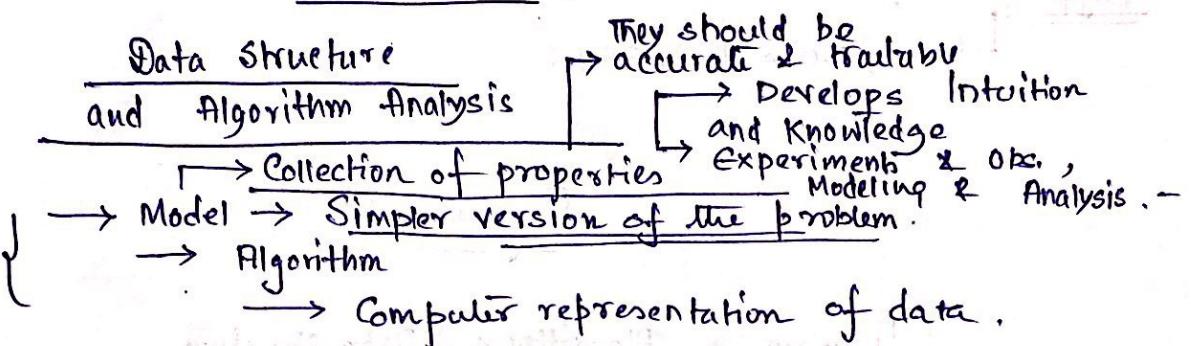


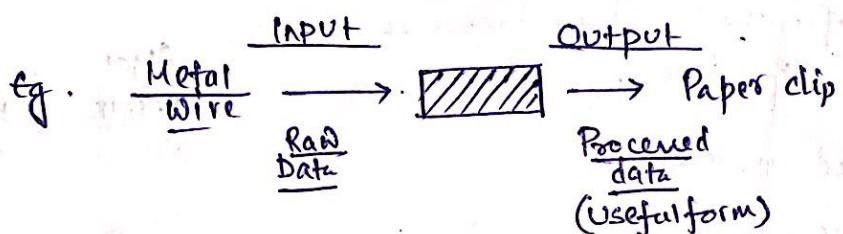
Data Structures and Algorithms

Data Structure and algorithm analysis

3 steps
to solve a
problem



Data Processor → Computer



Algorithm

→ Finite sequence of steps to solve a problem

↓
must be
feasible &
understandable

Information & Data → Abstraction of Information

Google → Circular Defn

→ Height of

Objective
Interpretation independent
of

Subjective

Human Bias
plays a role
Kamal is 6.5 ft.
↑ Data →

Data & Operation

Datatype → ① Range
 ② Type of operation.

integer → set of all
 whole numbers
as a mathematical entity → add, subtract, multiply

| int → subset of
 integer

+, -, *,

add +
overflow
can happen.

$$\text{Diff} \rightarrow 5/3 = 1.$$

$$\rightarrow 5.0/3.0 = 1.66667.$$

Program Structure

Program = Algorithm + Data Structure

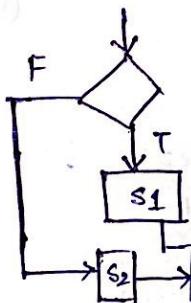
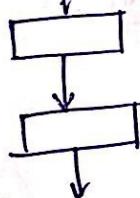
Programming Language provide facilities to represent algorithms & data.
+
structured constructs

- 1. Sequence
- 2. Conditional.
- 3. Subprogram
- 4. Iteration

Function

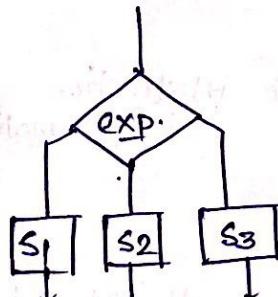
- 5. Recursion
- 6. Control transfer

↓
considered harmful.
goto, break,
continue, return,



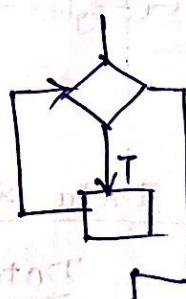
conditional

if (condition)
 S1
else
 S2



while (cond).

do
 {
 ...
 }
while (cond);



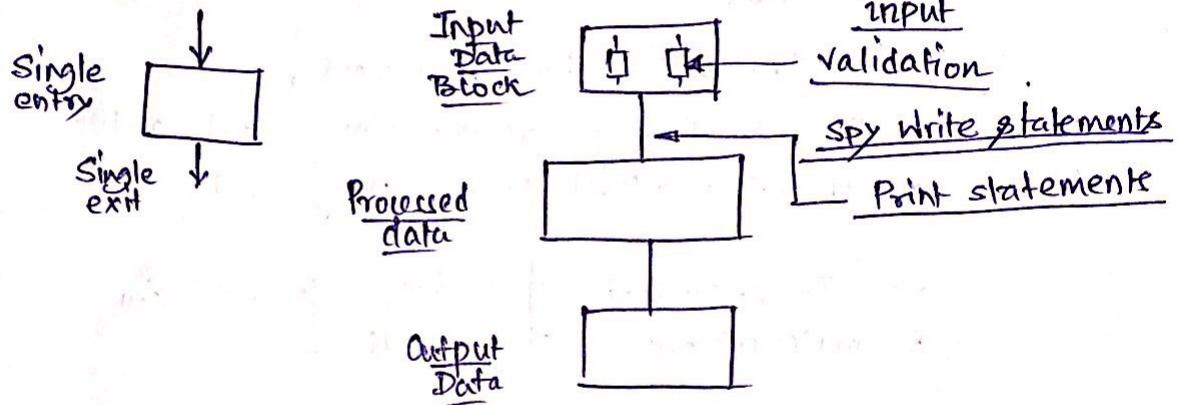
FORTRAN,
→ Formula Translation.
Algol
PASCAL
C
BASIC,
COBOL

04/08/2019

Data Structures & Algorithms

Page No.
Chandan
Mazumder

①



Data Types

Scalar

Integer Real character ...
Pointer subrange

LISP } AI
Prolog

Data Aggregation Facilities

Arrays Records Sets
↑ ↑ ↑
finite sequence of values of the same type Record of structs

Structured data types

- ① Component
- ② Defined by the set of rules that put them together.
- ③ set of operations

int x;
↑
Memory allocated during compile time (static allocation)

mydata x, y, z;

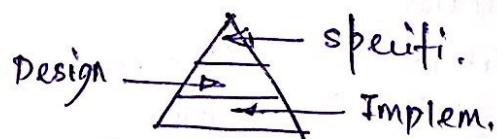
- Create
- Enter the element
- Read the element
- take out the element
- Destroy

Abstract Data Type (ADT)

(2)

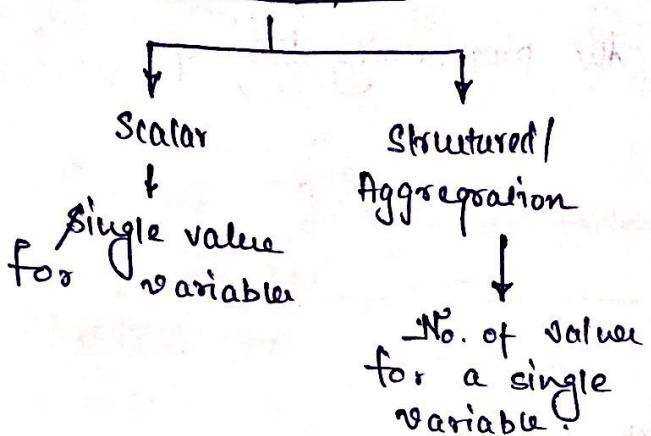
- Conceptual model of information structure
- Specifies the components, their structuring relationships and a list of operations allowed to be performed.
- Independent of data representation & implementation of operation.
- Just a specification & no design / implementation info.
↑ what?
now?

Components themselves
are other ADTs.

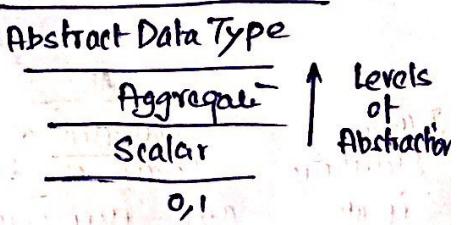


- No assumption is made of the range.
- Generalization of primitive data types.

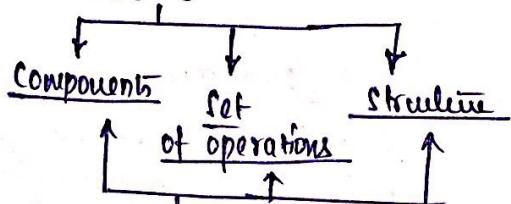
Data Type



Object Orientation



Aggregate → Arrays, Records, Sets



- ADT defines , independent of data repr. & implementation of operation , just a specification and no design/implementation is included .

- No assumption is made about the range .
- Specification involves the 'what' & not 'how' .
- Generalization of primitive data types .
- Encapsulate data values .

- Data - Structure is the design representation of an ADT .
 - e.g. 1. $\langle \text{int}, \text{int} \rangle$
 - 2. $\langle \text{int} \rangle, \langle \text{int} \rangle$

can be rep.
by several
data structures
- Many data structures correspond to ADT .
- operations on data structures are ~~refer~~ referred to as algorithms .

→ Relation among data structure

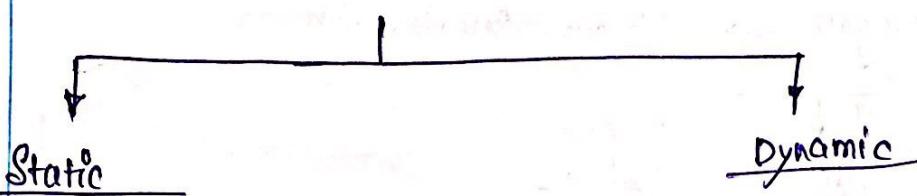
Data Structure

→ Facilitated using the programming language

Data Structure
is an
ADT

→ restricted at the
level of design
and implementation

Type of Data Structure



Static

→ Memory requirement
is fixed at the compile
time

'OFF BY 1' ERROR

Dynamic

→ They can shrink &
grow at the run-time.
thus, special
memory allocation &
deallocation functions
from the programming
system.

→ Algorithms & Memory
Usage
must be as efficient as
possible

(Time & memory management
is key)

Algorithm Design Methods

→ Divide & Conquer

→ Greedy Algorithm

They may not always provide the best possible result. However they produce good soln with high probability. They can produce a good soln within a short time.

e.g. Coin change,
Dijkstra's shortest path algo.

Coin change (exception)

1p 5p 11p 25p 50p

15p → 11p + 4 × 1p (Greedy).
5p × 3. ✓

Slowly filling up the table

Dynamic Programming

Mathematicians named Table tilling & look up as Instead of sub-dividing, we find the smaller problems whose soln lead to the original problem.

A table is created in which the entries are filled in a particular order.

e.g. Fibonacci Series, Factorial

Creating a factorial

0 1 2 3 7

eg. 2! 1 | 1 | 2 | 6 | 24 | 1

$$\text{fact}(n) = n \times \text{fact}(n-1).$$

Backtracking

Ex:- n-queens Algorithm Problem
(Please check)

Sir's Slider or Google.

Check:- Closed form soln for the n Queen problem.

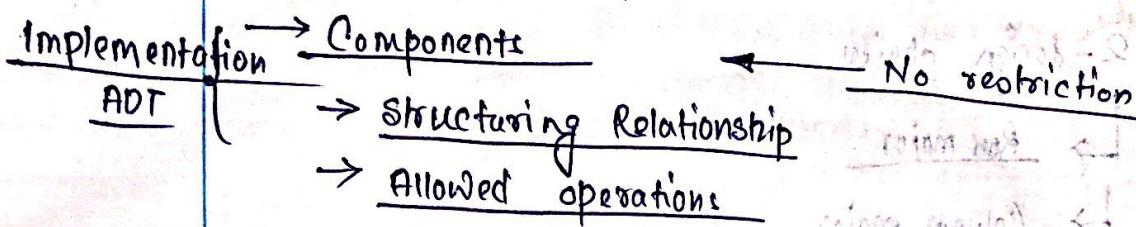
Branch & Bound

In a chess n-queens, the computer uses static Evaluation Function

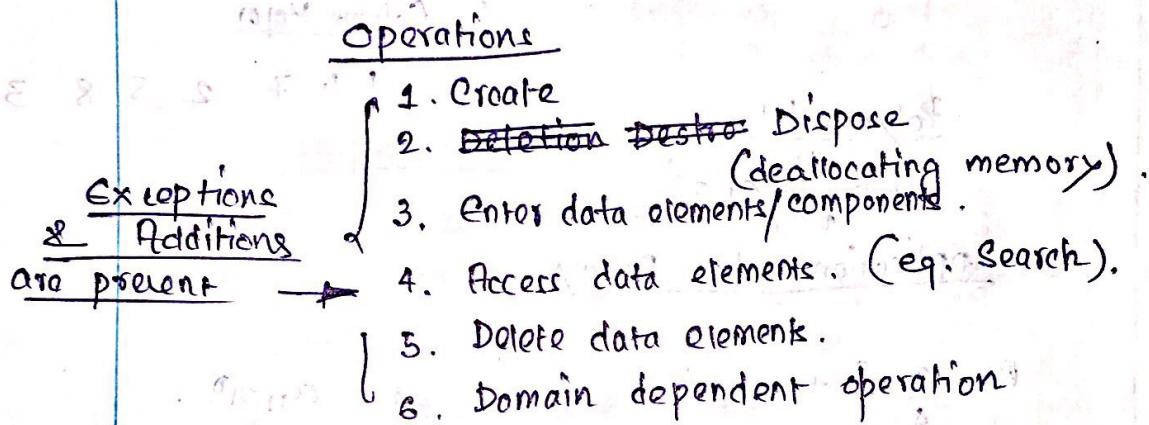
Qualities

- Memory consumed
- ~~Execution time~~ Execution time

factors affecting execution time (check Sir's slider).



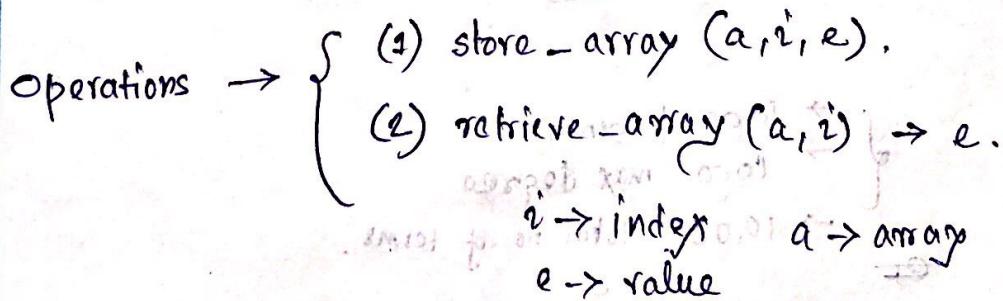
- * Also understand the limitations and explicitly mention them
- Static ~~datatype~~ datastructure (Memory allocated at compile time)
- ~~Compt~~ Dynamic ~~datatype~~ datastructure (Memory allocated dynamically).



Array as Data Structure

Elements of the same type arranged in a sequence.

ORDINAL → finite sequence



Design

→ Required no. of memory locations are statically allocated consecutively.

Implementation

MAX-INT
 → Built into language. (index size).

Higher Dimensional Arrays

2-design choice

Row major

→ Column major

Compiler generates code for calculating the physical address of the element.

$$\begin{matrix} & 1 & 2 & 3 \\ \text{row 1} & 4 & 5 & 6 \\ \text{row 2} & 7 & 8 & 9 \end{matrix}$$

The diagram shows two ways to store a 4x4 matrix in memory:

- Row Major:** The matrix is stored row by row. The first row contains elements 1 through 4. The second row contains elements 5 through 8. The third row contains elements 9 through 12. The fourth row contains elements 13 through 16.
- Column Major:** The matrix is stored column by column. The first column contains elements 1, 5, 9, and 13. The second column contains elements 2, 6, 10, and 14. The third column contains elements 3, 7, 11, and 15. The fourth column contains elements 4, 8, 12, and 16.

147 258 369

Polynomial

$$\sum_{i=0}^n a_i \cdot x^i$$

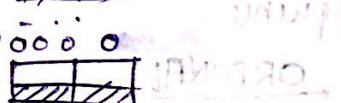
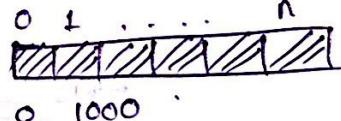
Application of Array

$$P = a_0 x^0 + a_1 x^1 + \dots + a_n x^n.$$

floating point
coefficients

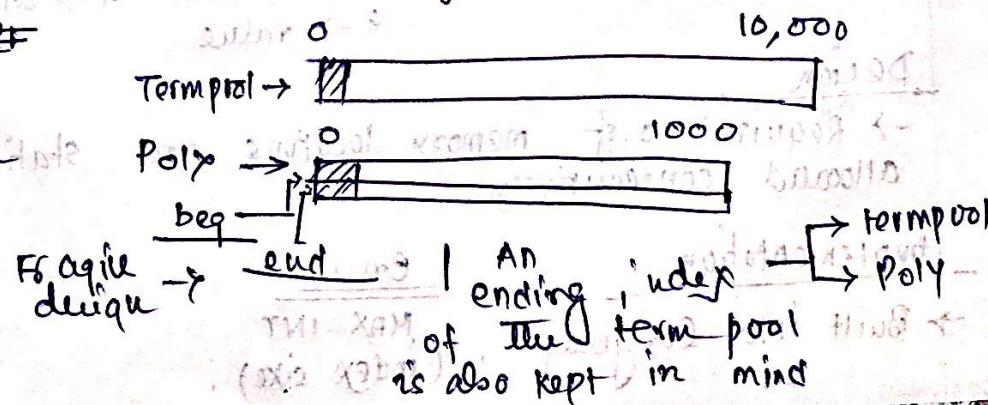
(2) →

Degree → ()
of the polynomial



- 1000 polynomials
- 1000 max degree
- 10,000 total no. of terms

Attaching
a new term
is difficult



Factor of Safety

(9)

→ (1) 130%. (30% increase)

→ (2) When memory utilisation
is close to 70%, add more
resources.

Sparse Matrix

Randomly distributed, non-zero elements
among a pool of zero elements

$$\begin{bmatrix} 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 9 & 0 & 11 & 0 \\ 0 & 0 & 15 & 0 \end{bmatrix}$$

Approach

→ Store the non-zero elements of a sparse matrix in the form of 3 tuples (i, j, val) in an array.

→ The first tuple contains (m, n, t) for a $m \times n$ matrix having t non-zero values.

→ Storage &
Time &

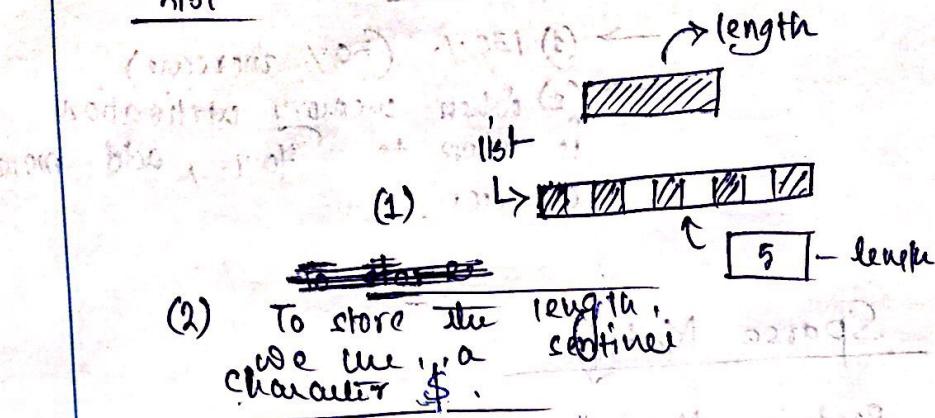
<u>m</u>	<u>n</u>	<u>t</u>
4	4	4
<u>i</u>	<u>j</u>	<u>val</u>
0	1	2
2	0	9
2	2	11
3	2	15

using list (1)

using set (2)

⑩

hist



Pointers (Recap)

Refer notes sks sir's

$i = 20;$

associated
with the
type of the
value, it
is pointing to
 \downarrow

pointer ptr \rightarrow
variable that
holds the address
of another variable

address
of i

~~Ex~~

Ex. Solve CM sir's
pointer question
in his slides.

char *s = "abcdefg";

\rightarrow Heap

st[20] = "..."

\rightarrow stored in stack

- (1) void pointer
- (2) ptr to fn.

26/8/19

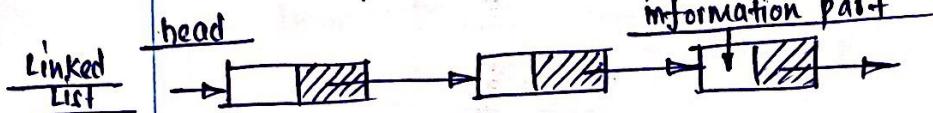
Data Structures and Algorithms

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Mazumder

①

Linked List

- Finite no. of nodes having the same type
- Special variable that joins them together



ADT
Single
Linked List

Operations (Check CM sir's slides),

init-l, empty-l, atend-l, insert-front, insert-after
delete-front, delete-after

→ Read integers from a file & arrange them in a linked list

Note → Check the existence of a file & whether the file is empty or not.

Equality

- 2 empty lists are equal
- Each corresponding node will have same information

→ Check whether list is empty or not.

Hint :- Always check (boundary cases or corner) || very Important
~~Put pen & paper~~

→ C pointer implementation of single Linked List. (CM sir's slide).
→ Self-referential Structure

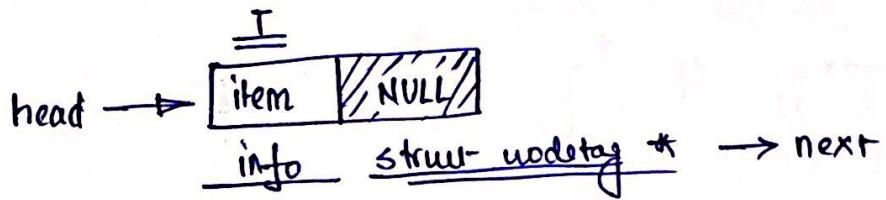
→ Check for memory allocation
& exit/return if it fails

27/8/2019

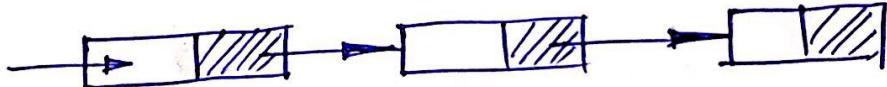
Data Structures
and Algorithms

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Creating Node



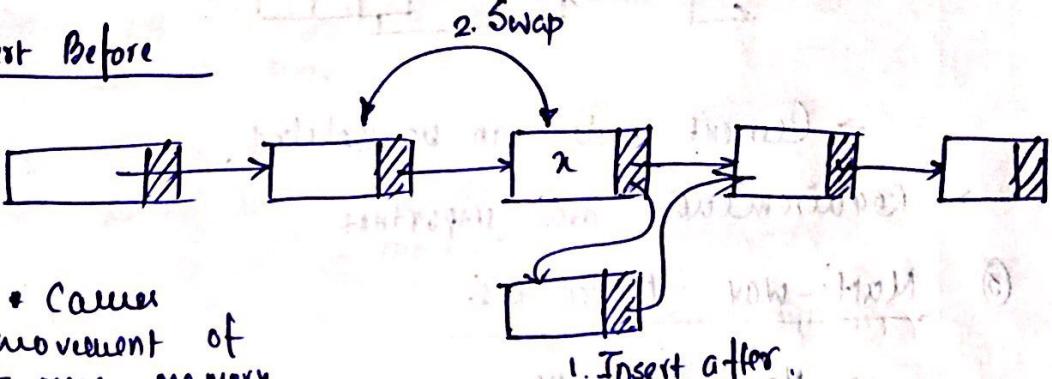
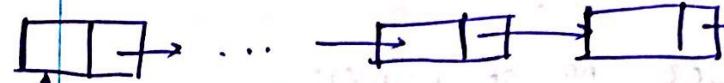
* + Please check the C implementation
from CM sir's slides * *



Note :- nodetyp* delete-front (nodetyp **phead)

```
{  
    cur->next = NULL;  
    return (cur);  
}
```

else, we might
have a security violation.

Single Linked ListInsert Before① Head NodeVariations of Single Linked List

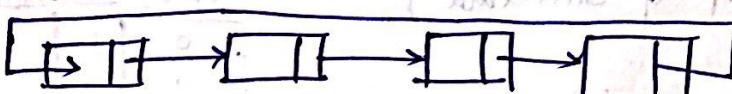
• List with a special head node containing the length as info.

② Dummy first node

→ uniform insertion and deletion operation
insert-front & delete-front are not necessary.

Dummy →

→ Depends on the environment.

③ Circular Single linked list

→ Any node to any node traversal is possible

→ Tail pointer can be used.
However, when adding elements to the end of tail, remember to traverse tail, else it would be a insert-front operation.

② → Doubly Linked List

Frequent to R from movement.



→ Current node can be deleted.

→ Requirements are important

③ Multi-way Linked List

→ More than one sequence of dirn. the list is made.

→ Double linked list can also be implemented inside the DLL

→ Due to higher no. of pointers, security can be a problem.

(4) Sparse Matrix using ~~tiny~~ linked list

(Refer to Six's slides)

→ Single Linked List using array.

→ 'Available nodes' also maintains all the bt | unused nodes.

(Array of Structure)

Creating a node

→ on node creation,
the cursor has to be
modified.



03/09/2019

Data Structures and algorithms

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Algorithm Design Methods

- Algorithms are developed on the basis of the choice of data structure.
- The first intuitive algorithm may not be the best one as per memory & time.
- There are general techniques for development of algorithms.

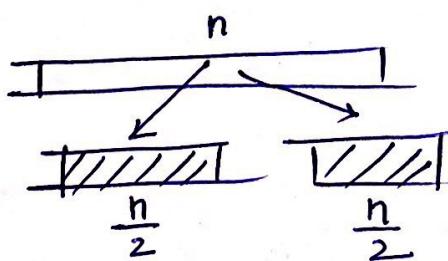
Divide & Conquer

- Break the problem of size n into smaller problems & combine them.

D&C
Problem
Solving



1. Data Input
2. Data / Input Validation
3. Data Processing → Result
4. Result Validation
5. Form the Report.



Greedy Algorithm

- Performance Analysis (Mathematical analysis of memory usage & time of execution)
- Performance Measurement (Mathematical Measurement of actual memory usage & time of execution)

Algorithm complexity

- Space complexity (Main memory needed)
- Time complexity (Amount of Computer Time needed)
- 1. Fixed Space Requirement
 2. Variable Space Requirement

Time Complexity

- Compile time does not depend on the program instance.
- Execution time depends on the kind of algorithm and the size of the input data.

Time Complexity Analysis Objective

→ Machine Model

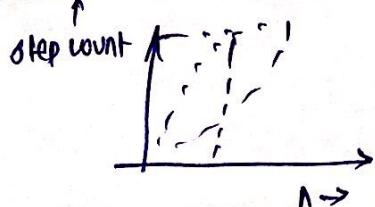
Normal single processor sequential computer where all instructions take equal amount of time, & have infinite memory

Time Complexity

eg. $=$ OR Count the syntactically / semantic statement

ticks of 18 bits \neq
finite accuracy

Time Complexity (Step Count)



$$\text{tq. } 2n + 100, \\ 30n^2 + 15n - 20.$$

(2)

Estimation

- Worst case estimate is taken as it provides an upper bound for all inputs particularly the bad ones.
- Average ~~analysis~~, case analysis.

Rules for computing running time

- Sequence
- Alternative structures
- Loops
- Nested Loops

Subprogram

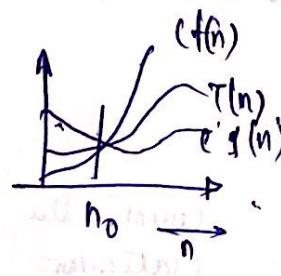
Recursive Subprograms

Order of $T(n)$

→ $T(n) = O(f(n))$ if there are positive constants $c \neq 0$ & n_0 s.t. $T(n) \leq c f(n)$ when $n \geq n_0$.

→ $f(n)$ is at least as large as $T(n)$.

→ Growth ratio of $T(n) \leq$ that of $f(n)$. $\frac{n}{n_0}$ is valid



$$T(n) = \Omega(g(n))$$

$T(n) \geq c g(n)$ for $n \geq n_0$.

$$T(n) = \Theta(h(n))$$

Properties

$$T_1(n) = O(f(n))$$

$$T_2 = O(g(n))$$

$$T_1(n) + T_2(n) = O(\max(f(n), g(n)))$$

$$T_1(n) \cdot T_2(n) = O(f(n) \cdot g(n))$$

$$T_1(n) \leq c_1 f(n)$$

$$T_2(n) \leq c_2 g(n)$$

$$\frac{c_1 f(n) + c_2 g(n)}{n > n_0}$$

$$\Rightarrow T_1 + T_2 \leq (c_1 + c_2) \max(f(n), g(n)) \quad n > n_0 = \max(n_0, n_0)$$

$$\Rightarrow T_1 + T_2 \leq C \max(f(n), g(n))$$

$$\Rightarrow T_1 + T_2 = O(\max(f(n), g(n)))$$

\downarrow
Max. time complexity that counts

Growth Functions

Computation

$$\lim_{n \rightarrow \infty} \left(\frac{f(n)}{g(n)} \right)$$

20/9/19

Algorithm Analysis

Proof
Chandan
Mazumder

(1)

$\text{read } n; \text{ sum} = 0; i = 0;$
 $\text{while } (x \neq 0) \text{ do}$
 $i = i + 1;$ (2)
 $\text{sum} = \text{sum} + x;$ (2)
 $\text{read } n;$ (1)
 $\text{avg} = \text{sum} / i;$ (2)
 write avg; (1)

$\frac{5n+6}{O(5n+6)} = O(n)$
 ~~$O(5n+6) = O(n)$~~
 OR $O(1) + O(n) = O(n).$

Correctness of an Algorithm① Partial Correct-

If the algorithm terminates it gives correct result.

② Termination

for all inputs, the algorithm terminates

Read $A[i]$ → (2)

$O(n)$ { $\text{for } (i=1 \text{ to } n) \text{ do}$
 $\quad \text{read } A[i] \rightarrow (2+1) \cdot](n) \text{ cond.}$
 $\quad \text{for } (i=1 \text{ to } n-1) \text{ do}$
 $\quad \quad \text{for } (i=n \text{ to } i+1) \text{ do, } - (1)$
 $\quad \quad \quad \text{if } (A[j] > A[j+1]) \quad (3)$
 $\quad \quad \quad \quad \text{then swap}(A[j+1], A[j]) (3)$



$$\sum_{i=1}^{n-1} \sum_{j=i}^{i+1} q$$

$$\begin{aligned}
 &= q \sum_{i=1}^{n-1} (n-i) = q[(n-1) + (n-2) + \dots + (1)] \\
 &= \frac{q}{2}(n^2 - n).
 \end{aligned}$$

Rough Calculation.

$$T(n) = O(n^2).$$

②

Logarithmic Time Algorithm

$$O(\log n)$$

Cuts the

problem by a

fraction (e.g. 1/2).

Binary Search

Iteration

No. of element

1

$$\frac{n}{2}$$

2

$$\frac{n}{2^2}$$

n

$$\frac{n}{2^n}$$

number of iteration

number of element

Time Analysis

$$n = 1024.$$

$$\text{Linear Search} = 1023.$$

$$\text{Binary search} = 10$$

Egypt's GCD algorithm

No. of iteration

$$2 \log n$$

After 2 iterations,

remainder < original value.

30/09/2019

Data Structures
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Logarithmic time Algorithm

(Refer to Sir's slides)

CT: 21st October,
2019

Fibonacci Series

$$\checkmark T(n) = T(n-1) + T(n-2) + 2$$

$$T(n) \geq fib(n). \\ \therefore fib(n) = fib(n-1) + fib(n-2),$$

$$fib(n) \geq \left(\frac{3}{2}\right)^{n-1}$$

→ Prove by induction

$$fib(0) = 1 \geq \left(\frac{3}{2}\right)^{0-1} = \frac{2}{3}.$$

$$fib(1) = 1 \geq \left(\frac{3}{2}\right)^{1-1} = 1.$$

$$fib(2) = 2 \geq \left(\frac{3}{2}\right)^1 = \frac{3}{2}.$$

Taking Induction hypothesis

$$fib(m) \geq \left(\frac{3}{2}\right)^{m-1} \quad [m=0, 1 \dots m]$$

$$\begin{aligned} fib(m+1) &= fib(m) + fib(m-1) \\ &\geq \left(\frac{3}{2}\right)^{m-1} + \left(\frac{3}{2}\right)^{m-2} \\ &= \left(\frac{3}{2}\right) \left[\frac{2}{3} + \frac{4}{9} \right] \\ &= \left(\frac{3}{2}\right)^m \left[\frac{10}{9} \right] \end{aligned}$$

$$\Rightarrow \boxed{fib(m+1) \geq \left(\frac{3}{2}\right)^m}$$

$$\boxed{T(n) = \Omega\left(\left(\frac{3}{2}\right)^{n-1}\right)}$$

Proved by
Induction

Even time
lower-bound is
exponential

~~05/11/2019~~

05/11/2019

Data Structures and Algorithms

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①

ADT Stack

→ finite seq. of elements of same type, where insertion & deletion can happen only at one end.

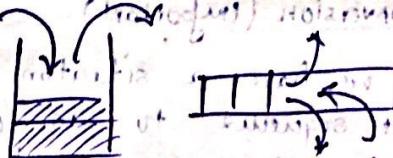
→ s-create(s)

→ s-dispose(s)

→ s-empty(s)

→ s-full(s)

→ push(s,e) → pop(s)



Stack is LIFO (Last In First Out structure),

Array Implementation of Stack

Note Element might be still accessed by array[index] however they cannot be accessed using stack operations.

linked List Implementation of Stack

Note check stack full while pushing

(??) → Physically nodes may not be created, so stack full case may arise.

Logically stack cannot overflow when using linked list.

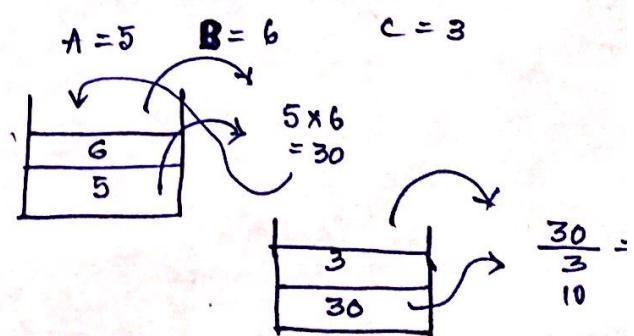
Infix to Postfix

① A + B / C

 A B + C / \$

② A/B * C + D * E - A * C

 ABC * / DE * + AC * - \$



Postfix Evaluation

Important Example

Observations

- Refer to sir's slides.
- has to be done in a sequential manner.

Conversion (Important)

If we have a situation of converting an input sequence to an output sequence, stack notation should be preferred.

- Refer to sir's slides.

All books will do it this way.

(1) god

(2) dad

(3) dad

Point to additional point

[Probability of getting heads] = 1/2

and tails will remain

and always point to one side

Point to additional point

and will slide on both sides

and can point either direction

not point to one side

so both sides are

entirely same just

one has value zero

0

8 : 0

0 : 8

6 : A



X : { 0 : 8 }

Recursion

(1)

- fn defined in terms of itself.
- Power lies in defining a potentially infinite set of objects by a finite set of statements.
- sub-programs can be defined recursively.
- $f(x_0) = \text{expression}$

$$f(x_i) = g_i(x_i, f(x_j)) \quad j < i$$

Types of recursion

- Linear Recursion can also be expressed in terms of. \rightarrow Simple iterative soln

```

fNL(...)

if (base cond is satisfied)
then return its value
else
    perform some actions
}
    
```

Binary Recursion

Algorithm that makes 2 internal calls to itself.

Non-linear Recursion

e.g. Sample Generation,
Combination Generation,
Permutation Generation.

(Algorithm using a number of internal recursive calls within the body of procedure).

```

    ()+
    ()* { + } *
    [ ] *
    { } *
    
```

Mutual Relationship

Rules for Writing correct & eff. algorithms.

- 1) Bare Case
 - 2) Making progress
 - 3) Design Rule
 - 4) Compound Interest Rule

Recursive calls are particularly appropriate when the underlying data is defined in recursive terms.
and NOT ALL RECURSIVE ALGORITHMS ARE EFFICIENT.

Bad example of Recursion

→ charge No. of calls. (if we have 1M calls, then 1M calls will be active).

In linear recursion, we can remove recursion.

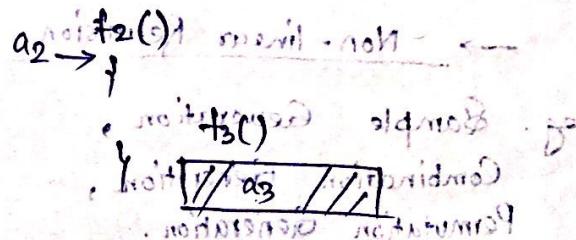
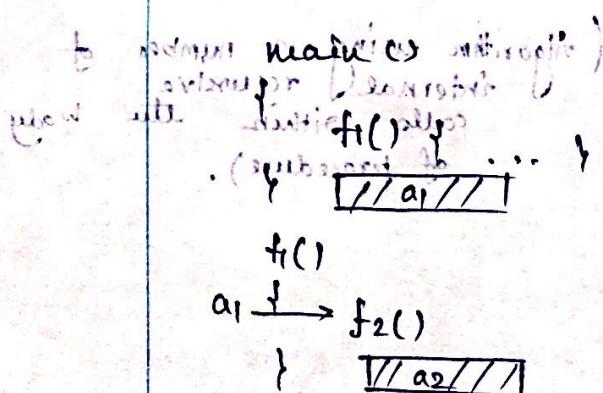
Non-terminating Recursive Programs

$$\rightarrow f(x) = f(x+1) + f(x+2)$$

↑ ↑
 ARRY

Application of States

\rightarrow LIFO Data Structure

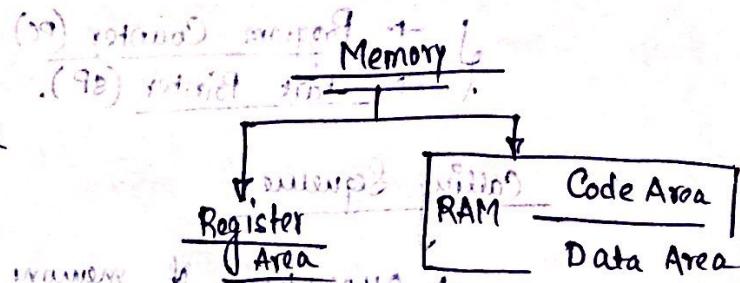


- fn call (An top level overview)
 → fn calls & The Runtime Stack

① Runtime Environment

② C Language

use a stack based runtime environment



Entry point
for procedure 1

Entry point
for procedure n

→ Code for Procedure 1

↓
⋮
→ Code for Procedure n

→ Data Area

→ Dynamic Memory



Memory Organisation

code/area

global/static area

↑
stack

↑
free space

↑
Heap.

Procedure activation Record

→ Concept of stack frames

When activation records are kept on stack, they are called stack frames.

↑
arguments

bookkeeping info

local data

local temporaries

Registers

→ When the processor has many registers, the entire static area & whose activation records may be kept in the registers.

- Program Counter (PC)
- Stack Pointer (SP).

Calling Sequence

- Allocation of memory for the activation record
- Computation & storing the arguments
- ...

Return Sequence

Stack-based Runtime Environment

Global Procedures

A view of the Runtime Stack

Access to Variables

Calling sequence

Prologue

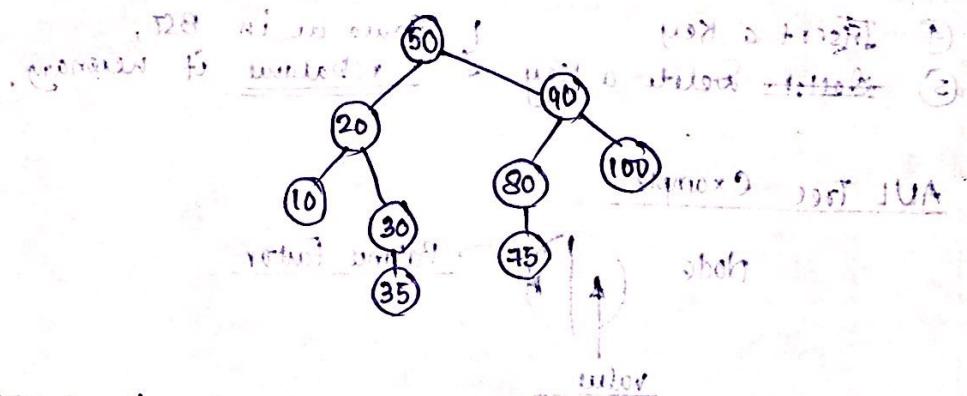
Object code

Epilogue

Compiler works on KERNEL

Binary Search Tree

- Defn. BST :-
 a) Key of every node in the right
 sub-tree of T is greater than key at root
 b) " "
 c) All keys are distinct.

BST operations (Refer to slides)

- Recursive Search (Linear Recursion) → Iterative
- Non-Recursive Search
- Insertion Example
- Deletion Example

Deleting the root node, (eg. 50). [using Inorder successor / Inorder predecessor, and checking whether the node has a left or right child].

Problem of BST.

Height depends on insertion & deletion of keys.

Tree may degenerate into a linked list.

Remedy:- Balanced Tree

Height Balanced Tree (AVL Tree)

100% 60
80% 80% 70%

Adel'son - Velskii, Landi.

$$|h_L - h_R| \leq 1$$

Left sub tree Right sub tree

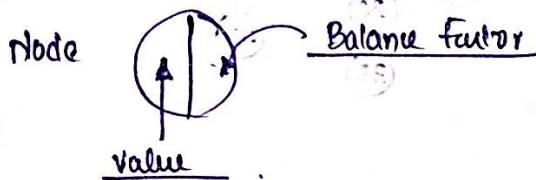
$$BF = h_L - h_R$$

if $BF > 1$ then left child is high for $BF = 1$ \rightarrow left high
if $BF < -1$ then right child is high for $BF = -1$ \rightarrow right high
" " $= 0$ \rightarrow equal. Right.

AVL Tree Operations

- ① Search
- ② Max & Min } Same as BST.
- ③ k^{th} max & k^{th} min }
- ④ Insert a Key } Same as in BST,
⑤ ~~Delete~~ Delete a Key } rebalance if necessary.

AVL Tree Example



Rebalancing needs Rotation (Refer to Sir's slides) →

<u>Right - Rotation</u>	Constant time algorithm
<u>Left - Rotation</u>	

Insertion Examples

Tree remains unbalanced even after rotation. If right child has left high / left child has right high.

Remedy :- Double rotation

Deletion Example

After deletion we arrange right
to go with left
left child has right high
left child has left high

last balanced - please

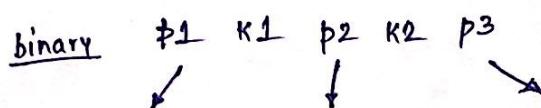
$\left[\begin{matrix} (c_1, c_2) \\ (c_3, c_4) \end{matrix} \right] \times \left[\begin{matrix} (c_1, c_2) \\ (c_3, c_4) \end{matrix} \right] \in \mathbb{R}^2 \times \mathbb{R}^2$

2-3 Tree

- All leaf nodes are at the same level
- All non-leaf nodes are either binary / ternary.

$$\begin{array}{l} \text{Keys } \{K_1 \rightarrow P_1\} \\ \text{Keys } \{K_2 \rightarrow P_2\} \\ \text{Keys } \{K_3 \rightarrow P_3\} \\ K_1 < K_2 < K_3 \end{array}$$

time complexity CT
Sorting
Stair & Queue
Recursion

Tree NodeOperations

$$\begin{aligned} 2^{n-1} &\leq n \leq 3^n - 1 \\ \Rightarrow \log_3(n+2) &\leq h \leq \log_2(n+2) \end{aligned}$$

Insert a node (Inversion Examples)Deletion ExamplesB-TreeDefinition

- Each node contains almost d records & at least $[d/2]$ records.
- Root node can have almost d records & as few as one record.
- $1 \leq k \leq d$ with key values $k_1 \leq k_2$ have pointers to $k+1$ subintervals of keys stored in sub-trees.
- Leaf node has empty sub-tree below it. All leaf nodes are at the same level.

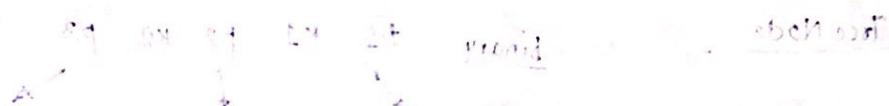
B-Tree Node

$$\lceil \log_{\frac{d}{2}} n+1 \rceil \geq h \geq \lceil \log_{d+2} (n+1) \rceil$$

2-3 Tree is a B-tree of order 2.
X is root of order 2. i.e. max 2 children.

Insertion Examples on a B-Tree of order 4

Deletion Example



21 34 45 56 parent
1 2 3 4 child
(child) of 21 & 2 (child) of 34
(child) of 45 & 1 (child) of 56

(empty) child

empty child

will be the answer to question 1. (empty child)

Ans 1. If root is empty then answer is empty [child]

Ans 2. If root is not empty then answer is not empty [child]

Ans 3. If root is empty then answer is empty [child]

Ans 4. If root is not empty then answer is not empty [child]

Ans 5. If root is empty then answer is empty [child]

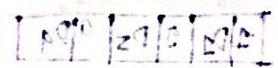
Ans 6. If root is not empty then answer is not empty [child]

→ Root node has atmost d records and at few or one record.

→ All leaves are at the same level.

→ Height becomes extremely small. \rightarrow Super fast search.

shortest
height



$$\frac{d=1}{\log d}$$

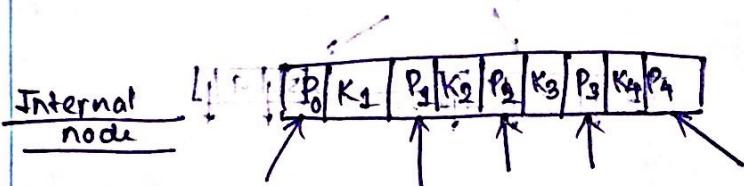
B+ Tree

Internal nodes → Keys are stored just like a B-Tree.

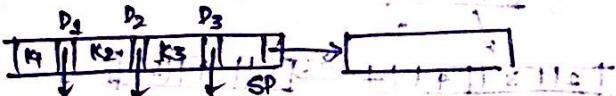
leaf nodes → Within them, keys are unique and sorted.

One key value can have a max of 2^r occurrences.

Other than the root, we can have d keys & $(d+1)$ pointers.



$< k_1 \quad k_1 \leq k_2 \leq k_3 \quad > k_2 \quad k_2 \leq k_3 \dots$

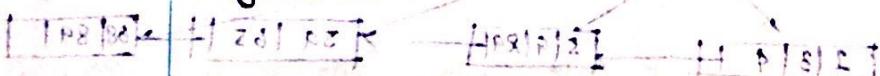


Helps to prevent Graph Traversal.

two types of Search

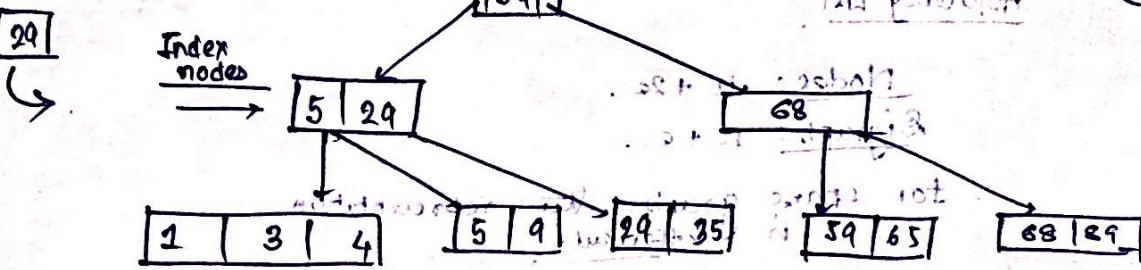
→ Exact Search

→ Range Query.



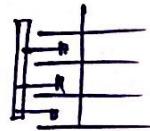
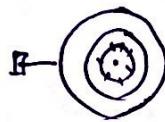
avg \sqrt{N} nodes per level





Data stored in hard disk (which part is addressable) Address level address <-
[Side Track Sector] Address level address <-

Front-view



Front view

multiple platters

multiple tracks per platter

multiple sectors per track

multiple cylinders

multiple recording surface

in order & always after previous

Graph

$G = (V, E)$ edges.
 vertex

$$e_i = (v_m, v_n),$$

Ordered pair \Rightarrow directed graph (DiGraph).
 Unordered \Rightarrow un-directed graph

$(v_m, v_n) \subseteq V \times V$
 $\Rightarrow v_m \& v_n$ are said to be adjacent

SubGraph

Path from v_1 to v_n
 in a sequence v_1, v_2, \dots, v_k .

Paths may have cycles
 & edges may have associated graphs.

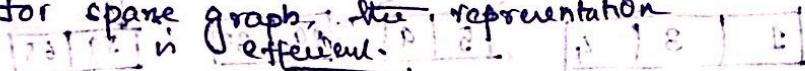
Adjacency Matrix

Adjacency List.

Nodes: $n + 2e$.

Digraph: $n + e$.

for sparse graph, tree representation is efficient.



Graph Traversal

- Depth First Search (Recursion & stack based).
- Breadth First Search.

Spanning Tree

Kruskal's Algorithm

Prim's Algorithm

Dijkstra's shortest path algorithm

Transitive Closure Matrix

All pair shortest path

Digraph with cycles & negative wt

(Algorithm) Strongly connected digraph
Strongly connected digraph
transitive set of edges and acyclic is

$\forall x \in S$ contains

transitive set of edges and acyclic is

algorithm
implementation
complexity
algorithm

algorithm

or at the most diff
the number of edges is

most of the edges
belong to a single component
and from graphs to
graphs

minimum weight

Hash Table → list of record structures with distinct keys.

- Insert
- Search
- Delete
- Modify

Goal

Example

Linear Probing

→ Collision Resolution Strategies

$$\alpha = \frac{n}{M} \rightarrow \text{load factor}$$

$$0.5 - 0.8$$

Problem of Linear Probing

→ May lead to Primary Clustering. → degrades the performance of the hash.

Quadratic Probing

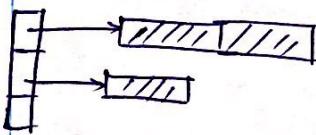
Small clusters will be formed instead of very large cluster.
→ Secondary clustering

Double Hashing

→ No. of collisions greatly reduced & cluster formation can be also avoided.

Chaining

a)



b) Coalesced Chaining ✓

Linked lists are maintained in the same array.

Bucket Hashing

Expected no. of buckets for random input

$$N = M * \lceil [n/M] / b \rceil$$

Expected no. of buckets in each chain

$$= \lceil [n/b] / b \rceil \quad \text{ceil fn. ✓}$$

Filtering based on Set membership

Filtering with One Hash fn

Bloom Filter

Analysis

Online process: Streaming data comes

False positive Analysis