Ex/CSE/Math/T/222/80/2011

Bachelor of Computer Science Engineering Examination, 2011

(2nd Year, 2nd Semester)

Mathematics - VI D

Paper : Probability

Time : Three Hours Full Marks - 100

Attempt any **Five** questions.

Each question carries 20 makrs

1. Define the following terms (giving an example to each term) :

   a) Random Experiment.

   b) Sample space — finite, infinite.

   c) Events, null event, sure event, elementary event.

   d) Axioms of probability (give example on a concrete sample space, defining a probability obeying these axioms).

   2+4+6+8=20

2. a) On a certain day, it rains with probability 0.4 and it snows with probability 0.8. If the probability to rain or snow is 1, find the probability of both raining and snowing.

   b) If $P(A)=0.5, P(B)=P(C)=0.3$, $P(A \cup B \cup C)=1$ and $P(A \cap B)=P(B \cap C)=P(C \cap A) = P(A \cap B \cap C) = x$, find x.

   10+10=20

[Turn over

3. State the classical Matching problem and find the probability of exactly k matches where k is any non-negative integer. 20

4. a) Motivate and define conditional probability. State and prove Bayes' theorem.

   b) There are 3 Physics books, each with 2 volumes and 5 Mathematics books, each with 3 volumes. Find the probability of placing these 21 books on a shelf with increasing volume numbers, given that Mathematics books are placed before Physics books if both of them have identical volume numbers. 10+10=20

5. a) Define a geometric random variable through coin tossing, hence, find its mean, variance and characteristic function.

   b) Let x be a continuous random variable with density (p.d.f.) given by

$$f(x)=\begin{cases}\frac{1}{2}e^{-\frac{x}{2}} & \text{if } x>0\\ 0 & \text{otherwise}\end{cases}$$

   Find the mean, variance and characteristic function of x.

   10+10=20

6. a) Define a Hypergeometric random variable by occupancy problems (distribution of balls in urns). Hence find its mean, median and mode.

   b) Find the mean, median and mode of a binomial random variable. 10+10=20

7. Let $\underset{\sim}{X}_{2x1}$ be a two dimensional random vector with joint density (jt.p.d.f.)

$$f\begin{pmatrix}x\\y\end{pmatrix}=K.e^{-\frac{1}{2}\begin{pmatrix}x\\y\end{pmatrix}^{t}\begin{pmatrix}1 & \rho\\ \rho & 1\end{pmatrix}(x,y)}$$

   where K^{ER} is a constant and x.y

   Find K_0, $E(\underset{\sim}{X})_{2x1}$ and the dispersion (variance-covariance) matrix of $\underset{\sim}{X}_{2x1}$. 20