1. Let X be random variable whose first two raw moments exist. Then show that
   (a) $\text{Var}(X) \geq 0$
   (b) $E\left(X^2\right) \geq (E(X))^2$
   (c) $V(a+bX) = b^2V(X)$, for two real constants a and b.

2. From a box containing N identical tickets, numbered $1, 2, \ldots, N, n(\leq N)$ tickets are drawn with replacement. Let X be the largest number drawn. Find $E(X)$
   $\left(Ans : E(X) = N - \frac{1}{N^n}\sum_{i=1}^{N-1} i^n\right)$

3. Let X be a CRV such that $E\left(|X|^\beta\right) < \infty$ for some $\beta > 0$. Then show that $E\left(|X|^\alpha\right) < \infty$ for all $\alpha \in (0, \beta]$

4. Consider the following joint PMF of the random vector (X, Y)

$x\backslash y$	1	2	3	4
4	0.08	0.11	0.09	0.03
5	0.04	0.12	0.21	0.05
6	0.09	0.06	0.08	0.04

   (a) Find the probabilities $P(X+Y<8), P(X+Y>7), P(XY \leq 14)$
   (b) Find the $\text{Corr}(X, Y)$

5. Let X and Y be jointly distributed random variables with $E(X) = 0, E\left(X^2\right) = E\left(Y^2\right) = 2$, and $\text{Corr}(X, Y) = 1/3$. Find $\text{Corr}\left(\frac{X}{3} + \frac{2Y}{3}, \frac{2X}{3} + \frac{Y}{3}\right)$

6. Let X and Y are RVs such that $|\rho(X, Y)| = 1$. Show that $Y = a + bX$ for some real constants a and b