1. Let S be a sample space of a random experiment. Let A, B, and C be three events. What is the event that only A occurs? What is the event that at least two of A, B, C occur? What is the event that both A, B, but not C occur? What is the event of at most one of the A, B, C occurs?

2. Let $A_1, A_2, \ldots$ be a sequence of events. Then prove that
$$P\left(\bigcup_{i=1}^{\infty} A_i\right) \leq \sum_{i=1}^{\infty} P\left(A_i\right)$$

3. Let A, B, C, and D be four events such that $P(A)=0.6, P(B)=0.5, P(C)=0.4, P(A \cap B)=0.3, P(A \cap C)=0.2, P(B \cap C)=0.2, P(A \cap B \cap C)=0.1, P(B \cap D)=P(C \cap D)=0, P(A \cap D)=0.1$, and $P(D)=0.2$. Find
   (a) $P(A \cup B \cup C)$ and $P\left(A^c \cap B^c \cap C^c\right)$. (Ans: 0.9 and 0.1)
   (b) $P((A \cup B) \cap C)$ and $P(A \cup(B \cap C))$. (Ans: 0.3 and 0.7)
   (c) $P\left(\left(A^c \cup B^c\right) \cap C^c\right)$ and $P\left(\left(A^c \cap B^c\right) \cup C^c\right)$.(Ans: 0.4 and 0.7)
   (d) $P(D \cap B \cap C)$ and $P(A \cap C \cap D)$.(Ans: 0 and 0)
   (e) $P(A \cup B \cup D)$ and $P(A \cup B \cup C \cup D)$. (Ans: 0.9 and 1.0)
   (f) $P((A \cap B) \cup(C \cap D))$.(Ans: 0.3)

4. Let $(\Omega, \mathcal{F}, P)$ be a probability space and let $A, B \in \mathcal{F}$. Show that $P(A \cap B)-P(A) P(B)=P(A) P\left(B^c\right)-P(A \cap\ B^c)=P\left(A^c\right) P(B)-P\left(A^c \cap B\right)=P\left((A \cup B)^c\right)-P\left(A^c\right) P\left(B^c\right)$

5. Suppose that $n(\geq 3)$ persons $P_1, \ldots, P_n$ are made to stand in a row at random. Find the probability that there are exactly r persons between P_1 and P_2; here $r \in\{1,2, \ldots, n-2\}$.(Ans: $2(n-r-1) /(n(n-1))$.)

6. A class consisting of four graduate and twelve undergraduate students is tivided into four groups of four. What is the probability that each group includes a graduate student? [Ans: $\left(2 \times 3 \times 4^3\right) /(15 \times 14 \times 13)$]

7. Find the probability that among three random digits there appear exactly two different digits. (Ans: 0.27)

8. Let $S=(0,1)$ and $P(I)=$ length of I, where I is an interval in S . Let $A=(0,1/2), B=(1/4,1)$ and $C=(1/4,11)$). Show that $P(A \cap B \cap C)=P(A) P(B) P(C)$, but $P(A \cap B) \neq P(A) P(B)$.

9. Consider two independent fair coin tosses, in which all four possible outcomes are equally likely. Let $H_1=\{1 \text{ st toss is a head }\}, H_2=\{2n\text{nd toss is a head }\}, D=\{\text{ the two tosses have different results }\}$. Find $P\left(H_1\right) P\left(H_2\right), P\left(H_1 \cap H_2\right), P\left(H_1 \mid D\right), P\left(H_2 \mid D\right)$, and $P\left(H_1 \cap H_2 \mid D\right)$
$(Ans: P\left(H_1\right)=0.5, P\left(H_2\right)=0.5, P\left(H_1 \cap H_2\right)=0.25, P\left(H_1 \mid D\right)=0.5, P\left(H_2 \mid D\right)=0.5$, $P\left(H_1 \cap H_2 \mid D\right)=0)$ (Note: Independent does not imply conditionally independent.)

10. There are two coins, a blue and a red one. We choose one of the two at random, each being chosen with probability 1/2, and proceed with two independent tosses. The coins are biased. With the blue coin, the probability of heads in any given toss is 0.99 , whereas for the red coin it is 0.01 . Let D be the event that the blue coin was selected. Let $H_i, i = 1, 2$, be the event that the i th toss resulted in head. Find $P(H_1), P(H_2), P(H_1 \cap H_2), P(H_1|D), P(H_2|D)$, and $P(H_1 \cap H_2|D)$.
(Ans: $P(H_1) = 0.5, P(H_2) = 0.5, P(H_1 \cap H_2) = 0.4901, P(H_1|D) = 0.99, P(H_2|D) = 0.99$ and $P(H_1 \cap H_2|D) = 0.9801$.) (Note: Conditional independence does not imply independence.)

11. A student is taking a course and at the end of each week, she can be either up-to-date or she may have fallen behind. If she is up-to-date in a given week, the probability that she will be up-to-date in the next week is 0.8 . If she is behind in a week, the that she will be up-to-date in the next week is 0.4 . She is up-to-date when she starts the class. Find the that she is up-to-date after three weeks. (Ans: 0.668 .)

12. (The Monty Hall problem) Suppose you're on a game show, and you're given the choice of three doors: Behind one door is a car; behind the others are goats. You pick a door, say No. 1, and the host, who knows what's behind the doors, opens another door, which has a goat. He then asks to you, "Do you want to pick the other closed door r " What should be your answer? (Ans: Yes, as the probability of wining the car is $\frac{2}{3}$ if I pick the other closed door.)

13. A laboratory blood test is 95% effective in detecting a certain disease when it is, in fact, present. However, the test also yields a "false positive" result for 1% of the healthy persons tested. If 0.5% of the population actually has the disease, what is the probability a person has the disease given his test result is positive? (Ans: 95/294)

14. An individual uses the following gambling system. He bets Re. 1. If he wins, he quits. If he loses, he makes the same bet a second time only time he bets Rs. 2 , and then regardless the result of the second match he quits the game. Assuming that he has a probability 0.5 to win each bet, find the probability that he goes home a winner. (Ans: 3/4)