B.E. Second Year First Semester Examination 2022

Mathematics-III

Full Marks: 70 | Time: 3 Hours

The figures in the margin indicate full marks
Notations and symbols have their usual meanings

Group A
(35 Marks)
Attempt all questions

1. Answer any three questions:-

(a) i. Show by vector method $\sin(A+B) = \sin A \cos B + \cos A \sin B$. (2)

ii. Prove that $\left[\alpha\vec{a}+\beta\vec{b} \quad \vec{c} \quad \vec{d}\right] = \alpha\left[\vec{a}\ \vec{c}\ \vec{d}\right] + \beta\,[\vec{b}\ \vec{c}\ \vec{d}\ ]$. (2)

iii. If $\vec{F} = x^2y\,\hat{i} + y^2z\hat{j} + z^2\hat{k}$, find $\vec{\nabla}\times(\vec{\nabla}\times\vec{F})$ at the point (1,-1,0). (3)

(b) i. Find the equations of the tangent plane and normal line to the surface $xyz = 4$ at the point (1,2,2). (2)

ii. Show that $\vec{\nabla}\times\left(\frac{\vec{a}\times\vec{r}}{r^3}\right) = -\frac{\vec{a}}{r^3} + \frac{3(\vec{a}\,.\,\vec{r}\,)}{r^5}\vec{r}$ where $\vec{a}$ is constant and $\vec{r} = x\hat{i}+y\hat{j}+z\hat{k}$, $r = \left|\vec{r}\right|$. (3)

iii. If $\vec{u} = (xy\sin z)\,\hat{i} + (y^2\sin x\,)\,\hat{j} + (z^2\sin y)\,\hat{k}$, find $\vec{\nabla}.\,\vec{u}$ at $\left(0, \frac{\pi}{2}, \frac{\pi}{2}\right)$. (2)

(c) i. Show that $\vec{\nabla}\,.\left(\vec{A}\times\vec{B}\right) = \vec{B}.\left(\vec{\nabla}\times\vec{A}\right) - \vec{A}\,.\,(\vec{\nabla}\times\vec{B}\,)$. (3)

ii. Show that the vector $\vec{f} = (2x-yz)\hat{i} + (2y-zx)\hat{j} + (2z-xy)\hat{k}$ is irrotational. Find the scalar potential φ such that $\vec{f} = grad\ \varphi$. (4)

(d) Evaluate the surface integral $\iint_S (yz\,\hat{i} + zx\,\hat{j} + xy\,\hat{k}\,).\,d\vec{s}$ where S is the surface of the sphere $x^2+y^2+z^2 = 1$ in the first octant. (7)

(e) Verify Green's theorem in the plane for $\int_C (x^2-xy^2)dx + (y^2-2xy)\,dy$ where C is the square with vertices (0,0), (2,0), (2,2), (0,2). (7)

(f) Verify the divergence theorem of Gauss for $\vec{F} = 2x^2\,\hat{i} + y\,\hat{j} - z^2\,\hat{k}$ where S is the closed surface consisting of the curved surface of the cylinder $x^2+y^2 = 16$ between the plane $z = 0$ and $z = 2$ together with the circular ends of those planes. (7)

2. Answer any two questions:-

   (a) Establish the formula $r = \frac{{\sigma_x}^2+{\sigma_y}^2-{\sigma_{x-y}}^2}{2\sigma_x\sigma_y}$, hence calculate r from the following data: (7)

x	21	23	30	54	57	58	72	78	87	90
y	60	71	72	83	110	84	100	92	113	135

   (b) In a bolt factory, there are four machines A, B, C, D manufacturing 20%, 15%, 25% and 40% of the total output respectively. Of their outputs 5%, 4%, 3% and 2% in the same order are defective bolts. A bolt is chosen at random from the factory's production and is found defective. What is the probability that the bolt was manufactured by machine A or machine D? (7)

   (c) X is a Poisson variable and it is found that the probability that X=2 is two-thirds of the probability that X=1. Find the probability that X=0 and the probability that X=3. What is the probability that X exceeds 3? (7)

Group B

(35 Marks)

Attempt all questions

3. Find the expressions for x and y as function of t separately from the simultaneous equations, (5)

$$\begin{aligned} 2\frac{dx}{dt} - \frac{dy}{dt} + 2x + y &= 11t, \\ 2\frac{dx}{dt} + 3\frac{dy}{dt} + 5x - 3y &= 2. \end{aligned}$$

4. Find general integral of the partial differential equation, (5)

$$(x^2 - y^2 - z^2)\frac{\partial z}{\partial x} + 2xy\frac{\partial z}{\partial y} = 2xz.$$

5. Solve the following equation : (5)

$$(3x+2)^2\frac{d^2y}{dx^2} + 3(3x+2)\frac{dy}{dx} - 36y = 3x^2 + 4x + 1.$$

6. Apply the method of variation of parameters to solve the equation (5)

$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = \frac{1}{1+e^x}.$$

7. Find the solution of (5)

$$(D^2 - D')(D - 2D')z = e^{2x+y} + xy, \text{ where } D \equiv \frac{\partial}{\partial x} \text{ and } D' \equiv \frac{\partial}{\partial y}.$$

8. Use separation of variables method to solve (10)

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}, \quad 0 < x < \pi, \ t > 0,$$

subject to
$u(0,t) = u(\pi,t) = 0$ for $t \geq 0$,
$u(x,0) = 2(\sin x + \sin 3x)$ and $u_t(x,0) = 0$ for $0 \leq x \leq \pi$.