UNIT-3

PROBABILITY

INTRODUCTION:

Probability theory was originated from gambling theory. A large number of problems exist even today which are based on the game of chance, such as coin tossing, dice throwing and playing cards.

The probability is defined in two different ways,

- Mathematical (or a priori) definition
- Statistical (or empirical) definition

SOME IMPORTANT TERMS &CONCEPTS:

- RANDOM EXPERIMENTS:

  Experiments of any type where the outcome cannot be predicted are called random experiments.
- SAMPLE SPACE:

  A set of all possible outcomes from an experiment is called a sample space.

  Eg: Consider a random experiment E of throwing 2 coins at a time. The possible outcomes are HH, TT, HT, TH.

  These 4 outcomes constitute a sample space denoted by, S ={ HH, TT, HT, TH}.
- TRAIL & EVENT:

  Consider an experiment of throwing a coin. When tossing a coin, we may get a head(H) or tail(T). Here tossing of a coin is a trail and getting a hand or tail is an event.

  In otherwords, "Every non-empty subset of A of the sample space S is called an event".
- NULL EVENT:

  An event having no sample point is called a null event and is denoted by $\emptyset$.
- EXHAUSTIVE EVENTS:

  The total number of possible outcomes in any trail is known as exhaustive events.

  Eg: In throwing a die the possible outcomes are getting 1 or 2 or 3 or 4 or 5 or 6. Hence we have 6 exhaustive events in throwing a die.
- MUTUALLY EXCLUSIVE EVENTS:

  Two events are said to be mutually exclusive when the occurrence of one affects the occurrence of the other. In otherwords, if A & B are mutually exclusive events and if A happens then B will not happen and viceversa.

  Eg: In tossing a coin the events head or tail are mutually exclusive, since both tail & head cannot appear in the same time.

- EQUALLY LIKELY EVENTS:

  Two events are said to be equally likely if one of them cannot be expected in the preference to the other.

  Eg: In throwing a coin, the events head & tail have equal chances of occurrence.

- INDEPENDENT & DEPENDENT EVENTS:

  Two events are said to be independent when the actual happening of one doesnot influence in any way the happening of the other. Events which are not independent are called dependent events.

  Eg: If we draw a card in a pack of well shuffled cards and again draw a card from the rest of pack of cards (containing 51 cards), then the second draw is dependent on the first. But if on the other hand, we draw a second card from the pack by replacing the first card drawn, the second draw is known as independent of the first.

- FAVOURABLE EVENTS:

  Mathematical or classical or a priori definition of probability,

  Probability (of happening an event E) $= \frac{Number\ of\ favourable\ cases}{Total\ number\ of\ exhaustive\ cases}$

  $= \frac{m}{n}$

  Where m = Number of favourable cases

  n = Total number of exhaustive cases.

<u>PROBLEMS:</u>

1. In tossing a coin, what is the prob. of getting a head.

Sol: Total no. of events = {H, T}= 2

Favourable event = {H}= 1

Probability $= \frac{Number\ of\ favourable\ cases}{Total\ number\ of\ exhaustive\ cases}$

$= \frac{1}{2}$

2. In throwing a die, the prob. of getting 2.

Sol: Total no. of events = {1,2,3,4,5,6}= 6

Favourable event = {2}= 1

Probability $= \frac{Number\ of\ favourable\ cases}{Total\ number\ of\ exhaustive\ cases}$

$= \frac{1}{6}$

3. Find the prob. of throwing 7 with two dice.

Sol: Total no. of possible ways of throwing a dice twice = 36 ways

Number of ways of getting 7 is, (1,6), (2,5), (3,4), (4,3), (5,2), (6,1) = 6

$$\text{Probability} = \frac{Number\ of\ favourable\ cases}{Total\ number\ of\ exhaustive\ cases}$$
$$= \frac{6}{36}$$
$$= \frac{1}{6}$$

4. A bag contains 6 red & 7 black balls. Find the prob. of drawing a red ball.

Sol: Total no. of possible ways of getting 1 ball = 6 + 7

Number of ways of getting 1 red ball = 6

$$\text{Probability} = \frac{Number\ of\ favourable\ cases}{Total\ number\ of\ exhaustive\ cases}$$
$$= \frac{6}{13}$$

5. Find the prob. of a card drawn at random from an ordinary pack, is a diamond.

Sol: Total no. of possible ways of getting 1 card = 52

Number of ways of getting 1 diamond card is 6

$$\text{Probability} = \frac{Number\ of\ favourable\ cases}{Total\ number\ of\ exhaustive\ cases}$$
$$= \frac{13}{52}$$
$$= \frac{1}{4}$$

6. From a pack of 52 cards, 1 card is drawn at random. Find the prob. of getting a queen.

Sol: A queen may be chosen in 4 ways.

Total no. of ways of selecting 1 card = 52

$$\text{Probability} = \frac{Number\ of\ favourable\ cases}{Total\ number\ of\ exhaustive\ cases}$$
$$= \frac{4}{52} = \frac{1}{13}$$

7. Find the prob. of throwing: (a) 4, (b) an odd number, (c) an even number with an ordinary die (six faced).

Sol: a) When throwing a die there is only one way of getting 4.

$$\text{Probability} = \frac{Number\ of\ favourable\ cases}{Total\ number\ of\ exhaustive\ cases}$$
$$= \frac{1}{6}$$

b) Number of ways of falling an odd number is 1, 3, 5 = 3

$$\text{Probability} = \frac{Number\ of\ favourable\ cases}{Total\ number\ of\ exhaustive\ cases} = \frac{3}{6} = \frac{1}{2}$$

c) Number of ways of falling an even number is 2, 4, 6 = 3

Probability $= \frac{Number\ of\ favourable\ cases}{Total\ number\ of\ exhaustive\ cases} = \frac{3}{6} = \frac{1}{2}$

8. From a group of 3 Indians, 4 Pakistanis, and 5 Americans, a sub-committee of four people is selected by lots. Find the probability that the sub-committee will consist of
   i) 2 Indians and 2 Pakistanis.
   ii) 1 Indians, 1 Pakistanis and 2 Americans.
   iii) 4 Americans.

Sol: Total no. of people = 3 + 4 + 5 = 12

∴ 4 people can be chosen from 12 people = $12C_4$ ways

$= \frac{12 \times 11 \times 10 \times 9}{1 \times 2 \times 3 \times 4}$ = 495 ways

i) 2 Indians can be chosen from 3 Indians = $3C_2$ ways

2 Pakistanis can be chosen from 4 Pakistanis = $4C_2$ ways

∴ No. of favourable cases = $3C_2 \times 4C_2$

∴ Prob. $= \frac{3C_2 \times 4C_2}{495} = \frac{2}{55}$

ii) 1 Indian can be chosen from 3 Indians = $3C_1$ ways

1 Pakistani can be chosen from 4 Pakistanis = $4C_1$ ways

2 Americans can be chosen from 5 Americans = $5C_2$ ways

Favourable events = $3C_1 \times 4C_1 \times 5C_2$

∴ Prob. $= \frac{3C_1 \times 4C_2 \times 5C_2}{495} = \frac{8}{33}$

iii) 4 Americans can be chosen from 5 Americans = $5C_4$ ways

∴ Prob. $= \frac{5C_4}{495} = \frac{1}{99}$

9. A bag contains 7 white, 6 red & 5 black balls. Two balls are drawn at random. Find the prob. that they both will be white.

Sol: Total no. of balls = 7 + 6 + 5

= 18

From there 18 balls, 2 balls can be drawn in $18C_2$ ways

i.e) $\frac{18 \times 17}{1 \times 2} = 153$

2 white balls can be drawn from 7 white balls = $7C_2$ ways

$= 21$

$\therefore$ Favourable cases = 21

P(drawing 2 white balls) = $\frac{21}{153} = \frac{7}{51}$

10. A bag contains 10 white, 6 red, 4 black & 7 blue balls. 5 balls are drawn at random. What is the prob. that 2 of them are red and one is black?

Sol: Total no. of balls = 10 + 6 + 4 + 7 =27

5 balls can be drawn from these 27 balls = $27C_5$ ways

$= \frac{27 \times 26 \times 25 \times 24 \times 23}{1 \times 2 \times 3 \times 4 \times 5}$

= 80730 ways

Total no. of exhaustive events = 80730

2 red balls can be drawn from 6 red balls = $6C_2$ ways

$= \frac{6 \times 5}{1 \times 2} = 15$ ways

1 black balls can be drawn from 4 black balls = $4C_1$ ways

$= 4$

$\therefore$ No. of favourable cases = $15 \times 4 = 60$

Probability $= \frac{Number\ of\ favourable\ cases}{Total\ number\ of\ exhaustive\ cases}$

$= \frac{60}{80730} = \frac{6}{8073}$

11. What is the prob. of having a king and a queen, when 2 cards are drawn from a pack of 52 cards?

Sol: 2 cards can be drawn from a pack of 52 cards = $52C_2$ ways

$= \frac{52 \times 51}{1 \times 2} = 1326$ ways

1 queen card can be drawn from 4 queen cards = $4C_1$ ways

1 king card can be drawn from 4 king cards = $4C_1$ ways

Favourable cases = 4 × 4 = 16 ways

P(drawing 1 queen & 1 king card) = $\frac{Number\ of\ favourable\ cases}{Total\ number\ of\ exhaustive\ cases}$

$$= \frac{16}{1326} = \frac{8}{663}$$

12. What is the prob. that out of 6 cards taken from a full pack, 3 will be black and 3 will be red?

Sol: A full pack contains 52cards. Out of 52 cards, 26 cards are red & 26 black cards .

6 cards can be chosen from 52 cards = $52C_6$ ways

3 black cards can be chosen from 26 black cards = $26C_3$ ways

3 red cards can be chosen from 26 red cards = $26C_3$ ways

Favourable cases = $26C_3 \times 26C_3$

Probability $= \frac{Number\ of\ favourable\ cases}{Total\ number\ of\ exhaustive\ cases}$

$$= \frac{26C_3 \times 26C_3}{52C_6}$$

13. Find the prob. that a hand at bridge will consist of 3 spades, 5 hearts, 2 diamonds & 3 clubs?

Sol: Total no. of balls = 3 + 5 + 2 + 3 = 13

From 52 cards, 13 cards are chosen in $52C_{13}$ ways

In a pack of 52 cards, there are 13 cards of each type.

3 spades can be chosen from 13 spades = $13C_3$ ways

5 hearts can be chosen from 13 hearts = $13C_5$ ways

2 diamonds can be chosen from 13 diamonds = $13C_2$ ways

3 clubs can be chosen from 13 clubs = $13C_3$ ways

Hence the total no. of favourable cases are = $13C_3 \times 13C_5 \times 13C_2 \times 13C_3$

Probability $= \frac{Number\ of\ favourable\ cases}{Total\ number\ of\ exhaustive\ cases}$

$= \frac{13C_3 \times 13C_5 \times 13C_2 \times 13C_3}{52C_{13}}$

OPERATIONS ON SETS:

If A & B are any two sets, then

i) UNION OF TWO SETS

$A \cup B = \{x: x \in A\ (or)\ x \in B\}$

In general, $A_1 \cup A_2 \cup \ldots..\cup A_n$ = $\{x: x \in A_1\ or\ x \in A_2\ or \ldots\ldots..or\ x \in A_n\}$

i.e) $\bigcup_{i=1}^{n} A_i = \{x: x \in A_i,$ for atleast one i$\}$

ii) INTERSECTION OF TWO SETS

$A \cap B = \{x: x \in A\ \&\ x \in B\}$

In general, $A_1 \cap A_2 \cap \ldots..\cap A_n$ = $\{x: x \in A_1\ and\ x \in A_2\ and \ldots\ldots..and\ x \in A_n\}$

i.e) $\bigcap_{i=1}^{n} A_i$ = $\{x: x \in A_i,$ for all i = 1,2,3…..n$\}$

iii) COMPLEMENT OF A SET

$A'\ or\ \bar{A} = \{x: x \notin A\}$

iv) DIFFERENCE OF TWO SETS

A – B = $\{x: x \in A\ but\ x \notin B\}$

COMMUTATIVE LAW:

$A \cup B = B \cup A\ \ \&\ A \cap B = B \cap A$

ASSOCIATIVE LAW:

$(A \cup B) \cup C = A \cup (B \cup C)\ \&\ (A \cap B) \cap C = A \cap (B \cap C)$

DISTRIBUTIVE LAW:

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

COMPLEMENTARY LAW:

$$A \cup A' = S \;\&\; A \cap A' = \emptyset$$

AXIOMATIC APPROACH TO PROBABILITY:

It is a rule which associates to each event a real number P (A) which satisfies the following three axioms.

AXIOM I : For any event A, P (A) $\geq$ 0.

AXIOM II : P (S) =1

AXIOM III: If $A_1, A_2, \ldots, A_n$ are finite number of disjoint event of S, then

$$P(A_1, A_2, \ldots, A_n) = P(A_1) + P(A_2) + \ldots + P(A_n)$$

$$= \sum P(A_i)$$

THEOREMS ON PROBABILITY:

THEOREM 1: Probability of an impossible event is zero. i.e) P ($\emptyset$) = 0

THEOREM 2: Probability of the complementary event $\bar{A}$ of A is given by, P ($\bar{A}$) =1 – P(A).

THEOREM 3: For any two events A & B, P ($\bar{A} \cap B$) = P (B) – P($A \cap B$).

THEOREM 4: If A and B are two events such that $A \subset B$, then P ($B \cap \bar{A}$) = P (B) – P (A).

THEOREM 5: If $B \subset A$, then P (A) $\geq$ P (B).

THEOREM 6: If $A \cap B = \emptyset$, then P (A) $\leq$ P ($\bar{B}$).

LAW OF ADDITION OF PROBABILITIES:

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$, where A & B are any two events and are not disjoint.

PROBLEMS:

1. If from a pack of cards a single card is drawn. What is the prob. that it is either a spade or a king?

Sol: $P(A) = P(\text{a spade card}) = \frac{13}{52}$

$P(B) = P(\text{a king card}) = \frac{4}{52}$

P (either a spade or a king card) = P (A or B)

$$= P(A \cup B)$$

$$= P(A) + P(B) - P(A \cap B)$$

$$= \frac{13}{52} + \frac{4}{52} - \left[\frac{13}{52} \times \frac{4}{52}\right]$$

$$= \frac{4}{13}$$

2. A person is known to hit the target in 3 out of 4 shots, whereas another person is known to hit the target in 2 out of 3 shots. Find the probability of the targets being hit at all when they both person try.

Sol: The prob. that the first person hit the target = P (A) = $\frac{3}{4}$

The prob. that the second person hit the target = P (B) = $\frac{2}{3}$

The two events are not mutually exclusive, since both persons hit the same target.

$$P(A \text{ or } B) = P(A \cup B)$$

$$= P(A) + P(B) - P(A \cap B)$$

$$= \frac{3}{4} + \frac{2}{3} - \left[\frac{3}{4} \times \frac{2}{3}\right]$$

$$= \frac{11}{12}$$

MULTIPLICATION LAW OF PROBABILITY (INDEPENDENT EVENTS):

If A & B are two independent events, then

$$P(A \cap B) = P(\text{Both A \& B will happen})$$

$$= P(A) \times P(B)$$

PROBLEMS:

1. If P (A) = 0.35, P (B) = 0.73, P ($A \cap B$) = 0.14. Find P ($\bar{A} \cup \bar{B}$)

Sol: Using Demargon's Law,

$$\bar{A} \cup \bar{B} = \overline{A \cup B}$$

$$P(\bar{A} \cup \bar{B}) = P(\overline{A \cup B})$$

$$P(\bar{A} \cup \bar{B}) = 1 - P(A \cap B)$$

$$= 1 - 0.14 = 0.86$$

2. A bag contains 8 white and 10 black balls. Two balls are drawn in succession. What is the prob. that first is white and second is black.

Sol: Total no. of balls = 8 + 10 = 18

P (drawing one white ball from 8 balls) = $\frac{8}{18}$

P (drawing one black ball from 10 balls) = $\frac{10}{18}$

P (drawing first white & second black) = $\frac{8}{18} \times \frac{10}{18}$

$$= \frac{20}{81}$$

3. Two persons A & B appear in an interview for 2 vacancies for the same post. The probability of A's selection is $\frac{1}{7}$ and that of B's selection is $\frac{1}{5}$. What is the probability that, i) both of them will be selected, ii) none of them will be selected.

Sol: P (A selected) = $\frac{1}{7}$

P (B selected) = $\frac{1}{5}$

P (A will not be selected) = 1 - $\frac{1}{7} = \frac{6}{7}$

P (B will not be selected) = 1 - $\frac{1}{5} = \frac{4}{5}$

i) P (Both of them will be selected) = P (A) × P (B)

$$= \frac{1}{7} \times \frac{1}{5}$$

$$= \frac{1}{35}$$

ii) P (none of them will be selected) = P (A) × P (B)

$$= \frac{6}{7} \times \frac{4}{5}$$

$$= \frac{24}{35}$$

4. A problem in mathematics is given to 3 students A, B, C whose chances of solving it are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ respectively. What is the prob. that the problem will be solved?

Sol: P (A will not solve the problem) = $1- \frac{1}{2} = \frac{1}{2}$

P (B will not solve the problem) = $1- \frac{1}{3} = \frac{2}{3}$

P (C will not solve the problem) = $1- \frac{1}{4} = \frac{3}{4}$

P (all three will not solve the problem) = $\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4}$

$= \frac{1}{4}$

$\therefore$ P (all the three will solve the problem) = $1- \frac{1}{4} = \frac{3}{4}$

5. What is the chance of getting two sixes in two rolling of a single die?

Sol: P (getting a six in first rolling) = $\frac{1}{6}$

P (getting a six in second rolling) = $\frac{1}{6}$

Since two rolling are independent.

$\therefore$ P (getting two sixes in 2 rolls) = $\frac{1}{6} \times \frac{1}{6}$

$= \frac{1}{36}$

6. An article manufactured by a company consists of two parts A & B. In the process of manufacture of part A, 9 out of 100 are likely to be defective. Similarly, 5 0ut of 100 are likely to be defective in the manufacture of part B. Calculate the prob. that the assembled article will not be (assuming that the events of finding the part A non-defective and that of B are independent).

Sol: Prob. that part A will be defective = $\frac{9}{100}$

$\therefore$ P (A will not be defective) = $1- \frac{9}{100}$

$= \frac{100-9}{100}$

$= \frac{91}{100}$

Prob. that part B will be defective = $\frac{5}{100}$

$\therefore$ P (A will not be defective) = 1- $\frac{5}{100}$

$= \frac{100-5}{100}$

$= \frac{95}{100}$

$\therefore$ P (the assembled article will not be defective) = P (A will not be defective) $\times$

P (B will not be defective)

$= \frac{91}{100} \times \frac{95}{100}$

= 0.86

7. From a bag containing 4 white and 6 black balls, two balls are drawn at random. If the balls are drawn one after the other without replacement, find the probability that
   i) both balls are white.
   ii) both balls are black.
   iii) the first ball is white and the second ball is black.
   iv) one ball is white and the other is black.

Sol: Total no. of balls = 4 + 6 = 10

i) P (first ball is white) = $\frac{4}{10}$

P (second ball is white) = $\frac{3}{9}$

$\therefore$ P (both balls are white) = $\frac{4}{10} \times \frac{3}{9}$

$= \frac{2}{15}$

ii) P (first ball is black) = $\frac{6}{10}$

P (second ball is black) = $\frac{5}{9}$

$\therefore$ P (both balls are black) = $\frac{6}{10} \times \frac{5}{9}$

$= \frac{1}{3}$

iii) P (first ball is white) = $\frac{4}{10}$

P (second ball is black) = $\frac{6}{9}$

$\therefore$ P (first ball is white & second ball is black) = $\frac{4}{10} \times \frac{6}{9}$

$= \frac{4}{15}$

iv) a) P (first ball is white & second ball is black) = $\frac{4}{10} \times \frac{6}{9}$

$= \frac{24}{90}$

b) P (first ball is black & second ball is white) = $\frac{6}{10} \times \frac{4}{9}$

$= \frac{24}{90}$

Hence both events (a) & (b) are mutually exclusive.

$\therefore$ P (one ball is white & the other is black) = $\frac{24}{90} + \frac{24}{90}$

$= \frac{8}{15}$

8. Find the probability in each of the below four cases, if the balls are drawn one after the other with replacement. A bag containing 4 white & 6 black balls, 2 balls are drawn at random.
   i) both balls are white.
   ii) both balls are black.
   iii) the first ball is white and the second ball is black.
   iv) one ball is white and the other is black.

Sol: Total no. of balls = 4 + 6 = 10

i) P (first ball is white) = $\frac{4}{10}$

P (second ball is white) = $\frac{4}{10}$

$\therefore$ P (both balls are white) = $\frac{4}{10} \times \frac{4}{10}$

$= \frac{4}{25}$

ii) P (first ball is black) = $\frac{6}{10}$

P (second ball is black) = $\frac{6}{10}$

∴ P (both balls are black) = $\frac{6}{10} \times \frac{6}{10}$

$= \frac{9}{25}$

iii) P (first ball is white) = $\frac{4}{10}$

P (second ball is black) = $\frac{6}{10}$

∴ P (first ball is white & second ball is black) = $\frac{4}{10} \times \frac{6}{10}$

$= \frac{6}{25}$

iv) P (first ball is white & second ball is black) = $\frac{4}{10} \times \frac{6}{10}$

$= \frac{6}{25}$

CONDITIONAL PROBABILITY:

The conditional probability of event A, when the event B has already happened is defined as,

$$P\,(A/B) = \frac{P\,(A \cap B)}{P\,(B)} \quad , P\,(B) \neq 0 \ \ \text{(OR)}\ P\,(A \cap B) = P\,(A/B)\,.\,P\,(B)$$

If A & B are mutually exclusive events then,

$$P\,(B/A) = \frac{P\,(A \cap B)}{P\,(A)} \quad , P\,(A) \neq 0$$

PROBLEMS:

1. A bag contains 3 red & 4 white balls. Two draws are made without replacement. What is the prob. that both the balls are red.

Sol: P (drawing a red ball in the first draw) = $\frac{3}{7}$

i.e) P (A) = $\frac{3}{7}$

P (drawing a red ball in the first draw given that first ball drawn is red) = $\frac{2}{6}$

i.e) $P\,(B/A) = \frac{2}{6}$

$\therefore P\ (A \cap B) = P\ (B/A)\ \times P\ (A)$

$= \frac{2}{6} \times \frac{3}{7}$

$= \frac{1}{7}$

2. Find the prob. of drawing a queen and a king from a pack of cards in two consecutive draws, the cards drawn not being replaced.

Sol: P (drawing a queen card) = $\frac{4}{52}$

i.e) P (A) = $\frac{4}{52}$

P (drawing a king after a queen has been drawn) = $\frac{4}{51}$

i.e) $P\ (B/A) = \frac{4}{51}$

$\therefore P\ (A \cap B) = P\ (B/A)\ \times P\ (A)$

$= \frac{4}{51} \times \frac{4}{52}$

$= \frac{4}{663}$

3. In a box there are 100 resistors having resistance and tolerance as shown in the following table. Let a resistor be selected from the box and assume each resistor has the same likelihood of being chosen. Define three events A as draw a 47Ω resistor, B as draw a resistor with 5% tolerance and C as draw a 100Ω resistor. Find $P\ (A/B), P\ (A/C), P\ (B/C)$.

Resistance Ω	5%	10%	Total
22	10	14	24
47	28	16	44
100	24	8	32
Total	62	38	100

Sol: P (A) = P (47Ω) = $\frac{44}{100}$

P (B) = P (5%) = $\frac{62}{100}$

$$P\ (C) = P\ (100\Omega) = \frac{32}{100}$$

The joint probabilities are,

$$P\ (A \cap B) = P\ (47\Omega \cap 5\%)$$

$$= \frac{28}{100}$$

$$P\ (A \cap C) = P\ (47\Omega \cap 100\Omega)$$

$$= 0$$

$$P\ (B \cap C) = P\ (5\% \ \cap 100\Omega)$$

$$= \frac{24}{100}$$

$$\therefore P\ (A/B) = \frac{P\ (A \cap B)}{P\ (B)} = \frac{28/100}{62/100}$$

$$= \frac{28}{62}$$

$$P\ (A/C) = \frac{P\ (A \cap C)}{P\ (C)} = \frac{0}{32/100}$$

$$= 0$$

$$P\ (B/C) = \frac{P\ (B \cap C)}{P\ (C)} = \frac{24/100}{32/100}$$

$$= \frac{24}{32}$$

4. The Hindu newspaper publishes three columns entitled politics (A), books(B), cinema(C). Reading habits of a randomly selected reader with respect to three columns are,

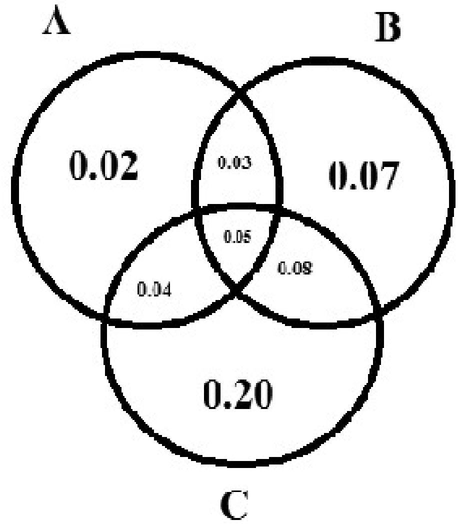
Read Regularly	A	B	C	A ∩ B	A ∩ C	B ∩ C	A ∩ B ∩ C
Probability	0.14	0.23	0.37	0.08	0.09	0.13	0.05

Find P (A/B), P (A/B∪C), P (A/reads atleast one), P (A∪B /C).

Sol: $$P\ (A/B) = \frac{P\ (A \cap B)}{P\ (B)}$$

$$= \frac{0.08}{0.23}$$

$$= 0.348$$



$$P\,(A/B\ \cup C) = \frac{P\,[A \cap (B \cup C)]}{P\,(B \cup C)}$$

$$= \frac{0.04+0.05+0.03}{0.47}$$

$$= 0.255$$

P (A / reads atleast one) = P [A / (A$\cup$ B $\cup$ C)]

$$= \frac{P\,[A\cap(A\cup B\cup C)]}{P\,(A\cup B\cup C)}$$

$$= \frac{P\,(A)}{P\,(A\cup B\cup C)}$$

$$= \frac{0.14}{0.49}$$

$$= 0.286$$

$$P\,(A \cup B\,/\,C) = \frac{P\,[(A\cup B)\cap C]}{P\,(C)}$$

$$= \frac{0.04+0.05+0.08}{0.37}$$

$$= 0.459$$