Ref. No.: Ex/CSE/PC/B/T/325/2022

B.E. COMPUTER SCIENCE AND ENGINEERING THIRD YEAR SECOND SEMESTER -2022

Subject: DESIGN AND ANALYSIS OF ALGORITHMS

Time: 3 hours

Full Marks: 100

Answer Question No. **1** ***(compulsory)*** *and question no.* ***2 or 3*** *from Group A and question no.* ***4 or 5*** *from Group B and question no.* ***6 or 7*** *from Group C*

1. State whether the following statements is TRUE or FALSE **with proper justifications.**

a) Insertion sort is a slow sorting algorithm for any value of n

b) For a divide and conquer algorithm, the division step is always costlier than the combination step.

c) Dynamic programming approach achieves better performance by repeatedly evaluating the function calls to the similar type of sub-problems.

d) If a problem is NP-hard, it may not be NP-complete. But if the problem is NP-complete, it is NP-hard

e) For a connected weighted undirected graph, there is a unique minimum spanning tree.

f) Prim's Algorithm is a greedy algorithm that finds minimum spanning tree from a weighted, connected, and directed or undirected graph.

g) $\text{Max}(f(n),g(n)) = \theta(f(n) + g(n))$, where f and g are two functions of n.

h) The greedy algorithm always gives an optimal solution to an optimization problem.

i) $2n^3 - 7n + 1 = \Omega(n^3)$

j) The total running time for the following code segment is O(logn)

```
sum=0;
for (i=1;i<n; i=i+2)
  {for (j=n;  j>n/2; j=j-2)
    {for(k=1; k<n/2; k=k*2)
      { sum++;
      }
    }
  }
```

$4 \times 10 = 40$ marks

Group A

(Answer question no. 2 or 3 from this group)

2. Write the Mergersort algorithm and find how many key comparisons are done by this algorithm if the keys are already in order when the sort begins.

Does Mergesort algorithm follow the basic characteristics of the Divide and Conquer Approach?- Explain. What is the asymptotic order of the worst case running time for this algorithm?

What is called memoization? How does it help in improving running time of a dynamic programming algorithm- elaborate with a suitable example.

10+5+5=20 marks

3. Consider the following version of insertion sort: for $1 \leq i < n$, to insert the element E[i] among $E[0] \leq E[1] \leq \ldots \leq E[i-1]$, do a Binary Search to find the correct position for E[i].
Write the outline of this algorithm. Find how many key comparisons would be done in worst case? What is the asymptotic order of the worst case running time?

Compare and contrast the "Divide and Conquer" and Dynamic programming approaches.

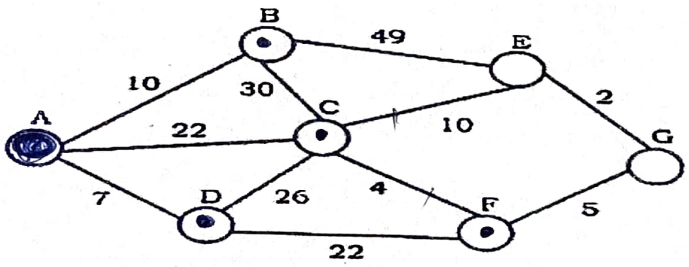
(5+5 +5)+5=20 marks

Group B

(Answer question no. 4 or 5 from this group)

4. a) Consider the connected, weighted and undirected graph below: Using Prim's algorithm, construct a minimum spanning tree (MST) starting with node A. Show at each step what is in the priority queue and which edge is added to construct the minimum spanning tree.

The greedy criterion used by the Prim's algorithm is to choose the lightest edge crossing the cut. From the following graph, when the edge AB is added to the MST, highlight how the idea of choosing the lightest edge crossing the cut is implemented with the help of a priority queue.



b) How Prim's algorithm can be useful in clustering a set of data points? Write an outline of the clustering algorithm that uses the Prim's algorithm.

10+5+5=20 marks

5. What is the input and output of the Dijktra's shortest path finding algorithm? What is its input size?

Write the priority queue implementation of this algorithm. Compute the running time of this algorithm (Show line by line analysis).

The greedy criterion used by the Dijktra's algorithm is to select the next unprocessed vertex which is closest to the source node. Highlight where in the algorithm and how this idea is implemented.

5+ 10+ 5=20 marks

Group C

(Answer question no. 6 or 7 from this group)

6. Use the dynamic programming approach to find minimum edit distance and the optimal alignment between the strings "A L G O R I T H M" and "A L T R U I S T I C". Show the table of computations, and find min edit distance and final optimal alignment. If more than one possible optimal alignment exists, find all possible optimal alignments.

For the class NP, the name NP comes from "Nondeterministic polynomially bounded". What is meant by " Nondeterministic polynomially bounded"?-Elaborate

Does "NP-hard" mean "in NP and hard"? -elaborate with an example.

10+5+5=20 marks

7. Derive dynamic programming algorithm for solving string alignment problem. Instead of using dynamic programming approach, if we use an recursive algorithm for solving this problem, how many recursive calls are needed to align "DO" with "GO" ?

If any NP-complete Problem is in P, then P=NP? Justify this statement.

If X and Y are two different problems, X is polynomially reducible to Y, and Y is in the class P, then X is in the class P - justify this statement.

10 + 5 +5 =20 marks