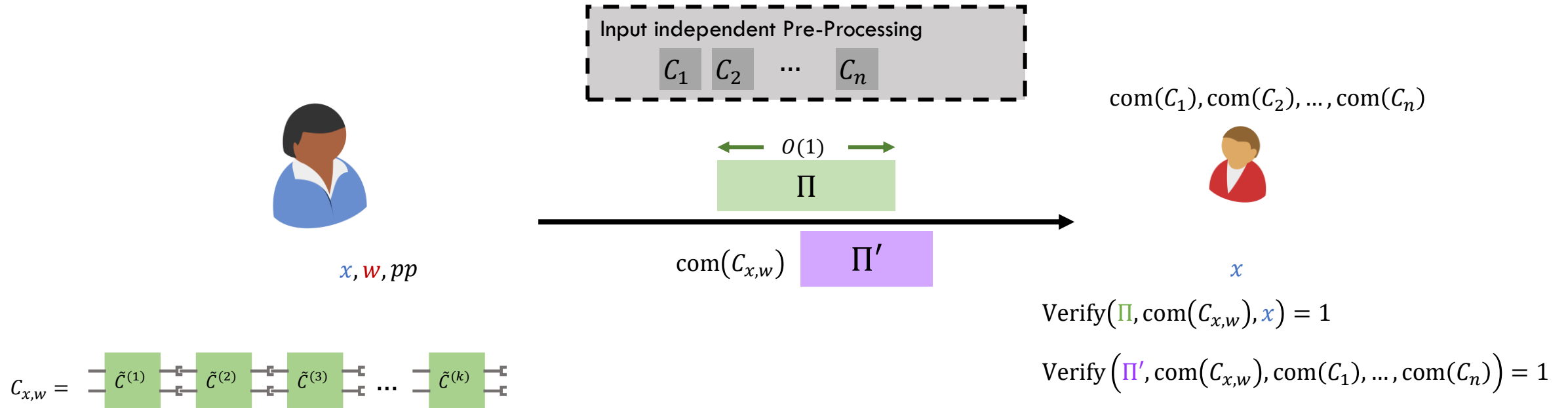


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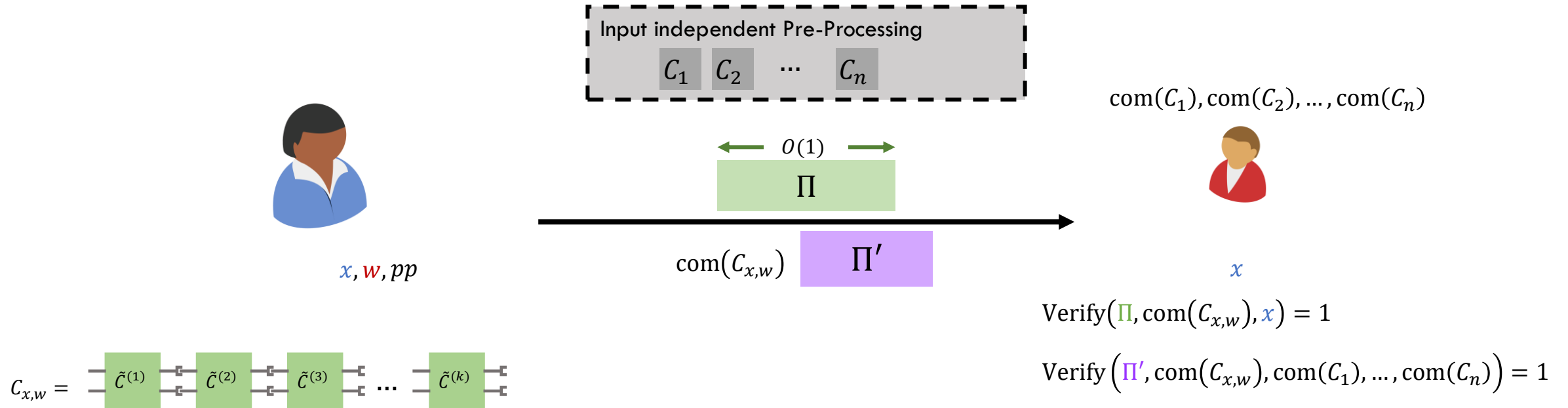
Sublinear Prover \mathcal{PlonK}



Sublinear Prover \mathcal{PlonK}



Sublinear Prover \mathcal{P}_{onK}



Tool to generate Π' : Table Lookups

Table Lookup

Table

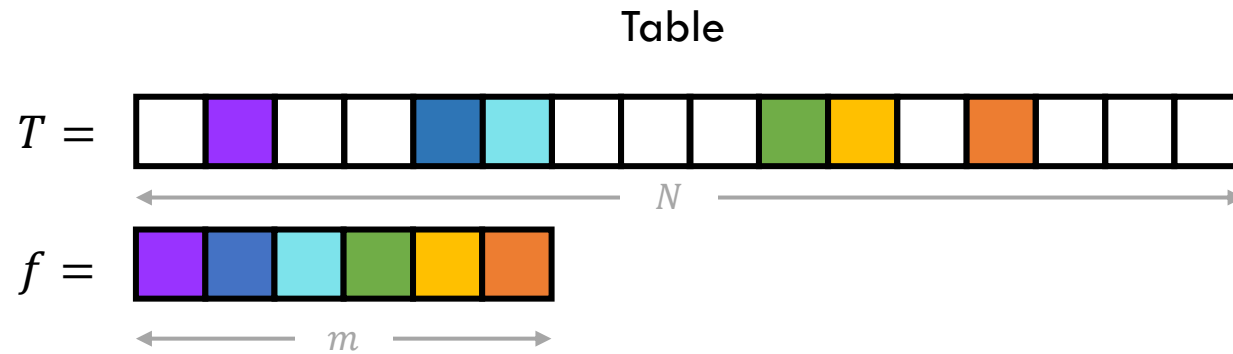


One-time Table Pre-processing



$\text{Com}(T)$

Table Lookup

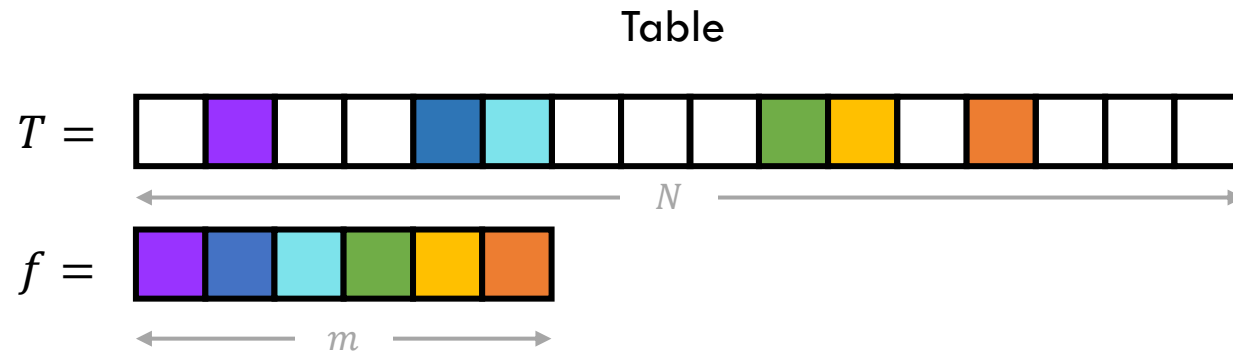


One-time Table Pre-processing



$\text{Com}(T)$

Table Lookup



$\forall i \in [m], \exists j \in [N]$ such that
 $f[i] = T[j]$



One-time Table Pre-processing

Com(f)

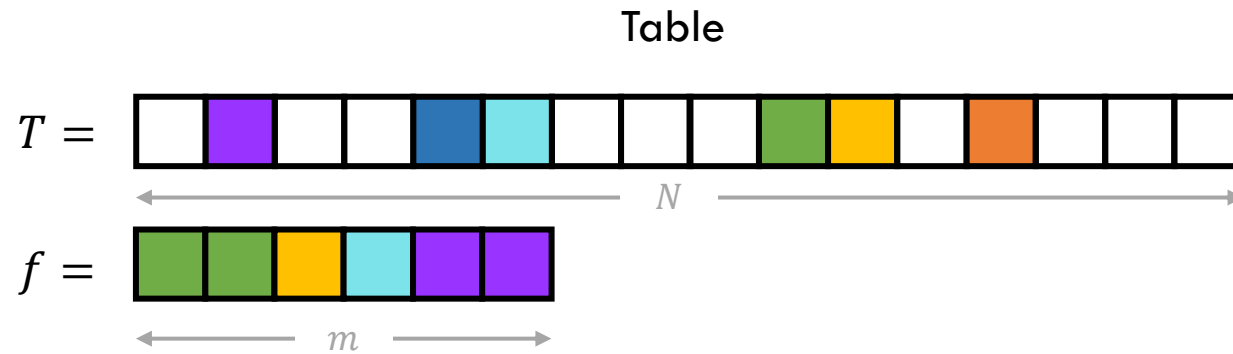
Π



Com(T)

Verify(Π , com(f), com(T)) = 1

Table Lookup



$\forall i \in [m], \exists j \in [N]$ such that

$$f[i] = T[j]$$

Allows **out of order** and **repetitions**.



Com(f)

Π



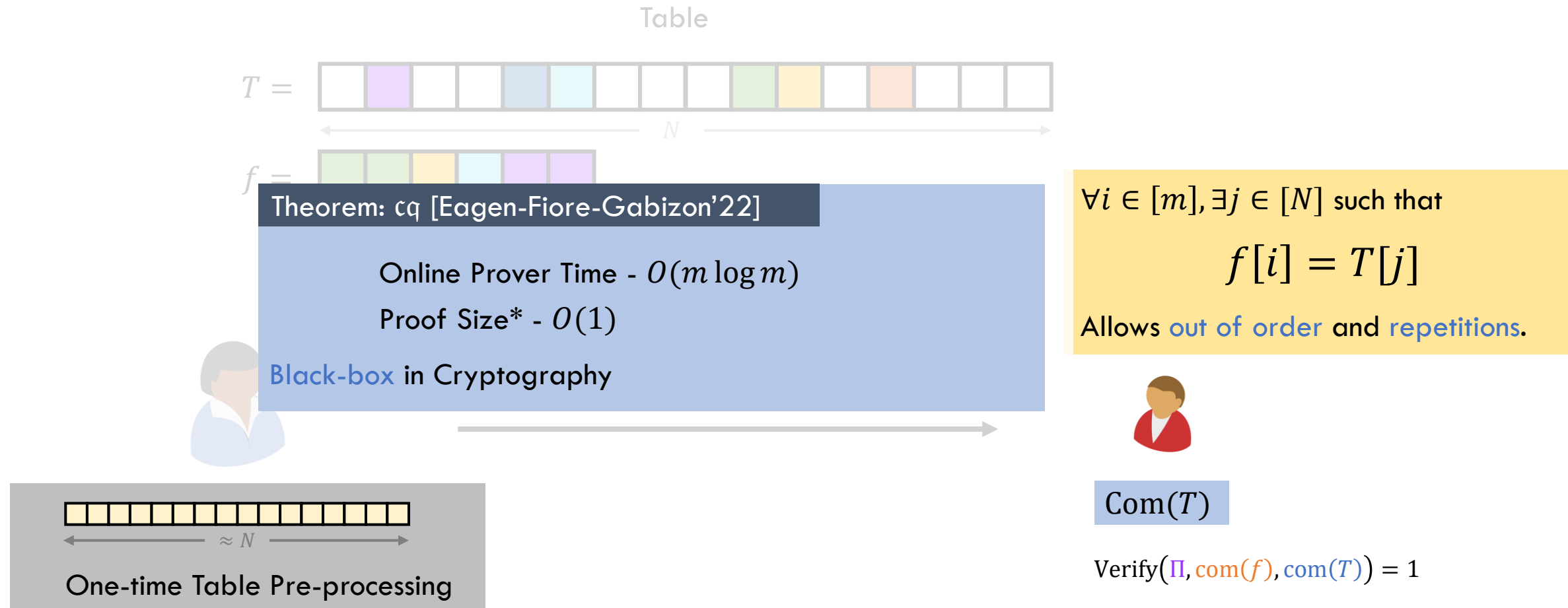
Com(T)

$$\text{Verify}(\Pi, \text{com}(f), \text{com}(T)) = 1$$



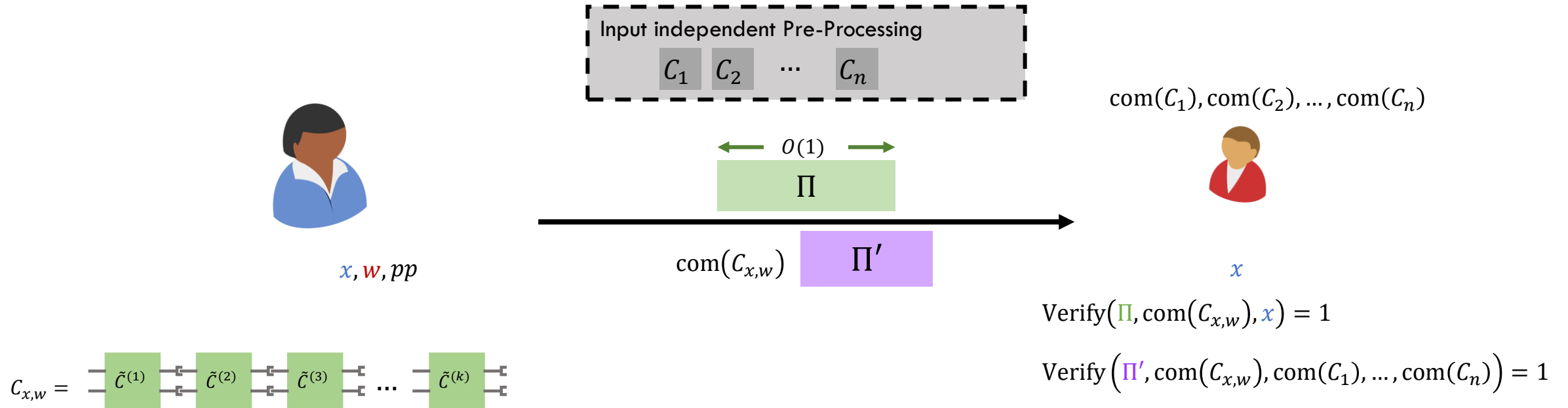
One-time Table Pre-processing

Table Lookup



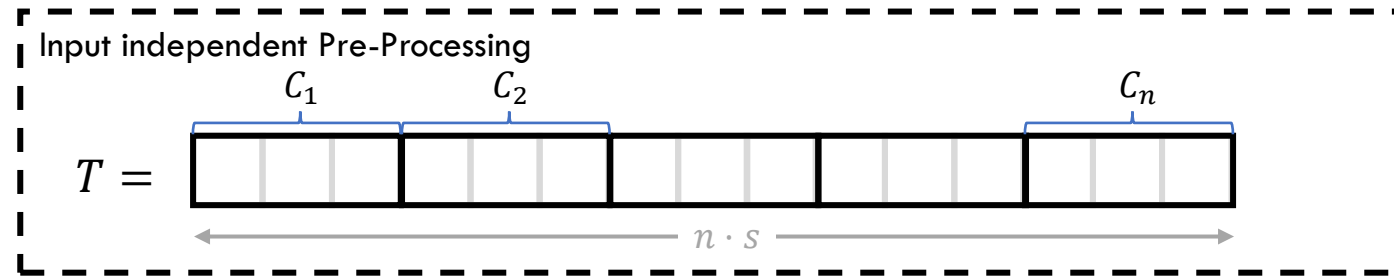
*Proof Size includes size of $\text{com}(f)$

Sublinear Prover \mathcal{PlonK}



Tool to generate Π' : Table Lookups

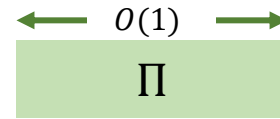
Sublinear Prover \mathcal{P} on \mathcal{K}



$\text{com}(C_1), \text{com}(C_2), \dots, \text{com}(C_n)$



x, w, pp



$\text{com}(C_{x,w})$

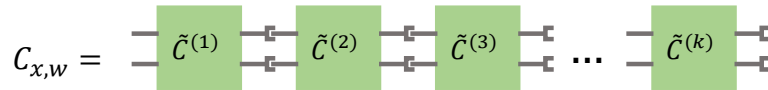
Π'



x

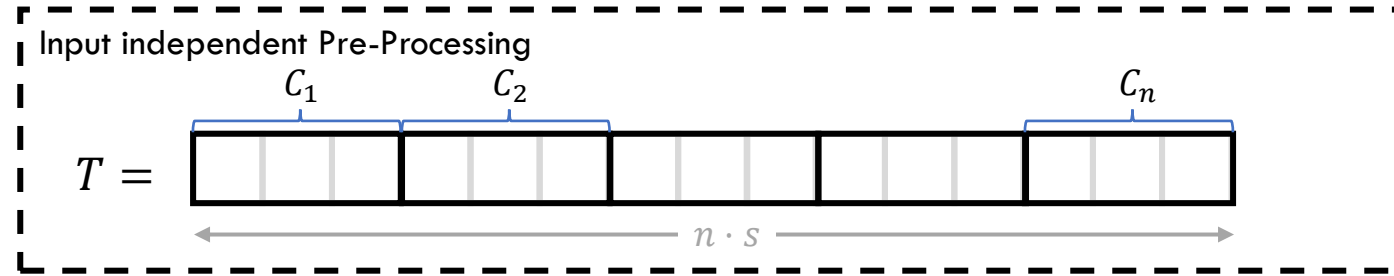
$\text{Verify}(\Pi, \text{com}(C_{x,w}), x) = 1$

$\text{Verify}(\Pi', \text{com}(C_{x,w}), \text{com}(C_1), \dots, \text{com}(C_n)) = 1$

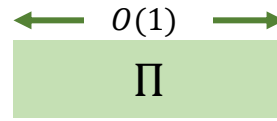


One-time Table Pre-processing

Sublinear Prover \mathcal{P} on \mathcal{K}



x, w, pp



$\text{com}(C_{x,w})$

Π'

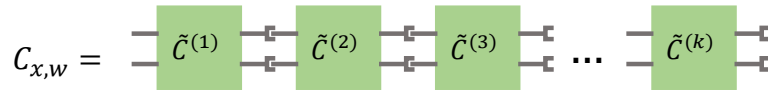
$\text{com}(T)$



x

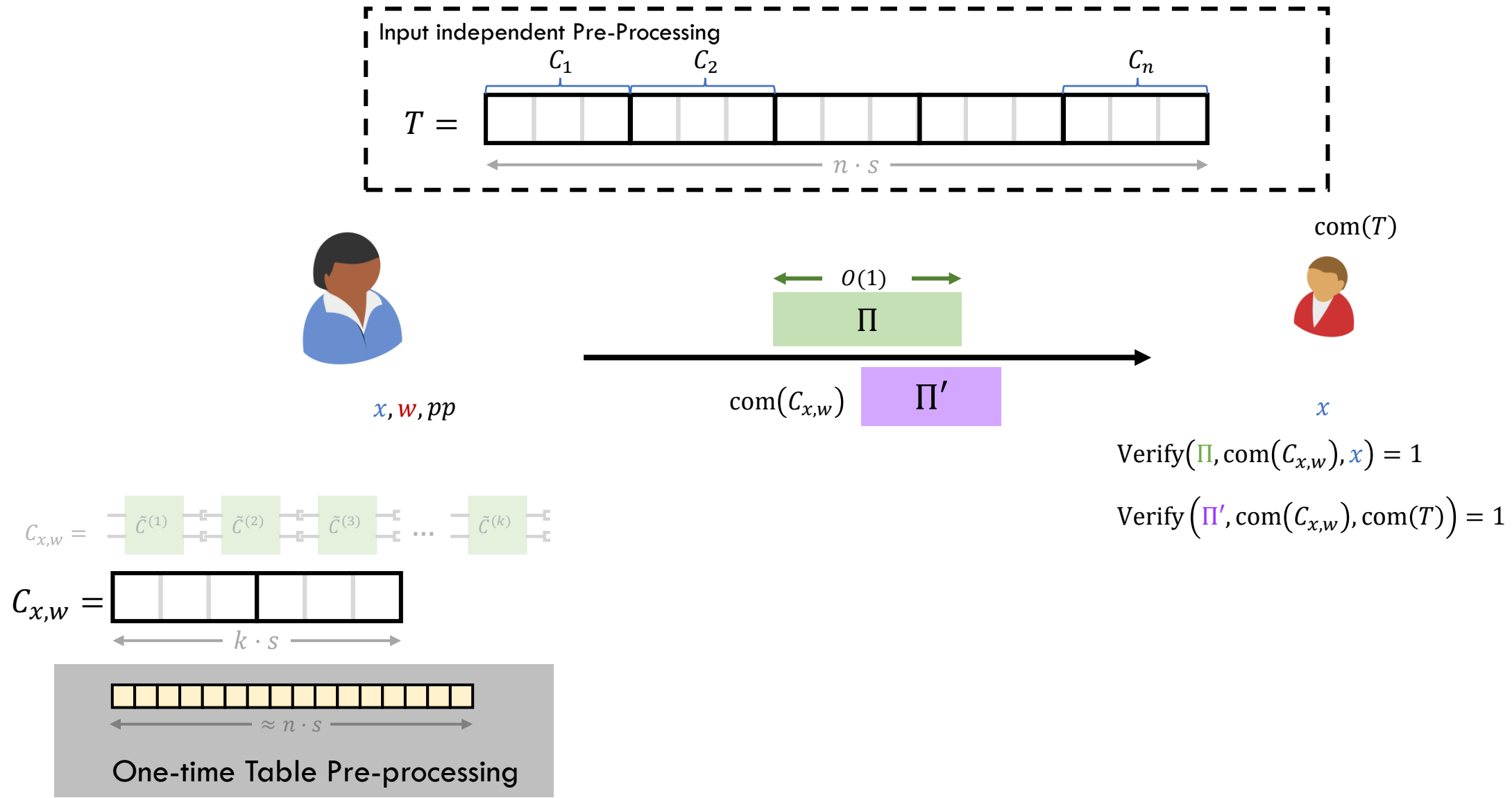
$\text{Verify}(\Pi, \text{com}(C_{x,w}), x) = 1$

$\text{Verify}(\Pi', \text{com}(C_{x,w}), \text{com}(T)) = 1$

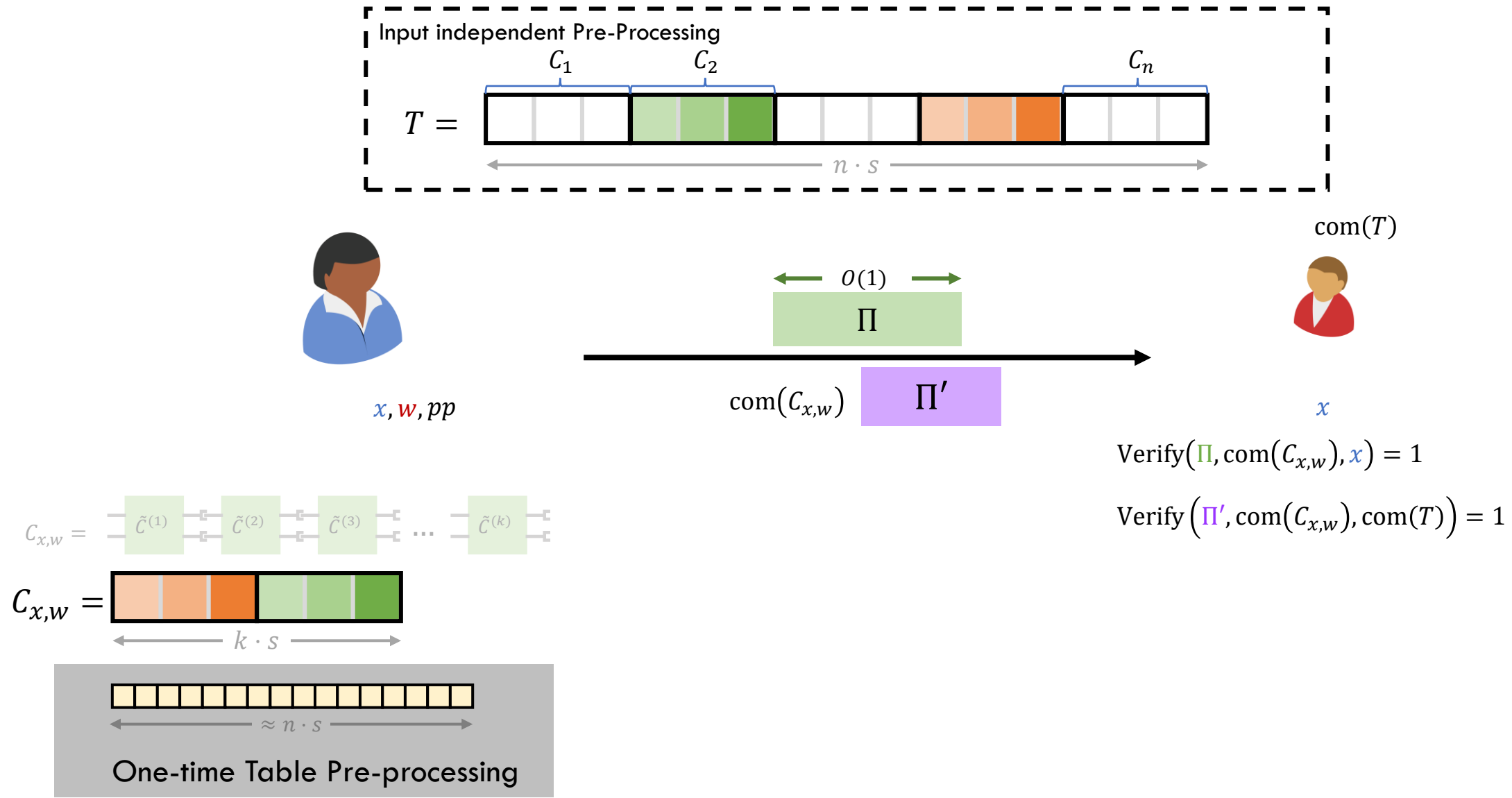


One-time Table Pre-processing

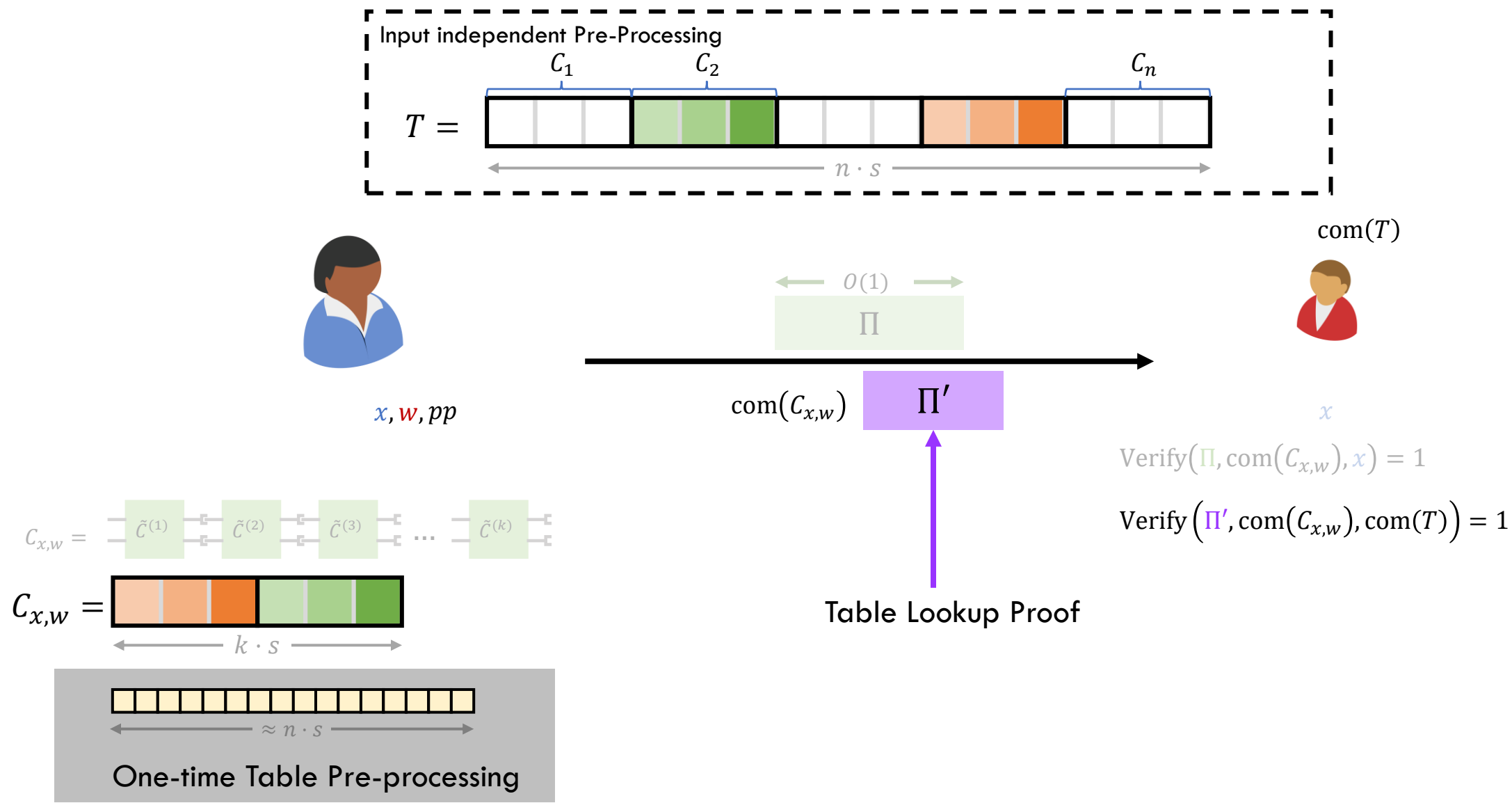
Sublinear Prover \mathcal{P} on \mathcal{K}



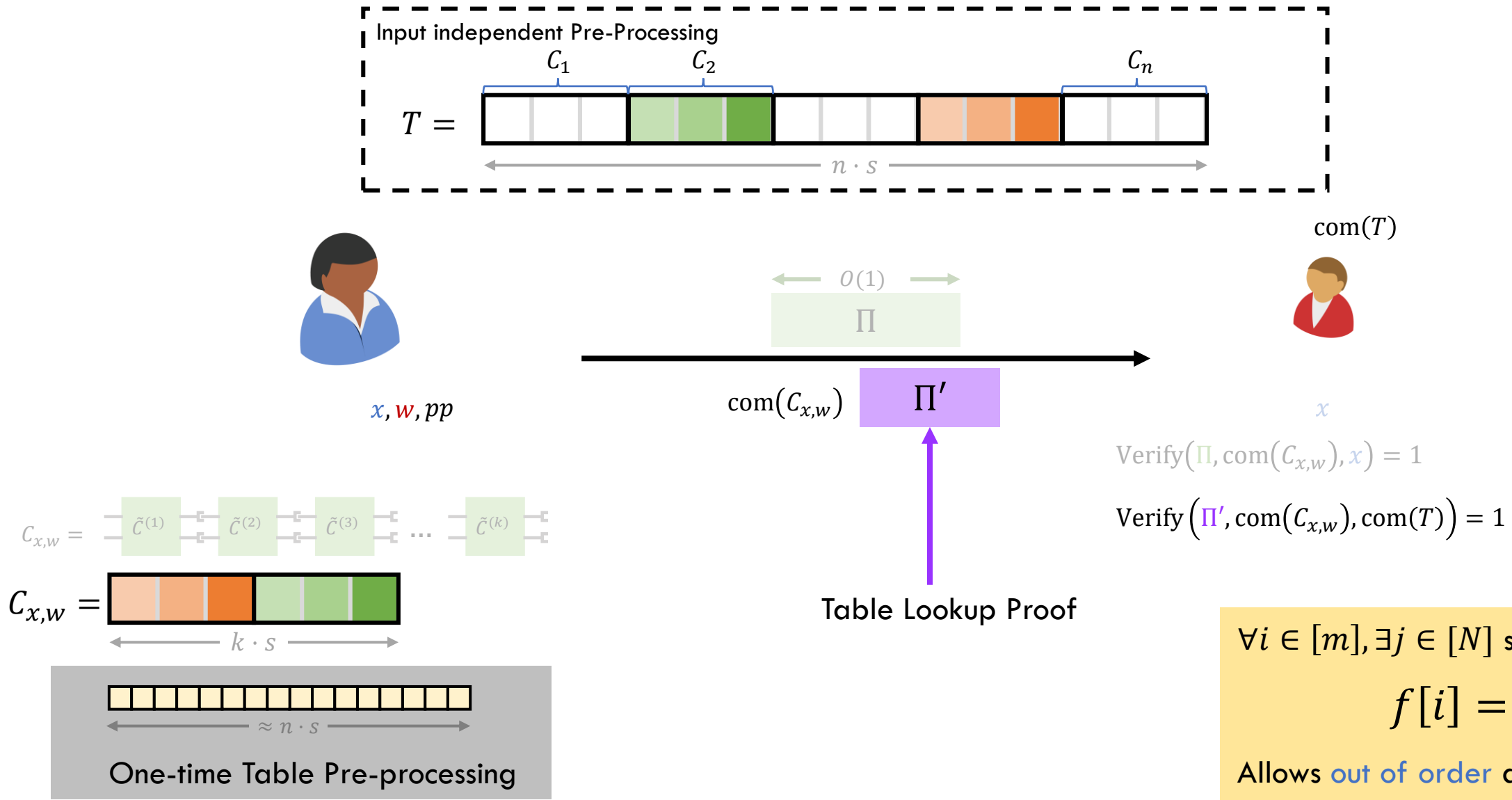
Sublinear Prover \mathcal{PlonK}



Sublinear Prover \mathcal{P}_{OnK}



Sublinear Prover \mathcal{P} on \mathcal{K}

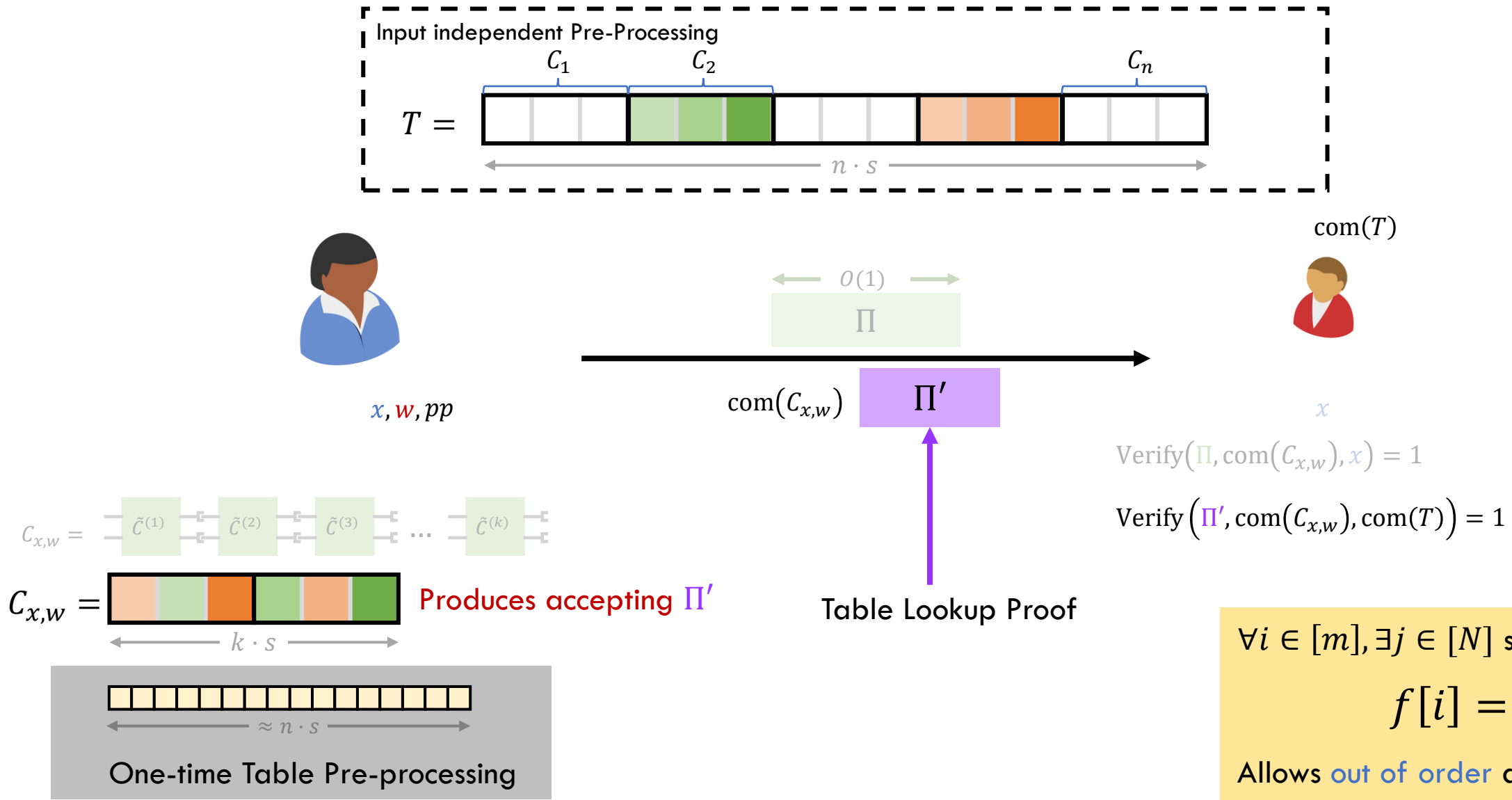


$\forall i \in [m], \exists j \in [N]$ such that

$$f[i] = T[j]$$

Allows **out of order** and **repetitions**.

Sublinear Prover \mathcal{PlonK}



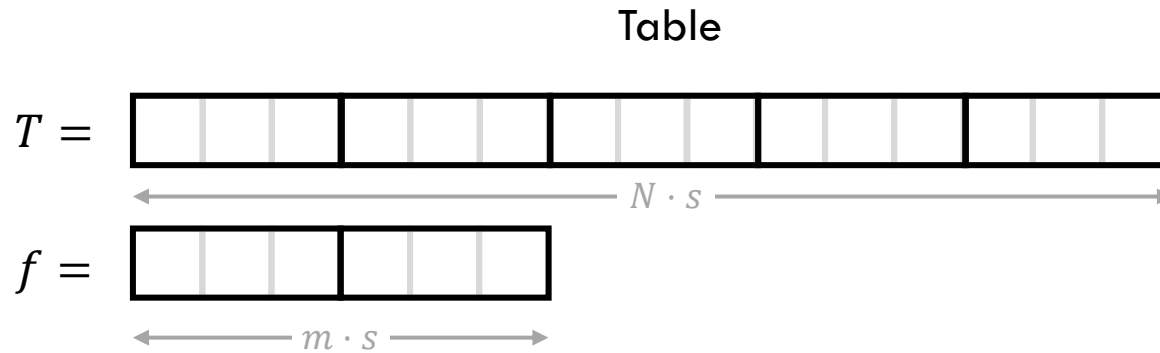
$\forall i \in [m], \exists j \in [N]$ such that

$$f[i] = T[j]$$

Allows out of order and repetitions.

Our Contribution: Segment Lookup

N segments each
of size s



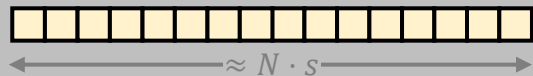
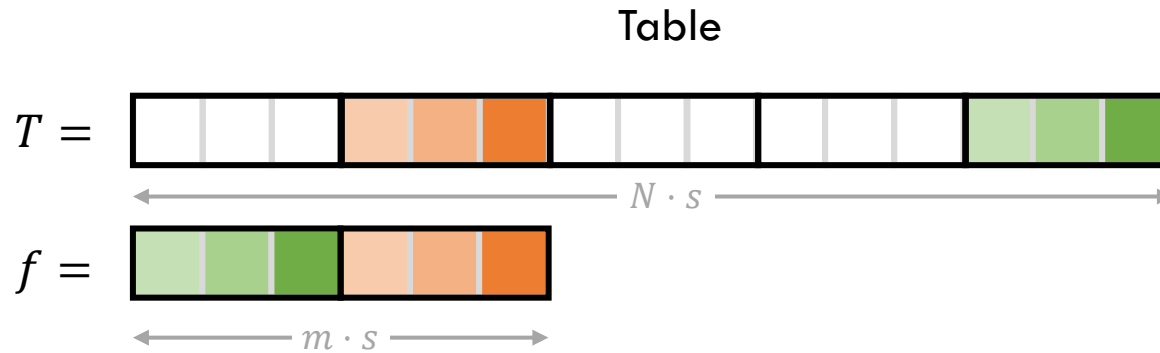
One-time Table Pre-processing



$\text{Com}(T)$

Our Contribution: Segment Lookup

N segments each
of size s



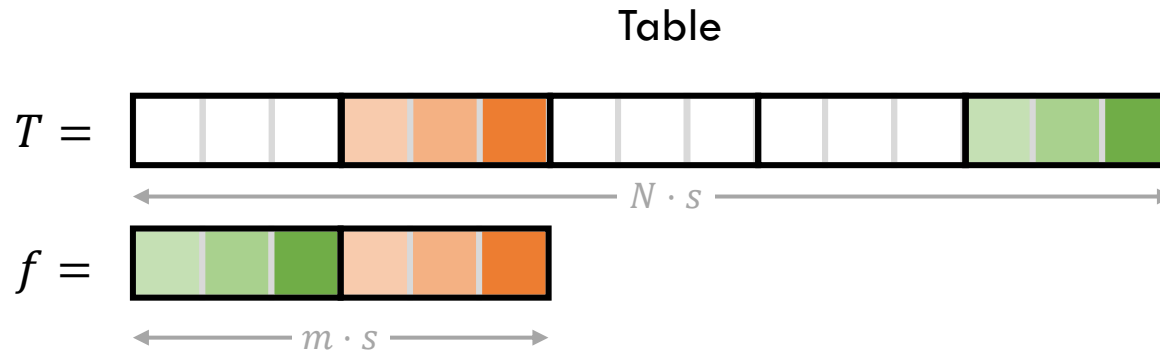
One-time Table Pre-processing



$\text{Com}(T)$

Our Contribution: Segment Lookup

N segments each
of size s



$\forall i \in [m], \exists j \in [N]$ such that

$$\begin{aligned} f[is + 1] &= T[js + 1], \\ f[is + 2] &= T[js + 2] \\ &\vdots \\ f[(i + 1)s] &= T[(j + 1)s] \end{aligned}$$


One-time Table Pre-processing

$\text{Com}(f)$

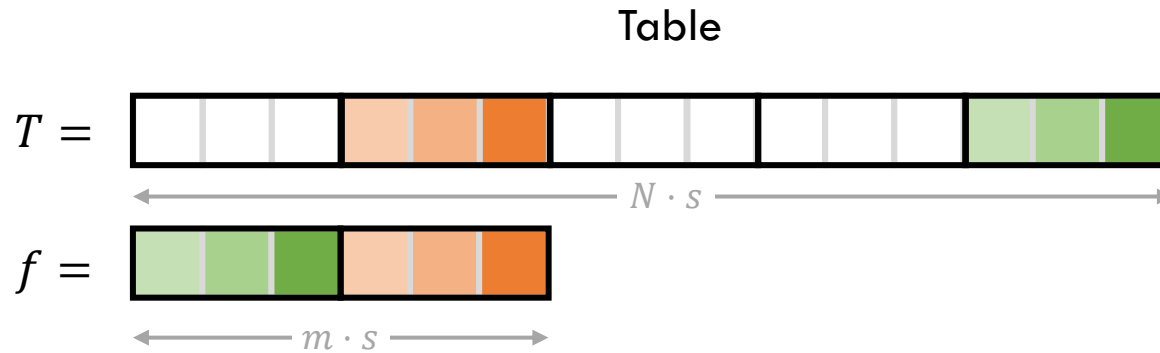
Π



$\text{Com}(T)$

Our Contribution: Segment Lookup

N segments each
of size s



$\forall i \in [m], \exists j \in [N]$ such that

$$f[is + 1] = T[js + 1],$$

$$f[is + 2] = T[js + 2]$$

\vdots

$$f[(i + 1)s] = T[(j + 1)s]$$

Allows **out of order** and **repeated**
segments.



$\text{Com}(f)$

Π



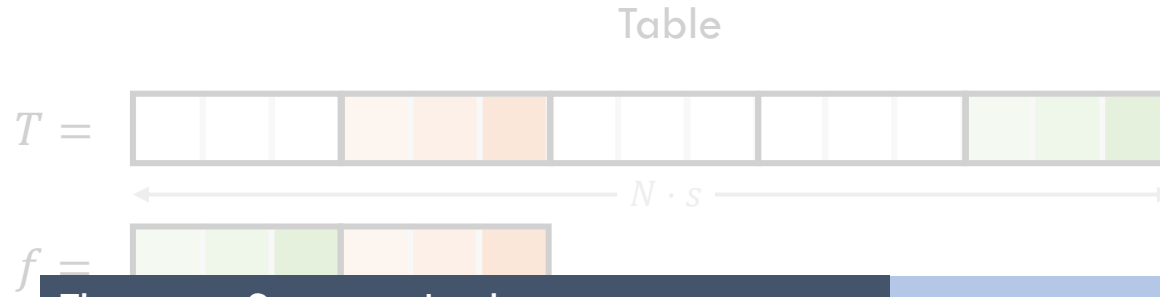
$\text{Com}(T)$



One-time Table Pre-processing

Our Contribution: Segment Lookup

N segments each
of size s



Theorem: Segment Lookup

Online Prover Time - $O(ms (\log ms + \log N))$

Proof Size* - $O(1)$

Black-box in Cryptography

$\forall i \in [m], \exists j \in [N]$ such that

$$f[is + 1] = T[js + 1],$$

$$f[is + 2] = T[js + 2]$$

\vdots

$$f[(i + 1)s] = T[(j + 1)s]$$

Allows out of order and repeated
segments.



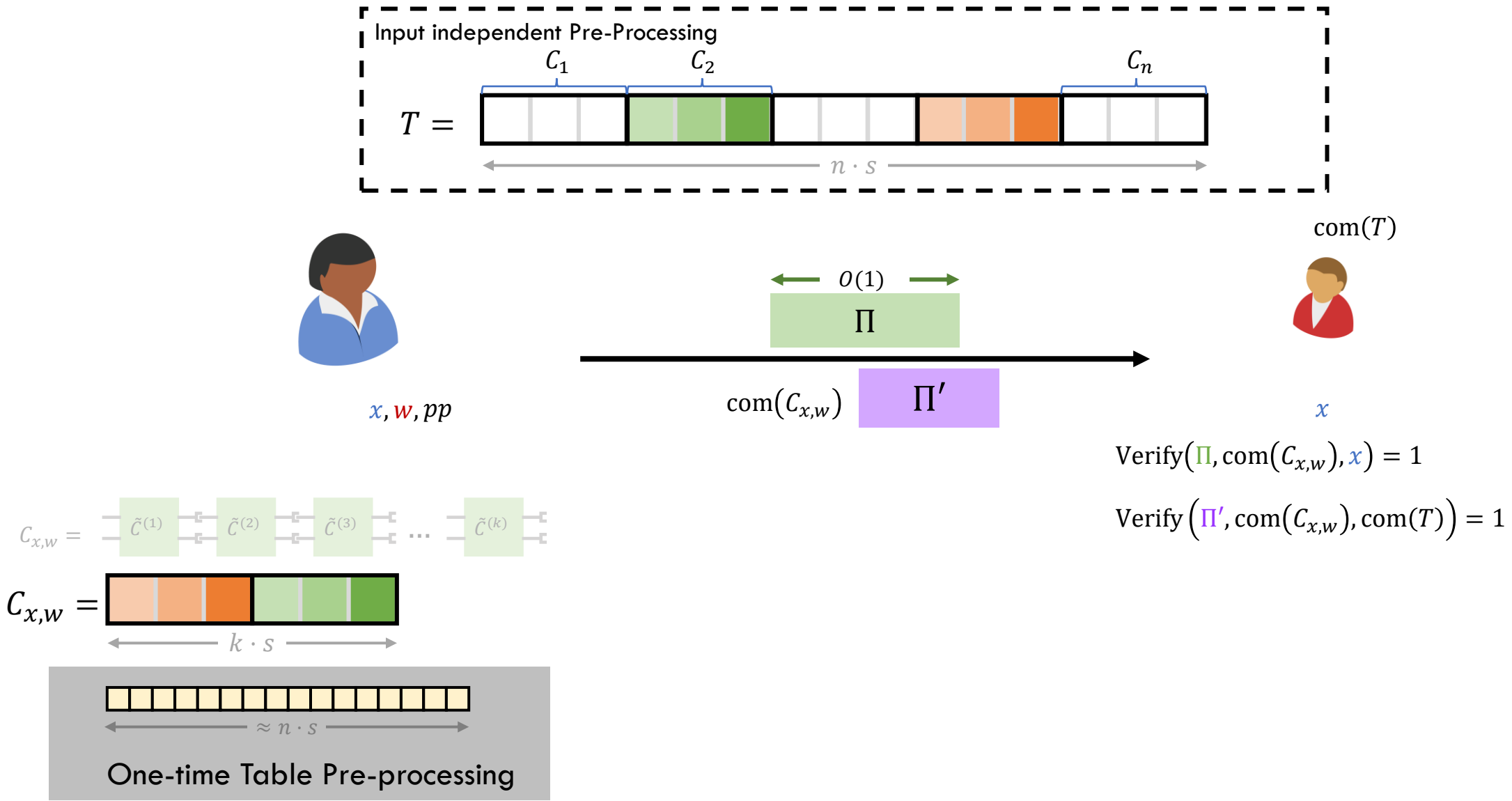
One-time Table Pre-processing



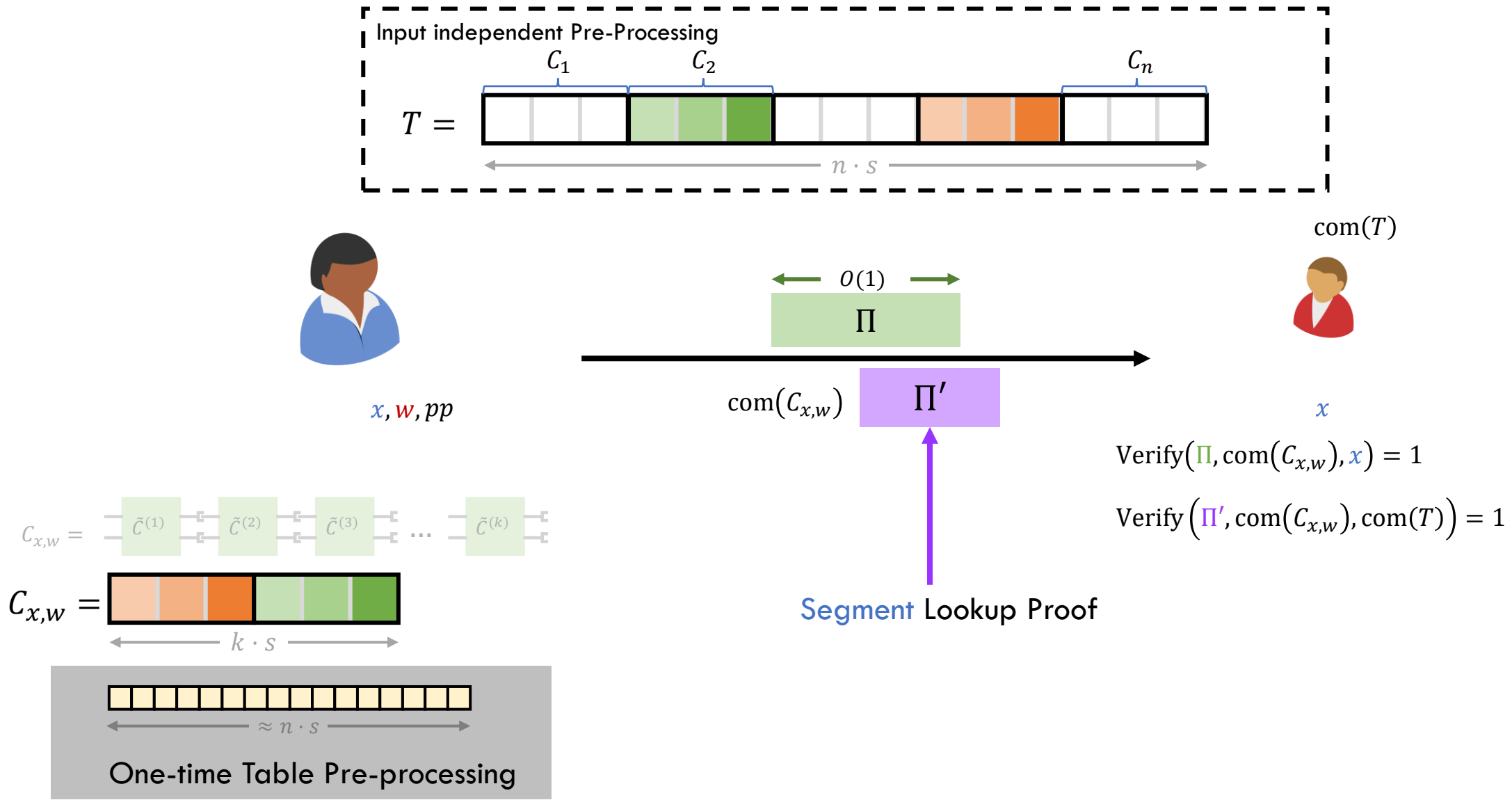
$\text{Com}(T)$

*Proof Size includes size of $\text{com}(f)$

Sublinear Prover \mathcal{PlonK}



Sublinear Prover \mathcal{P}_{OnK}



Sublinear Prover $\mathcal{P}_{\text{IonK}}$

Theorem: Sublinear Prover $\mathcal{P}_{\text{IonK}}$

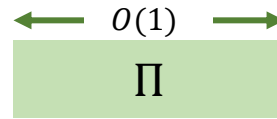
Online Prover Time - $O(ks (\log ks + \log n))$

Proof Size* - $O(1)$

Black-box in Cryptography with **input independent** pre-processing



x, w, pp



$\text{com}(C_{x,w})$

Π'

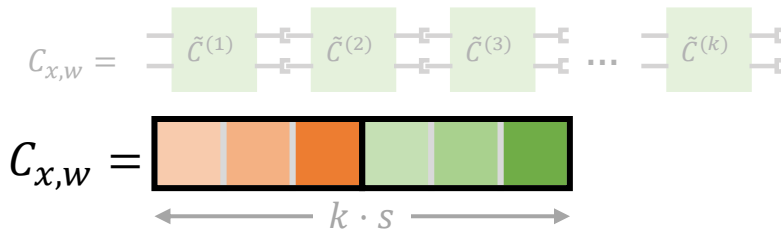


x

$\text{Verify}(\Pi, \text{com}(C_{x,w}), x) = 1$

$\text{Verify}(\Pi', \text{com}(C_{x,w}), \text{com}(T)) = 1$

Segment Lookup Proof



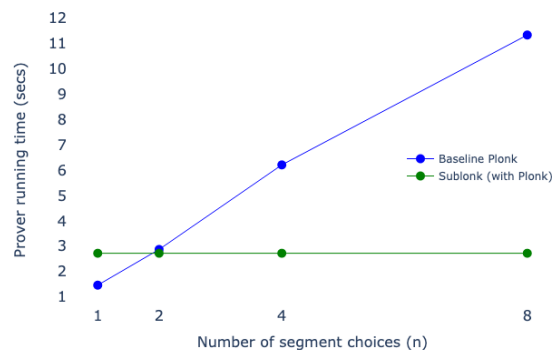
One-time Table Pre-processing

Experimental Results

Prover Time (Comparison to Baseline \mathcal{PlonK})

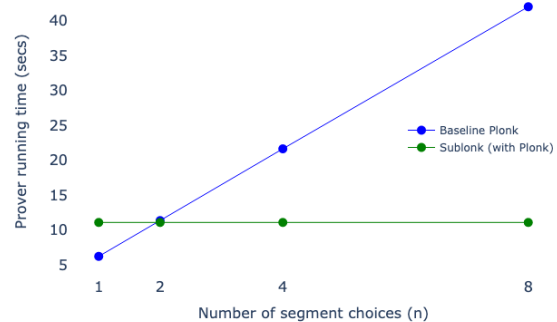
$$s = 2^{10}$$

Prover time with segments of size 2^{10}



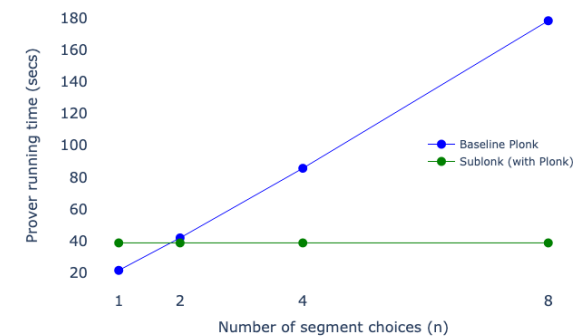
$$s = 2^{12}$$

Prover time with segments of size 2^{12}



$$s = 2^{14}$$

Prover time with segments of size 2^{14}



$$k = 128$$

Experimental Results

Proof Size and Verification Cost (Comparison to Baseline $\mathcal{P}\text{IonK}$)

	<u>$\mathcal{P}\text{IonK}$</u>	<u>SubIonK</u>
Proof size	9 \mathbb{G} and 6 \mathbb{F}	42 \mathbb{G} and 12 \mathbb{F}
Verification Cost	18 \mathbb{G} and 2 Pairings	27 \mathbb{G} and 23 Pairings

Thank you. Questions?

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