HASHING

HASH TABLES

Hash Table - A list of records/structures with distinct keys.

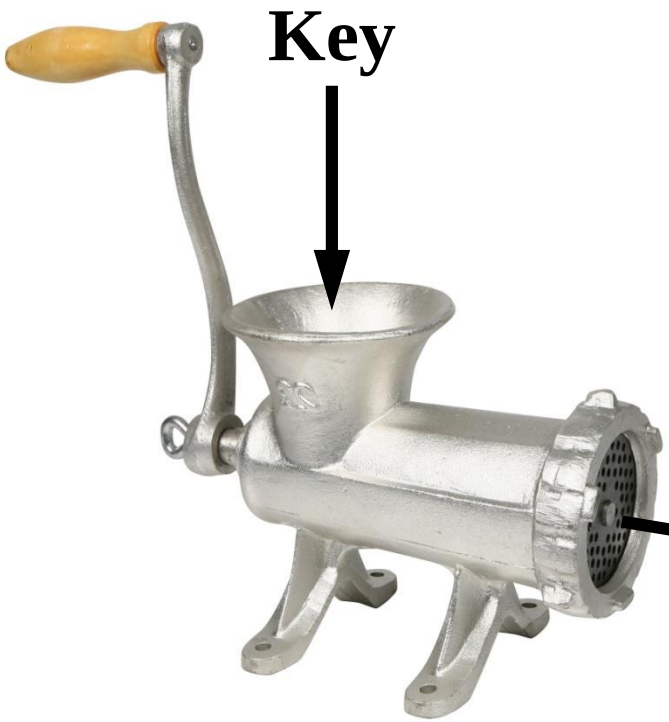
Operations -

- Insert
- Search
- Delete
- Modify [fields other than the key field]

Goal of Hashing

- Search or insertion in array, list or tree is based on key comparison.
- In hash table, searching/insertion is guided by address of the record having a key value k, where the address is computed by applying the so-called Hash function on the key value.
- Objective is to perform search/insertion operation in constant bounded time, i.e., O(1) time.

The idea



A simple example

n elements are to be stored in a hash table of size M, M >> n. An element with key k, will be put in slot i of the hash table, where i = h(k), h being the hash function.

h(k) = k mod M, such that 0 <= h(k) <= M-1 for all key k.

Properties of Hashing

- Two keys k_1 and k_2 are said to be **synonyms** with respect to a hash function h if $h(k_1) = h(k_2)$.
- Thus if two synonyms k_1 and k_2 are likely to be inserted into a hash table in that sequence, they will hash to the same slot in the table and a **collision** is said to occur.
- In the event of a collision, a **Collision Resolution Strategy** has to be called for, to find out a free slot in the table.
- A *good* hashing function would **minimize collisions**.
- A *good* Collision Resolution Strategy would locate a free slot **quickly**.

Hash Functions

- For any key value k, h(k), the hashed value of k is an integer between 0 & M-1, M being the size of the hash table.

Objectives:

- Computation of h should be **efficient**.
- **No. of collisions** should be relatively **small**.

Hash Function Examples

1. **Division:-** Assume k is some integer representing the key; h(k)=k mod M. A good choice of M is a prime number. (Why?)

2. **Mid-square:-** Square the key value, take appropriate no. of middle digits.

   k = 637, M = 100, k^2 = 40**57**69, h(k) = 57

Hash Function Examples …

3. **Folding:-** Assume k is a big number. In most cases, you can represent k in the form of a big number. Partition k into several parts, add the parts modulo M.

   Example: key – "INDIA", M = 1000.

   Taking the ASCII codes, key = 7378687365.

   Partitioning with 3 digits, the key becomes
   7/378/687/365.

   h(k) = (7+378+687+365) mod 1000 = 437

Hash Function Examples …

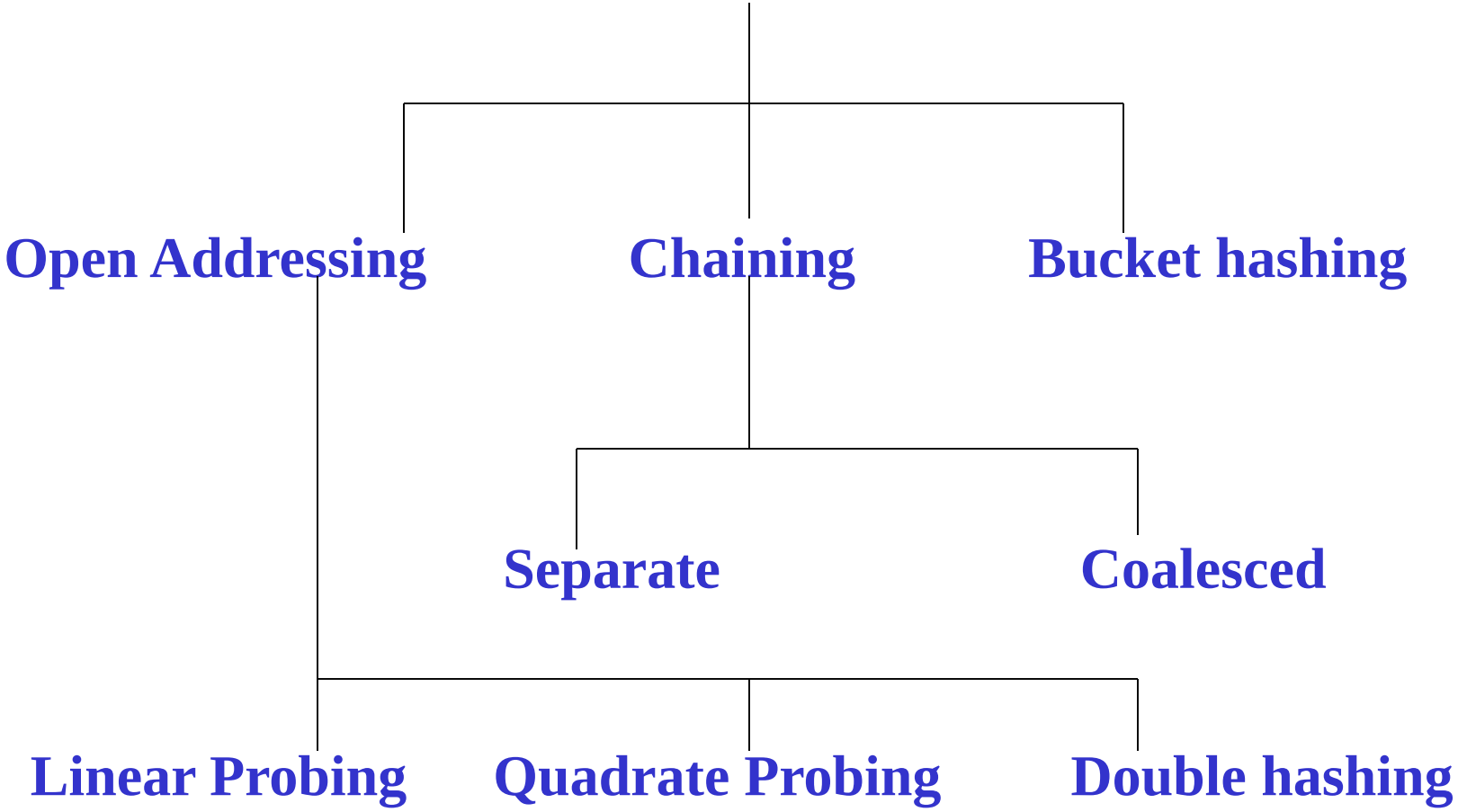
4. **Radix Transformation:-**Let k be given in decimal system. Consider that the number has been given in a different radix and convert it into decimal equivalent and finally take modulus M.

   Suppose k = 123456, M = 1000

   $h(k) = (123456)_{11} \bmod 1000 = 195231 \bmod 1000 = 231$

5. **Multiplication:-** $h(k) = \lfloor M*((c*k) \bmod 1)) \rfloor$ where c is a constant fraction.

Collision Resolution Strategies



Collision Resolution Strategies

Open Addressing:- The hash table is organized as an array of records and the hash function is used to find out the index in the array corresponding to the given key. Collision occurs when the position where a new key is to be inserted is already filled up.

Collision Resolution Strategies …

Linear Probing:- Probe sequence -- (h(k)+i) mod M

i = 0, 1, 2, …, M-1

Example: M = 11, h(k) = k mod M

Key sequence- 8, 20, 98, 17, 25, 66, 45, 67

$\alpha = n/M$ is known as the load factor.

Considering equal probability of arrival of keys, expected no. of probes for searching an existing key is

$C = (1+1/(1- \alpha))/2$, $0< \alpha < 1$ for large n & M.

Expected no. of probes for insertion is

$C' = (1+1/(1- \alpha)^2)/2$.

C and C' gives two figures of merit for hashing.

Collision Resolution Strategies ...

Problem of linear probing:- Once a cluster is formed, there are more chances that subsequent insertions will land up in the same cluster and thereby increase the length of cluster.

This phenomenon is known as **Primary Clustering**. It degrades the performance of hash.

Collision Resolution Strategies ...

Quadratic Probing:-

Probe sequence -- $(h(k) + c_1 i + c_2 i^2) \bmod M$,

c_1 and c_2 are constants.

Small clusters will be formed instead of a very large cluster **- secondary clustering**.

Collision Resolution Strategies ...

Double hashing:-

Probe sequence (h(k) + i*h'(k)) mod M,

where h and h' are two hashing functions.

If h and h' are chosen properly, it is very unlikely that k_1 & k_2 will be synonyms for both h and h'. Therefore no. of collisions can be greatly reduced. Cluster formation can also be avoided.

Example: k = 123456, M = 701, h(k)=k mod M = 80

h'(k) = 1 + k mod(M-1) = 257

Probe sequence = 80 + 257*i , different for different k

Deletion of data

- In order to delete data from the hash table, you need to keep a flag (at least 2 bits) to indicate whether the entry is empty, full or deleted.
- This will allow the reuse of the entry
- Also, the search or insertion algorithms can determine the status of the entry for stopping or taking some other relevant decisions

Collision Resolution Strategies ...

Chaining

a) **Separate Chaining:-** The hash table is a collection of M linear lists. Each key k hashes to one of these M lists. Storage requirement – n records and M+n pointers.

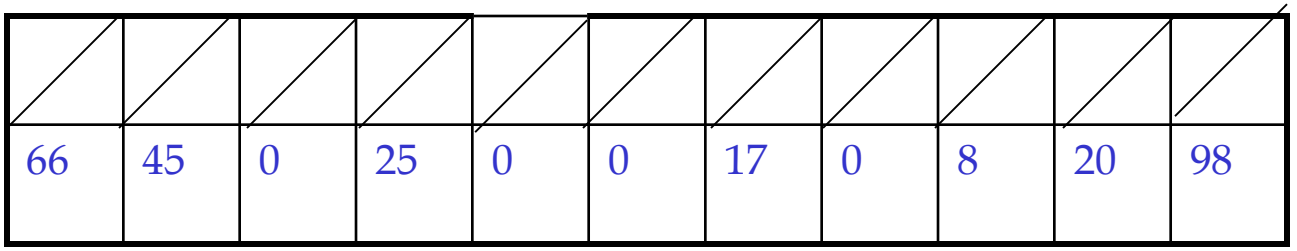
Collision Resolution Strategies ...

b) **Coalesced chaining:-** The linked lists are maintained in the same array using cursors. Different chains may be coalesced (merged) -- chain headers are not maintained separately.

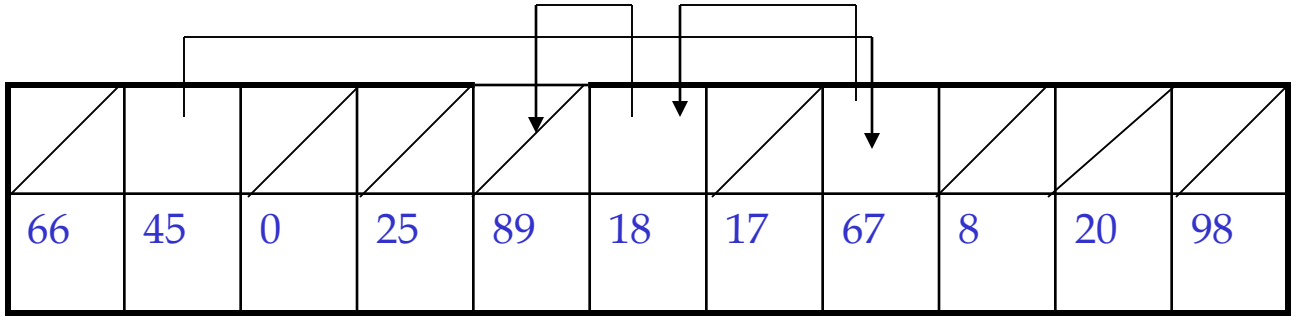
Example: After initialization

0	0	0	0	0	0	0	0	0	0	0

After entering 8, 20, 98, 17, 25, 66, 45



After entering 67, 18, 89



Bucket hashing

- Useful for external searching
- Each bucket can hold b records
- A bucket is stored in a disk block
- When a bucket overflows, another bucket is allocated and linked to the former.
- There is a bucket header which has M slots, each slot heading a bucket list.

Bucket hashing …

Example: M = 7, b = 3, n = 20.

insertion sequence – 10, 5, 20, 8, 4, 3, 19, 45, 71, 29, 84, 49, 54, 42, 73, 24, 18, 94, 84, 59.

Expected number of buckets in each chain is

$$\lceil \lceil n/M \rceil / b \rceil$$

Expected number of buckets for random inputs

n>M, $N = M * \lceil \lceil n/M \rceil / b \rceil$

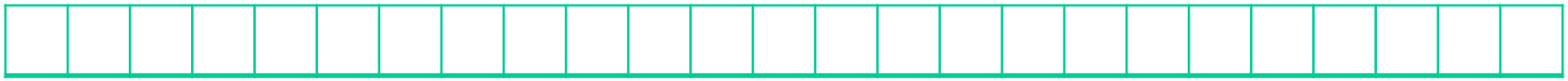
Expected no. of probes [disk reads] = $1/2 \lceil \lceil n/M \rceil / b \rceil$

Filtering based on Set membership

- Challenge: The criteria involves a membership lookup
  - Simplified example: Emails <email address, email> stream
  - Task: Filter emails based on email addresses
  - Have *S* = Set of 1 billion email addresses which are not spam
  - Keep emails from addresses in *S*, discard others
- Each email ~ 20 bytes or more. Total > 20GB
  - Not to keep in main memory
  - Option 1: make disk access for each stream element and check ------- high time complexity and absolute time
  - Option 2: Bloom filter, use 1GB main memory ----- extremely fast and memory efficient

Filtering with One Hash Function

- Available memory: *n* bits (e.g. 1GB ~ 8 billion bits)
- Use a bit array of *n* bits (in main memory), initialize to all 0s
- A hash function *h*: maps an email address → one of the *n* bits
- Pre-compute hash values of *S*
- Set the hashed bits to 1, leave the rest to 0



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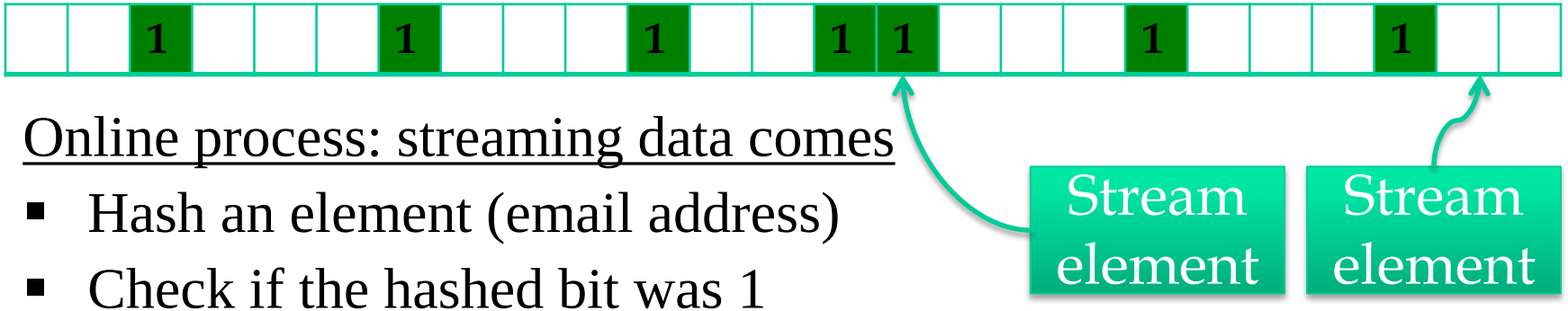

Online process: streaming data comes

- Hash an element (email address)
- Check if the hashed bit was 1
- If yes, accept the email, otherwise discard
- Note: $x = y$ implies $h(x) = h(y)$, but not vice versa
- So, there would be false positives

The Bloom Filter

- Available memory: n bits
- Use a bit array of n bits (in main memory), initialize to all 0s
- Want to minimize probability of false positives
- Use k hash functions $h_1, h_2, \ldots, h_k$
- Each h_i maps an element → one of the n bits
- Pre-compute hash values of S for all h_i
- Set a bit to 1 if any element is hashed to that bit for any h_i
- Leave the rest of the bits to 0

Online process: streaming data comes

- Hash an element with all hash functions
- Check if the hashed bit was 1 for all hash functions
- If yes, accept the element, otherwise discard

The Bloom Filter: Analysis

Let $|S| = m$, bit array is of n bits, k hash functions $h_1, h_2, \ldots, h_k$

- Assumption: the hash functions are independent and they map one element to each bit with equal probability
- P[a particular h_i maps a particular element to a particular bit] = $1/n$
- P[a particular h_i does not map a particular element to a particular bit] = $1 - 1/n$
- P[No h_i maps a particular element to a particular bit] = $(1 - 1/n)^k$
- P[After hashing m elements of S, one particular bit is still 0] = $(1 - 1/n)^{km}$
- P[A particular bit is 1 after hashing all of S] = $1 - (1 - 1/n)^{km}$

False positive analysis

- Now, let a new element x not be in S. Should be discarded.
- Each $h_i(x) = 1$ with probability $1 - (1 - 1/n)^{km}$
- $P[h_i(x) = 1 \text{ for all } i] = (1 - (1 - 1/n)^{km})^k$
- $= (1 - ((1 - 1/n)^n)^{km/n})^k$
- This probability is $\approx (1 - e^{-km/n})^k$
- Optimal number k of hash functions: $\log_e 2 \times n/m$

$(1-\varepsilon)^{1/\varepsilon} \approx 1/e$
for small ε

Conclusion

- Hashing has become very important mechanism for today's era of Big Data

- In many problem areas, there are multiple hash algorithms being used with very high acceptable performance