ii) Let X have density

$$f(x)=\begin{cases} ax^2, & \text{if} \quad 0 \le x \le b \\ 0 & \text{otherwise} \end{cases}$$

where b is a positive real no. Find a and the density of the random variable $Y = X^3$. 10+10=20

7. i) Define a multivariate normal random vector. Let $\underset{\sim}{X}$ and $\underset{\sim}{Y}$ be two multivariate normal vectors with respective dispersion matrices Σ_X and Σ_Y prove that $\underset{\sim}{X}$ and $\underset{\sim}{Y}$ are independent if and only if the dispersion matrix of the augmented random vector $\begin{bmatrix} \underset{\sim}{X} \\ \underset{\sim}{Y} \end{bmatrix}$ is given by $\begin{bmatrix} \Sigma_X & \underset{\sim}{O} \\ \underset{\sim}{O} & \Sigma_Y \end{bmatrix}$

ii) Let (X, Y) be a bivariate normal random vector with parameters $(\mu_x, \mu_y, \sigma_x^2, \sigma_y^2, \rho)$ Find the regression line of Y on X from a random sample of size n from this bivariate population. 10+10=20

Ex/CSE/Math/T/221A/2019

Bachelor of Engineering in Computer Science Engineering Examination, 2018

(2nd Year, 2nd Semester)

Mathematics - V

Time : Three hours Full Marks : 100

Answer ***any five*** questions. Each question carries 20 marks.

1. a) Define a random expt and the sample space associated with it. What is an event ?

   b) Define Probability classically and point out its various draw backs.

   c) Give the 'Frequency' definition of Probability along with its short comings.

   d) Write down the 'axioms' of Probability and explain how such an axiomatic definition removes the draw backs of (b) and (c) above. 5+5+5+5=20

2. Let there be N tickets numbered 1, 2, ... N in a box. Tickets are drawn one by one from the box, at random without replacement. If on the 'i' th draw, ticket no. 'i' is drawn, we say that there is a match at the i-th position, i = 1, 2, ..., N. After all the tickets are drawn, let X denote the total no of matches.

   a) Find Prob (X = N – 1)

b) Find Prob $(X = O)$

c) Find Prob $(X = k)$, for all $k \varepsilon \mathbb{N}$. 3+8+9=20

3. i) Define independence of two events.

ii) Prove that 'A and B are independent' implies

a) A and B^C are independent.

b) A^C and B^C are independent too.

iii) Let A and B be two non-null events. If they are independent, can they be disjoint ? Explain your answer.

iv) Box 1 conatins 5W and 3B balls and Box 2 contains 3W and 5B balls. A ball is chosen at random from Box 1 and transferred to Box 2, and then, a ball is drawn at random from Box 2.

a) Given that the transferred ball is B, find the conditional probability that the ball drawn from Box 2 is also B.

b) Given that the ball drawn from box 2 is B, find the conditional probability that the transferred ball is B.

2+8+2+8=20

4. i) Buddha repeatedly tosses a coin independently and scores one point for a Head, and two points for a Tail. If r_n denotes the probability of scoring n points, $n \varepsilon \mathbb{N}$,

show that $2r_n = r_{n-1} + r_{n-2}$, for all $n \geq 3$, Find r_1, r_2 and an expression for r_n as a function of n only. Also, find $\lim_{n \to \infty} r_n$. 10

ii) If there are A males and $B(<A)$ females in a group and N sweets are distributed uniformly among this group of people, find the probability that the females obtain an odd no. of sweets. Show that as $N \to \infty$, this probability tends to 50%. 10

5. Let a biased coin with Prob (+1 end) = p $(0 < p < 1)$ be tossed repeatedly and independently until a total of 100 Head come up, at which point, the experiment is stopped and let X denote the total number of tosses needed to get 100 Heads. Then X is a random variable. What is the name of such random variable. Find the mean and variance of X.

6. i) Let X, Y and Z be independent and identically distributed continuous random variables with common density function

$$f(x) = \begin{cases} 6x^5, & \text{if} \quad 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Find the density, mean and variance of the random variable W = minimum (X, Y, Z). 10+10=20