LR Parsers

All grammars are not LL(1)

Z → d
Z → XYZ
Y → ε
Y → c
X → Y
X → a

	a	c	d	$
X	X → a X → Y	X → Y	X → Y	
Y	Y → **ε**	Y → c Y → **ε**	Y → **ε**	
Z	Z→XYZ	Z→XYZ	Z → d Z→XYZ	Z→XYZ

FIRST (X) = {a, c, ε}
FIRST (Y) = {c, ε}}
FIRST (Z) = {d, a, c}
FOLLOW(X) = {c, d, a}
FOLLOW(Y) = {a, c, d}
FOLLOW(Z) = {$}

All grammars are not LL(1)

A grammar is an LL(1) grammar if all productions conform to the following conditions:

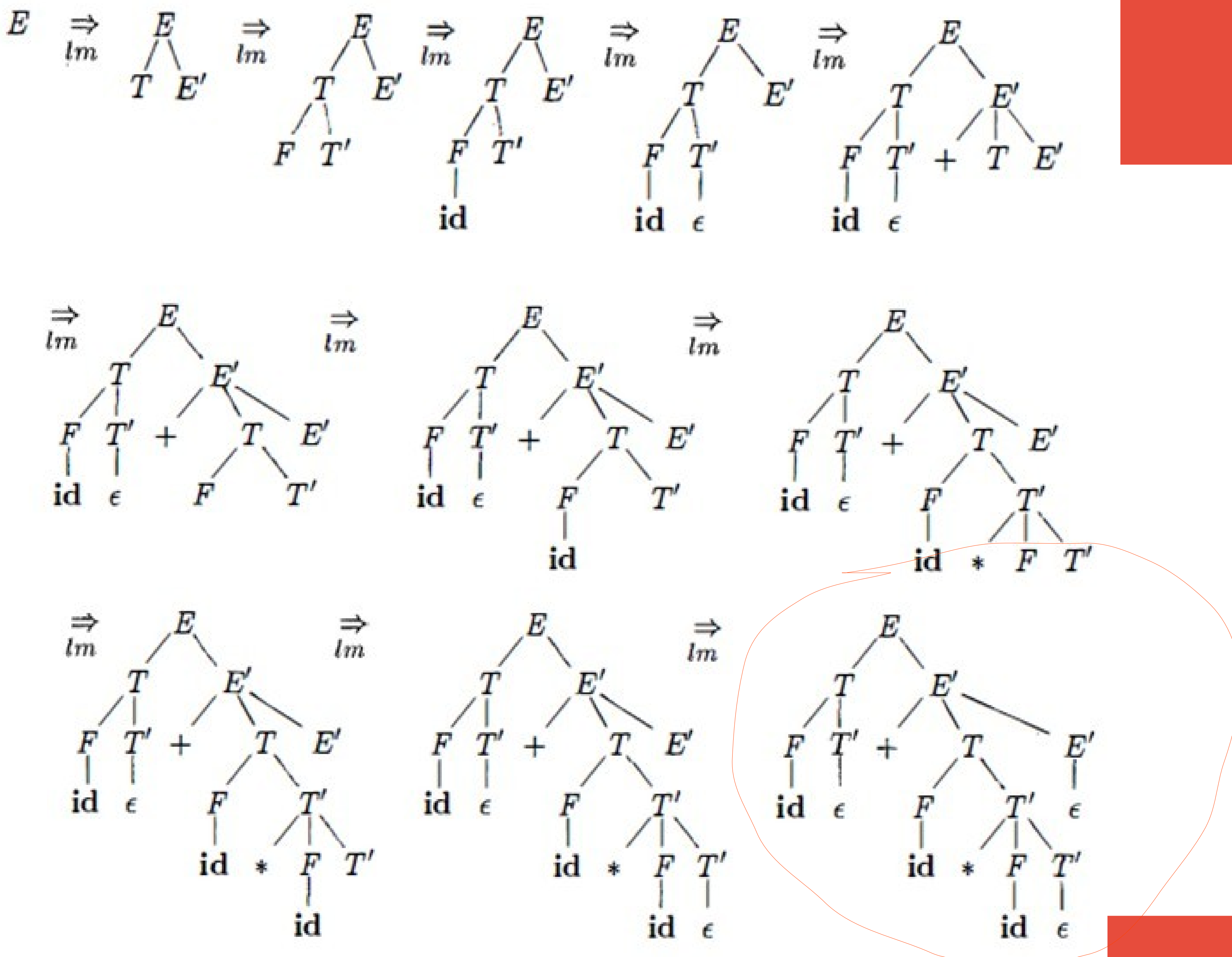
1. For each production $A \rightarrow \sigma 1 \mid \sigma 2 \mid \sigma 3 \ldots \mid \sigma n$,

$\text{FIRST}(\sigma_i) \cap \text{FIRST}(\sigma_j) = \emptyset$, for all i, j, i <> j

2. If nonterminal X derives an empty string, then

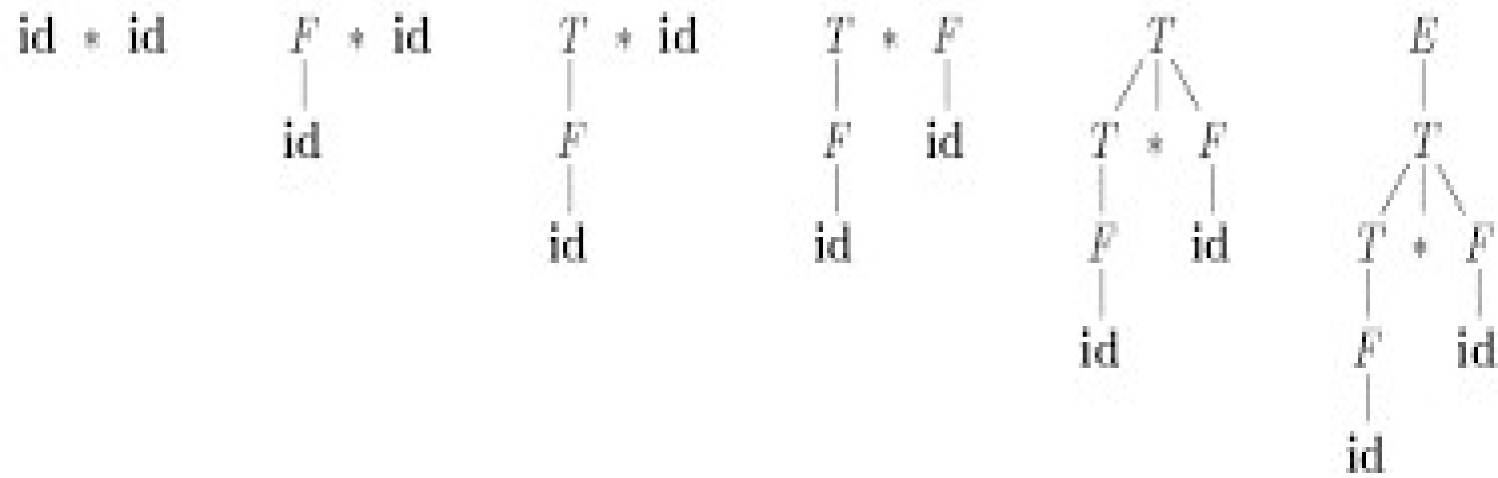
$\text{FIRST}(X) \cap \text{FOLLOW}(X) = \emptyset$

TOP DOWN PARSING



Bottom up parsing

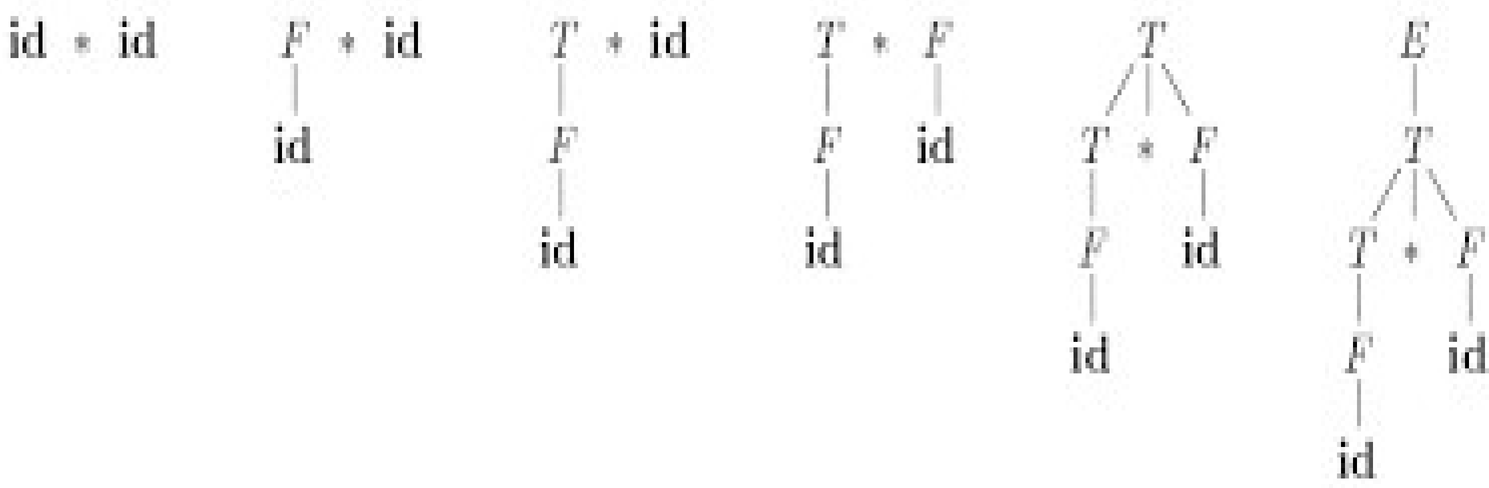
E → E + T | T
T → T * F | F
F → (E) | id



E => T => T * F => T * id => F * id => id * id

Bottom up parsing

$E \rightarrow E + T \mid T$
$T \rightarrow T * F \mid F$
$F \rightarrow (E) \mid id$



$E \Rightarrow \underline{T} \Rightarrow \underline{T * F} \Rightarrow T * \underline{id} \Rightarrow \underline{F} * id \Rightarrow \underline{id} * id$

Bottom up parsing

A handle of a string is a substring that matches the right side of a production, and whose reduction to the non-terminal on the left side of the production represents one step along the reverse of a rightmost derivation.

(The string, together with its position in the right sentential form where it occurs and the production used to reduce it).

Reducing β to A in $\alpha\beta w$ is termed as "pruning the handle"

Where w contains only terminal symbols.

Shift-reduce parsing

Step 1: Locating a substring

Step 2: Choosing a production

	Stack	Input
Initial condition	$	w$
Final condition	$S	$

Viable Prefix- The sequence of symbols on the parsing stack

Stack	Input	Action
$	id * id $	shift
$ id	* id $	reduce F → id
$ F	* id $	shift
$ F	* id $	reduce T → F
$ T	* id $	shift
$ T *	id $	shift
$ T * id	$	reduce F → id
$ T * F	$	reduce T→T* F
$ T	$	reduce E → T
$ E	$	Accept

LR Parsing

- **LR(k) parsing, introduce by D. Knuth in 1965**
  - L is for Left-to-right scanning of the input
  - R is for reverse Rightmost derivation
  - k is the number of lookahead tokens
- **Most general non-backtracking shift-reduce parsing method**
- **Can be constructed to recognise virtually all programming language constructs**
- **The class of grammar is a proper superset of the class of grammars that can be parsed with *predictive parsers***

LR Parsing

- **Different types:**
  - *Simple LR or SLR, the easiest method for constructing shift-reduce parsers*
  - *Canonical LR*
  - *LALR*
- **LR-parsers represent the DFA as a 2D – table**
- **Rows correspond to DFA states**
- **Columns correspond to terminals (action table) and non-terminals (goto table)**

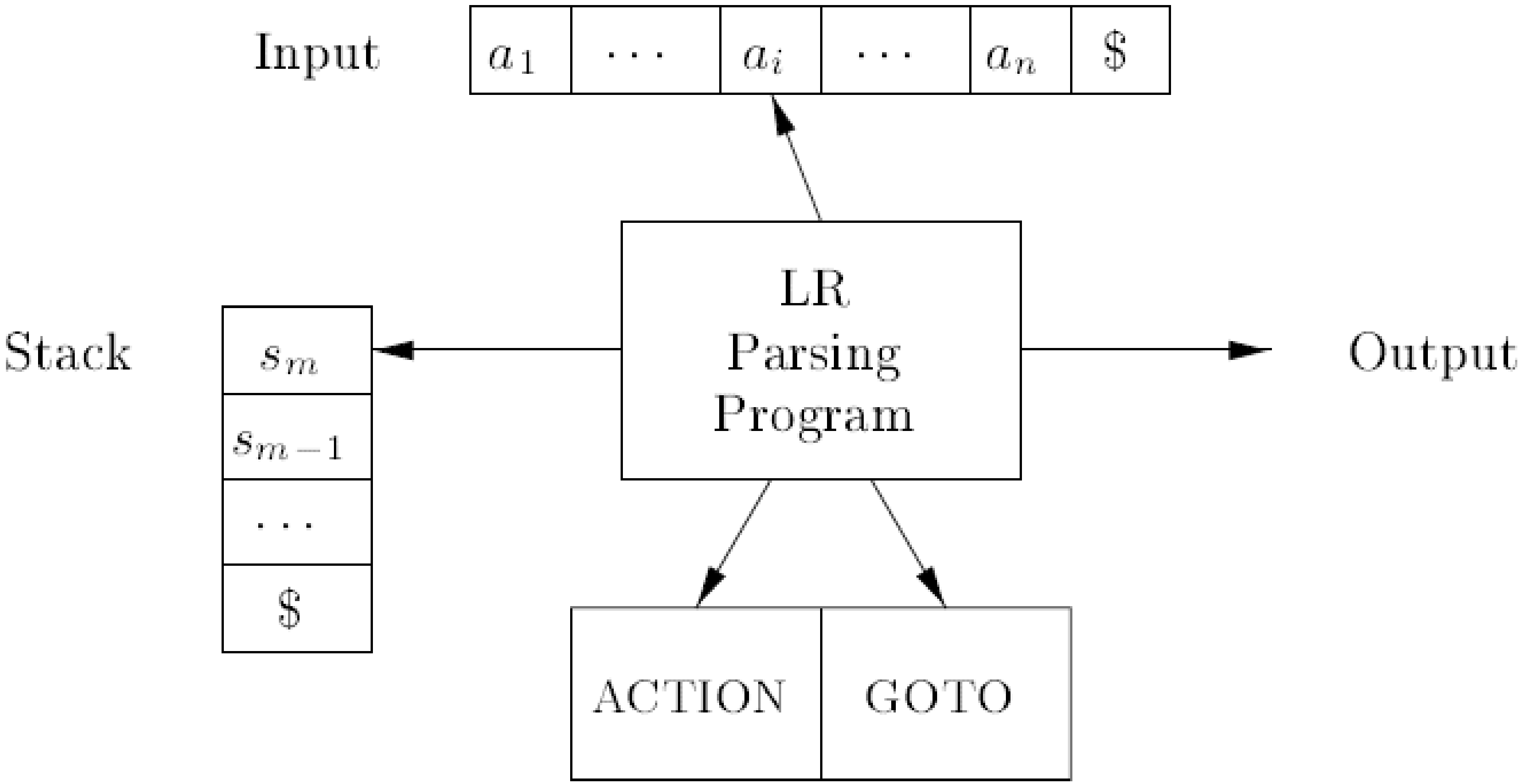
Transition between states

$$A \to a\,.\,X\,n \xrightarrow{\;X\;} A \to a\,X\,.\,n$$

Two possibilities

1. If X is a terminal, then push X onto the stack
2. If X is a non-terminal, X will be pushed onto the stack during a reduction by X → β.
3. Thus, we need to first recognize β
4. So for every production X → β, we need to indicate that X can be produced by reducing (recognising) any of the right hand sides of such productions.
5. This is basically the following transition.

$$A \to a\,.\,X\,n \xrightarrow{\;\varepsilon\;} X \to \beta$$



Input
a_1
$\cdots$
a_i
$\cdots$
a_n
\$
LR Parsing Program
Stack
s_m
s_{m-1}
$\cdots$
\$
Output
ACTION
GOTO

SLR Parsing

LR(0) - zero tokens of look-ahead

SLR - Simple LR: like LR(0), but uses FOLLOW sets to build more “precise” parsing tables

First we augment the grammar G with a new start symbol S’ and a production

S’ → S

Closure operation -

For a set of items I, we construct closure(I) as follows:

Every item in I is added to closure (I)

If A → α . B β is in closure (I) and B → γ is a production, then B → . γ is added to closure (I)

Goto operation -

Goto(I, X) is the closure of the set of all items [A → α X . β], such that [A→ α . X β] is in I.

Example

Augmented Grammar:

E’ → E

E → E + T | T

T → T* F | F

F → (E) | id

If I = { E’ → • E },

then CLOSURE(I) is

E’ → • E

E → • E + T

E → • T

T → • T * F

T → • F

F → • (E)

F → • id

Example

Augmented Grammar:

$E' \rightarrow E$

$E \rightarrow E + T \mid T$

$T \rightarrow T^* F \mid F$

$F \rightarrow (E) \mid id$

$I = \boxed{\begin{array}{l} E' \rightarrow E \bullet \\ E \rightarrow E \bullet + T \end{array}}$

GOTO(I, +)

$$\boxed{\begin{array}{l} E \rightarrow E + .\, T \\ T \rightarrow .\, T * F \\ T \rightarrow .\, F \\ F \rightarrow .\, (E) \\ F \rightarrow .\, \mathbf{id} \end{array}}$$

The Canonical LR(0) Collection -- Example

I_0: E' → .E
E → .E+T
E → .T
T → .T*F
T → .F
F → .(E)
F → .id

I_1: E' → E.
E → E.+T

I_2: E → T.
T → T.*F

I_3: T → F.

I_4: F → (.E)
E → .E+T
E → .T
T → .T*F
T → .F
F → .(E)
F → .id

I_5: F → id.

I_6: E → E+.T
T → .T*F
T → .F
F → .(E)
F → .id

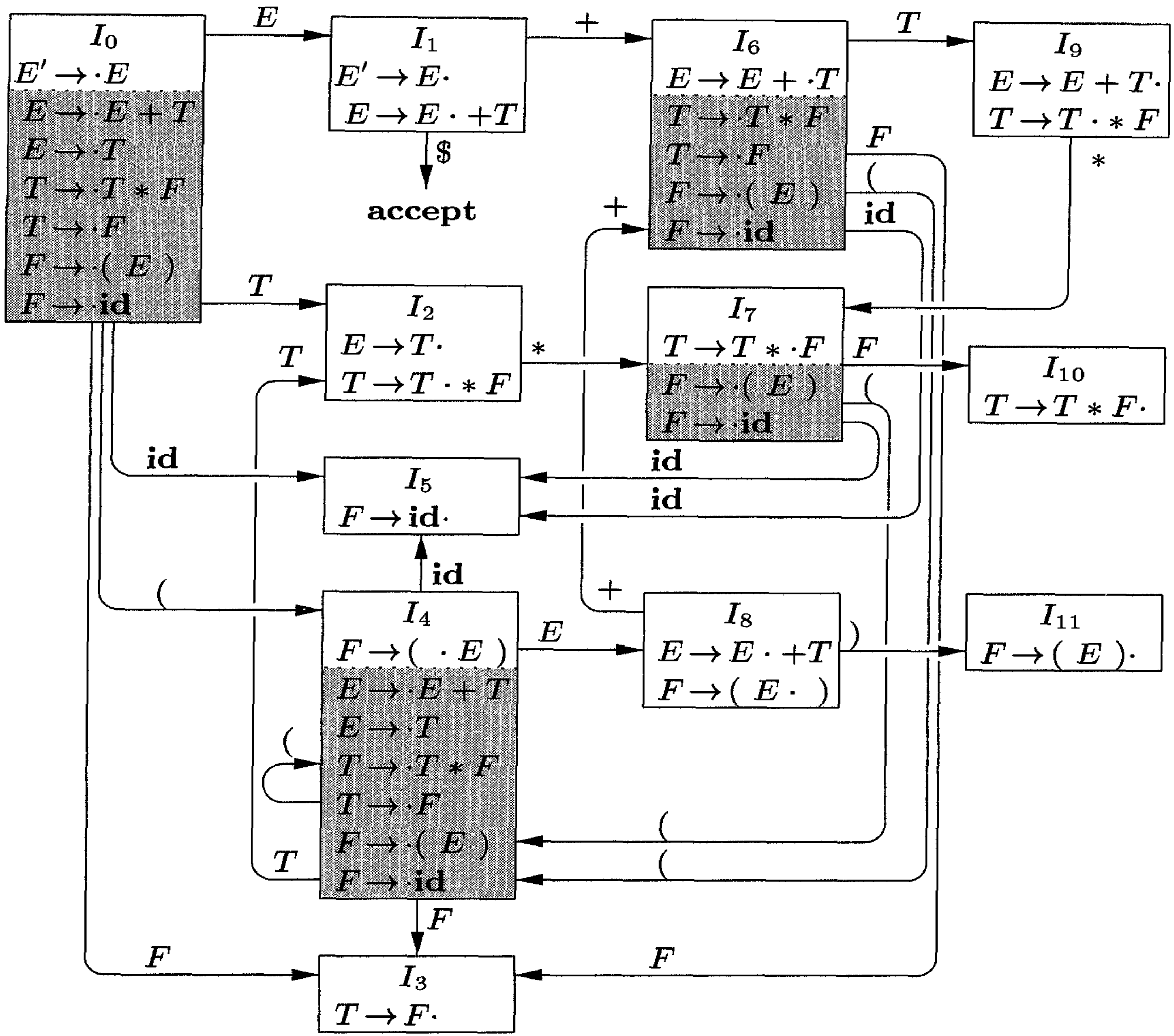
I_7: T → T*.F
F → .(E)
F → .id

I_8: F → (E.)
E → E.+T

I_9: E → E+T.
T → T.*F

I_{10}: T → T*F.

I_{11}: F → (E).



I0
E' → ·E
E → ·E + T
E → ·T
T → ·T * F
T → ·F
F → ·(E)
F → ·id
I1
E' → E·
E → E · +T
accept
I2
E → T·
T → T · * F
I3
T → F·
I4
F → (· E)
E → ·E + T
E → ·T
T → ·T * F
T → ·F
F → ·(E)
F → ·id
I5
F → id·
I6
E → E + ·T
T → ·T * F
T → ·F
F → ·(E)
F → ·id
I7
T → T * ·F
F → ·(E)
F → ·id
I8
E → E · +T
F → (E ·)
I9
E → E + T·
T → T · * F
I10
T → T * F·
I11
F → (E)·
E
T
F
id
(
)
+
*
$

Constructing SLR Parsing Table

Consider the grammar G'

1)Construct the canonical collection of sets of LR(0) items for G',

2)C ← {I_0, I_1 . . . I_n}

3)Create the parsing <u>action</u> table as follows:

1)If a is a terminal, A → α . a β in I_i, and goto(I_i,a) = I_j , then action[i,a] = shift j

2)If A → α . is in I_i , then action[i,a] is reduce A → α for all a in FOLLOW(A), where A <> S'

3)If S' → S is in I_{I}, then **action [I, $] is accept**

4)Create the parsing <u>goto</u> table

1)For all non-terminals A, if goto(I_i,A) = I_j , then goto[i,A] = j

5) **All other entries will be marked as error**

6)Initial state of the parser contains S' → S

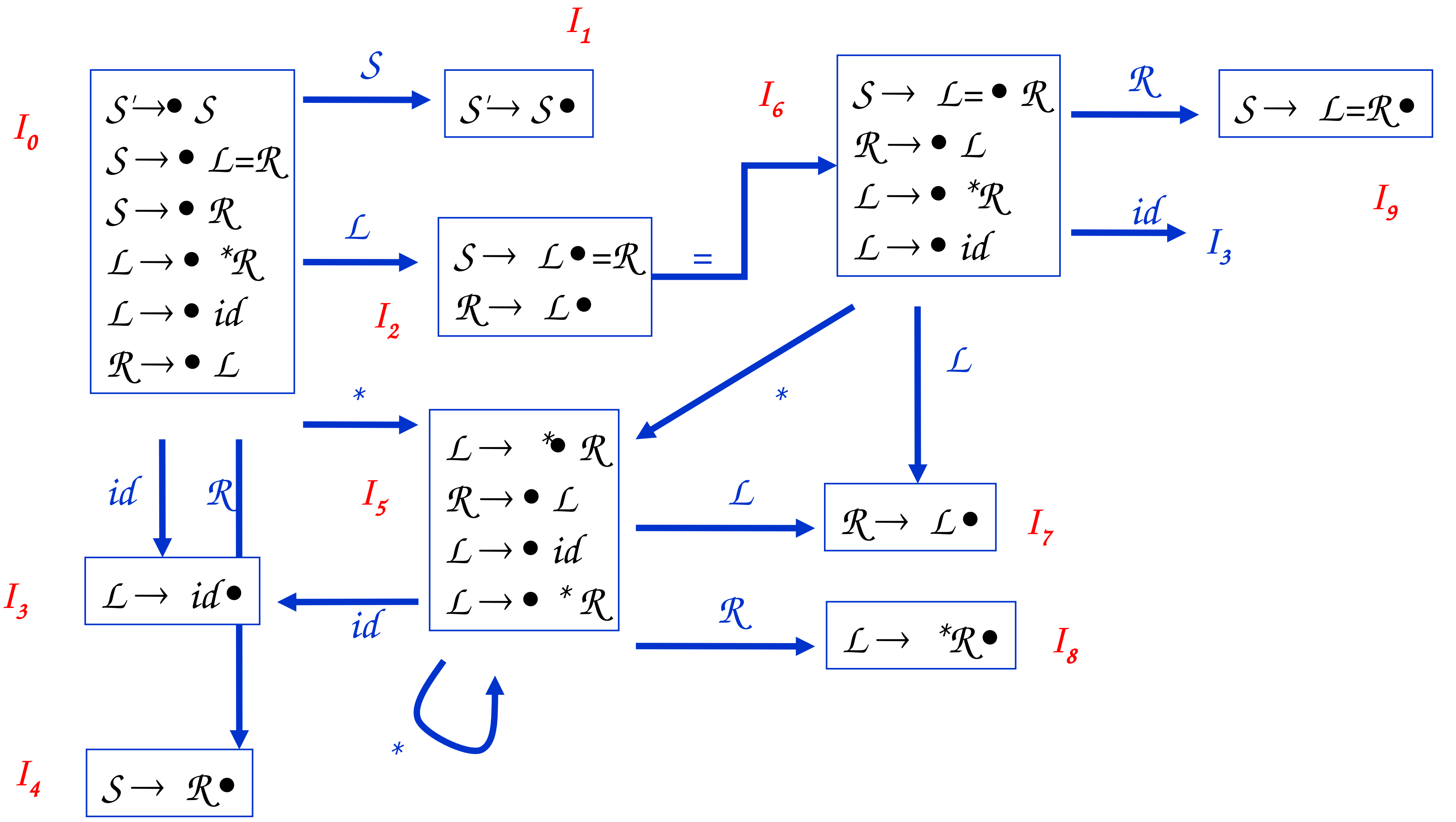
Parsing Tables of Expression Grammar

1. E' → E
2. E → E + T
3. E → T
4. T → T* F
5. T → F
6. F → (E)
7. F → id

	Action Table						Goto Table		
state	**id**	**+**	*****	**(**	**)**	**$**	**E**	**T**	**F**
0	s5			s4			1	2	3
1		s6				acc			
2		r2	s7		r2	r2			
3		r4	r4		r4	r4			
4	s5			s4			8	2	3
5		r6	r6		r6	r6			
6	s5			s4				9	3
7	s5			s4					10
8		s6			s11				
9		r1	s7		r1	r1			
10		r3	r3		r3	r3			
11		r5	r5		r5	r5			

Parsing

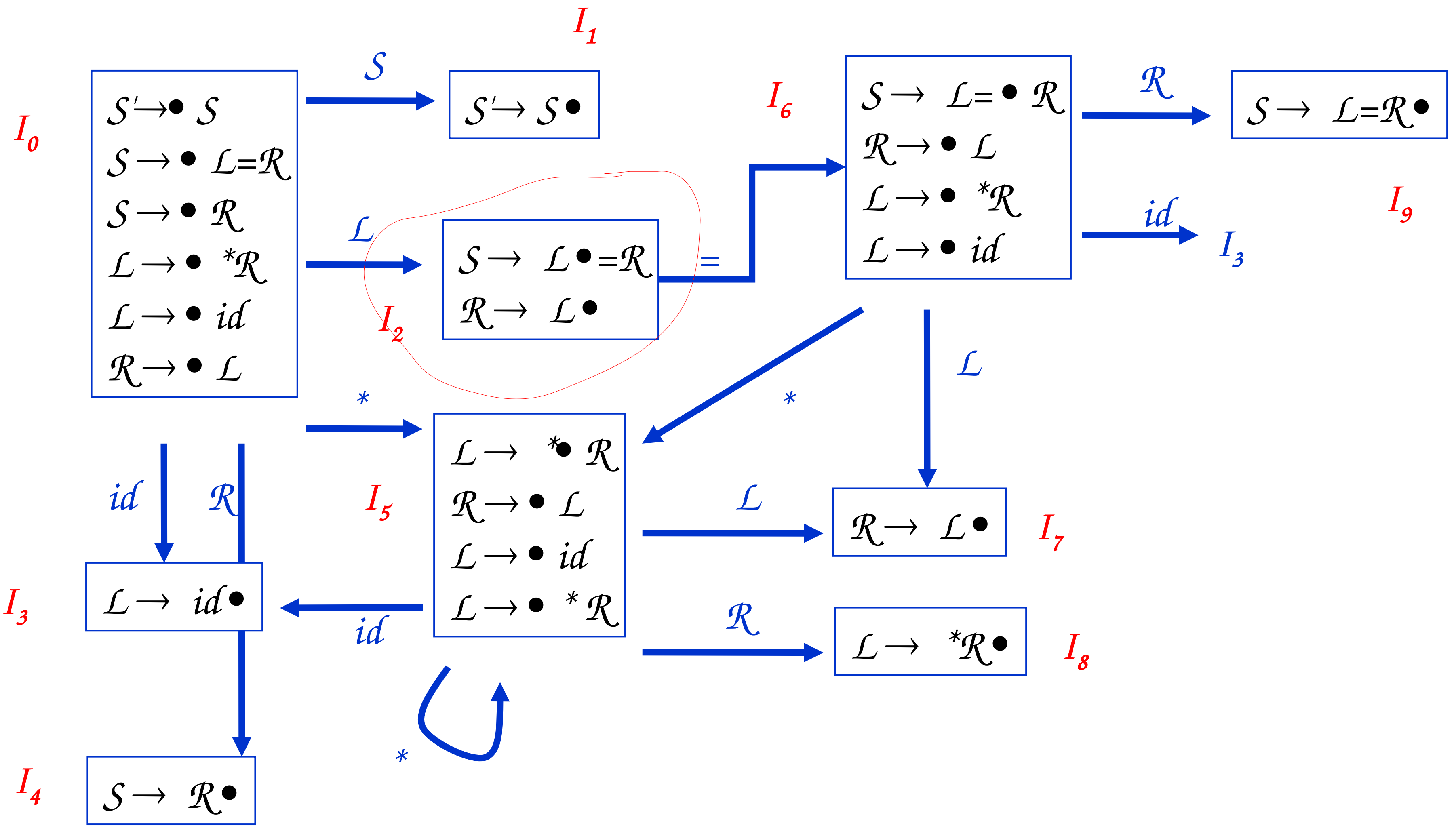
Stack	Input	Parser Action
S0	id+id $	Shift and move to state 5
S0 id S5	+id $	Reduce F → id
S0 F S3	+id $	Reduce T→ id
S0 T S2	+id $	Reduce E → T
S0 E S1	+id $	Shift and move to state 6
S0 E S1 + S6	id $	Shift and move to state 5
S0 E S1 + S6 id S5	$	Reduce F → id
S0 E S1 + S6 F S3	$	Reduce T → F
S0 E S1 + S6 T S9	$	Reduce E → E+T
S0 E S1	$	Accept



I0
S'→• S
S→• L=R
S→• R
L→• *R
L→• id
R→• L
S
I1
S'→ S•
L
I2
S→ L•=R
R→ L•
=
I6
S→ L=• R
R→• L
L→• *R
L→• id
R
S→ L=R•
I9
id
I3
*
L
I5
L→ *• R
R→• L
L→• id
L→• *R
L
R→ L•
I7
R
L→ *R•
I8
id
R
I3
L→ id•
id
*
I4
S→ R•

state		action	goto				
	id	=	*	$	S	L	R
0	s3		s5		1	2	4
1				accept			
2		s6/r(R→L)					
3		r(L→id)		r(L→id)			
4				r(S→R)			
5	s3		s5			7	8
6	s3		s5			7	9
7		r(R→L)		r(R→L)			
8		r(L→ *R)		r(L→ *R)			
9				r(S→L=R)			

state		action	goto				
	id	=	*	$	S	L	R
0	s3		s5		1	2	4
1				accept			
2		s6/r(R→L)					
3		r(L→id)		r(L→id)			
4				r(S→R)			
5	s3		s5			7	8
6	s3		s5			7	9
7		r(R→L)		r(R→L)			
8		r(L→ *R)		r(L→ *R)			
9				r(S→L=R)			



I0
S'→• S
S→• L=R
S→• R
L→• *R
L→• id
R→• L
S
I1
S'→ S•
L
S→ L•=R
R→ L•
I2
=
I6
S→ L=• R
R→• L
L→• *R
L→• id
R
S→ L=R•
I9
id
I3
*
L
*
id
R
I5
L→ *• R
R→• L
L→• id
L→• *R
L
R→ L•
I7
R
L→ *R•
I8
id
I3
L→ id•
*
I4
S→ R•

Two possible scenarios

1)S’ => S => L=R => L=L => L=id => id=id

2)S’ => S => L=R => L=L => L=id => *R=id

=> *L=id => *id=id

$S' \rightarrow S$

$S \rightarrow L\mathbf{=}R$

$S \rightarrow R$

$L \rightarrow \mathbf{*}R$

$L \rightarrow \mathbf{id}$

$R \rightarrow L$

$FOLLOW(S') = \{\mathbf{\$}\}$

$FOLLOW(S) = \{\mathbf{\$}\}$

$FOLLOW(L) = \{\mathbf{\$}, \mathbf{=}\}$

$FOLLOW(R) = \{\mathbf{\$}, \mathbf{=}\}$

Two possible scenarios

1)S’ => S => L=R => L=L => L=id => id=id

2)S’ => S => L=R => L=L => L=id => *R=id

=> *L=id => *id=id

$S' \rightarrow S$

$S \rightarrow L=R$

$S \rightarrow R$

$L \rightarrow {*}R$

$L \rightarrow \mathbf{id}$

$R \rightarrow L$

$FOLLOW(S') = \{\$\}$

$FOLLOW(S) = \{\$\}$

$FOLLOW(L) = \{\$, =\}$

$FOLLOW(R) = \{\$, =\}$

Two possible scenarios

1)S’ => S => L=R => L=L => L=id => id=id

2)S’ => S => L=R => L=L => L=id => *R=id

=> *L=id => *id=id

$S' \rightarrow S$

$S \rightarrow L = R$

$S \rightarrow R$

$L \rightarrow * R$

$L \rightarrow \textbf{id}$

$R \rightarrow L$

$FOLLOW(S') = \{\$\}$

$FOLLOW(S) = \{\$\}$

$FOLLOW(L) = \{\$, =\}$

$FOLLOW(R) = \{\$, =\}$

Two possible scenarios

1)S’ => S => L=R => L=L => L=id => id=id

2)S’ => S => L=R => L=L => L=id => *R=id

=> *L=id => *id=id

$S' \rightarrow S$

$S \rightarrow L = R$

$S \rightarrow R$

$L \rightarrow *R$

$L \rightarrow \mathbf{id}$

$R \rightarrow L$

$FOLLOW(S') = \{\$\}$

$FOLLOW(S) = \{\$\}$

$FOLLOW(L) = \{\$, =\}$

$FOLLOW(R) = \{\$, =\}$

Two possible scenarios

1)S’ => S => L=R => L=L => L=id => id=id

2)S’ => S => L=R => L=L => L=id => *R=id

=> *L=id => *id=id

$S' \rightarrow S$

$S \rightarrow L = R$

$S \rightarrow R$

$L \rightarrow * R$

$L \rightarrow id$

$R \rightarrow L$

$FOLLOW(S') = \{\$\}$

$FOLLOW(S) = \{\$\}$

$FOLLOW(L) = \{\$, =\}$

$FOLLOW(R) = \{\$, =\}$

Two possible scenarios

1)S’ => S => L=R => L=L => L=id => id=id

2)S’ => S => L=R => L=L => L=id => *R=id

=> *L=id => *id=id

$$S' \rightarrow S$$
$$S \rightarrow L = R$$
$$S \rightarrow R$$
$$L \rightarrow *R$$
$$L \rightarrow \textbf{id}$$
$$R \rightarrow L$$

$$FOLLOW(S') = \{\$\}$$
$$FOLLOW(S) = \{\$\}$$
$$FOLLOW(L) = \{\$, =\}$$
$$FOLLOW(R) = \{\$, =\}$$

Two possible scenarios

1)S' => S => L=R => L=L => L=id => id=id

2)S' => S => L=R => L=L => L=id => *R=id

=> *L=id => *id=id

$S' \rightarrow S$

$S \rightarrow L = R$

$S \rightarrow R$

$L \rightarrow * R$

$L \rightarrow id$

$R \rightarrow L$

$FOLLOW(S') = \{\$\}$

$FOLLOW(S) = \{\$\}$

$FOLLOW(L) = \{\$, =\}$

$FOLLOW(R) = \{\$, =\}$

Two possible scenarios

S' => S => L=R => L=L => L=id => id=id

Stack	Input	• Action
$id	=id$	Reduce
$L	=id$	Shift
$L=	id$	Shift
$L=id	$	Reduce
$L=L	$	Reduce
$L=R	$	

S' => S => L=R => L=L => L=id => *R=id => *L=id => *id=id

Stack	Input	• Action
$*id	=id$	Reduce
$*L	=id$	Reduce

*FOLLOW(L) = {**$**, **=**}*

Two types of Conflicts in LR parsing

- shift/reduce conflict
  - On some particular lookahead it is possible to shift or reduce
  - The if/else ambiguity would give rise to a shift/reduce conflict

- reduce/reduce
  - This occurs when a state contains more than one handle that may be reduced on the same lookahead.

Two types of Conflicts in LR parsing

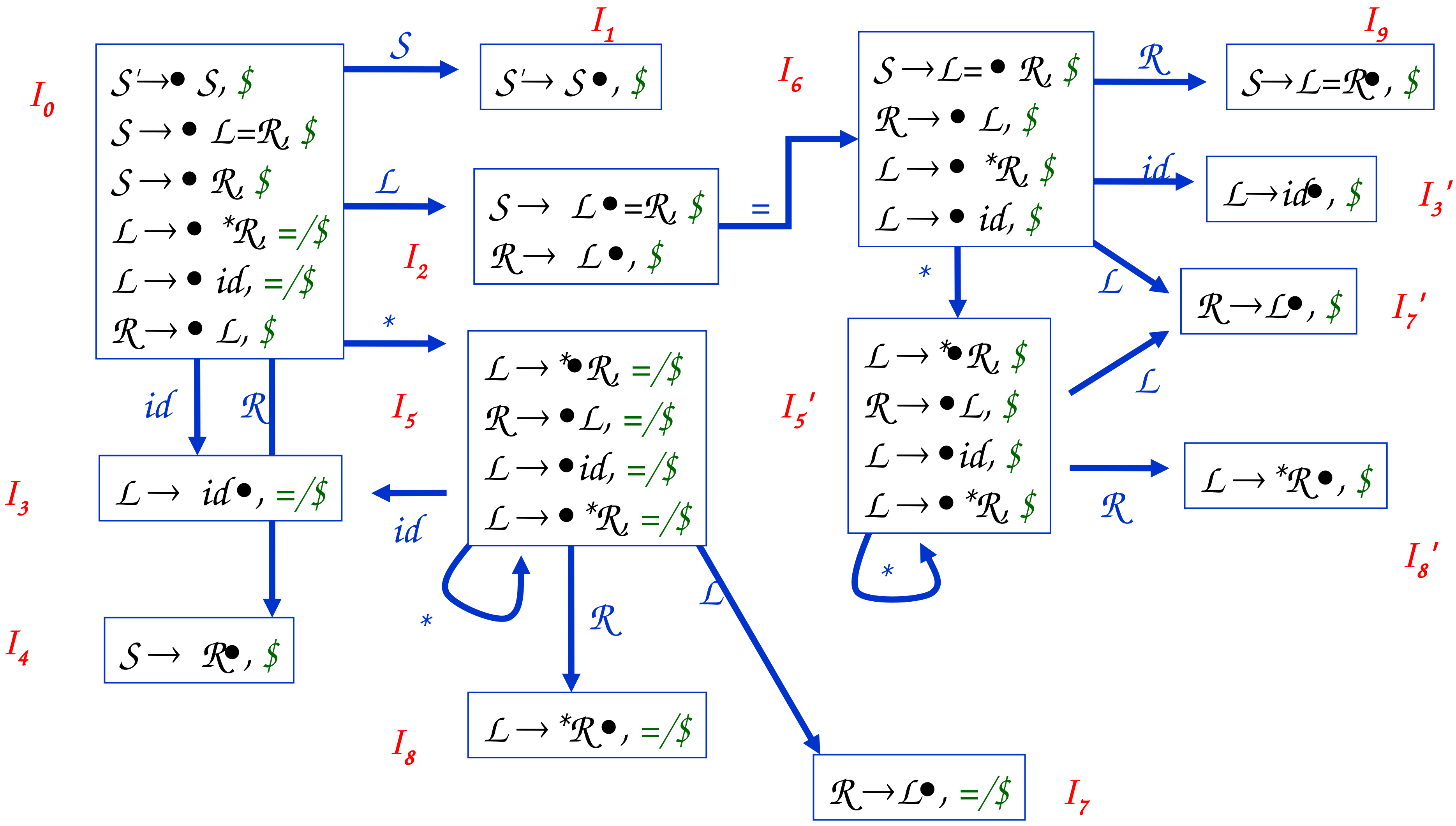
- The conflict occurred because we made a decision about when to reduce based on what token *may* follow a non-terminal at any time.
- However, the fact that a token t may follow a non-terminal N in some derivation does not necessarily imply that t will follow N in some other derivation.
- SLR parsing does not make a distinction.
- Solution : instead of using general FOLLOW information, try to keep track of exactly what tokens many follow a non-terminal in each possible derivation and perform reductions based on that knowledge.

LR(1) items

- In LR(1) items, we also save all possible lookaheads
- The closure function for LR(1) items is defined as follows:

For each item $A \to \alpha \bullet B\beta$, x in state I,
each production $B \to \gamma$ in the grammar,
and each terminal b in FIRST(βx),
add $B \to \bullet\gamma$, b to I

If a state contains core item $B \to \bullet\gamma$ with multiple possible lookaheads b_1, b_2,..., we write $B \to \bullet\gamma$, b_1/b_2 as shorthand for $B \to \bullet\gamma$, b_1 and $B \to \bullet\gamma$, b_2


I0
S'→• S, $
S→• L=R, $
S→• R, $
L→• *R, =/$
L→• id, =/$
R→• L, $
S
I1
S'→ S•, $
L
S→ L•=R, $
R→ L•, $
I2
=
I6
S→L=• R, $
R→• L, $
L→• *R, $
L→• id, $
R
I9
S→L=R•, $
id
L→id•, $
I3'
L
R→L•, $
I7'
*
id
R
I3
L→ id•, =/$
I4
S→ R•, $
I5
L→ *•R, =/$
R→•L, =/$
L→•id, =/$
L→• *R, =/$
I5'
L→ *•R, $
R→•L, $
L→•id, $
L→• *R, $
L→ *R•, $
I8'
I8
L→ *R•, =/$
R→L•, =/$
I7

Canonical LR(1) parsing

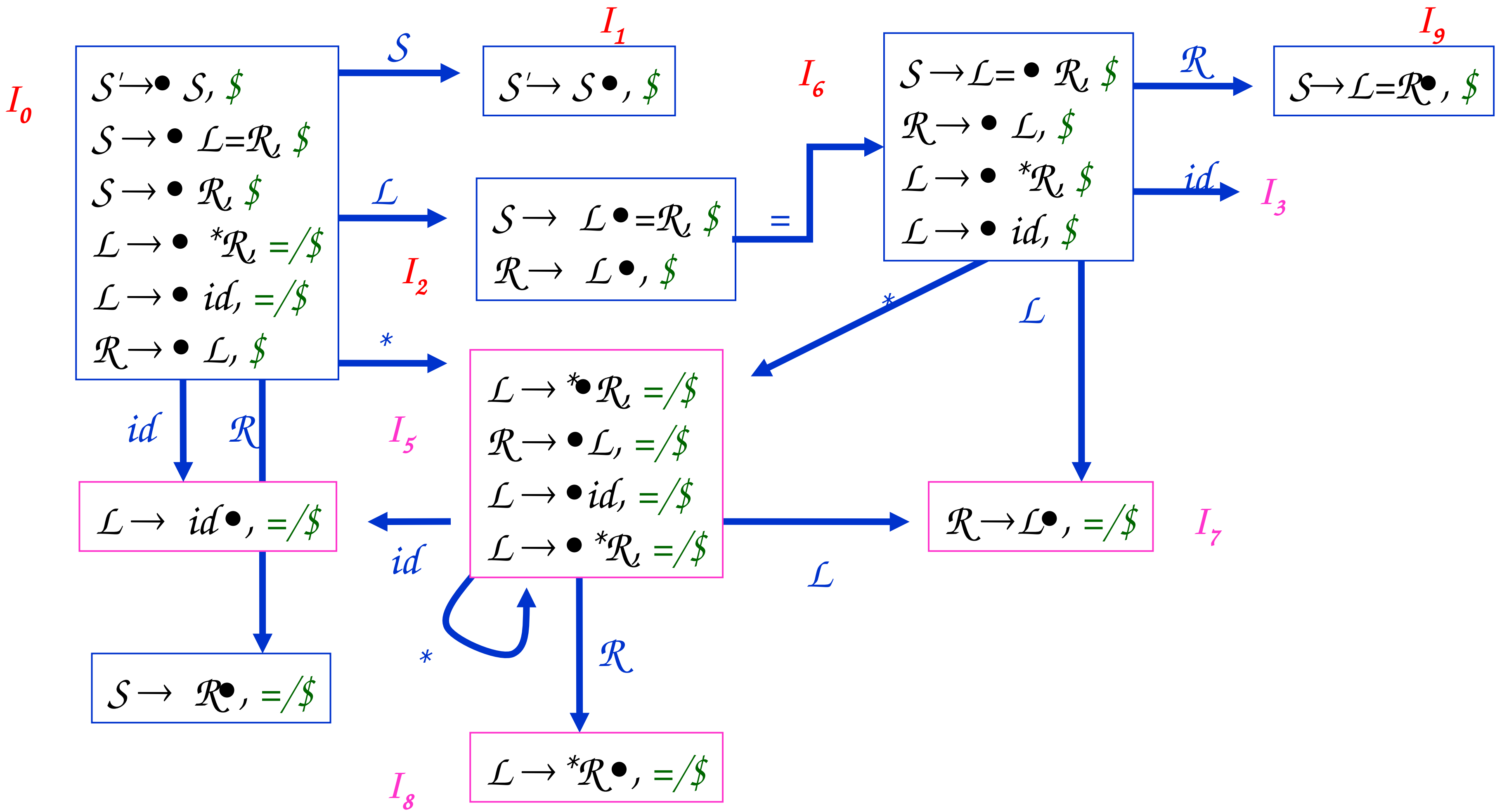
- Rules for creating the table is same as SLR, except we now use the possible lookahead tokens saved in each state, instead of the FOLLOW sets.
- The conflicts that appeared in the SLR parser is now resolved.
- However, the LR(1) parser has many more states. This is not very practical.

Canonical LR Parsing Table

	id	*	=	$		S	L	R
0	S3	S5				1	2	4
1				accept				
2			S6	R(R->L)				
3			R(L->id)	R(L->id)				
4				R(S->R)				
5	S3	S5					7	8
6	S3’	S5’					7’	9
7			R(R->L)	R(R->L)				
8			R(L->*R)	R(L->*R)				
9				R(S->L=R)				
3’				R(L->id)				
5’	S3’	S5’						
7’				R(R->L)				
8’				R(L->*R)				

LALR Parsing

- It is required to reduce the number of states in an LR(1) parser.
- Some states in the LR(1) automaton have the same core items and differ only in the possible lookahead information.
- They also have similar transitions.
- For example, states I_3 and I_3', I_5 and I_5', I_7 and I_7', I_8 and I_8' are states with same core items and similar transitions.
- Such states are merged in LALR parser.
- In our example - SLR : 10 states, LR(1): 14 states, LALR(1) : 10 states


I0
S'→• S, $
S→• L=R, $
S→• R, $
L→• *R, =/$
L→• id, =/$
R→• L, $
I1
S'→ S•, $
I2
S→ L•=R, $
R→ L•, $
I6
S→L=• R, $
R→• L, $
L→• *R, $
L→• id, $
I9
S→L=R•, $
I3
I5
L→ *•R, =/$
R→•L, =/$
L→•id, =/$
L→• *R, =/$
L→ id•, =/$
S→ R•, =/$
R→L•, =/$
I7
I8
L→ *R•, =/$
S
L
*
=
R
id

	id	*	=	$		S	L	R
0	S3	S5				1	2	4
1				accept				
2			S6	R(R->L)				
3			R(L->id)	R(L->id)				
4				R(S->R)				
5	S3	S5					7	8
6	S3’	S5’					7’	9
7			R(R->L)	R(R->L)				
8			R(L->*R)	R(L->*R)				
9				R(S->L=R)				

Conflicts in LALR(1) parsing

- LALR(1) parsers cannot introduce shift/reduce conflicts.
  - Such conflicts are caused when a lookahead is the same as a token on which we can shift.

- LALR(1) parsers can introduce reduce/reduce conflicts.
- Here's a situation when this might happen:

$$\boxed{\begin{array}{l} A \rightarrow B\bullet,\ x \\ A \rightarrow C\bullet,\ y \end{array}} \quad \textit{merge with} \quad \boxed{\begin{array}{l} A \rightarrow B\bullet,\ y \\ A \rightarrow C\bullet,\ x \end{array}} \quad \textit{to get:} \quad \boxed{\begin{array}{l} A \rightarrow B\bullet,\ x/y \\ A \rightarrow C\bullet,\ x/y \end{array}}$$