Exam Roll - CSE214021

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Part-1

1.

Let, suppose $\exists\, \alpha \in C_A(B)$

and- $B \not\subset B \cup \{\alpha\} \subseteq A$

The inclusion mappings:

$i_B : B \to A : \forall B \in B : i(B) = B$

$i_{B\cup\{a\}} : B \cup \{\alpha\} \to A : \forall B \in B : i(B) = B$

give:

$|A| = |B| \leq |B| + 1 \leq |A|$

from which we get:

$|A| = |B| + 1 = |A| + 1$ $\therefore$ A is infinite

So, now, α be any object such that $\alpha \notin A$.

Then there is a bijection $f : A \cup \{\alpha\} \to A$

Then from injection to image is bijection:

$Img(f)_A = A \setminus \{f(\alpha)\} = B$

which is a proper subset of A.

2) The statement $A \Delta C = B \Delta C$ implies the following.

if x belongs to either A or C but not in both a and C, then it belongs to either B or C but not in both.

If, an element x belongs to C but not A. by the previous statement it belongs to C but not B [$x \notin (B-C)$ in this case because $x \in C$ and $x \notin C$ cannot both be true]

Hence,

$$C - A = C - B$$

$$\Rightarrow C - (A \cap C) = C - (B \cap C) \quad \cdots (i)$$

$$\Rightarrow A \cap C = B \cap C$$

[if $x \notin C$ then $x \notin A \cap C$ and $x \notin B \cap C$ and if $x \in (A \cap C)$, then it must belongs to $(B \cap C)$ because if it didnot; then $x \in C - (B \cap C)$ which would be a contradiction to statement (i)]

Also $A \Delta C = B \Delta C$

$$\Rightarrow (A \cup C) - (A \cap C) = (B \cup C) - (B \cap C)$$

$$\Rightarrow A \cup C = B \cup C \quad [\because A \cap C = B \cap C]$$

The above statement implies that if $x \in A$ or $x \in C$; then $x \in B$ or $x \in C$

Now, we can say $\underline{A = B}$ because if $x \in A$; then

1) $x \notin C$: then $x \in B$ because $A \cup C = B \cup C$

2) $x \in C$: then $x \in x \cap C$ because $A \cap C = B \cap C$. and we know $A = B \cup (B \cap C) = B$ (Hence proved)

3) A relation $R \subseteq (S \times S)$ on a set S is reflexive iff $(a,a) \in R$ ~~$\forall$~~ $\forall a \in S$. a

A relation R in a set S is symmetric iff $(a,b) \in R \Rightarrow (b,a) \in R \ \forall (a,b) \in R$

Let, $S = \{a, b, c, d, e\}$ The number of reflexive relations of S is the number of possible subsets of $(S \times S)$ which satisfy the reflexive property, Now,

number of elements in $S \times S$

$|S \times S| = 25$: out of which 5 elements $\{(a,a), (b,b), (c,c), (d,d), (e,e)\}$ must be present ~~any~~ in any reflexive relation.

Number of ways to choose any number of elements out of remaining $25 - 5 = 20$ elements $= {}^{20}C_0 + {}^{20}C_1 + \cdots + {}^{20}C_{20} = (1+1)^{20}$

$= 2^{20}$ = Number of possible reflexive relations on S

Number of ways to choose ~~unequal~~ unordered pair subset of the from $\{a,b\}$ out of S =

Number of ways to choose 2 unequal elements out of S + Number of ways to choose $\{a,a\}$ out of S

$= {}^5C_2 + {}^5C_1 = 15$

If we consider $S_2 = \{\{a,b\} \mid a,b \in S\}$; $|S_2| = 15$,
then the number of all possible subsets of S_2
is the number of symmetric relations,
Since each unordered pair $\{a,b\}$ present in
a particular subset of S_2 can be translated
to two ordered pairs (a,b) and (b,a) [or 1
ordered pair (a,a)] in a particular subset
of $(S \times S)$.
Hence no. of possible Symmetric relations
on $S = 2^{15}$

7. Truth table of $(p \to q) \wedge (q \to r)$

p	q	r	$p \to q$	$q \to r$	$(p \to q) \wedge (q \to r)$
T	T	T	T	T	T
T	T	F	T	F	F
T	F	T	F	T	F
T	F	F	F	T	F
F	T	T	T	T	T
F	T	F	T	F	F
F	F	T	T	T	T
F	F	F	T	T	T

9.

Let,

$P(n) = 5^n + 3$

now, for $P(1) = 5+3 = 8$ $P(2) = 28$

~~P(1)~~ P(1) is divisible by 4 and P(2) also divisible by 4

now, Let, $P(k) = 5^k + 3$ is divisible by 4.

$\therefore 5^k + 3 = 4m$ [where m = natural number] ...(i)

now, $P(k+1) = 5^{k+1} + 3$

$= 5^k \cdot 5 + 3$

$= 5^k \cdot (4+1) + 3$

$= 4 \cdot 5^k + (5^k + 3)$

$= 4 \cdot 5^k + 4m$ [by (i)]

$= 4(5^k + m)$

which is also divisible by 4

Now as P(1) and P(2) are divisible by 4 and P(k+1) is divisible by 4 if P(k) is divisible; so by theory of mathematical induction we can say that $P(n) = 5^n + 3$ is divisible by 4 /

5. A set A is countable if $|A| < \aleph_0$, or there exists $f : A \to \mathbb{N}$, which is one to one, In case f is also onto ($\exists$ a bijection from A to $\mathbb{N}$) then A is called countably infinite; otherwise it is (countably) finite (since all finite sets are countable).

Let, there exists a class of sets $\{A_1, A_2 \ldots A_n\}$ where $n \in \mathbb{N}$, the set of all possible subscripts i of the A_i's in this class be $I \subseteq \mathbb{N}$. without loss of generarality we can assume, $A_i \cap A_j = \phi$ for all $i, j \in I$ $(i \neq j)$. Now, $|\bigcup_{i=I} A_i| \leq |I \times \mathbb{N}|$

Since there are $|I|$ disjoint sets each contributing at most $|\mathbb{N}|$ elements to the union.

$$|\bigcup_{i \in I} A_i| \leq |I \times \mathbb{N}| \leq |\mathbb{N} \times \mathbb{N}|$$

Since, $|I| \leq |\mathbb{N}|$

$$\therefore |\bigcup_{i=I} A_i| \leq |\mathbb{N} \times \mathbb{N}| = |\mathbb{N}|$$

It is possible to count $|\mathbb{N} \times \mathbb{N}|$ because we can order all possible ~~pair~~ pairs (n_1, n_2) first by the sum and then by n_1 $[\{(1,1), (1,2), (2,1), (1,3), (2,2), (3,1), \ldots\}]$

Hence the countable union of countable sets are countable /