

Ex 4

The chairs of an auditorium are to be labeled with an uppercase english letter (A-Z) followed by a +ve integer not exceeding 100. What is the largest no. of chairs that can be labeled differently

Ans

Sol ①

$$\frac{26}{1} \times \frac{100}{1}$$

↳ 26 possibilities

$$\frac{0}{1} \times \frac{0}{1} \times \frac{0}{1}$$

$$\frac{1}{1} \times \frac{0}{1} \times \frac{0}{1}$$

$$\frac{1}{1} \times \frac{10}{1} \times \frac{10}{1}$$

Due to condition of 000

$$\text{Total } 26(1 \times 10 \times 10 - 1 + 1) = 2600$$

↳ disjoint sets

Sol ②

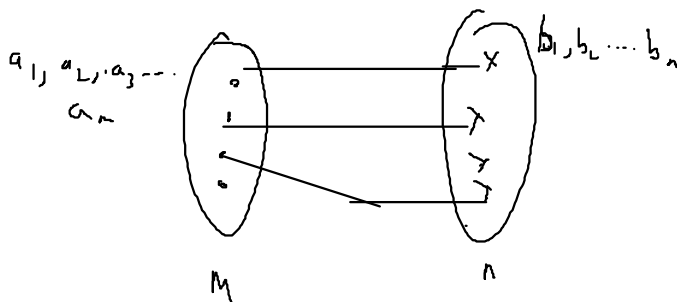
$$\frac{26}{1} \times \frac{100}{1}$$

$$\rightarrow 26 \times 100 = 2600$$

Ex 2

How many one-to-one functions are there from m elements to one with n elements.

Ans



Possible if $m \leq n$.

$$a_1 \quad a_2 \quad \dots \quad a_m$$

(n possibilities of b_1) ($n-1$ possibilities of b_2) ($n-m+1$ possibilities of b_i)

$$= n \times (n-1) \times (n-2) \times \dots \times (n-m+1)$$

Ex 3 A student can choose a project from one of 3 lists. The 3 lists contain 23, 15 & 19 possible projects respectively. No project is on more than one list. How many possible ways to choose.

Ans Sol 1 Since all projects are distinct, then all the lists can be combined & one can be chosen.
 \therefore combined lists contain
 $23 + 15 + 19.$

Sol 2 Students can take project either from first list or second list or third list.
from first list \rightarrow can take in 23 ways
2nd list \rightarrow . . . 15 ways.
3rd list \rightarrow . . . 19 ways.
 \therefore total ways = $23 + 15 + 19$
 \hookrightarrow either/or.

Question:

If there are

- a projects common in dest 1 & dest 2
- b " " " " " " dest 1 & dest 3
- c " " " " " " dest 2 & dest 3

Sol: $|dest 1| + |dest 2| + |dest 3| = a + b + c.$

Quest If 2 projects are in d_1, d_2, d_3
also the above.

Sol. $|L_1| + |L_2| + |L_3| = a + b + c + \underline{d}$.

Principle of inclusion-exclusion (Roxn).

(a) Subtraction rule: If a task can be done in either n_1 ways or in n_2 ways, then the number of ways to do the task is $n_1 + n_2$ minus the number of ways to do the task that are common to the two different ways.

② The subtraction rule is known as the principle of inclusion-exclusion especially when it is used to count the number of elements in the union of two sets.

94 $C \rightarrow$ strongly typed language.

Q2. Say In a weakly typed language, the name of a variable is a string of one or two

weekly typed (ph, ai)
language →
var. a & A → both are
same

var. integer } both mean same

strongly typed
language.
A & a \rightarrow both mean same.

(numbers or characters) alphanumeric characters, where uppercase & lowercase letters are not distinguished. Moreover, a variable name must begin with a letter & must be different from the 5 strings of two characters that are reserved for programming use. How many different variable names

Ans Let us say that the total number of variables be V .

Let the single character variable be $V_1 = 26$
 $2 \quad \dots \quad V_2 = 26 \cdot 36$

$$\therefore V = V_1 + V_2 \\ = 26 + 26 \cdot 36$$

\therefore there are 5 strings common.
 \therefore total no. of variables is
 $V = V_1 + V_2 - 5$

Ex A company has 350 applications out of which

$A_1 \rightarrow 200$ applicants from CS

$A_2 \rightarrow 147$ " " " MBA

51 applicants from CS + MBA

Question: How many applicants did neither in CS nor in MBA.

Ans. The total no. of candidates who did neither CS nor MBA,

$$A = |A_1 \cup A_2| \rightarrow |A_1| + |A_2|. \quad (\text{All person who did CS \& MBA.})$$

↓
This contains who did both CS & MBA.

∴ Total no. of candidates who did —

$$\begin{aligned} A = |A_1 \cup A_2| &= |A_1| + |A_2| - |A_1 \cap A_2| \\ &= 220 + 147 - 51. \\ &= 316. \quad (\text{Either CS/MBA} \\ &\quad \& \text{CS+MBA.}) \end{aligned}$$

∴ Total no. of applicants who did neither CS or MBA.

$$350 - 316 = \boxed{34}$$

⑥ Division Rule :

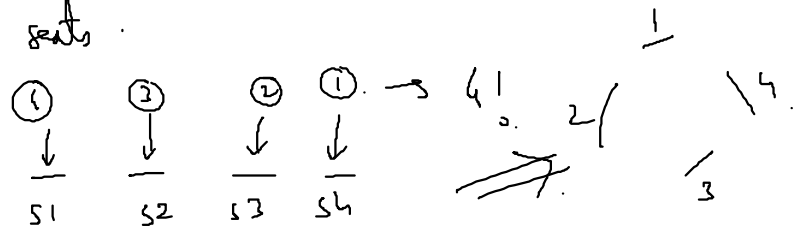
There are (n/d) ways to do a task if it can be done using a procedure that can be carried out in n ways, and for every way w , exactly d of the n ways correspond to way w .

⇒ If the finite set A is the union of n -pairwise disjoint subsets each with d elements, then $n = |A|/d$.

Q. How many different ways are there to seat 4 people around a circular table where two seatings are considered the same when each person has the same left neighbour & the same right neighbour.

Soln.

There can be a total of $4!$ arrangements for the 4 people in the seats.



Now, for seat 1, there are 4 choices.

but 2 ways where the assignment of left & right \equiv right & left.
2 arrangements.

for seat 2, there are 3 choices

but 2 ways where the assignment of left & right \equiv right & left.
2 arrangements

Total no. of ways = $\frac{24}{2 \times 2} = 6$ different seating arrangements of 4 people round the circular table.

Linear Diophantine Equation

(2 variable)

Let a, b & c be integers with a & b be both not zero.

Then the eq. $ax + by = c$ has an integral solution iff

d divides c , where $d = \gcd(a, b)$. Furthermore, if (x_0, y_0)

is a particular integral solution of this eq, then all the integral solutions for this eq are given by.

$$x = x_0 + \frac{b}{d}n, \quad y = y_0 - \frac{a}{d}n.$$

where n is any integer.

for multiple variable —

$$a_1 + a_2 + a_3 + \dots + a_n = r. \quad \text{--- (1)}$$

$a_i \in (0, 1)$, find the combination for which eq. (1) satisfies.

$$n > r.$$

eg

$$a_1 + a_2 + a_3 + a_4 = 3.$$

$$a_i \in (0, 1).$$

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