Lecture 07

Software Reliability

Software Reliability

- In the broad sense
  - S/w design will operate (execute) well for a substantial time period.
- In a narrow sense
  - Reliability is a metric which is probability of operational success of the s/w
- Probabilistic and Deterministic Model
  - Example – Winding of a motor bourns out
    - Due to temperature above melting point

(It will burns – it is certainly deterministic, But we are unable to predict when they will occur – it is probabilistic)

Software Reliability

- Thus the model of failures become a probabilistic one and the random element (random variable) is the time to failure

- Failure Modes
  - Hardware
  - Software
  - Skinware(Human)

- Reliability Theory
  - Modeling of failure and prediction of success probability
  - Based on the concept of random/continuous variable, probability density function, probability distribution function

Software Reliability

- Probability concept :

- Discrete random variable, X is a discrete random variable

  1) Probability Density Function :

     Probability of occurrence $P(x_i) = f(x_i)$

  2) Distribution Function : Defined in term of the probability that $X \leq x$:

$$P(X \leq x) \equiv F(x) = \sum_{X \leq x} f(x)$$

- Continuous random variable,

  The variable is continuous over some range of definition

  Density function, $P(t < x < t + \Delta t) = f(t)\Delta t$

Software Reliability

- Distribution Function: F(x)= $\int f(x)dx$
  - Let x take on all values between points a and b,

  So, $P(X \le x) = F(x) = \int_a^x f(x)dx \quad for\ a < x \le b$

  $dF(x)/dx = f(x)$

  - The Probability that X lies in an interval x < X <x+dx ,

$$P(x < X < x + dx) = P(X \le x + dx) - P(X \le x)$$

$$= \int_a^{x+dx} f(x)dx - \int_a^{x} f(x)dx$$

$$= \int_x^{x+dx} f(x)dx = F(x + dx) - F(x)$$

F(x) is a continuous, dx→0;P(X=x) is zero

Definition of Reliability

- R(t) =Reliability is the mathematical probability is function of time t, is the success.

R(t) = P(x>t) = 1- P(x<t) where x is continuous

= P(x>=t) = 1 – P(x<=t) where x is discrete

- Hazard rate (conitional failure rate), z(t)
  - Defined in term of the probability that a failure occurs in the same interval t to t+ Δt , given that system has survival up to time t

$$Z(t)\,\Delta t = P(t < x < t + \Delta t \mid x > t)$$

We know, F(t)=1-R(t)

Relationship R(t) and Z(t)

- We know, from definition of Hazard rate

$$Z(t)\,\Delta t = P(t < x < t + \Delta t \mid x > t) \qquad [P(AB)=P(A).P(B/A)]$$

$$Z(t) = P(t < x < t + \Delta t)/\Delta t.P(x > t) \qquad [P(AB)=P(B).P(A/B)]$$

$$= \{P(x < t + \Delta t) - P(x < t)\} / \Delta t.P(x > t)$$

$$= \{1 - P(x > t + \Delta t)\} - \{1 - P(x > t)\} / \Delta t.P(x > t)$$

$$= -1/R(t) \times [R(t + \Delta t) - R(t)] / \Delta t$$

$$= -R'(t)/R(t)$$

$$\Rightarrow \int_0^t Z(t)\,dt = -\int_0^t R'(t)/R(t)\,dt$$

$$R(t) = e^{-\int_0^t Z(t)\,dt}$$

Mean Time To Failure (MTTF)

- Estimation theory:
  - How one determines the parameters in a probabilistic model from statistical data taken on the items governed by the model
  - Specifically, in reliability work we place a group of components on life test and observe the sequence of failure times t1,t2,…,tn
  - On the basis of these data we compute time-to-failure models and hazard models
  - Estimation theory provides guidelines for efficient and accurate components

Mean Time To Failure (MTTF)

- $n \leq 5$ (few data) → result must be questioned
- $n \geq 100$ → result should be good
- $n \to \infty$ →many of the different computational scheme is need
- $10 \leq n \leq 50$ →best for estimation theory
- A point estimation formula, MTTF=(10+20+25+35+40)/5
- =26h
- It is often convenient to characterize a failure model using set of failure data by a single parameter (MTTF/MTBF)

Mean Time To Failure (MTTF)

- If we have life test information on a population of n items with failure times t1,t2,…tn
- MTTF= $1/n\sum_{i=1}^{n} t_i$
- Expected value, $E(x) = \sum_{i=1}^{n} x_i f(x_i);\ x = x_{1,}\ x_{2,\ldots,}\ x_n$ where x

is discrete random variable

$$E(x) = \int_a^b x_i f(x_i)dx;\ a \le x \le b$$

where x is continuous random variable

Mean Time To Failure (MTTF)

- Using Hazard model, the MTTF for the probability distribution defined by the model is,
- MTTF= $E(t) = \int_0^\infty t\, f(t)\, dt$
- Let, I= $\int t\, f(t)\, dt$

$$= t \int t\, f(t)\, dt - \int [d/dt(t) \int f(t)\, dt]\, dt$$

$$= -tR(t) + \int R(t)\, dt$$

$$E(t) = \int_0^\infty R(t)\, dt$$

f(t)=dF(t)/dt

=d(1-R(t))/dt

=-dR(t)/dt

Mean Time To Failure (MTTF)

- Case I :Constant hazard, $R(t) = e^{-\lambda t}$
- Case II : Linearly increasing hazard, $R(t) = e^{-kt^2/2}$
- Case III : Weibull distribution, $R(t) = e^{-kt^{(m+1)}/(m+1)}$