Prove that:

Q If X is a random variable with parameters (n, p), where $0<p<1$, then as k goes from 0 to n, $P\{X=k\}$ first increases monotonically & then decreases monotonically reaching its largest value when k is the largest integer less or equals to $(n+1)p$.

Q A point is chosen at random on a semi-circle having centre at the origin & radius unity & projected on the diameter. Prove that the distance of the point of projection from centre has the prob. density $\frac{1}{\pi\sqrt{1-x^2}}$ for $-1<x<1$ & 0 elsewhere.

Q A point X is chosen at random on a line segment AB whose middle pt. is O. Find the prob. that AX, BX & AO form the sides of a triangle.

Q The joint density func. of X & Y is given by:

$$f(x,y) = \begin{cases} k(x+y) & \text{for } 0<x<1,\ 0<y<1 \\ 0 & \text{elsewhere} \end{cases}$$

(a) Find values of k

(b) The marginal density func. $f_X(x)$ & $f_Y(y)$

Q The prob. density func. of a two-dimensional r.v. (X,Y) is given by:

$$f(x,y) = \begin{cases} k(x+y) & x>0,\ y>0,\ x+y<2 \\ 0 & \end{cases}$$

Find k & $P(X<1, Y>\frac{1}{2})$.

Q A point P is chosen at random on a line segment AB of length 21. Find expected values of

(i) $AP \cdot PB$

(ii) $|AP - PB|$.

[Q] The radius X of a circle has uniform distr. in $(1,2)$. Find mean & variance of the area of the circle

[Q] The length of bolts produced by a machine is normally distributed with parameters $m = 4$, $\sigma = 0.5$. A bolt is defective if its length does not lie in the interval $(3.8, 4.3)$. Find the percentage of defective bolts produced by the machine.

$$\left[\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{0.6} e^{-t^2/2}\, dt = 0.7257, \right.$$

$$\left. \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{0.4} e^{-t^2/2}\, dt = 0.6554 \right]$$