**Bachelors Of Computer Science and Engineering 2021**

**(2nd Year, 2nd Semester)**

**Mathematics IV**

Full Marks: 70

**USE SEPARATE FILES FOR PART 1 AND PART 2**

Symbols/notations used have their usual meaning

**Part1**

**FULL MARKS: 35**

**Answer question 1 any FIVE from the rest:**

1. Prove that every infinite set $A$ has a proper subset $B$ such that $|A| = |B|$. 5

2. Let $A, B, C$ be three subsets of a set $X$. If $A \Delta C = B \Delta C$, then prove that $A = B$. 6

3. Define *reflexive* and *symmetric relations* on a nonempty set. Let $S$ be a set with 5 elements. Find the number of reflexive and symmetric relations that can be defined on $S$. 6

4. Define a *well-ordered set*. Show that the set of all natural numbers is well-ordered. 6

5. Define a countable set. Prove that a countable union of countable sets is countable. 6

6. What is a *cardinal number* $|A|$ of a set $A$? Prove that the set of all real functions defined on the closed unit interval $[0, 1]$ has the cardinal number $2^c$, where $c$ is the cardinal number of the set $\mathbb{R}$ of all real numbers. 6

7. Find the truth table of $(p \to q) \wedge (q \to r)$. 6

8. Define a *surjective* function. Let $f : A \longrightarrow B$ and $g, h : B \longrightarrow C$ be functions, where $A, B, C$ be three nonempty sets. If $g \circ f = h \circ f$ and $f$ is surjective, then prove that $g = h$. 6

9. Prove that $5^n + 3$ is divisible by 4 for all natural numbers $n$. 6

**Part 2**

**FULL MARKS: 35**

**Answer question 1 any THREE from the rest:**

1. (a) Write down Chebyshev's inequality. 2

   (b) State Bayes' theorem on conditional probability. 2

   (c) Write down the density function of a uniform random variable. 1

2. (a) A box contains 20 tickets of identical appearnces, tickets are numbered as $1, 2, \cdots, 20$. If 3 tickets are drawn at random, find the probability that the numbers on drawn tickets are in A.P? 3

   (b) From an urn containing $N_1$ white and $N_2$ black balls, balls are drawn successively without replacement. What is the probability that $i$ black balls will preceede the 1st white ball? 4

   (c) Two fair dices are rolled 100 times. Find the probability of getting at least once a double six. 3

3. (a) The joint density of two random variables $X$ and $Y$ are given by,

$$f(x, y) = \begin{cases} e^{(x+y)} & 0 < x < \infty, \ 0 < y < \infty \\ 0 & \text{otherwise} \end{cases}$$

   Find the density function of the random variable $\frac{X}{Y}$. 5

(b) Suppose that it is known that the number of items produced in a factory during a week is a random variable with mean 50.

(i) What can be said about the probability that this week's production exceeds 75?

(ii) If the variance of a week's production is known to be 25, then what can be said about the probability that this week's production will be between 40 and 60? $2 + 3$

4. (a) State and prove Champman-Kolmogorov equation. 3

(b) Suppose that whether it rains tomorrow depends on previous weather conditions only through whether it is raining today. Suppose that if it rains today, then it will rain tomorrow with probability $\alpha$ and if it is not raining today then it will rain tomorrow with probabiity $\beta$. If we say the the system is in state 0 when it rains and in the state 1 if it does not rain, the calculate the limiting probabilities $\pi_0$ and $\pi_1$ for given value of $\alpha = 0.6$ and $\beta = 0.3$. 5

(c) Define absorbing and transient states for a finite Markov chain. 2

5. (a) Let $W_\alpha$ denote the amount of time an arbitrary customer spends in the $M/M/1$ queuing system. Find the distribution of $W_\alpha$. 5

(b) Customers arrive at a watch repair shop according to a Poisson process at a rate of one per every 10 minutes and the service time is an exponential random variable with mean 8 minutes.

(i) Find the average number of customers $L$, the average time a customer spends in the shop $W$ and average time a customer spends in the waiting for services $W_q$. 3

(ii) Suppose that the arrival rate of the customers increases by 10 percent. Find the corresponding changes in the values of $L$, $W$ and $W_q$. 2