

Digital Communication

Topic 2- Signals & Signal Analysis

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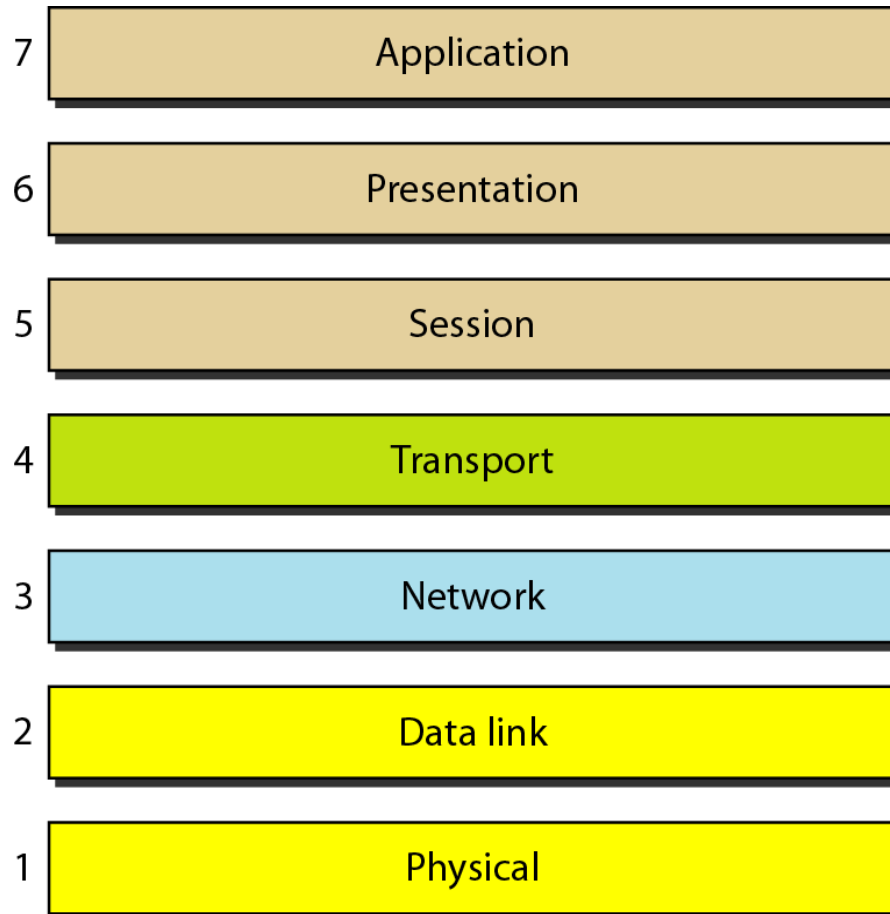
Doubts

- In the diagram of slide 71(interaction between layers in the OSI model), there is a peer-to-peer protocol in the transport and subsequent higher layers between the source and destination systems, I have not been able to understand that part, how is that established?
 - And why is it done so ?
 - Is it for SYN/ACK purpose?

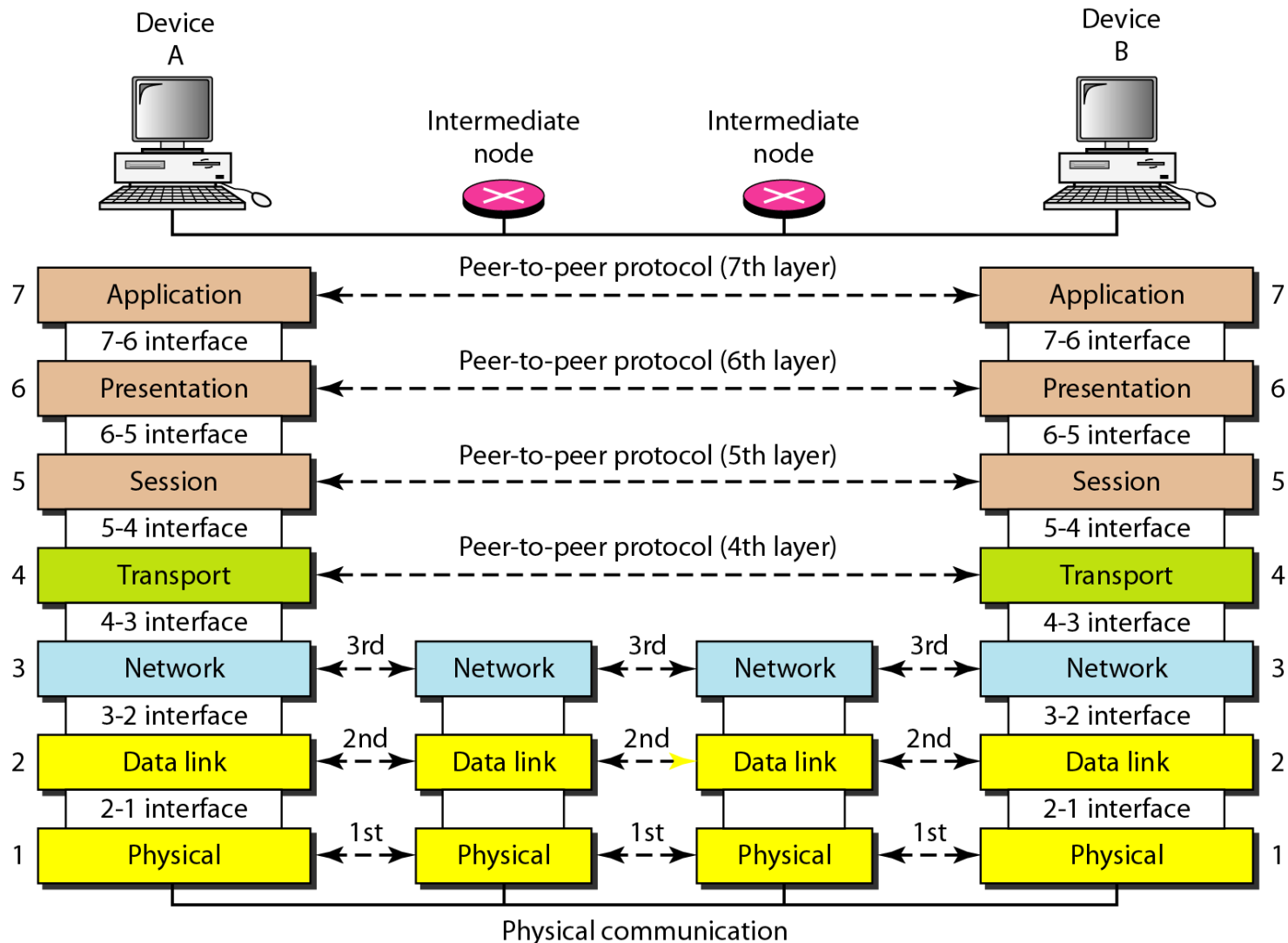
I tried googling but did not get satisfactory answers.

- Also why is the "Trailer" bit/flag at the end required in the target message?

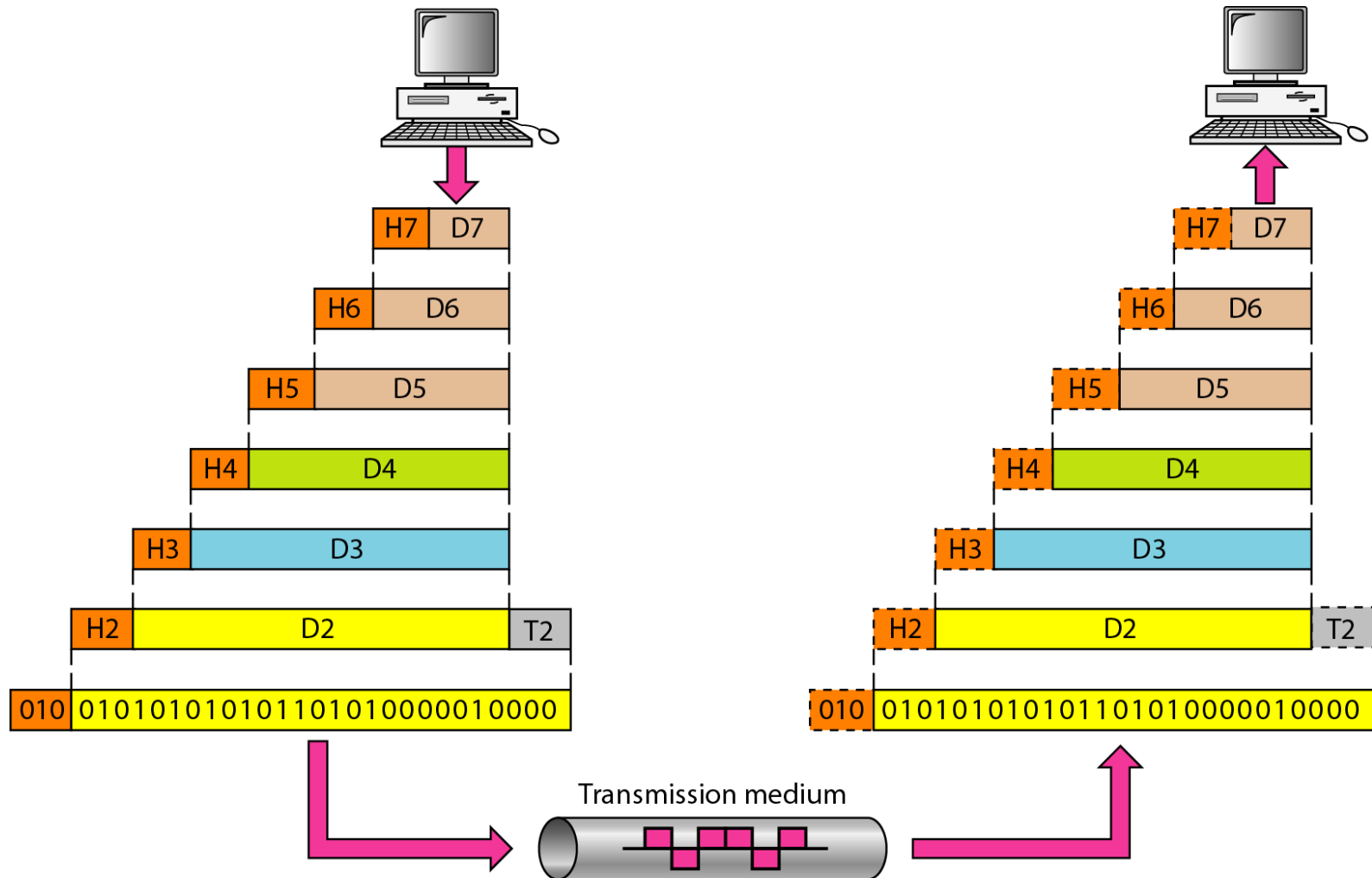
OSI Model



The interaction between layers in the OSI model



An exchange using the OSI model



Topics

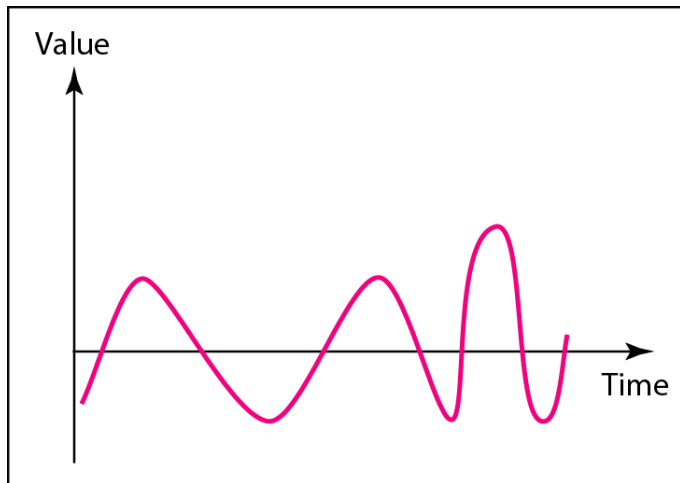
- Analog Signal
- Fourier
- Digital Signal

Data \Rightarrow Signals

To be transmitted, data must be transformed to electromagnetic signals.

Data

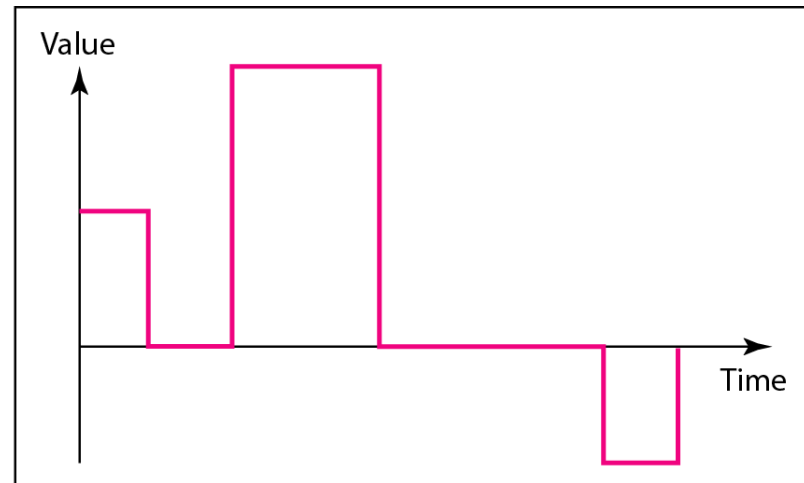
- Data can be **analog** or **digital**.
- **Analog data** are **continuous** and take continuous values.
- **Digital data** have **discrete** states and take discrete values.



a. Analog signal

Signals

- Signals can be **analog** or **digital**.
- Analog signals can have an **infinite** number of values in a range.
- Digital signals can have only a **limited** number of values.



b. Digital signal

Waves

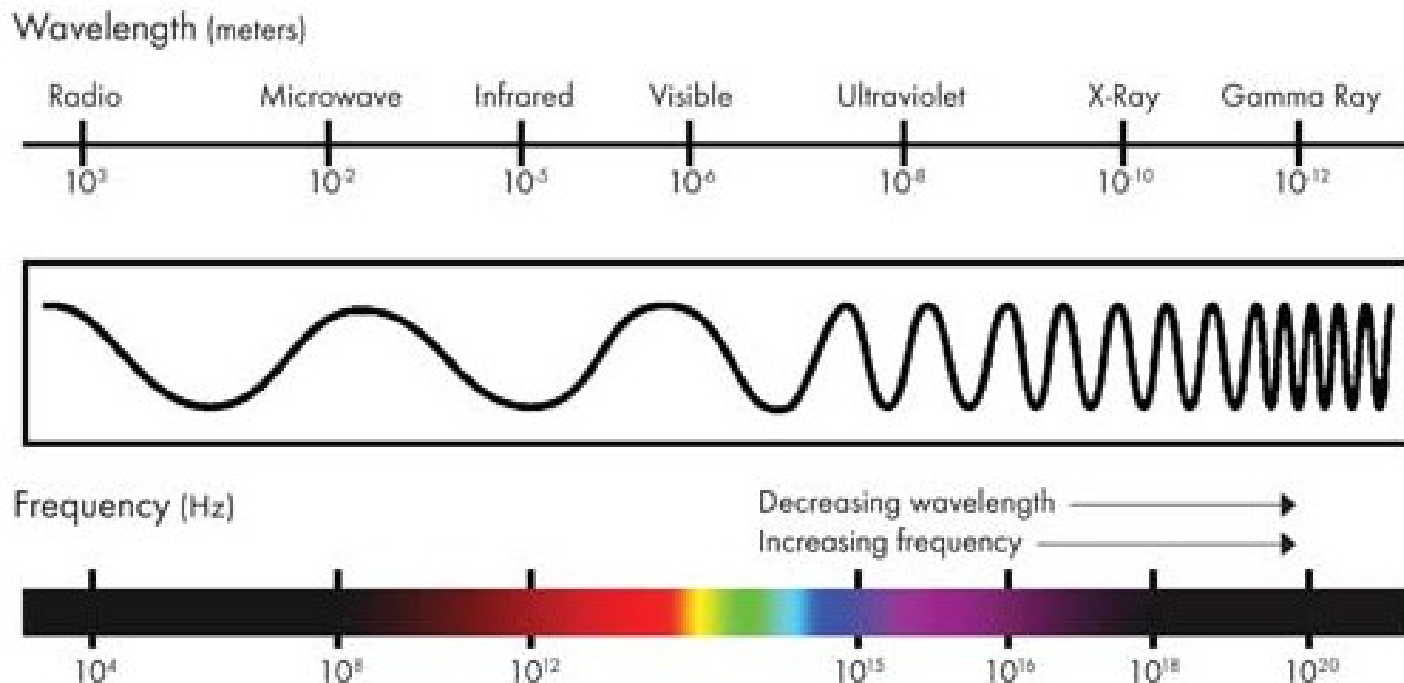
- Mechanical waves and electromagnetic waves are two important ways that energy is transported in the world around us. Two examples of mechanical waves are:
 - Waves in water and
 - sound waves in air
- Mechanical waves are caused by a disturbance or vibration in matter, whether solid, gas, liquid, or plasma.
- Matter that waves are traveling through is called a **medium**. Water waves are formed by vibrations in a liquid and sound waves are formed by vibrations in air.
- Sound waves cannot travel in the vacuum of space because there is no medium to transmit these mechanical waves.

Electro magnetic

- Electricity can be static, like the energy that can make your hair stand on end.
- Magnetism can also be static.
- A changing magnetic field will induce a changing electric field and vice-versa—the two are linked.
- These changing fields form electromagnetic waves.
- Electromagnetic waves differ from mechanical waves in that they do not require a medium to propagate.
- This means that electromagnetic waves can travel not only through air and solid materials, but also through the vacuum of space

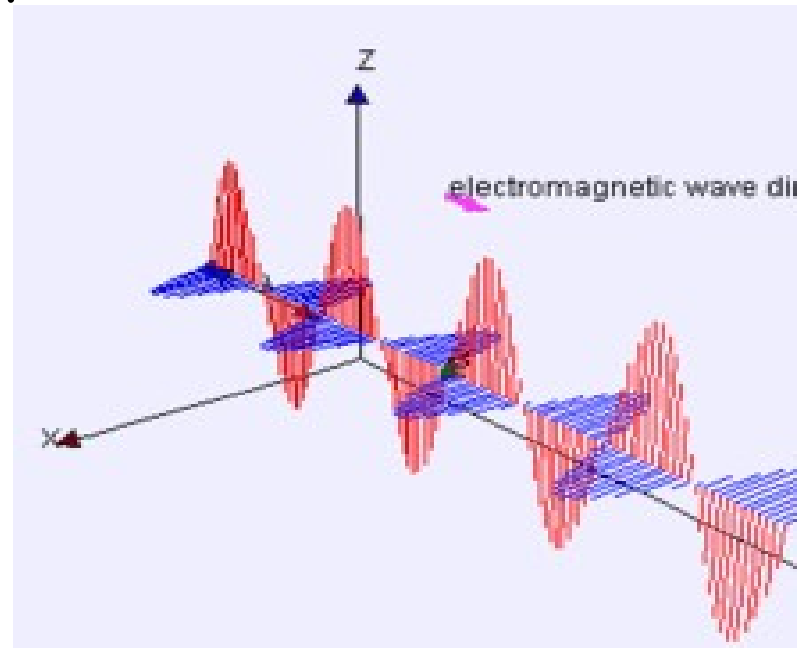
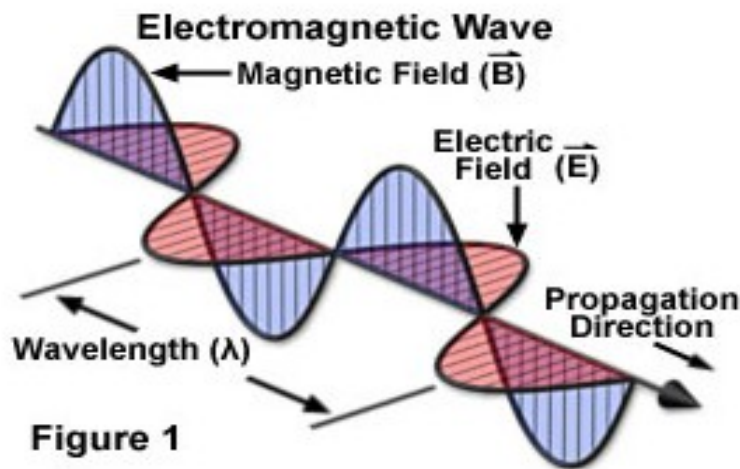
Electro-magnetic spectrum

- Electromagnetic waves form a continuous spectrum of wave energy ranging from very long radiowaves to very short gamma ray waves. Visible light represents only a very small portion of this spectrum. Below is a diagram that shows a portion of this electromagnetic spectrum.



EM

- Electromagnetic waves are created by the vibration of an electric charge. This vibration creates a wave which has both an electric and a magnetic component.
- An electromagnetic wave moves or **propagates** in a direction that is at right angles to the vibrations of both the electric and magnetic oscillating field vectors, carrying energy from its radiation source to undetermined final destination. The two fields are mutually perpendicular. The direction of propagation is the direction of $\mathbf{E} \times \mathbf{B}$.

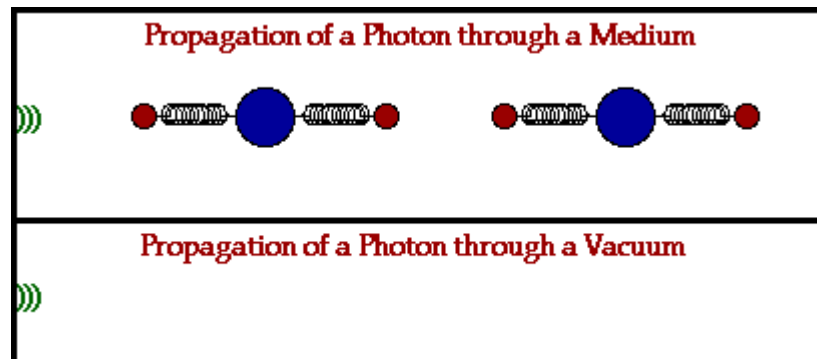


What is transmission media?

- A physical medium in data communications is the transmission path over which a signal propagates.
- Many transmission media are used as communications channel.
 - **Guided (or bounded (wired))**—waves are guided along a solid medium such as a transmission line. E.g., Copper wire, optical fiber etc
 - **Wireless (or unguided)**—transmission and reception are achieved by means of an antenna. E.g., radio frequencies, infrared, microwave, satellite.

Propagation of an EM

- The speed of any electromagnetic waves in **free space** is the **speed of light** $c = 3 \times 10^8 \text{ m/s}$.
- The **propagation of an electromagnetic wave through a material medium occurs at a net speed which is less than $3.00 \times 10^8 \text{ m/s}$.**
- The speed of any periodic wave is the product of its wavelength and frequency. $v = \lambda f$
- Electromagnetic waves can have any wavelength λ or frequency f as long as $\lambda f = c$.



Why speed of an EM wave is less than C?

- The mechanism of energy transport through a medium involves the absorption and reemission of the wave energy by the atoms of the material.
- When an electromagnetic wave impinges upon the atoms of a material, the energy of **that wave is absorbed**. The absorption of energy causes the electrons within the atoms to undergo vibrations.
- After a short period of vibrational motion, the vibrating electrons create a new electromagnetic wave **with the same frequency as the first electromagnetic wave**.
- While these vibrations occur for only a very short time, they **delay the motion** of the wave through the medium.
- Once the energy of the electromagnetic wave is reemitted by an atom, it travels through a small region of space between atoms. Once it reaches the next atom, the electromagnetic wave is absorbed, transformed into electron vibrations and then reemitted as an electromagnetic wave.
- While the electromagnetic wave will travel at a speed of c (3×10^8 m/s) through the vacuum of interatomic space, the absorption and reemission process causes the net speed of the electromagnetic wave to be less than c .

Wavelength can change but frequency remains same

- When electromagnetic waves travel through a medium, the speed of the waves in the medium is $v = c/n(\lambda_{\text{free}})$
where $n(\lambda_{\text{free}})$ is the **index of refraction** of the medium.
- The index of refraction n is a property of the medium, and it depends on the wavelength λ_{free} of the EM wave.
- If the medium absorbs some of the energy transported by the wave, then $n(\lambda_{\text{free}})$ is a complex number.
- For air n is **nearly equal to 1** for all wavelengths. When an EM wave travels from one medium with index of refraction n_1 into another medium with a different index of refraction n_2 , then its **frequency remains the same**, but its speed and wavelength change.

ANALOG SIGNALS

Analog Signals

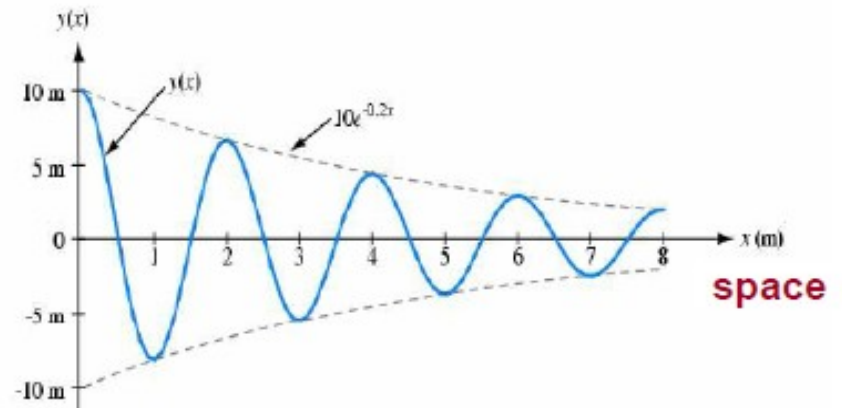
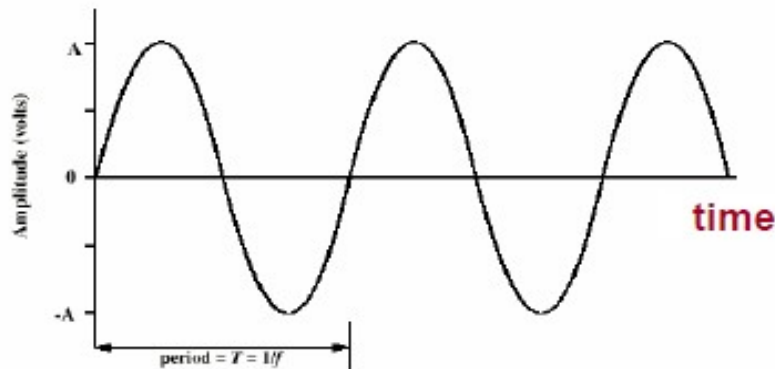
- In data communications, we commonly use **periodic** analog signals and **nonperiodic** digital signals.
 - Periodic analog signals can be classified as **simple** or **composite**. A simple periodic analog signal, a **sine wave**, cannot be decomposed into simpler signals.
 - A composite periodic analog signal is composed of multiple sine waves.

Periodic Function

- A function $f(x)$ is said to be periodic if there exists a number $T > 0$ such that $f(x + T) = f(x)$ for every x . The smallest such T is called the period of $f(x)$.
- Intuitively, periodic functions have repetitive behavior. A periodic function can be defined on a finite interval, then copied and pasted so that it repeats itself.
- Examples
 - $\sin x$ and $\cos x$ are periodic with period 2π
 - $\sin(\pi x)$ and $\cos(\pi x)$ are periodic with period 2
 - If L is a fixed number, then $\sin(2\pi x / L)$ and $\cos(2\pi x / L)$ have period L
- Sine and cosine are the most “basic” periodic functions!

Signal representation

- Signal representation: typically in 2D space, as a function of time, space or frequency
- When horizontal axis **is time**, graph displays the value of a signal at one particular point in space as a function of time
- when horizontal axis **is space**, graph displays the value of a signal at one particular point in time as a function of space



The time- and space- representation of a signal often resemble each other, though the signal envelope in the space-representation is different (signal attenuates over distance).

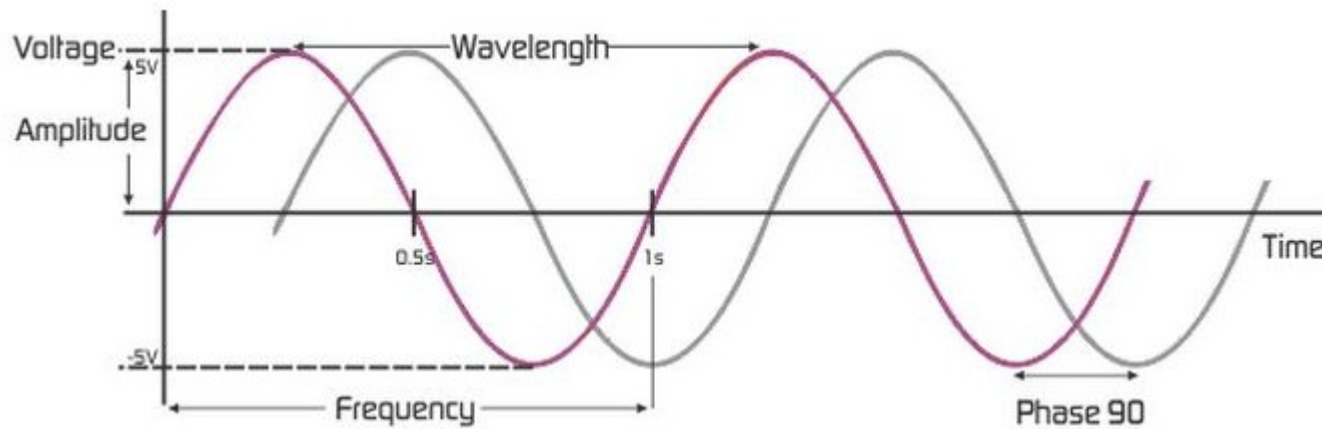
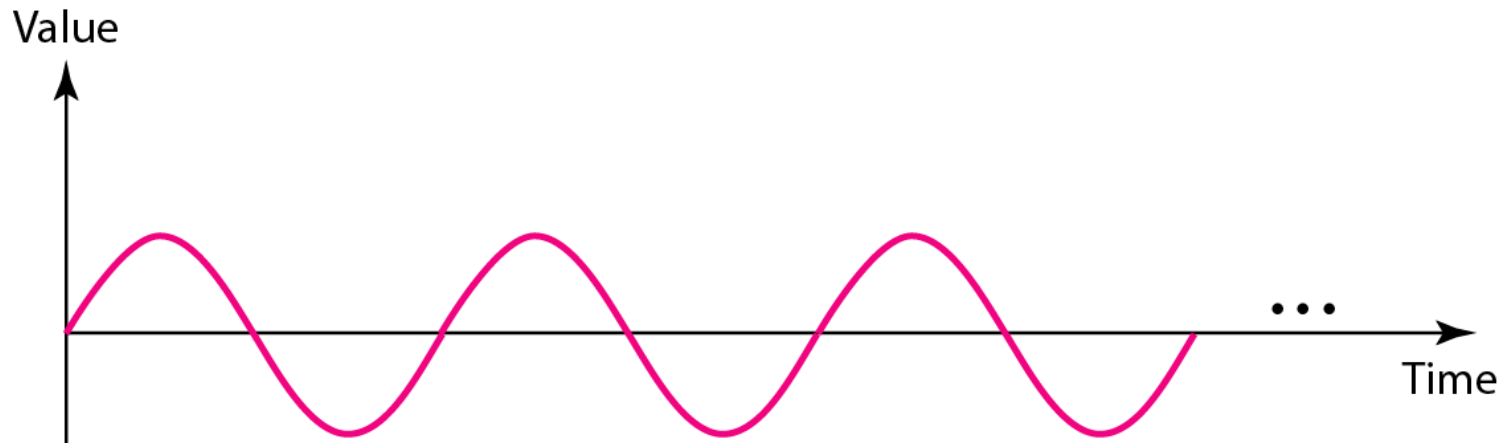
Sinewave

- Sinewave – most fundamental form of periodic analog signal – mathematically described with 3 parameters

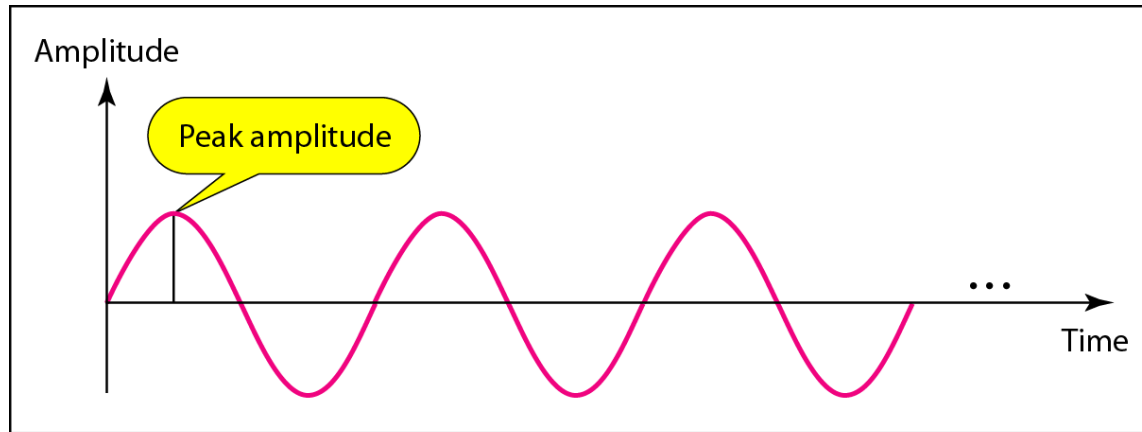
$$s(t) = A \cdot \sin(2\pi f t + \phi)$$

- **A** (Peak amplitude) – absolute value of signal's highest intensity – unit: volts [V]
- **f** (Frequency) – number of periods in one second – unit: hertz [Hz] = [1/s] – inverse of period (T)!
- **ϕ** (*Phase*) – *absolute position of the waveform* relative to an arbitrary origin – unit: degrees [°] or radians [rad]

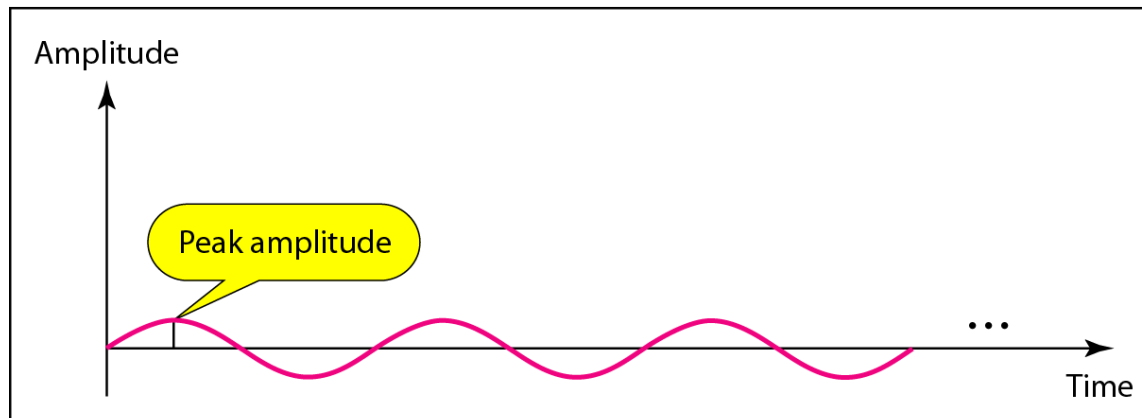
A sine wave



Two signals with the same phase and frequency, but different amplitudes



a. A signal with high peak amplitude

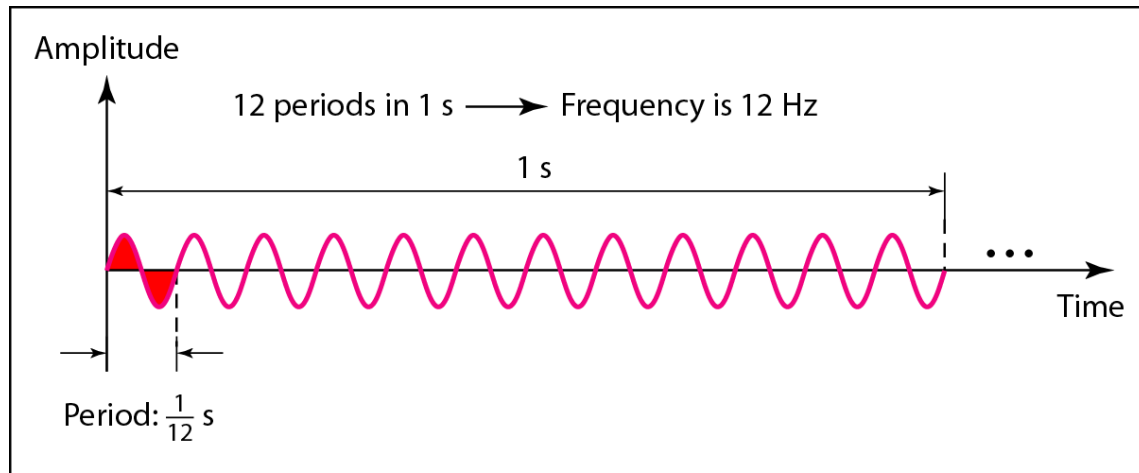


b. A signal with low peak amplitude

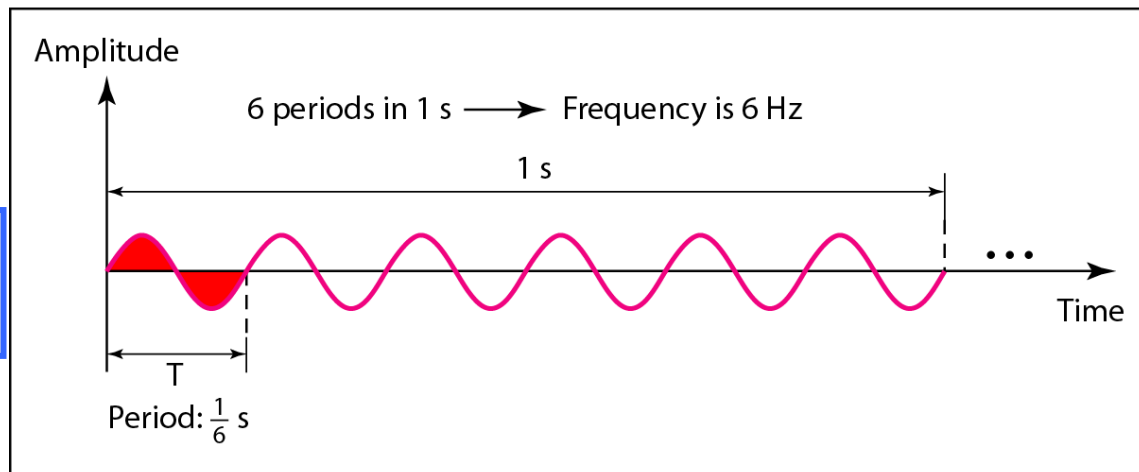
Frequency

- Two signals with the same amplitude and phase, but different frequencies
- Frequency and period are the inverse of each other.

$$f = \frac{1}{T} \quad \text{and} \quad T = \frac{1}{f}$$



a. A signal with a frequency of 12 Hz



b. A signal with a frequency of 6 Hz

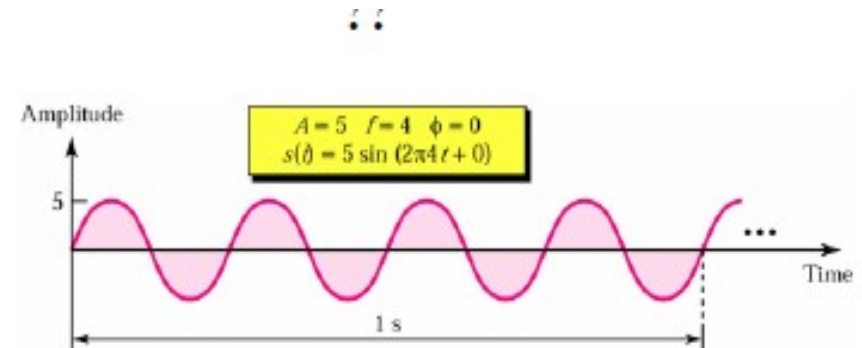
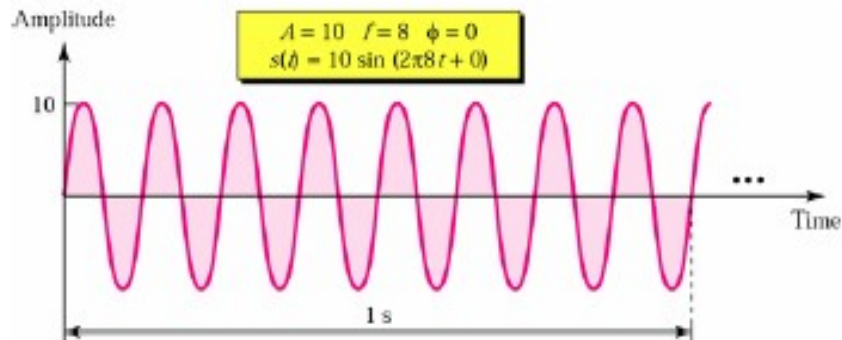
Units of period and frequency

<i>Unit</i>	<i>Equivalent</i>	<i>Unit</i>	<i>Equivalent</i>
Seconds (s)	1 s	Hertz (Hz)	1 Hz
Milliseconds (ms)	10^{-3} s	Kilohertz (kHz)	10^3 Hz
Microseconds (μ s)	10^{-6} s	Megahertz (MHz)	10^6 Hz
Nanoseconds (ns)	10^{-9} s	Gigahertz (GHz)	10^9 Hz
Picoseconds (ps)	10^{-12} s	Terahertz (THz)	10^{12} Hz

Some points..

- Frequency is the rate of change with respect to time.
- Change in a short span of time means high frequency.
- Change over a long span of time means low frequency.
- If a signal does not change at all, its frequency is zero.
- If a signal changes instantaneously, its frequency is infinite.

- Rate of signal change with respect to time
 - change in a short span of time \Rightarrow high freq
 - change over a long span of time \Rightarrow low freq



Problem 1

- The power we use at home has a frequency of 60 Hz. The period of this sine wave can be determined as follows:

$$T = \frac{1}{f} = \frac{1}{60} = 0.0166 \text{ s} = 0.0166 \times 10^3 \text{ ms} = 16.6 \text{ ms}$$

Problem 2

- The period of a signal is 100 ms. What is its frequency in kilohertz?

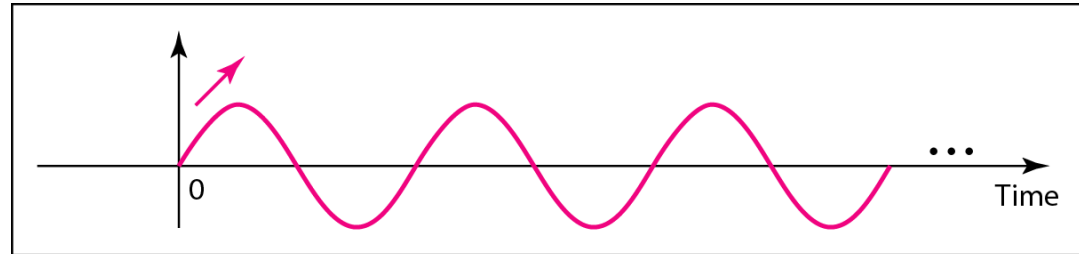
Solution

- First we change 100 ms to seconds, and then we calculate the frequency from the period (1 Hz = 10^{-3} kHz).

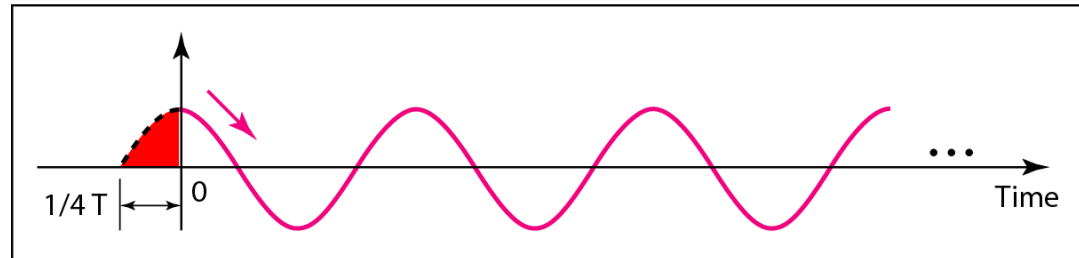
$$100 \text{ ms} = 100 \times 10^{-3} \text{ s} = 10^{-1} \text{ s}$$
$$f = \frac{1}{T} = \frac{1}{10^{-1}} \text{ Hz} = 10 \text{ Hz} = 10 \times 10^{-3} \text{ kHz} = 10^{-2} \text{ kHz}$$

Phase

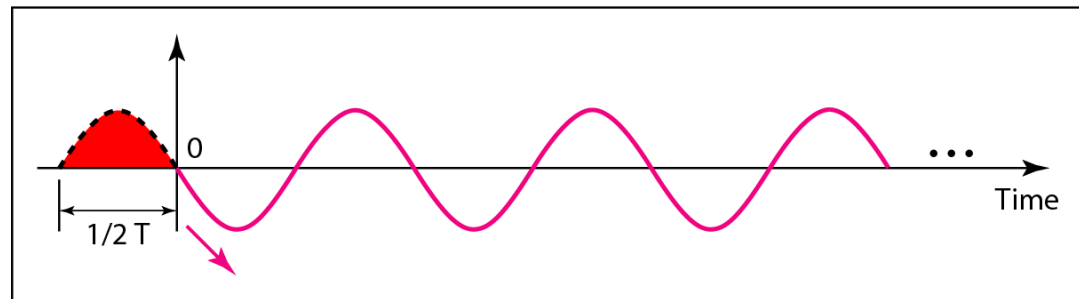
- Three sine waves with the same amplitude and frequency, but different phases \Rightarrow
- Phase describes the position of the waveform relative to time 0.



a. 0 degrees



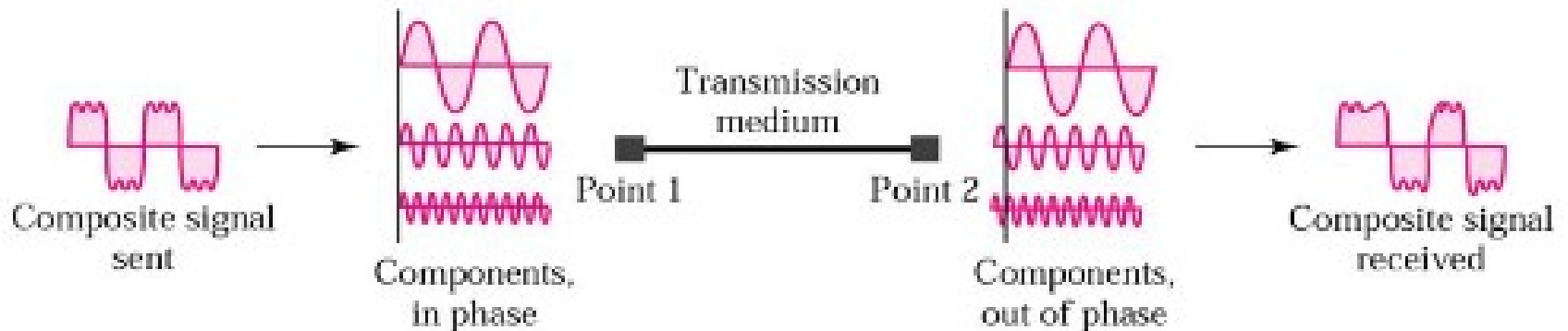
b. 90 degrees



c. 180 degrees

Phase difference due to distortion

- Composite signal made of different frequencies.
- Each signal component has its own propagation speed through the medium and therefore its own delay in arriving in the destination.
- Differences in delay may create a difference in phase.
- Thus the shape of the received signal is not the same.



Problem 3

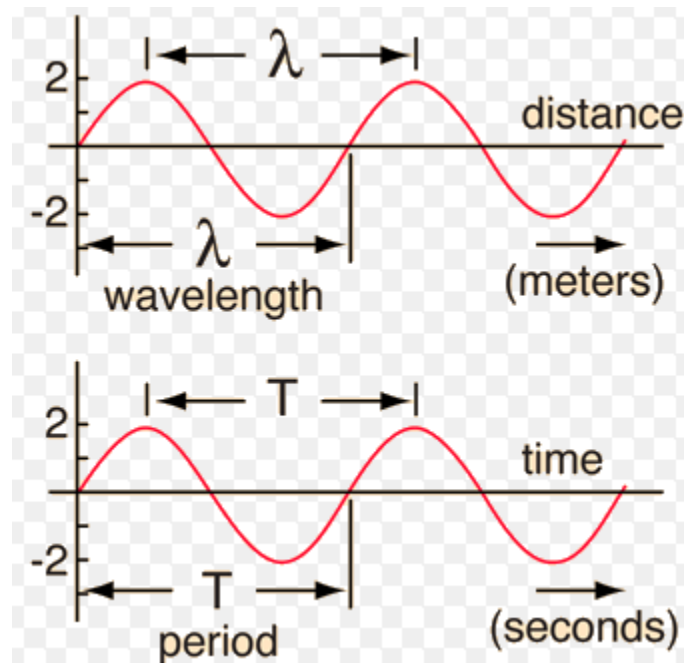
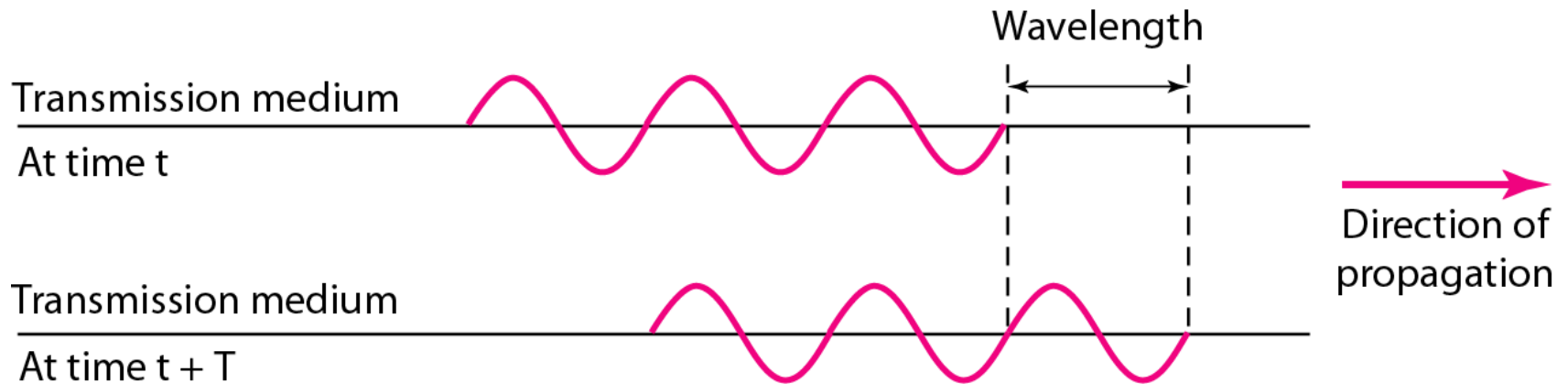
- A sine wave is offset 1/6 cycle with respect to time 0. What is its phase in degrees and radians?

Solution

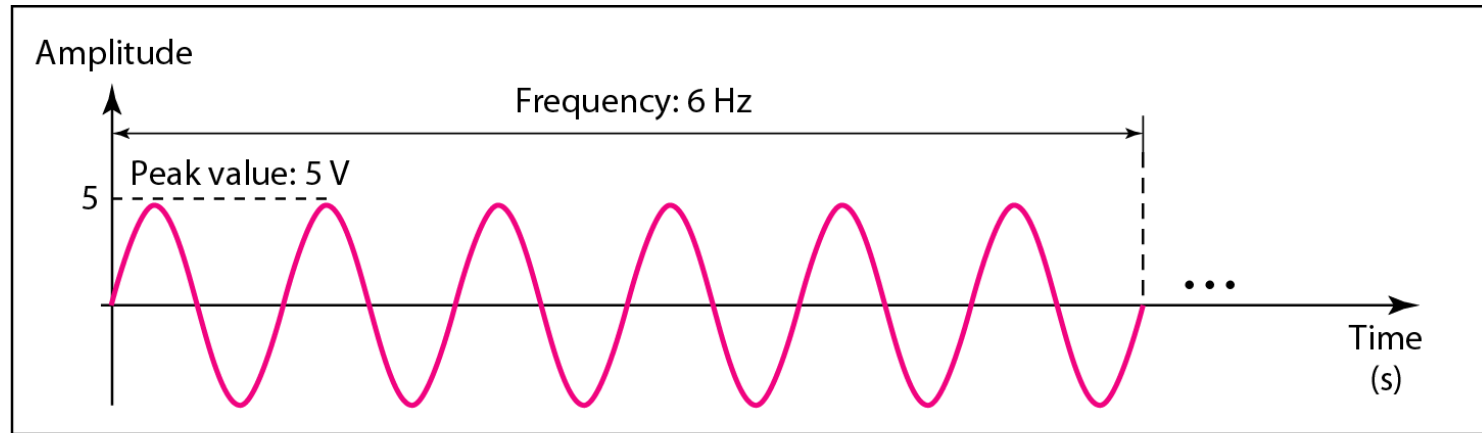
- We know that 1 complete cycle is 360°. Therefore, 1/6 cycle is

$$\frac{1}{6} \times 360 = 60^\circ = 60 \times \frac{2\pi}{360} \text{ rad} = \frac{\pi}{3} \text{ rad} = 1.046 \text{ rad}$$

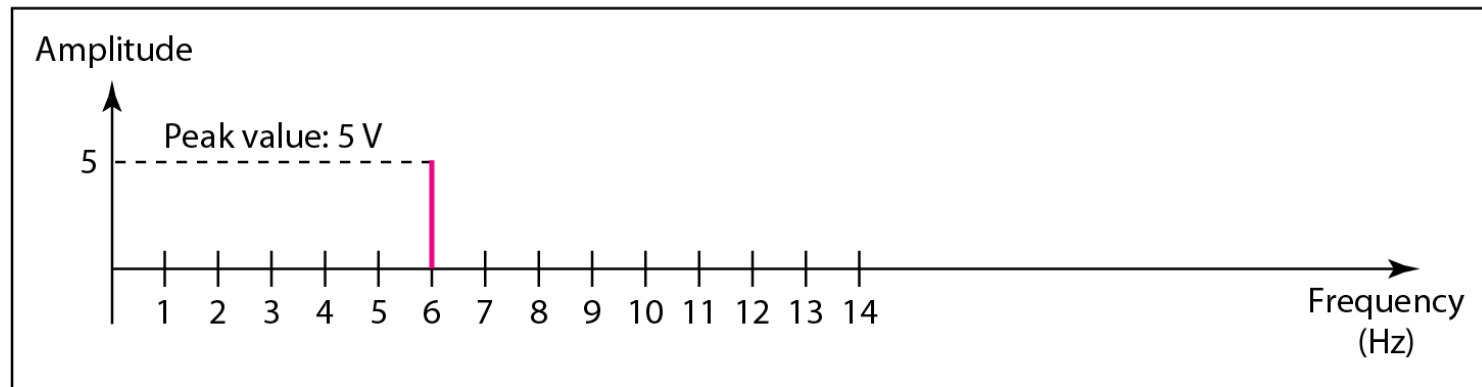
Wavelength and period



The time-domain and frequency-domain plots of a sine wave



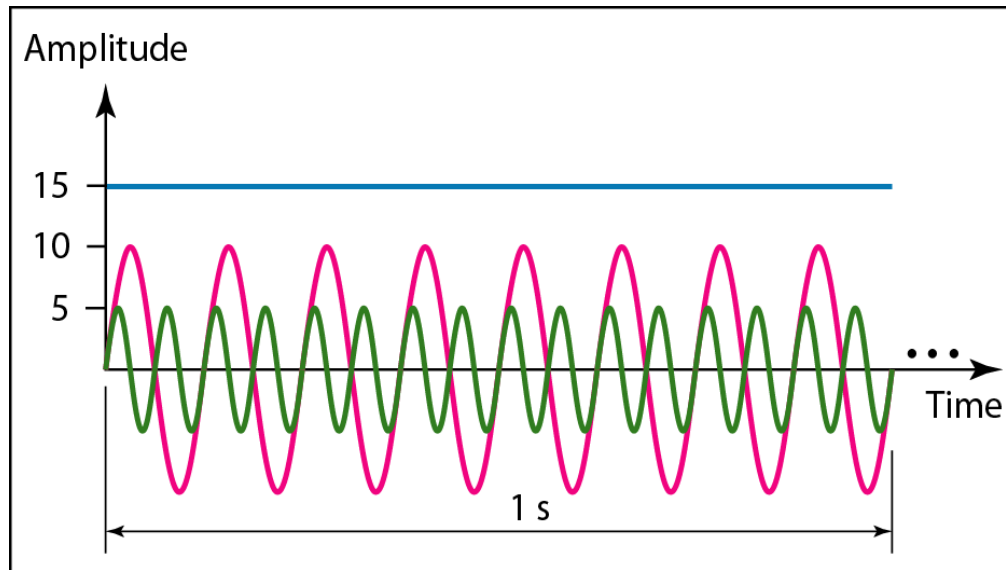
a. A sine wave in the time domain (peak value: 5 V, frequency: 6 Hz)



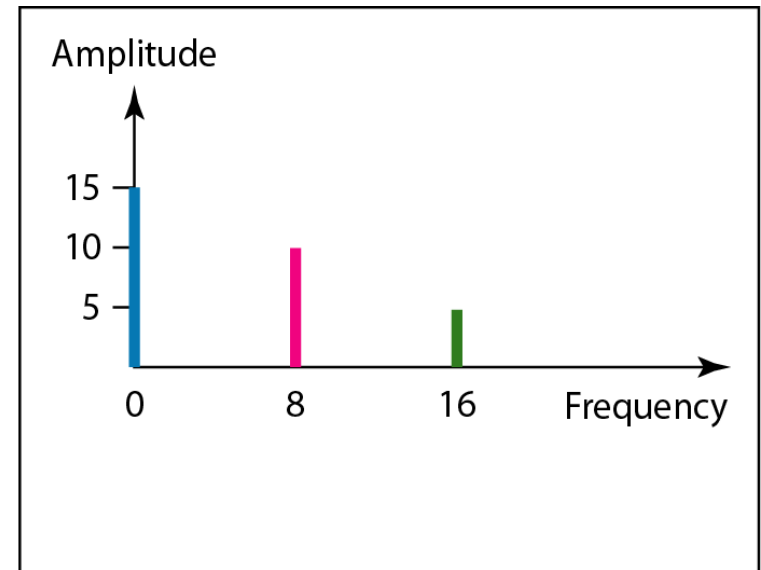
b. The same sine wave in the frequency domain (peak value: 5 V, frequency: 6 Hz)

Time domain and Frequency domain

- A complete sine wave in the time domain can be represented by one single spike in the frequency domain.
- The frequency domain is more compact and useful when we are dealing with more than one sine wave.
- Figure below shows three sine waves, each with different amplitude and frequency. All can be represented by three spikes in the frequency domain.



a. Time-domain representation of three sine waves with frequencies 0, 8, and 16



b. Frequency-domain representation of the same three signals

Signals and Communication

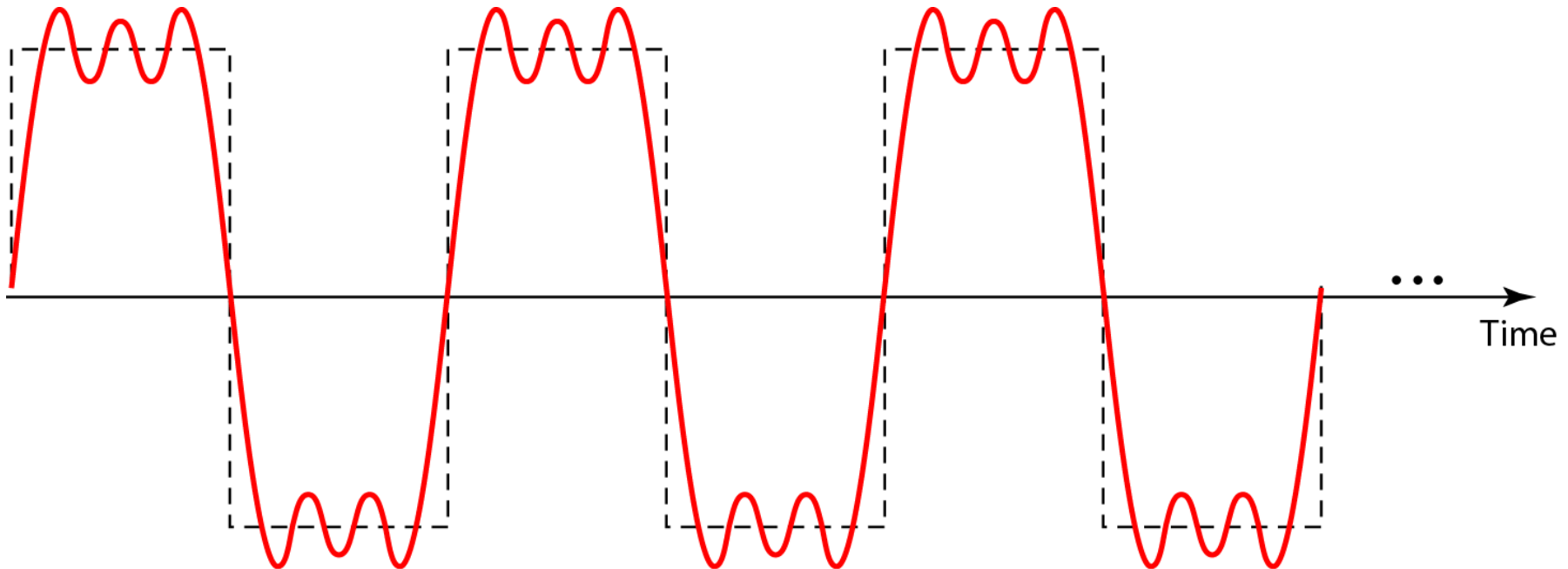
- A *single-frequency sine wave is not useful* in data communications
- We need to send a **composite signal**, a signal made of many simple sine waves.
- According to **Fourier analysis**, any composite signal is a combination of simple sine waves with different frequencies, amplitudes, and phases.

Composite Signals and Periodicity

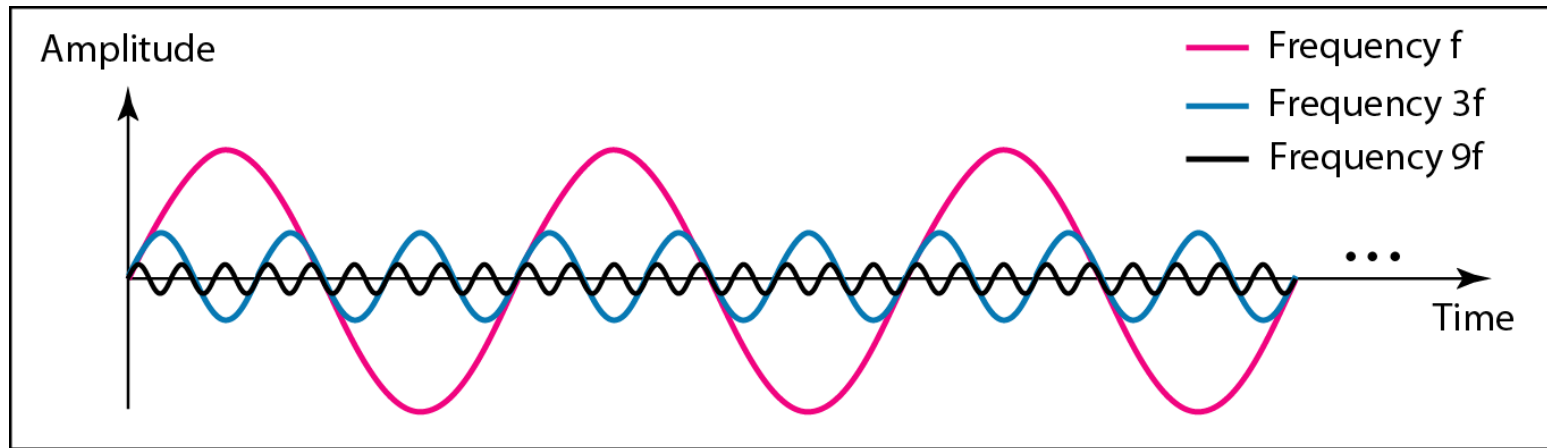
- If the composite signal is **periodic**, the decomposition gives a series of signals with **discrete** frequencies.
- If the composite signal is **nonperiodic**, the decomposition gives a combination of sine waves with **continuous** frequencies.

A composite periodic signal

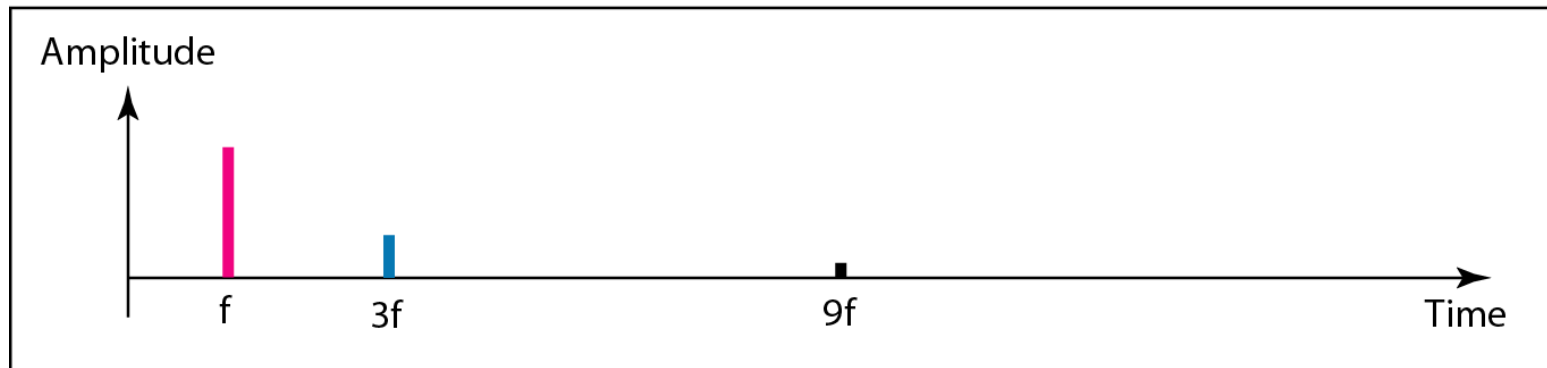
- Figure below shows a periodic composite signal with frequency f . This type of signal ***is not typical of those found in data communications***. We can consider it to be three alarm systems, each with a different frequency. The analysis of this signal can give us a good understanding of how to decompose signals.



Decomposition of a composite periodic signal in the time and frequency domains



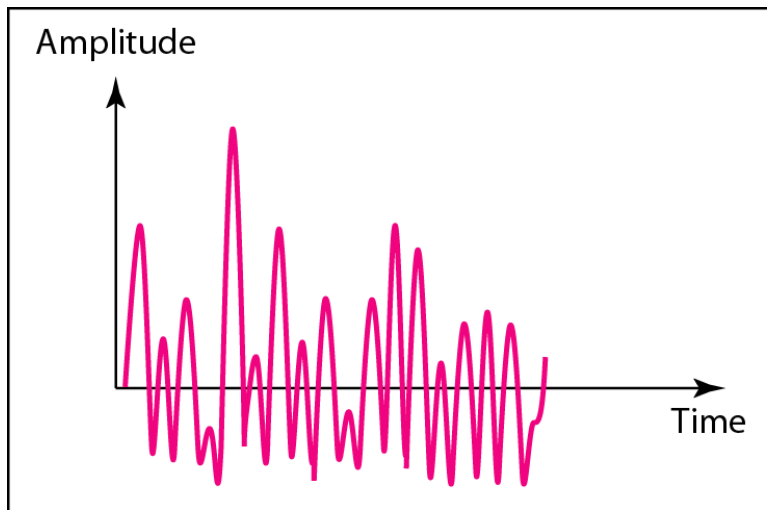
a. Time-domain decomposition of a composite signal



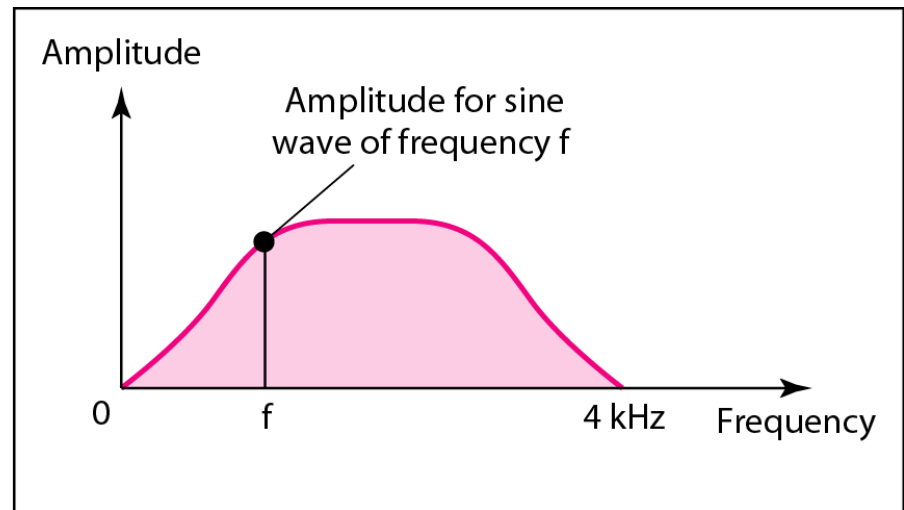
b. Frequency-domain decomposition of the composite signal

The time and frequency domains of a nonperiodic signal

- Figure below shows a nonperiodic composite signal. It can be the signal created by a microphone or a telephone set when a word or two is pronounced. In this case, the composite signal cannot be periodic, because that implies that we are repeating the same word or words with exactly the same tone.



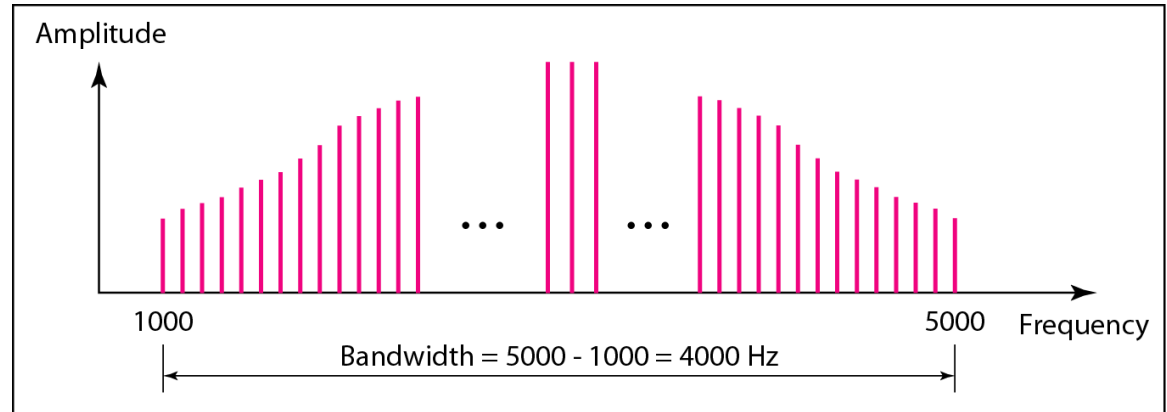
a. Time domain



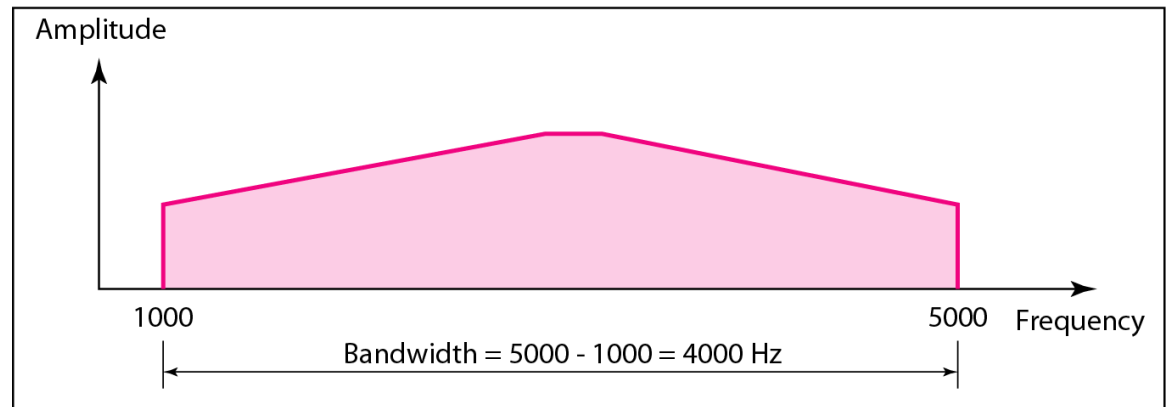
b. Frequency domain

Bandwidth and Signal Frequency

- The bandwidth of a composite signal is the **difference** between the highest and the lowest frequencies contained in that signal.



a. Bandwidth of a periodic signal



b. Bandwidth of a nonperiodic signal

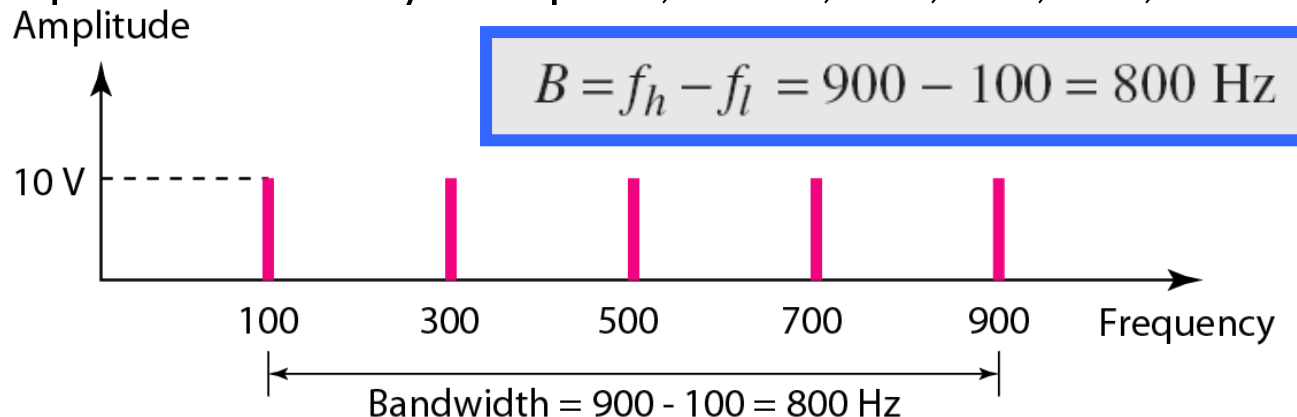
Problem 4

- If a periodic signal is decomposed into five sine waves with frequencies of 100, 300, 500, 700, and 900 Hz, what is its bandwidth? Draw the spectrum, assuming all components have a maximum amplitude of 10 V.

Solution

- Let f_h be the highest frequency, f_l the lowest frequency, and B the bandwidth. Then

The spectrum has only five spikes, at 100, 300, 500, 700, and 900 Hz



Problem 5

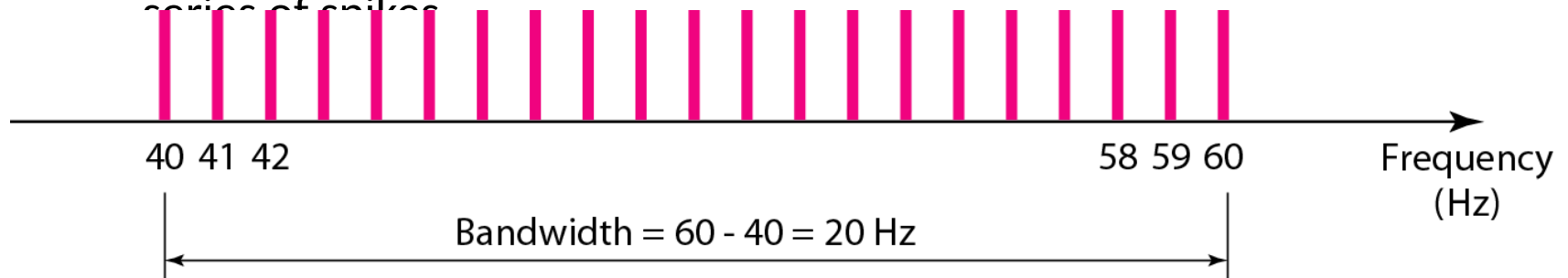
- A periodic signal has a bandwidth of 20 Hz. The highest frequency is 60 Hz. What is the lowest frequency? Draw the spectrum if the signal contains all frequencies of the same amplitude.

Solution

- Let f_h be the highest frequency, f_l the lowest frequency, and B the bandwidth. Then

$$B = f_h - f_l \Rightarrow 20 = 60 - f_l \Rightarrow f_l = 60 - 20 = 40 \text{ Hz}$$

- The spectrum contains all integer frequencies. Figure below shows a series of spikes

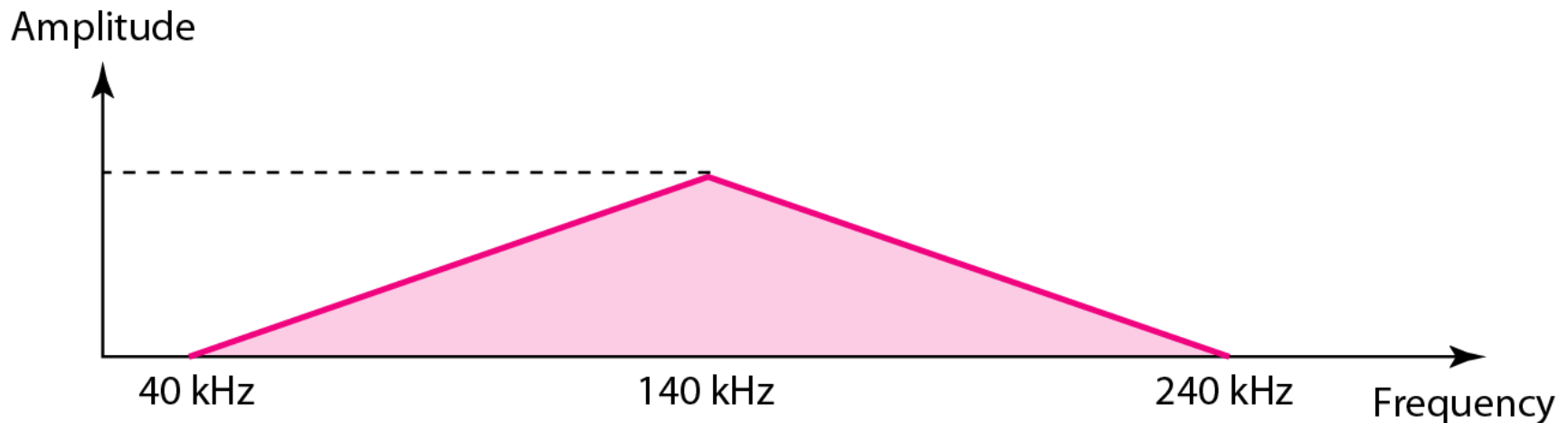


Problem 6

- A nonperiodic composite signal has a bandwidth of 200 kHz, with a middle frequency of 140 kHz and peak amplitude of 20 V. The two extreme frequencies have an amplitude of 0. Draw the frequency domain of the signal.

Solution

- The lowest frequency must be at 40 kHz and the highest at 240 kHz. Figure below shows the frequency domain and the bandwidth.



FOURIER ANALYSIS

Examples of nonperiodic composite signal

- An example of a nonperiodic composite signal is the signal propagated by an AM radio station. In the United States, each AM radio station is assigned a 10-kHz bandwidth. The total bandwidth dedicated to AM radio ranges from 530 to 1700 kHz.
- Another example of a nonperiodic composite signal is the signal propagated by an FM radio station. In the United States, each FM radio station is assigned a 200-kHz bandwidth. The total bandwidth dedicated to FM radio ranges from 88 to 108 MHz.
- Another example of a nonperiodic composite signal is the signal received by an old-fashioned analog black-and-white TV. A TV screen is made up of pixels. If we assume a resolution of 525×700 , we have 367,500 pixels per screen. If we scan the screen 30 times per second, this is $367,500 \times 30 = 11,025,000$ pixels per second. The worst-case scenario is alternating black and white pixels. We can send 2 pixels per cycle. Therefore, we need $11,025,000 / 2 = 5,512,500$ cycles per second, or Hz. The bandwidth needed is 5.5125 MHz.

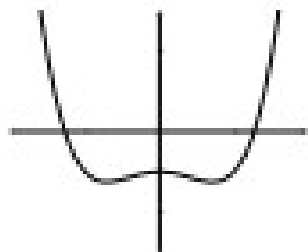
We will show the rationale behind bandwidth selection during lectures on analog transmission .

Fourier Analysis

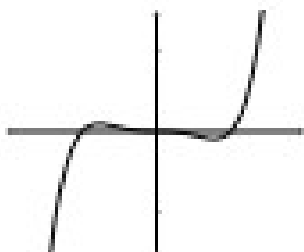
- Fourier analysis is a tool that changes a time domain signal to a frequency domain signal and vice versa.
- Every composite **periodic** signal can be represented with a series of sine and cosine functions.
 - The functions are integral harmonics of the fundamental frequency “f” of the composite signal.
 - Using the series we can decompose any periodic signal into its harmonics.

Even and odd functions

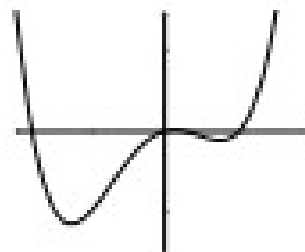
- A function $f(x)$ is said to be even if
$$f(-x) = f(x)$$
- The function $f(x)$ is said to be odd if
$$f(-x) = -f(x).$$
- The function $f(x)$ is said to be neither odd nor even if
$$f(-x) \neq f(x) \text{ and } f(-x) \neq -f(x)$$
- Graphically, even functions have symmetry about the y-axis, whereas odd functions have symmetry around the origin.



Even



Odd



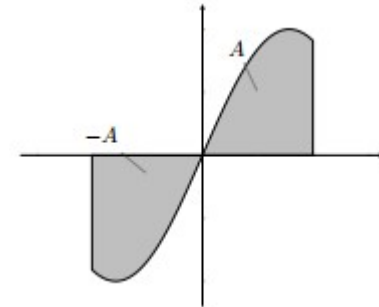
Neither

Integrating odd functions over symmetric domains

- Let $p > 0$ be any fixed number. If $f(x)$ is an odd function, then

$$\int_{-p}^p f(x) dx = 0.$$

Intuition: The area beneath the curve on $[-p, 0]$ is the same as the area under the curve on $[0, p]$, but opposite in sign. So, they cancel each other out!

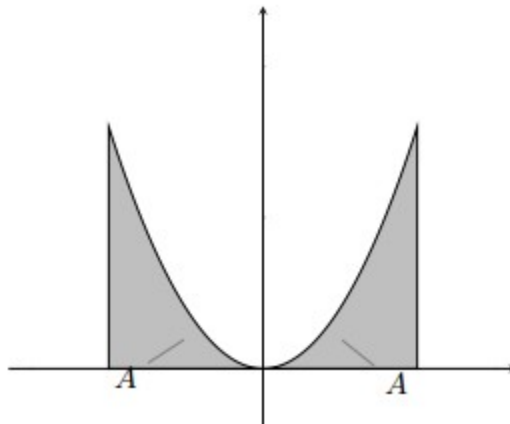


Integrating odd functions over symmetric domains

- Let $p > 0$ be any fixed number. If $f(x)$ is an even function, then

$$\int_{-p}^p f(x) \, dx = 2 \int_0^p f(x) \, dx.$$

Intuition: The area beneath the curve on $[-p, 0]$ is the same as the area under the curve on $[0, p]$, but this time with the same sign. So, you can just find the area under the curve on $[0, p]$ and double it!



Periodic Function

- A function $f(x)$ is said to be periodic if there exists a number $T > 0$ such that $f(x + T) = f(x)$ for every x . The smallest such T is called the period of $f(x)$.
- Intuitively, periodic functions have repetitive behavior. A periodic function can be defined on a finite interval, then copied and pasted so that it repeats itself.
- Examples
 - $\sin x$ and $\cos x$ are periodic with period 2π
 - $\sin(\pi x)$ and $\cos(\pi x)$ are periodic with period 2
 - If L is a fixed number, then $\sin(2\pi x / L)$ and $\cos(2\pi x / L)$ have period L
- Sine and cosine are the most “basic” periodic functions!

Composite signal

- Any composite signal can be represented as a combination of simple sine waves with different frequencies, phases and amplitudes

$$s(t) = A_1 \sin(2\pi f_1 t + \varphi_1) + A_2 \sin(2\pi f_2 t + \varphi_2) + \dots$$

- Periodic composite signal (period=T, freq. = $f_0=1/T$) can be represented as a sum of simple sines and/or cosines known as *Fourier series***

$$s(t) = \frac{A_0}{2} + \sum_{n=1}^{\infty} [A_n \cos(2\pi n f_0 t) + B_n \sin(2\pi n f_0 t)]$$

Fourier Series

- Let $p > 0$ be a fixed number and $f(x)$ be a periodic function with period $2p$, defined on $(-p, p)$. The Fourier series of $f(x)$ is a way of expanding the function $f(x)$ into an infinite series involving sines and cosines:

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{p}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{p}\right)$$

where a_0 , a_n , and b_n are called the Fourier coefficients of $f(x)$, and are given by the formulas

$$a_0 = \frac{1}{p} \int_{-p}^p f(x) dx, \quad a_n = \frac{1}{p} \int_{-p}^p f(x) \cos\left(\frac{n\pi x}{p}\right) dx, \\ b_n = \frac{1}{p} \int_{-p}^p f(x) \sin\left(\frac{n\pi x}{p}\right) dx,$$

Remarks

- To find a Fourier series, it is sufficient to calculate the integrals that give the coefficients a_0 , a_n , and b_n and plug them in to the big series formula, equation above.
- Typically, $f(x)$ will be piecewise defined.
- Big advantage that Fourier series have over Taylor series: the function $f(x)$ can have discontinuities!
- Useful identities for Fourier series: if n is an integer, then

$$\sin(n\pi) = 0$$

$$\text{e.g. } \sin(\pi) = \sin(2\pi) = \sin(3\pi) = \sin(20\pi) = 0$$

$$\cos(n\pi) = (-1)^n = \begin{cases} 1 & n \text{ even} \\ -1 & n \text{ odd} \end{cases}$$

$$\text{e.g. } \cos(\pi) = \cos(3\pi) = \cos(5\pi) = -1,$$

$$\text{but } \cos(0\pi) = \cos(2\pi) = \cos(4\pi) = 1.$$

Fourier Coefficients of an even function

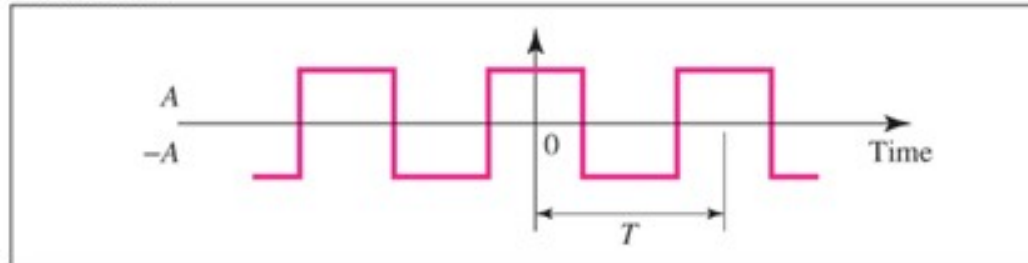
- If $f(x)$ is an even function, then the formulas for the coefficients simplify. Specifically, since $f(x)$ is even, $f(x) \sin(n\pi x/p)$ is an odd function, and thus

$$b_n = \frac{1}{p} \int_{-p}^p \underbrace{f(x)}_{\text{even}} \underbrace{\sin\left(\frac{n\pi x}{p}\right)}_{\text{odd}} dx = 0$$

- Therefore, for even functions, you can automatically conclude (no computations necessary!) that the b_n coefficients are all 0.

Example

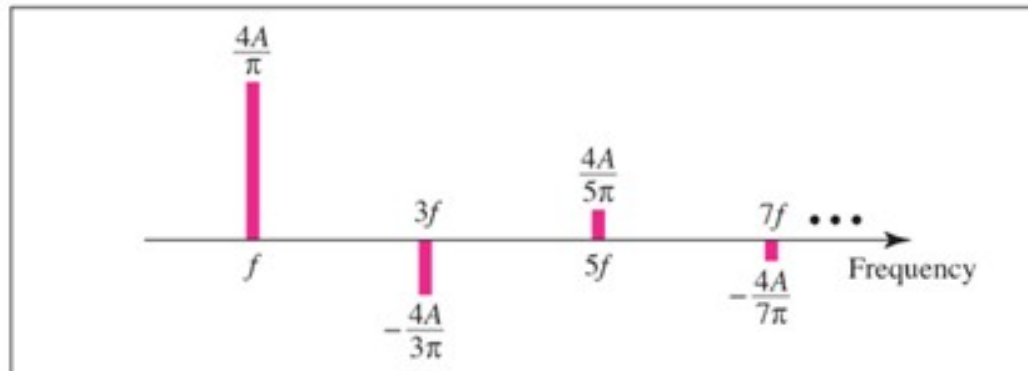
Time domain



f(x) i.e., s(t)
here

$$A_0 = 0 \quad A_n = \begin{cases} \frac{4A}{n\pi} & \text{for } n = 1, 5, 9, \dots \\ -\frac{4A}{n\pi} & \text{for } n = 3, 7, 11, \dots \end{cases} \quad B_n = 0$$

$$s(t) = \frac{4A}{\pi} \cos(2\pi ft) - \frac{4A}{3\pi} \cos(2\pi 3ft) + \frac{4A}{5\pi} \cos(2\pi 5ft) - \frac{4A}{7\pi} \cos(2\pi 7ft) + \dots$$



Frequency domain

Fourier Coefficients of an odd function

- If $f(x)$ is odd, then we get two freebies:

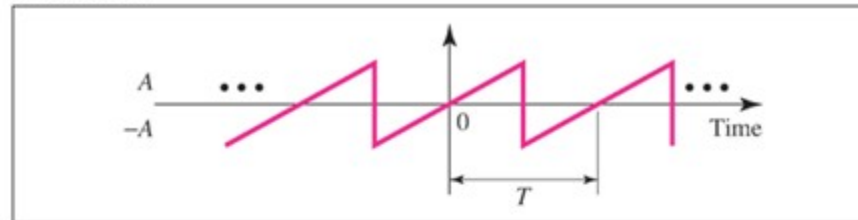
$$a_0 = \frac{1}{p} \int_{-p}^p \overbrace{f(x)}^{\text{odd}} dx = 0$$

$$a_n = \frac{1}{p} \int_{-p}^p \overbrace{\underbrace{f(x)}_{\text{odd}} \underbrace{\cos\left(\frac{n\pi x}{p}\right)}_{\text{even}}}_{\text{odd}} dx = 0$$

Example: Sawtooth Signal

$f(x)$ i.e., $s(t)$
here

Time domain

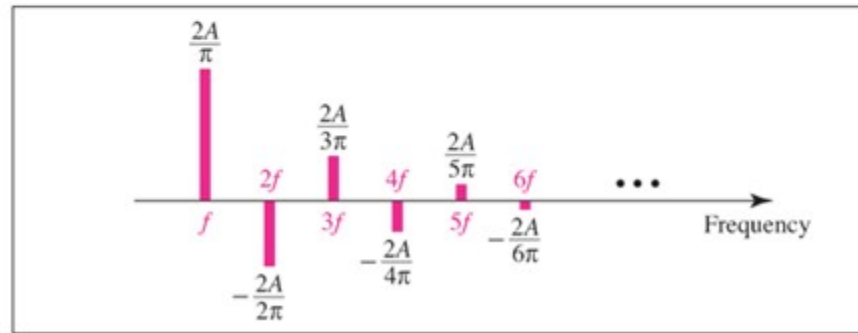


$$A_0 = 0$$

$$A_n = 0$$

$$B_n = \begin{cases} \frac{2A}{n\pi} & \text{for } n \text{ odd} \\ -\frac{2A}{n\pi} & \text{for } n \text{ even} \end{cases}$$

$$s(t) = \frac{2A}{\pi} \sin(2\pi ft) - \frac{2A}{2\pi} \sin(2\pi 2ft) + \frac{2A}{3\pi} \sin(2\pi 3ft) - \frac{2A}{4\pi} \sin(2\pi 4ft) + \dots$$



Frequency domain

Note

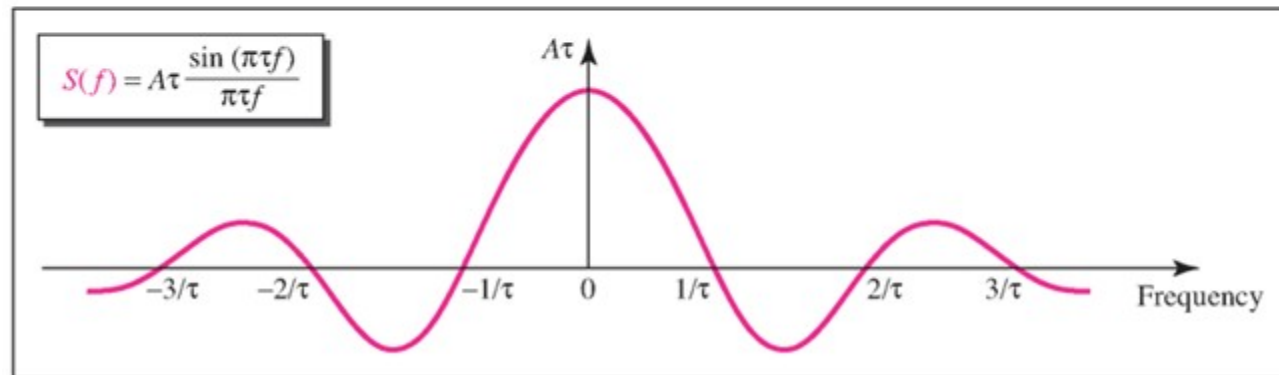
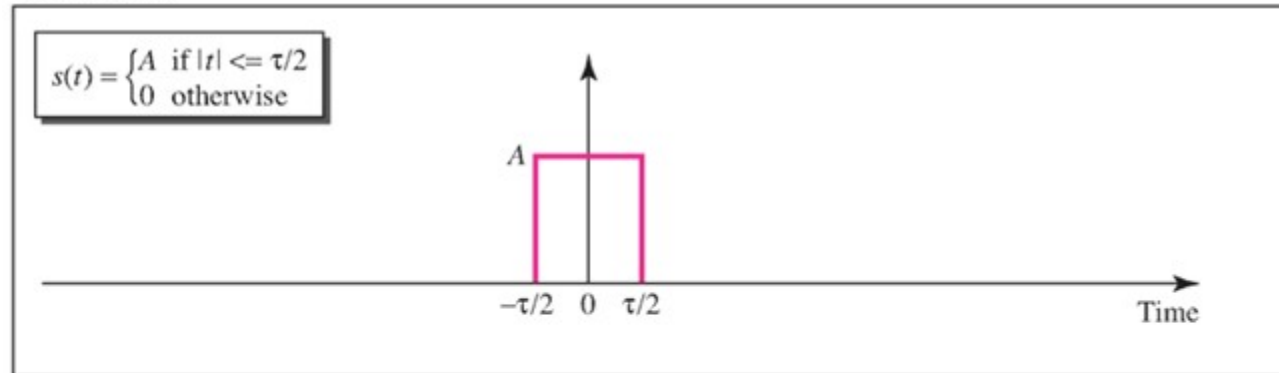
- Function may be neither even nor odd.
- In those cases, you should use the original formulas for computing Fourier coefficients

FOURIER TRANSFORM

Fourier Transform

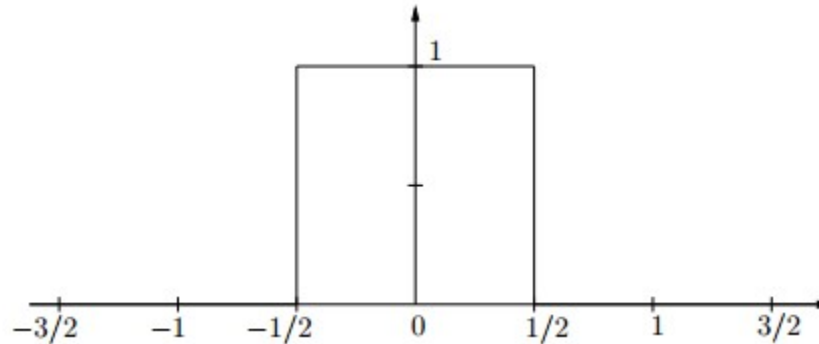
- Fourier Transform gives the frequency domain of a **nonperiodic** time domain signal
- A **time limited** signal is a signal for which the amplitude $s(t) = 0$ for $t > T_1$ and $t < T_2$
- A **band limited** signal is a signal for which the amplitude $S(f) = 0$ for $f > F_1$ and $f < F_2$

Time domain



Frequency domain

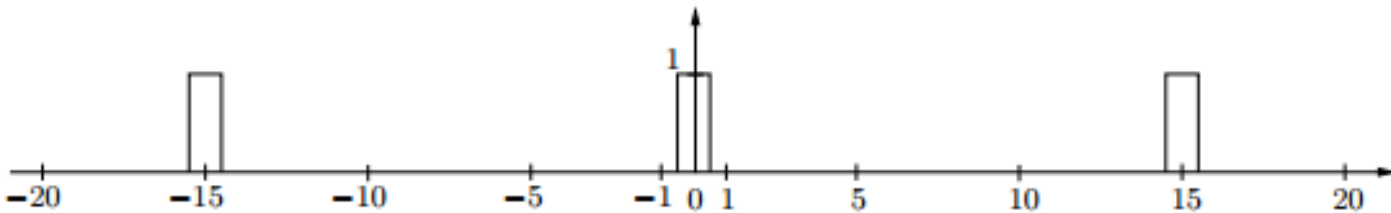
Example



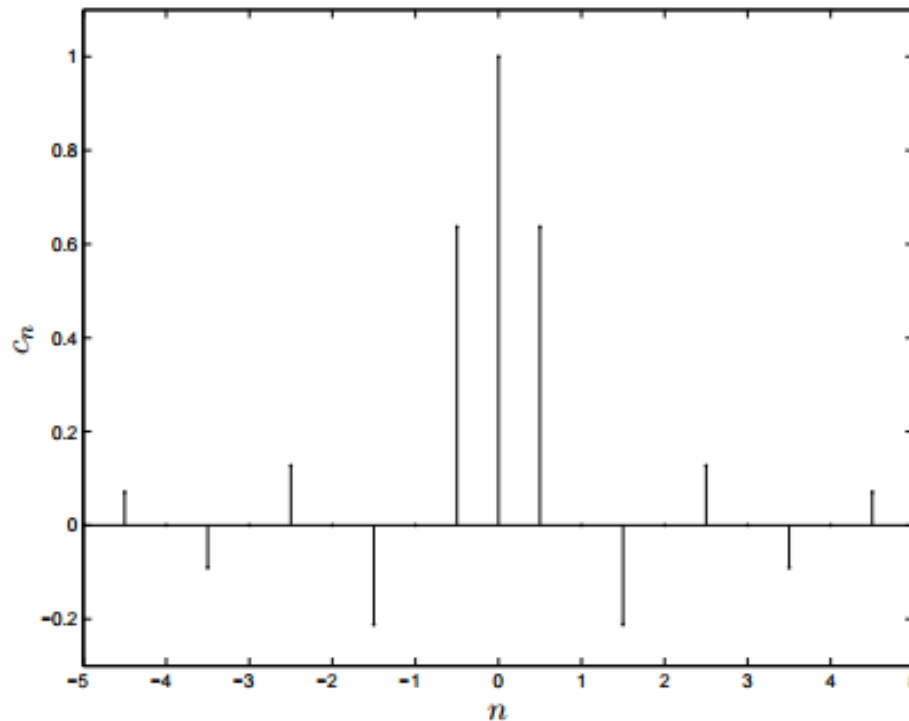
- $f(t)$ is not periodic. It doesn't have a Fourier series.
- $f(t)$ is even — centered at the origin — and has width 1.
- Later we'll consider shifted and scaled versions.
- You can think of $f(t)$ as modeling a switch that is on for one second and off for the rest of the time.

Periodic version

- As a periodic version of $f(t)$ we repeat the nonzero part of the function at regular intervals, separated by (long) intervals where the function is zero.
- We can think of such a function arising when we flip a switch on for a second at a time, and do so repeatedly, and we keep it off for a long time in between the times it's on.
- Here's a plot of $f(t)$ periodized to have period 15.

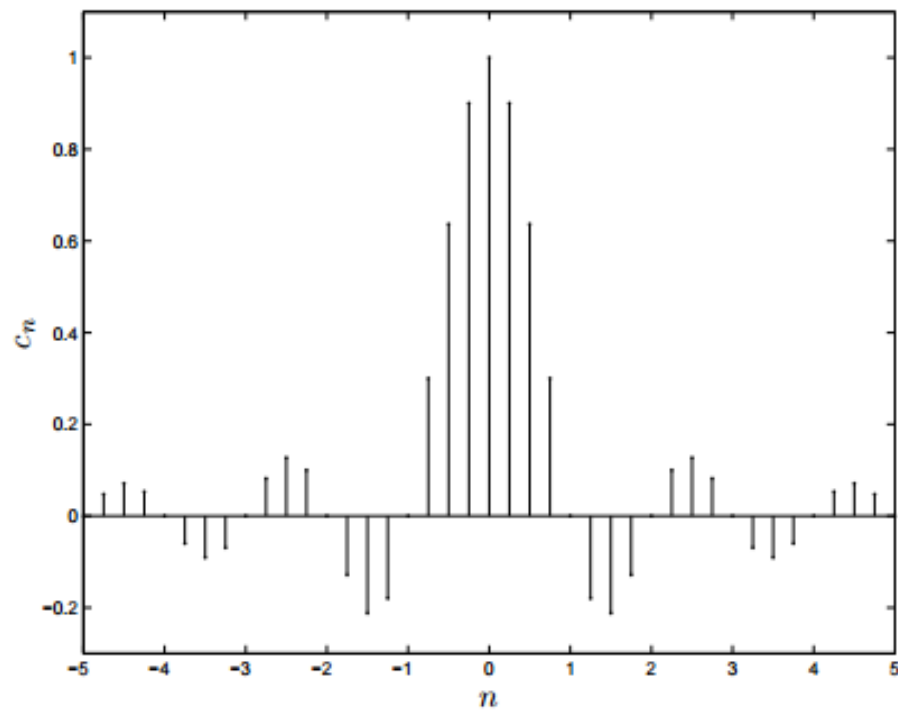


Fourier coefficients of periodized rectangle functions with period 2

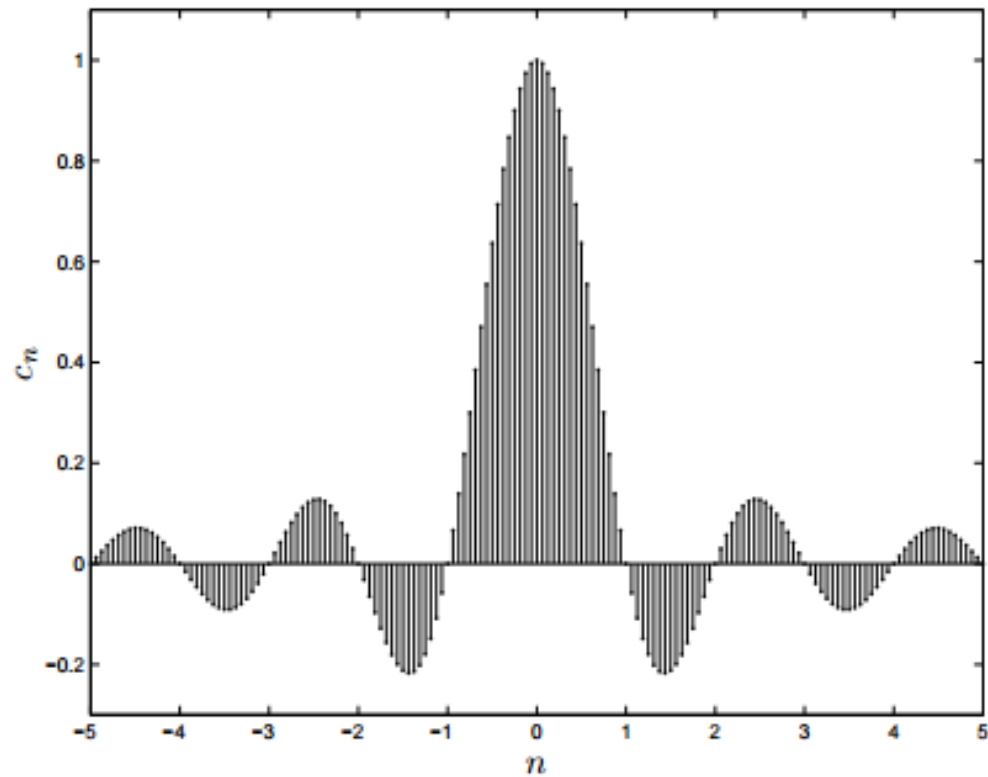


Because the function is real and even, in each case the Fourier coefficients are real, so this is plot of the actual coefficients, not their square magnitudes.

Fourier coefficients of periodized rectangle functions with period 4



Fourier coefficients of periodized rectangle functions with period 16.



- As the period increases the frequencies are getting closer and closer together and it looks as though the coefficients are tracking some definite curve.
- An important issue here is of vertical scaling.

$$S(f) = \int_{-\infty}^{\infty} s(t) e^{-j2\pi ft} dt$$

Fourier transform

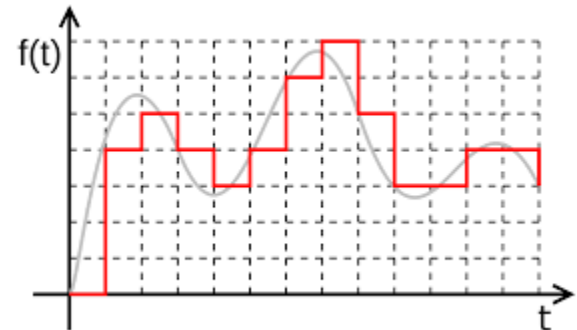
$$s(t) = \int_{-\infty}^{\infty} S(f) e^{j2\pi ft} df$$

Inverse Fourier transform

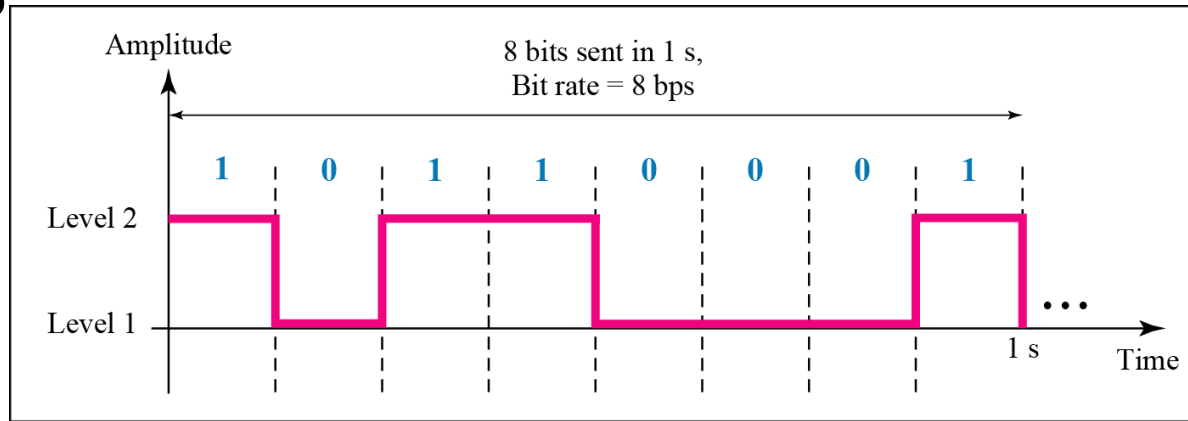
DIGITAL SIGNALS

Digital Signal

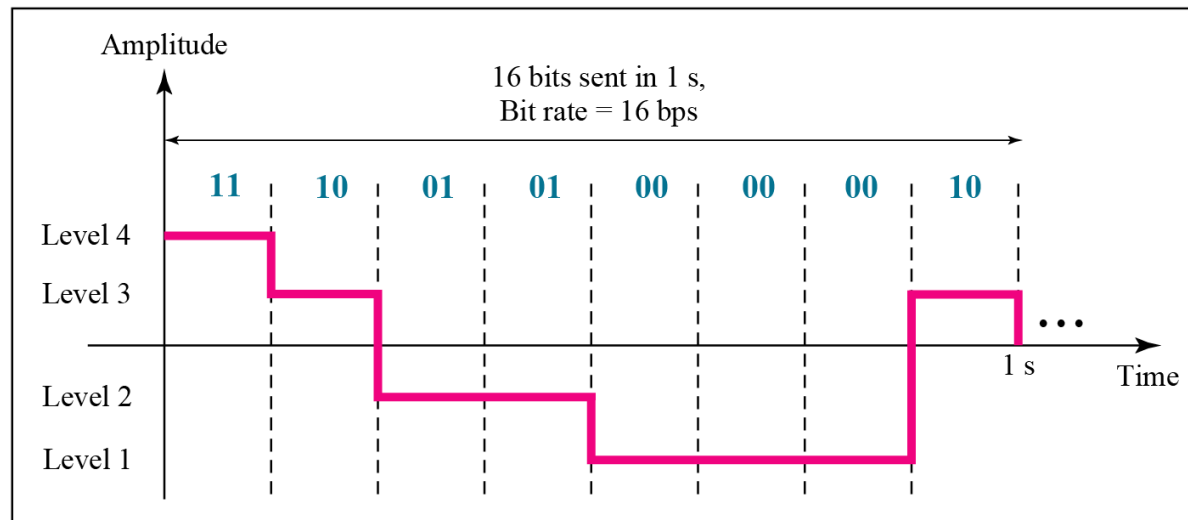
- A **digital signal** refers to an electrical **signal** that is converted into a pattern of bits.
- Unlike an analog **signal**, which is a continuous **signal** that contains time-varying quantities, a **digital signal** has a discrete value at each sampling point.



Example: Two digital signals: one with two signal levels and the other with four signal levels



a. A digital signal with two levels



b. A digital signal with four levels

Problem 1

- A digital signal has eight levels. How many bits are needed per level? We calculate the number of bits from the formula

$$\text{Number of bits per level} = \log_2 8 = 3$$

- Each signal level is represented by 3 bits.

- A digital signal has nine levels. How many bits are needed per level?

Problem 2

- A digital signal has nine levels. How many bits are needed per level?
 - We calculate the number of bits by using the formula. Each signal level is represented by 3.17 bits. **However, this answer is not realistic.**
 - The number of bits sent per level needs to be an integer as well as a power of 2. For this example, 4 bits can represent one level.

- Assume we need to download text documents at the rate of 100 pages per **sec**. What is the required bit rate of the channel?

Problem 3

- Assume we need to download text documents at the rate of 100 pages per sec. What is the required bit rate of the channel?
 - A page is an average of 24 lines with 80 characters in each line. If we assume that one character requires 8 bits (ascii), the bit rate is

$$100 \times 24 \times 80 \times 8 = 1,636,000 \text{ bps} = 1.636 \text{ Mbps}$$

- Bit rate: the number of bits per second that can be transmitted along a digital network.

- A digitized voice channel, is made by digitizing a 4-kHz bandwidth analog voice signal. We need to sample the signal at twice the highest frequency (two samples per hertz). We assume that each sample requires 8 bits. What is the required bit rate?

Problem 4

- A digitized voice channel, is made by digitizing a 4-kHz bandwidth analog voice signal. We need to sample the signal at twice the highest frequency (two samples per hertz). We assume that each sample requires 8 bits. What is the required bit rate?
 - The bit rate can be calculated as

$$2 \times 4000 \times 8 = 64,000 \text{ bps} = 64 \text{ kbps}$$

- What is the bit rate for high-definition TV (HDTV)?

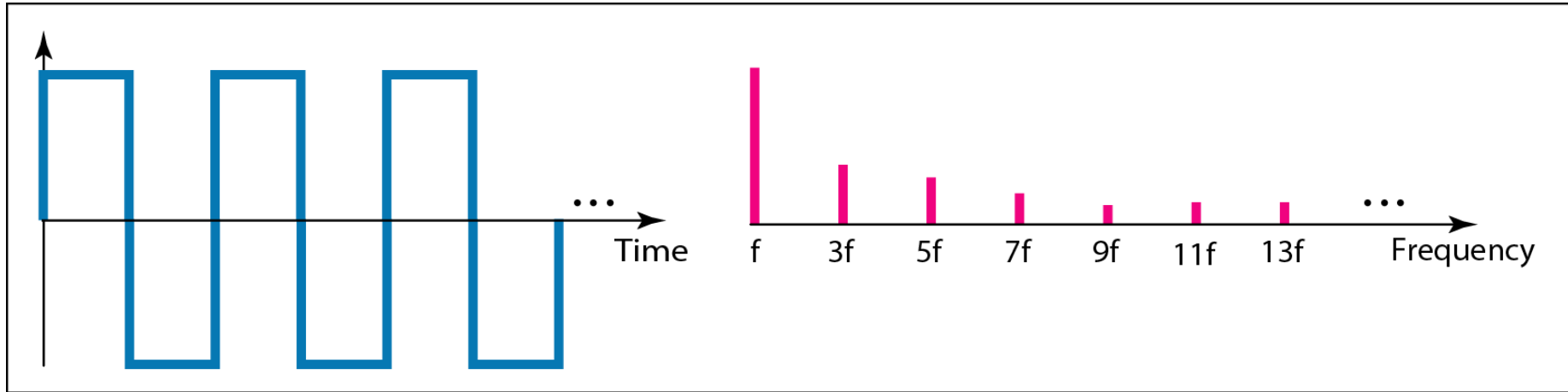
Problem 5

- What is the bit rate for high-definition TV (HDTV)?
 - HDTV uses digital signals to broadcast high quality video signals. The HDTV screen is normally a ratio of 16 : 9. There are 1920 by 1080 pixels per screen, and the screen is renewed 30 times per second. Twenty-four bits represents one color pixel.

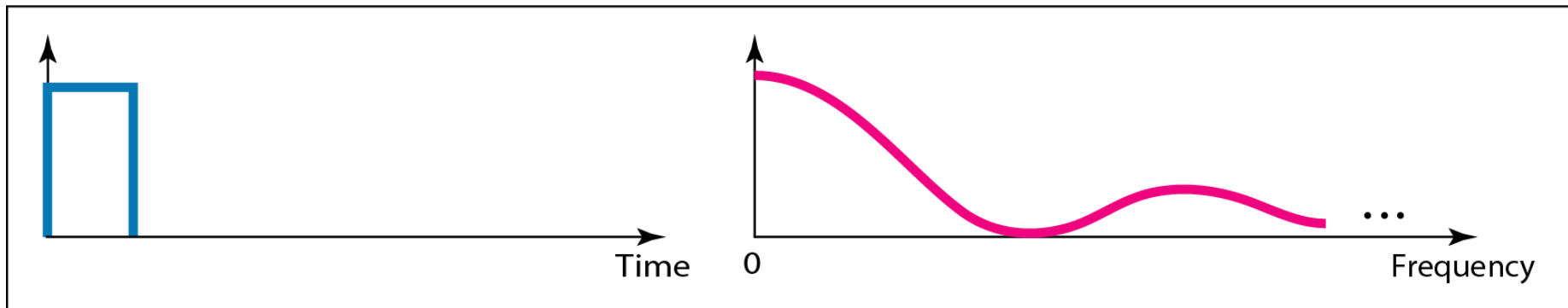
$$1920 \times 1080 \times 30 \times 24 = 1,492,992,000 \text{ or } 1.5 \text{ Gbps}$$

- The TV stations reduce this rate to 20 to 40 Mbps through compression.

The time and frequency domains of periodic and nonperiodic digital signals



a. Time and frequency domains of periodic digital signal

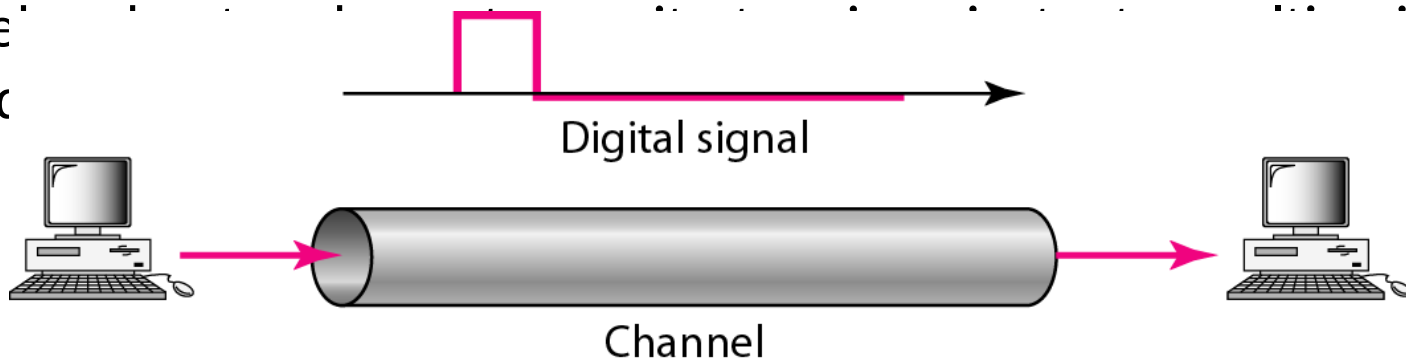


b. Time and frequency domains of nonperiodic digital signal

- So a periodic or non periodic digital signal is a composite analog signal with frequencies 0 and infinity.
- In data communications the case of non periodic digital signal is obvious.
- The fundamental question is ..How can we send a digital signal from point A to point B?
- Two approaches
 - Baseband Transmission
 - Broadband Transmission

Baseband transmission

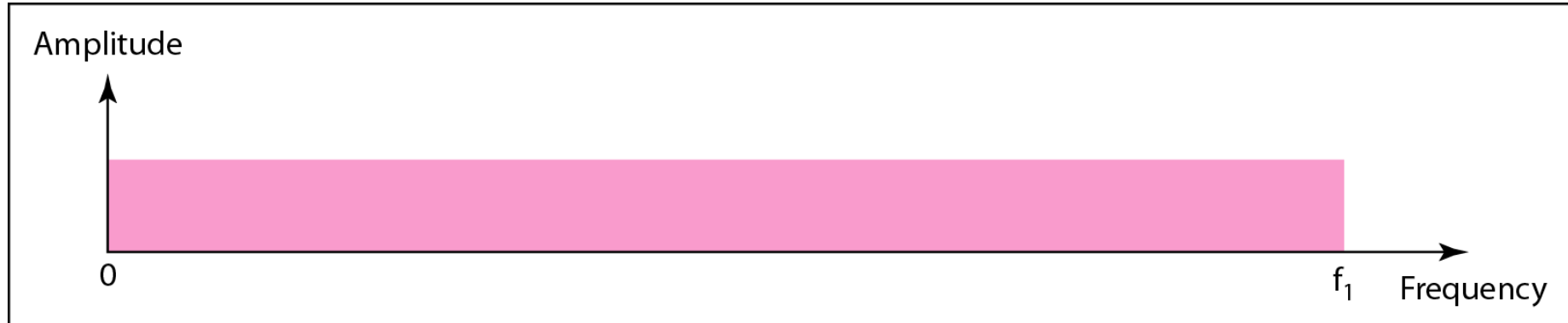
- Baseband transmission is a signaling technology that sends digital signals over a single frequency as discrete electrical pulses. Baseband transmission technologies do not use modulation
 - The entire bandwidth of a baseband system carries only one data signal and is generally less than the amount of bandwidth available on a broadband transmission system.
- LAN networking technologies such as Ethernet use baseband transmission technology. All stations on a baseband network share the same transmission medium, and they use the entire bandwidth of that medium for transmission. As a result, only one device on a baseband network can transmit at any given time in the network.



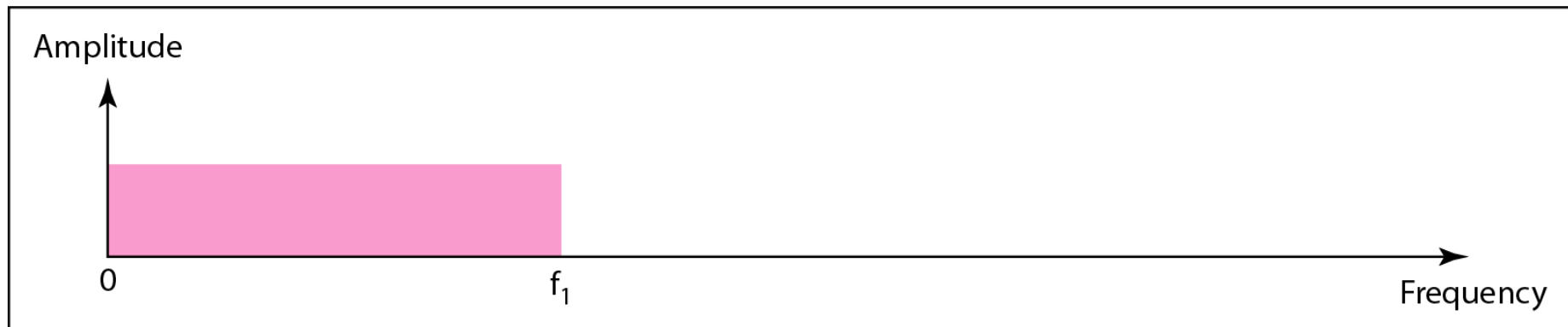
Baseband- Low pass channel

- Baseband transmission requires that we have a low pass channel, a channel with a bandwidth starts from 0.
- Example:
 - A dedicated medium with a bandwidth constituting only one channel i.e., the entire bandwidth of a cable connecting two computer in one single channel.
 - Another example, several computers connected to a bus , but not allow more than 2 stations to communicate at a time.
- Ideal case is low pass channel with infinite bandwidth, but we cannot have such channel in real life

Bandwidths of two low-pass channels

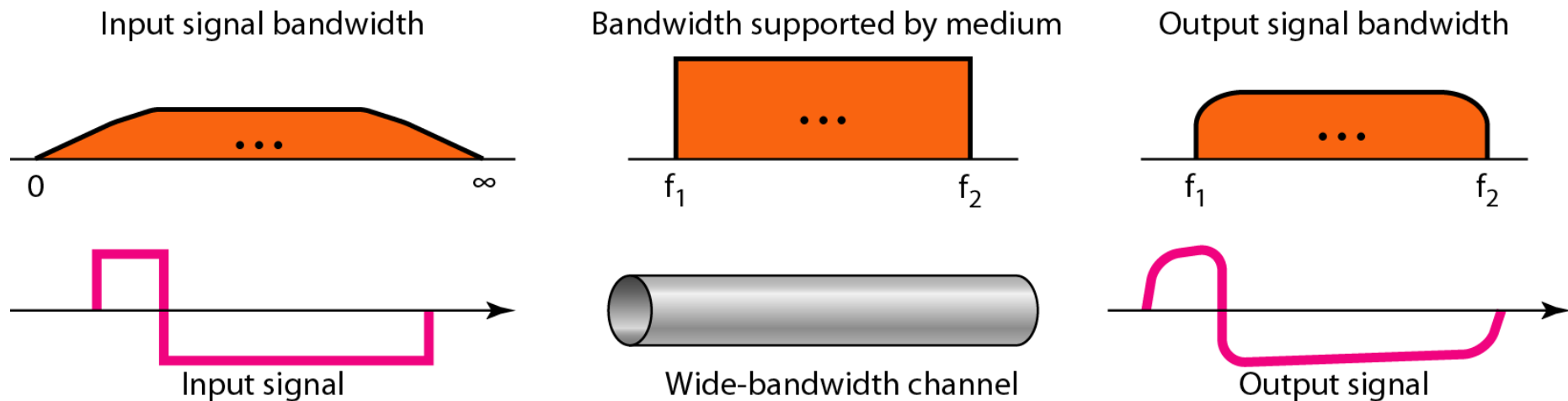


a. Low-pass channel, wide bandwidth



b. Low-pass channel, narrow bandwidth

Baseband transmission using a dedicated medium



Baseband transmission of a digital signal that preserves the shape of the digital signal is possible only if we have a low-pass channel with an infinite or very wide bandwidth.

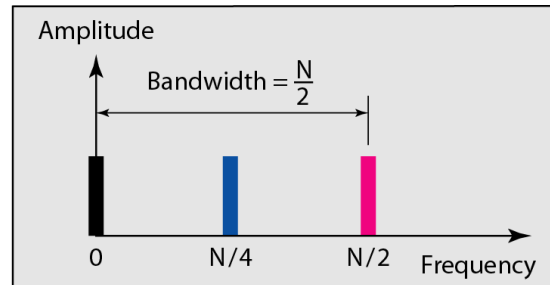
Example

- An example of a dedicated channel where the entire bandwidth of the medium is used as one single channel is a LAN.
 - Almost every wired LAN today uses a dedicated channel for two stations communicating with each other.
 - In a bus topology LAN with multipoint connections, only two stations can communicate with each other at each moment in time (timesharing); the other stations need to refrain from sending data.
 - In a star topology LAN, the entire channel between each station and the hub is used for communication between these two entities.

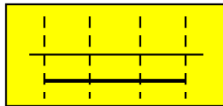
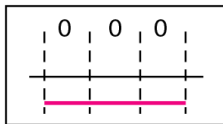
Low pass channel with limited bandwidth

- In a low pass channel with limited bandwidth, we approximate the digital signal with an analog signal.
- The level of approximation depends on the bandwidth available.

Rough approximation of a digital signal using the first harmonic for worst case

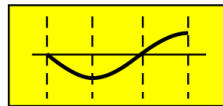
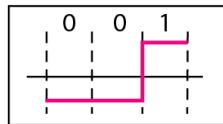


Digital: bit rate N



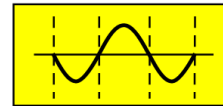
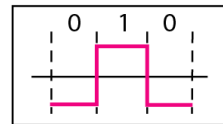
Analog: $f = 0$, $p = 180$

Digital: bit rate N



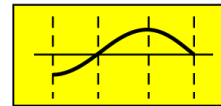
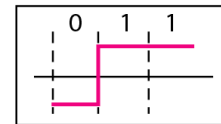
Analog: $f = N/4$, $p = 180$

Digital: bit rate N



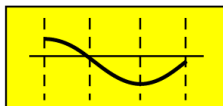
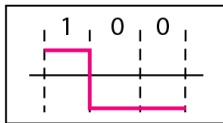
Analog: $f = N/2$, $p = 180$

Digital: bit rate N



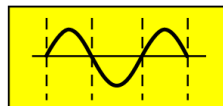
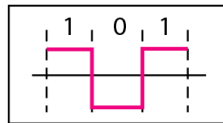
Analog: $f = N/4$, $p = 270$

Digital: bit rate N



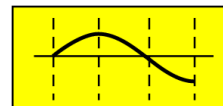
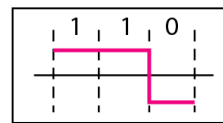
Analog: $f = N/4$, $p = 90$

Digital: bit rate N



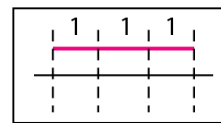
Analog: $f = N/2$, $p = 0$

Digital: bit rate N



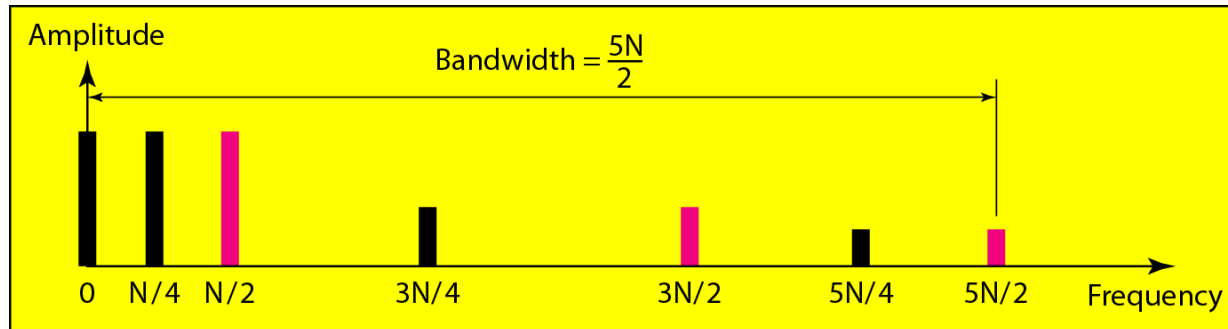
Analog: $f = N/4$, $p = 0$

Digital: bit rate N

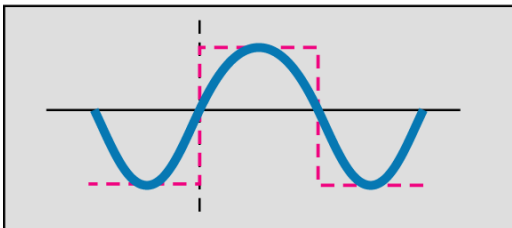
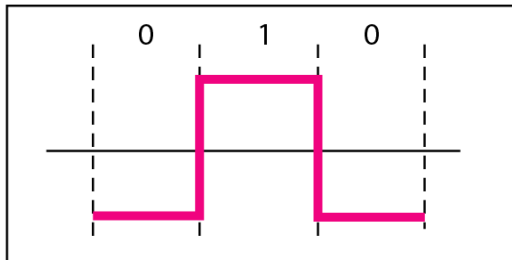


Analog: $f = 0$, $p = 0$

Simulating a digital signal with first three harmonics

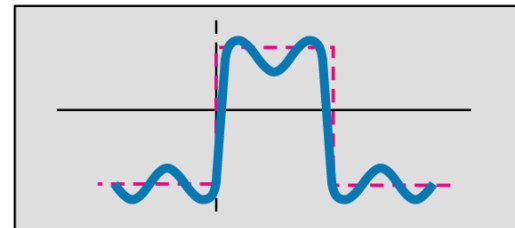
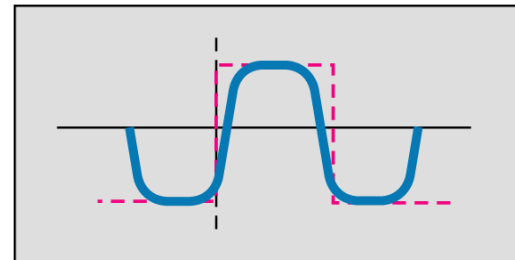


Digital: bit rate N



Analog: $f = N/2$

Analog: $f = N/2$ and $3N/2$



Analog: $f = N/2, 3N/2$, and $5N/2$

Bandwidth requirements

- In baseband transmission, the required bandwidth is proportional to the bit rate;
 - if we need to send bits faster, we need more bandwidth.

<i>Bit Rate</i>	<i>Harmonic 1</i>	<i>Harmonics 1, 3</i>	<i>Harmonics 1, 3, 5</i>
$n = 1 \text{ kbps}$	$B = 500 \text{ Hz}$	$B = 1.5 \text{ kHz}$	$B = 2.5 \text{ kHz}$
$n = 10 \text{ kbps}$	$B = 5 \text{ kHz}$	$B = 15 \text{ kHz}$	$B = 25 \text{ kHz}$
$n = 100 \text{ kbps}$	$B = 50 \text{ kHz}$	$B = 150 \text{ kHz}$	$B = 250 \text{ kHz}$

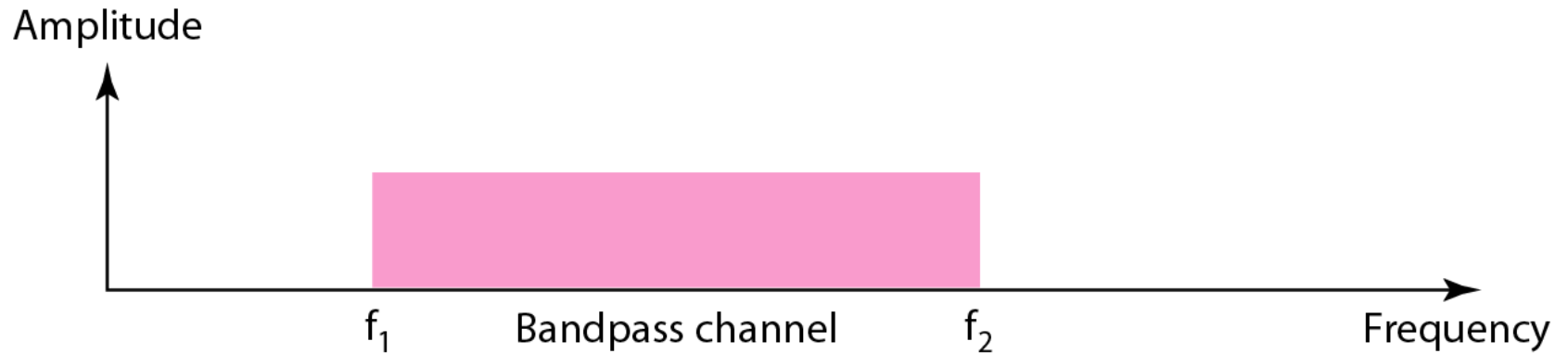
- What is the required bandwidth of a low-pass channel if we need to send 1 Mbps by using baseband transmission?
 - The answer depends on the accuracy desired.
 - 1. The minimum bandwidth, is $B = \text{bit rate} / 2$, or 500 kHz.
 - 2. A better solution is to use the first and the third harmonics with $B = 3 \times 500 \text{ kHz} = 1.5 \text{ MHz}$.
 - 3. Still a better solution is to use the first, third, and fifth harmonics with $B = 5 \times 500 \text{ kHz} = 2.5 \text{ MHz}$.

- We have a low-pass channel with bandwidth 100 kHz. What is the maximum bit rate of this channel?
 - The maximum bit rate can be achieved if we use the first harmonic. The bit rate is 2 times the available bandwidth, or 200 kbps.

Broadband Transmission

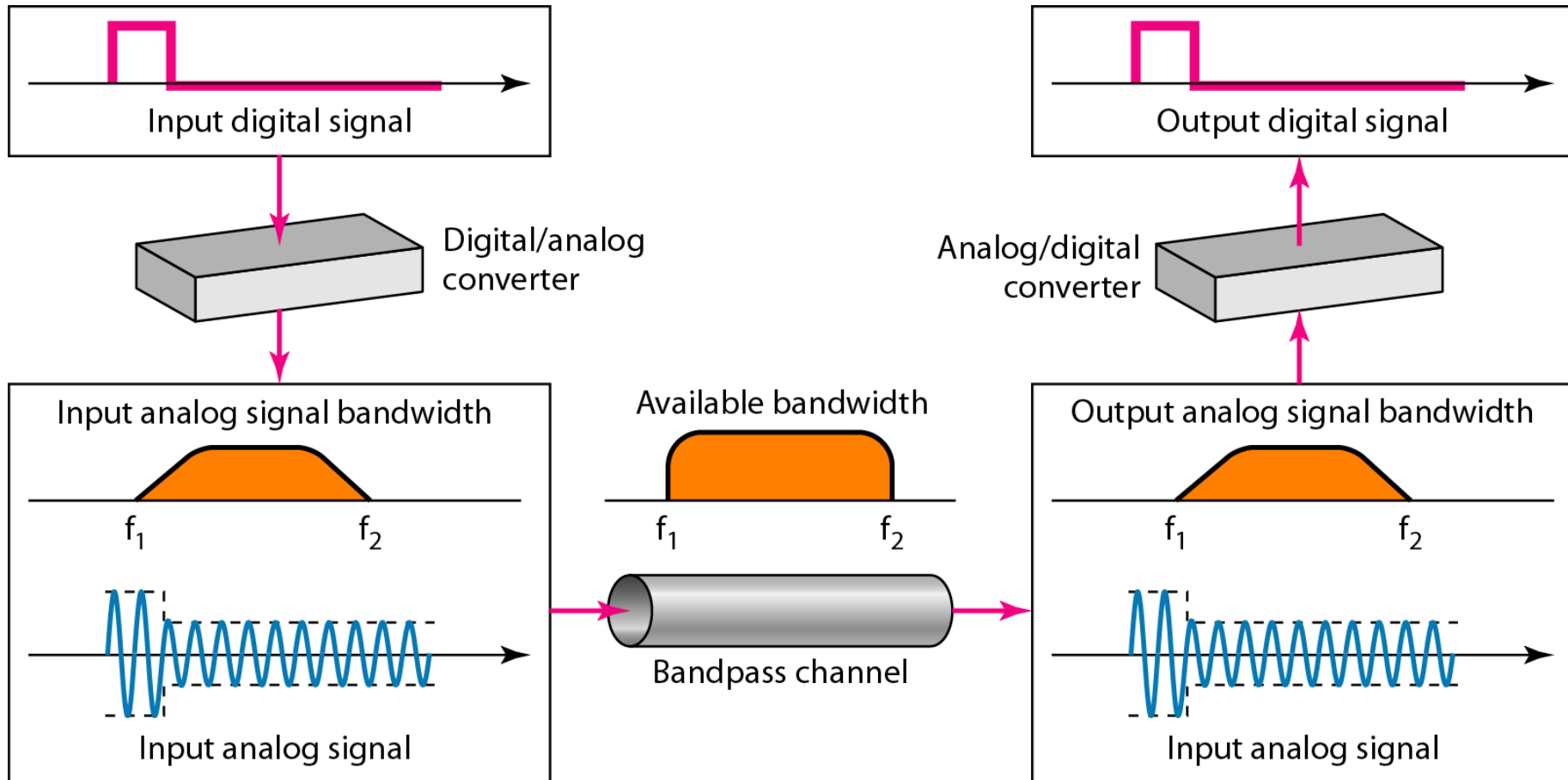
- Broadband transmission or modulation means changing the digital signal to an analog signal for transmission.
- Modulation allows us to use a bandpass channel – a channel with a bandwidth that does not start with 0.

Bandwidth of a bandpass channel



- If the available channel is a bandpass channel, we cannot send the digital signal directly to the channel;
 - we need to convert the digital signal to an analog signal before transmission.

Modulation of a digital signal for transmission on a bandpass channel



Modem

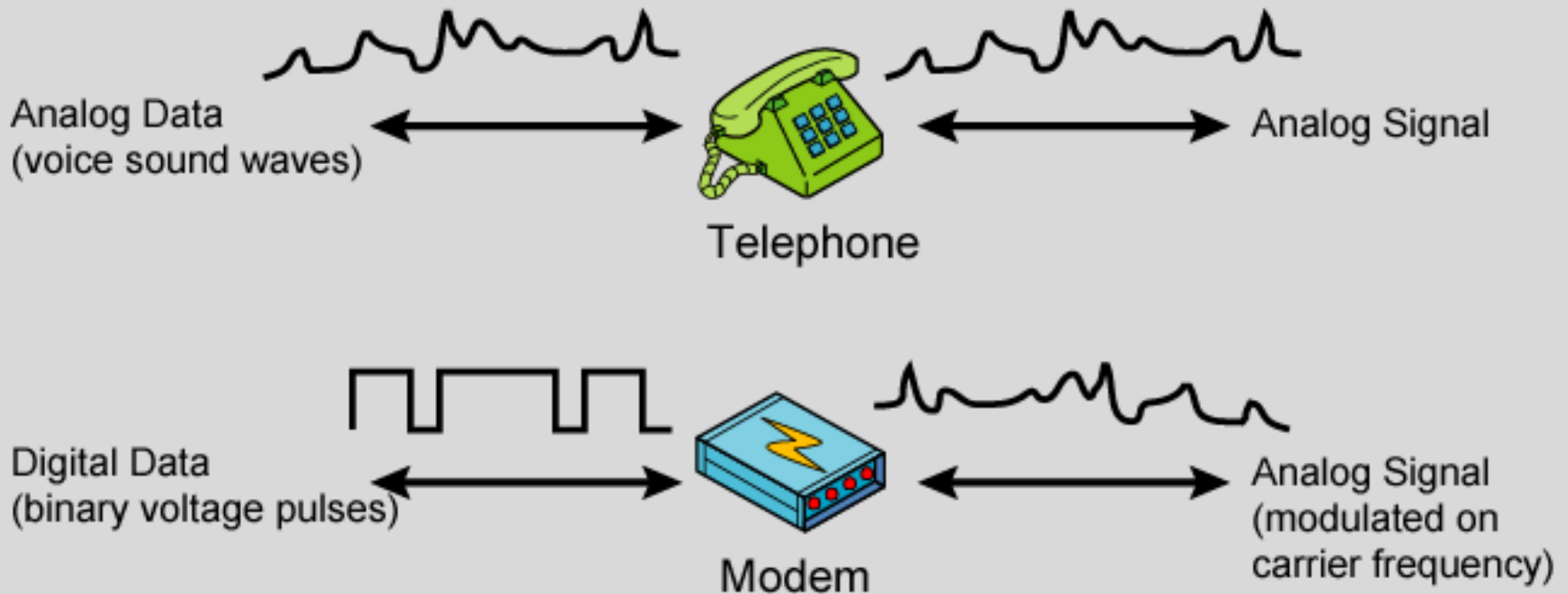
- An example of broadband transmission using modulation is the sending of computer data through a telephone subscriber line, the line connecting a resident to the central telephone office.
 - These lines are designed to carry voice with a limited bandwidth.
 - The channel is considered a bandpass channel.
 - We convert the digital signal from the computer to an analog signal, and send the analog signal.
 - We can install two converters to change the digital signal to analog and vice versa at the receiving end.
 - The converter, in this case, is called a **modem**

Digital cellular telephone

- A second example is the digital cellular telephone.
 - For better reception, digital cellular phones convert the analog voice signal to a digital signal.
 - Although the bandwidth allocated to a company providing digital cellular phone service is very wide, we still cannot send the digital signal without conversion.
 - The reason is that we only have a bandpass channel available between caller and callee.
 - We need to convert the digitized voice to a composite analog signal before sending.

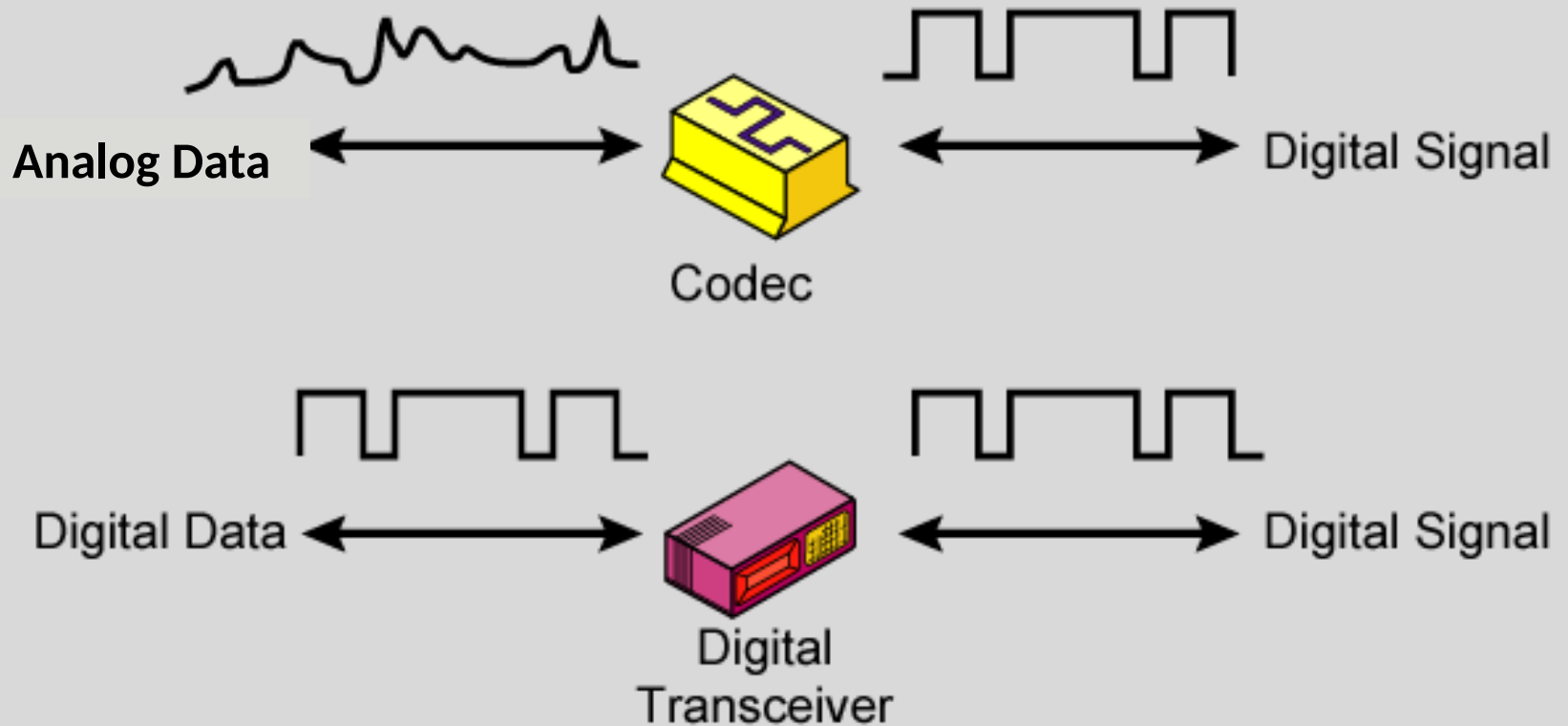
Analog Signals Carrying Analog and Digital Data

Analog Signals: Represent data with continuously varying electromagnetic wave



Digital Signals Carrying Analog and Digital Data

Digital Signals: Represent data with sequence of voltage pulses



Summary of data transmission

(a) Data and Signals

	Analog Signal	Digital Signal
Analog Data	Two alternatives: (1) signal occupies the same spectrum as the analog data; (2) analog data are encoded to occupy a different portion of spectrum.	Analog data are encoded using a codec to produce a digital bit stream.
Digital Data	Digital data are encoded using a modem to produce analog signal.	Two alternatives: (1) signal consists of a two voltage levels to represent the two binary values; (2) digital data are encoded to produce a digital signal with desired properties.

Summary of data transmission

(b) Treatment of Signals

	Analog Transmission	Digital Transmission
Analog Signal	Is propagated through amplifiers; same treatment whether signal is used to represent analog data or digital data.	Assumes that the analog signal represents digital data. Signal is propagated through repeaters; at each repeater, digital data are recovered from inbound signal and used to generate a new analog outbound signal.
Digital Signal	Not used	Digital signal represents a stream of 1s and 0s, which may represent digital data or may be an encoding of analog data. Signal is propagated through repeaters; at each repeater, stream of 1s and 0s is recovered from inbound signal and used to generate a new digital outbound signal.