# Jacobian Calculations for nls()

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## Jacobians in nls()

`nls()` needs Jacobians calculated at the current set of trial nonlinear model parameters to set up the Gauss-Newton equations. Unfortunately, `nls()` calls the Jacobian the "gradient", and uses function `numericDerivs()` to compute them. This document is an attempt to describe different ways to compute the Jacobian for use in nls() and related software, and to evaluate the performance of these approaches.

In evaluating performance, we need to know the conditions under which the evaluation was conducted. Thus the computations included in this document, which is built using `Rmarkdown`, are specific to the computer in which the document is processed. We will add tables that give the results for different computing environments at the bottom.

## An example problem

We will use the Hobbs weed infestation problem (@jncnm79, page 120).

```
# Data for Hobbs problem
ydat  <-  c(5.308, 7.24, 9.638, 12.866, 17.069, 23.192, 31.443,
            38.558, 50.156, 62.948, 75.995, 91.972) # for testing
tdat  <-  seq_along(ydat) # for testing

# A simple starting vector -- must have named parameters for nlxb, nls, wrapnlsr.
start1  <-  c(b1=1, b2=1, b3=1)
eunsc  <-   y ~ b1/(1+b2*exp(-b3*tt))
str(eunsc)
```

```
## Class 'formula'  language y ~ b1/(1 + b2 * exp(-b3 * tt))
##   ..- attr(*, ".Environment")=<environment: R_GlobalEnv>
```

```
# Can we convert a string form of this "model" to a formula
ceunsc <- " y ~ b1/(1+b2*exp(-b3*tt))"
str(ceunsc)
```

```
##  chr " y ~ b1/(1+b2*exp(-b3*tt))"
```

```
# Will be TRUE if we have made the conversion
print(as.formula(ceunsc)==eunsc)
```

```
## [1] TRUE
```

```
## LOCAL DATA IN DATA FRAMES
weeddata1  <-  data.frame(y=ydat, tt=tdat)
```

```r
## Put data in an Environment
weedenv <- list2env(weeddata1)
weedenv$b1 <- start1[[1]]
weedenv$b2 <- start1[[2]]
weedenv$b3 <- start1[[3]]
# Display content of the Environment
## Note that may need to do further commands to get everything
ls.str(weedenv)
```

```
## b1 :  num 1
## b2 :  num 1
## b3 :  num 1
## tt :  int [1:12] 1 2 3 4 5 6 7 8 9 10 ...
## y :  num [1:12] 5.31 7.24 9.64 12.87 17.07 ...
```

```r
# Generate the residual "call"
rexpr<-call("-",eunsc[[3]], eunsc[[2]])
# Get the residuals
r0<-eval(rexpr, weedenv)
print(r0)
```

```
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
```

```r
cat("Sumsquares at 1,1,1 is ",sum(r0^2),"\n")
```

```
## Sumsquares at 1,1,1 is  23520.58
```

```r
## Another way
ldata<-list2env(as.list(start1),envir=weedenv)
ldata
```

```
## <environment: 0x55792d521840>
```

```r
ls.str(ldata)
```

```
## b1 :  num 1
## b2 :  num 1
## b3 :  num 1
## tt :  int [1:12] 1 2 3 4 5 6 7 8 9 10 ...
## y :  num [1:12] 5.31 7.24 9.64 12.87 17.07 ...
```

```r
eval(rexpr,envir=ldata)
```

```
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
```

```r
## Do we need to get a model frame? How? and How to use it?

## Now ready to try things out.
```

# Tools for Jacobians

## numericDeriv()

`numericDeriv` is the R function used by `nls()` to evaluate Jacobians for its Gauss-Newton equations. The R source code is in the file `nls.R`. It calls a C function numeric_deriv in `nls.c`.

```r
## seems to work -- Note file ExDerivs.R has many "failures"
theta <- c("b1", "b2", "b3")
ndeunsc<-numericDeriv(rexpr, theta, rho=weedenv)
print(ndeunsc)
```

```
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##            [,1]          [,2]         [,3]
##  [1,] 0.7310585 -1.966119e-01 0.1966118813
##  [2,] 0.8807971 -1.049936e-01 0.2099871635
##  [3,] 0.9525741 -4.517674e-02 0.1355299950
##  [4,] 0.9820137 -1.766276e-02 0.0706508160
##  [5,] 0.9933071 -6.648064e-03 0.0332403183
##  [6,] 0.9975274 -2.466440e-03 0.0147991180
##  [7,] 0.9990890 -9.102821e-04 0.0063714981
##  [8,] 0.9996643 -3.356934e-04 0.0026817322
##  [9,] 0.9998765 -1.235008e-04 0.0011105537
## [10,] 0.9999547 -4.529953e-05 0.0004539490
## [11,] 0.9999828 -1.716614e-05 0.0001831055
## [12,] 0.9999943 -5.722046e-06 0.0000743866
```

```r
print(sum(ndeunsc^2))
```

```
## [1] 23520.58
```

```r
tndeunsc<-microbenchmark(ndeunsc<-numericDeriv(rexpr, theta, rho=weedenv))
print(tndeunsc)
```

```
## Unit: microseconds
##                                                      expr   min     lq     mean
##  ndeunsc <- numericDeriv(rexpr, theta, rho = weedenv) 9.264 9.5675 10.50526
##  median      uq    max neval
##  9.7625 10.0445 59.971   100
```

```r
## numericDeriv also has central difference option, as well as choice of eps parameter
## Central diff
ndeunsc2<-numericDeriv(rexpr, theta, rho=weedenv, central=TRUE)
print(ndeunsc2)
```

```
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##            [,1]          [,2]         [,3]
##  [1,] 0.7310586 -1.966119e-01 1.966119e-01
##  [2,] 0.8807971 -1.049936e-01 2.099872e-01
##  [3,] 0.9525741 -4.517666e-02 1.355300e-01
##  [4,] 0.9820138 -1.766271e-02 7.065082e-02
##  [5,] 0.9933071 -6.648057e-03 3.324028e-02
##  [6,] 0.9975274 -2.466509e-03 1.479906e-02
##  [7,] 0.9990889 -9.102211e-04 6.371548e-03
##  [8,] 0.9996647 -3.352378e-04 2.681902e-03
##  [9,] 0.9998766 -1.233799e-04 1.110414e-03
## [10,] 0.9999546 -4.539623e-05 4.539581e-04
## [11,] 0.9999833 -1.670090e-05 1.837134e-04
## [12,] 0.9999939 -6.143885e-06 7.372897e-05
```

```
print(sum(ndeunsc2^2))
```

```
## [1] 23520.58
```

```
tndeunsc2<-microbenchmark(ndeunsc2<-numericDeriv(rexpr, theta, rho=weedenv, central=TRUE))
print(tndeunsc2)
```

```
## Unit: microseconds
##                                                                     expr    min
##  ndeunsc2 <- numericDeriv(rexpr, theta, rho = weedenv, central = TRUE) 11.727
##       lq     mean median      uq    max neval
##  12.1265 12.71588 12.352 12.6265 38.646   100
```

```
## Forward diff with smaller eps
ndeunscx<-numericDeriv(rexpr, theta, rho=weedenv, eps=1e-10)
print(ndeunscx)
```

```
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##            [,1]          [,2]         [,3]
##  [1,] 0.7310597 -0.1966160568 0.1966071750
##  [2,] 0.8807977 -0.1049915710 0.2099920238
##  [3,] 0.9525714 -0.0451905180 0.1355182633
##  [4,] 0.9820056 -0.0176747506 0.0706457115
##  [5,] 0.9933032 -0.0066435746 0.0332534000
##  [6,] 0.9975309 -0.0024513724 0.0148148160
##  [7,] 0.9990941 -0.0009237056 0.0063593575
##  [8,] 0.9996626 -0.0003552714 0.0026290081
##  [9,] 0.9998757 -0.0001421085 0.0011368684
## [10,] 0.9999468  0.0000000000 0.0004973799
## [11,] 0.9998757 -0.0001421085 0.0001421085
## [12,] 1.0000178  0.0000000000 0.0001421085
```

```
print(sum(ndeunscx^2))
```

```
## [1] 23520.58
```

```
tndeunscx<-microbenchmark(ndeunscx2<-numericDeriv(rexpr, theta, rho=weedenv, eps=1e-10))
print(tndeunscx)
```

```
## Unit: microseconds
##                                                                   expr  min    lq
##  ndeunscx2 <- numericDeriv(rexpr, theta, rho = weedenv, eps = 1e-10) 8.13 8.366
##     mean median     uq    max neval
##  8.83646 8.4695 8.6435 31.639   100
```

```
## Central diff with smaller eps
ndeunscx2<-numericDeriv(rexpr, theta, rho=weedenv, central=TRUE, eps=1e-10)
print(ndeunscx2)
```

```
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##            [,1]          [,2]         [,3]
##  [1,] 0.7310597 -1.966116e-01 1.966116e-01
##  [2,] 0.8807977 -1.049916e-01 2.099876e-01
```

```
##  [3,] 0.9525714 -4.518164e-02 1.355271e-01
##  [4,] 0.9820145 -1.766587e-02 7.065459e-02
##  [5,] 0.9933032 -6.643575e-03 3.325340e-02
##  [6,] 0.9975309 -2.451372e-03 1.481482e-02
##  [7,] 0.9990941 -9.059420e-04 6.359357e-03
##  [8,] 0.9996981 -3.197442e-04 2.664535e-03
##  [9,] 0.9998757 -1.421085e-04 1.136868e-03
## [10,] 0.9999468 -3.552714e-05 4.618528e-04
## [11,] 0.9999468 -7.105427e-05 2.131628e-04
## [12,] 1.0000178  0.000000e+00 7.105427e-05
```

```
print(sum(ndeunscx2^2))
```

```
## [1] 23520.58
```

```
tndeunscx2<-microbenchmark(ndeunscx2<-numericDeriv(rexpr, theta, rho=weedenv, central=TRUE, eps=1e-10))
print(tndeunscx2)
```

```
## Unit: microseconds
##                                                                                                 expr
##  ndeunscx2 <- numericDeriv(rexpr, theta, rho = weedenv, central = TRUE,      eps = 1e-10)
##     min      lq     mean  median     uq    max neval
##  10.919 11.2505 11.99077 11.4305 11.828 39.708   100
```

**We have not tried the dir parameter (probably allows backward differences)**

## Symbolic methods from `nlsr`

The package `nlsr` has a function `model2rjfun()` that converts an expression describing how the residual functions are computed into an R function that computes the residuals at a particular set of parameters and sets the **attribute** "gradient" of the vector of residual values to the Jacobian at the particular set of parameters.

```
# nlsr has function model2rjfun. We can evaluate just the residuals
res0<-model2rjfun(eunsc, start1, data=weeddata1, jacobian=FALSE)
res0(start1)
```

```
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
```

```
# or the residuals and jacobian
## nlsr::model2rjfun forms a function with gradient (jacobian) attribute
funsc <- model2rjfun(eunsc, start1, data=weeddata1) # from nlsr: creates a function
tmodel2rjfun <- microbenchmark(model2rjfun(eunsc, start1, data=weeddata1))
print(tmodel2rjfun)
```

```
## Unit: microseconds
##                                          expr    min      lq     mean median
##  model2rjfun(eunsc, start1, data = weeddata1) 82.555 84.6795 88.05889 85.602
##       uq     max neval
##  86.7365 218.214   100
```

```
print(funsc)
```

```
## function (prm)
## {
##     if (is.null(names(prm)))
##         names(prm) <- names(pvec)
##     localdata <- list2env(as.list(prm), parent = data)
```

```
##     eval(residexpr, envir = localdata)
## }
## <bytecode: 0x55792b120d18>
## <environment: 0x55792c44a750>
```

```
print(funsc(start1))
```

```
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##              b1            b2           b3
##  [1,] 0.7310586 -1.966119e-01 1.966119e-01
##  [2,] 0.8807971 -1.049936e-01 2.099872e-01
##  [3,] 0.9525741 -4.517666e-02 1.355300e-01
##  [4,] 0.9820138 -1.766271e-02 7.065082e-02
##  [5,] 0.9933071 -6.648057e-03 3.324028e-02
##  [6,] 0.9975274 -2.466509e-03 1.479906e-02
##  [7,] 0.9990889 -9.102212e-04 6.371548e-03
##  [8,] 0.9996646 -3.352377e-04 2.681901e-03
##  [9,] 0.9998766 -1.233793e-04 1.110414e-03
## [10,] 0.9999546 -4.539581e-05 4.539581e-04
## [11,] 0.9999833 -1.670114e-05 1.837126e-04
## [12,] 0.9999939 -6.144137e-06 7.372964e-05
```

```
print(environment(funsc))
```

```
## <environment: 0x55792c44a750>
```

```
print(ls.str(environment(funsc)))
```

```
## data : <environment: 0x55792c43fc18>
## jacobian :  logi TRUE
## modelformula : Class 'formula'  language y ~ b1/(1 + b2 * exp(-b3 * tt))
## pvec :  Named num [1:3] 1 1 1
## residexpr :   expression({  .expr3 <- exp(-b3 * tt)  .expr5 <- 1 + b2 * .expr3  .expr10 <- .expr5^2
## rjfun : function (prm)
## testresult :  logi TRUE
```

```
print(ls(environment(funsc)$data))
```

```
## [1] "tt" "y"
```

```
eval(eunsc, environment(funsc))
```

```
## y ~ b1/(1 + b2 * exp(-b3 * tt))
```

```
vfunsc<-funsc(start1)
print(vfunsc)
```

```
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##              b1            b2           b3
##  [1,] 0.7310586 -1.966119e-01 1.966119e-01
##  [2,] 0.8807971 -1.049936e-01 2.099872e-01
##  [3,] 0.9525741 -4.517666e-02 1.355300e-01
##  [4,] 0.9820138 -1.766271e-02 7.065082e-02
##  [5,] 0.9933071 -6.648057e-03 3.324028e-02
```

```
##  [6,] 0.9975274 -2.466509e-03 1.479906e-02
##  [7,] 0.9990889 -9.102212e-04 6.371548e-03
##  [8,] 0.9996646 -3.352377e-04 2.681901e-03
##  [9,] 0.9998766 -1.233793e-04 1.110414e-03
## [10,] 0.9999546 -4.539581e-05 4.539581e-04
## [11,] 0.9999833 -1.670114e-05 1.837126e-04
## [12,] 0.9999939 -6.144137e-06 7.372964e-05
```

```
tfunsc<-microbenchmark(funsc(start1))
print(tfunsc)
```

```
## Unit: microseconds
##           expr    min      lq     mean  median      uq    max neval
##  funsc(start1) 13.285 13.6535 15.00195 13.9555 14.3095 65.374   100
```

## `numDeriv` package

The package `numDeriv` includes a function `jacobian()` that acts on a user function `resid()` to produce the Jacobian at a set of parameters by several choices of approximation.

```
# We use the residual function (without gradient attribute) from nlsr
jeunsc<-jacobian(res0, start1)
jeunsc
```

```
##            [,1]          [,2]         [,3]
##  [1,] 0.7310586 -1.966119e-01 1.966119e-01
##  [2,] 0.8807971 -1.049936e-01 2.099872e-01
##  [3,] 0.9525741 -4.517666e-02 1.355300e-01
##  [4,] 0.9820138 -1.766271e-02 7.065082e-02
##  [5,] 0.9933071 -6.648057e-03 3.324028e-02
##  [6,] 0.9975274 -2.466509e-03 1.479906e-02
##  [7,] 0.9990889 -9.102212e-04 6.371548e-03
##  [8,] 0.9996647 -3.352378e-04 2.681902e-03
##  [9,] 0.9998766 -1.233791e-04 1.110414e-03
## [10,] 0.9999546 -4.539572e-05 4.539580e-04
## [11,] 0.9999833 -1.670116e-05 1.837129e-04
## [12,] 0.9999939 -6.144205e-06 7.373002e-05
```

```
tjeunsc<-microbenchmark(jeunsc<-jacobian(res0, start1))
print(tjeunsc)
```

```
## Unit: microseconds
##                               expr     min      lq    mean  median      uq
##  jeunsc <- jacobian(res0, start1) 351.696 362.146 405.217 374.965 446.216
##       max neval
##  1554.042   100
```

Note that the manual pages for `numDeriv` offer many options for the functions in the package. At 2021-5-27 we have yet to explore these.

## Comparisons

In the following, we are comparing to `vfunsc`, which is the evaluated residual vector at `start1=c(1,1,1)` with "gradient" attribute (jacobian) included, as developed using package `nlsr`. This is taken as the "correct" result.

`numericDeriv` computes a similar structure (residuals with "gradient" attribute): `ndeunsc`: the forward difference result with default `eps` (1e-07 according to manual) `ndeunsc2`: Central difference with default `eps` `ndeunscx`: Forward difference with smaller eps=1e-10 `ndeunscx2`: Central difference with smaller eps=1e-10

`jeunsc`: numDeriv::jacobian() result with default settings.

```
## Matrix comparisons
attr(ndeunsc, "gradient")-attr(vfunsc,"gradient")
```

```
##                  b1            b2            b3
##  [1,] -4.066995e-08 -7.619266e-09 -5.198538e-08
##  [2,]  1.016833e-08  3.631656e-09 -7.263312e-09
##  [3,]  7.050552e-09 -8.473015e-08  1.577186e-08
##  [4,] -8.764533e-08 -5.738229e-08 -8.889419e-09
##  [5,] -3.542825e-08 -6.988878e-09  3.494439e-08
##  [6,] -1.592723e-08  6.909055e-08  6.229383e-08
##  [7,]  5.380365e-08 -6.095489e-08 -5.015294e-08
##  [8,] -3.432289e-07 -4.556886e-07 -1.691883e-07
##  [9,] -1.062480e-07 -1.214742e-07  1.395936e-07
## [10,]  9.833867e-08  9.627771e-08 -9.102750e-09
## [11,] -4.647158e-07 -4.649948e-07 -6.071033e-07
## [12,]  4.221287e-07  4.220910e-07  6.569545e-07
```

```
attr(ndeunsc2, "gradient")-attr(vfunsc,"gradient")
```

```
##                  b1            b2            b3
##  [1,] -5.513268e-11  1.371020e-11  5.962686e-11
##  [2,]  6.850076e-13 -3.831403e-11  3.291006e-12
##  [3,] -2.144829e-11 -1.208666e-10  6.925160e-11
##  [4,] -2.665634e-11  4.123477e-11 -1.826495e-11
##  [5,]  2.175706e-10 -7.825720e-11  9.793778e-11
##  [6,] -2.416068e-10 -1.666460e-10 -1.735172e-10
##  [7,] -8.350343e-11  5.415257e-11 -8.571976e-11
##  [8,]  3.255913e-10 -9.864836e-11  2.024904e-10
##  [9,]  1.379652e-11 -5.685668e-10 -1.631673e-10
## [10,] -9.296786e-11 -4.173983e-10  6.710748e-11
## [11,] -4.266865e-11  2.415372e-10  8.632699e-10
## [12,] -2.738492e-10  2.515432e-10 -6.717320e-10
```

```
attr(ndeunscx, "gradient")-attr(vfunsc,"gradient")
```

```
##                  b1            b2            b3
##  [1,]  1.078625e-06 -4.123528e-06 -4.758257e-06
##  [2,]  5.790545e-07  2.014411e-06  4.852963e-06
##  [3,] -2.771694e-06 -1.385826e-05 -1.171591e-05
##  [4,] -8.202081e-06 -1.204434e-05 -5.113350e-06
##  [5,] -3.931620e-06  4.482091e-06  1.311668e-05
##  [6,]  3.569890e-06  1.513685e-05  1.576029e-05
##  [7,]  5.191947e-06 -1.348438e-05 -1.219078e-05
##  [8,] -2.074929e-06 -2.003370e-05 -5.289324e-05
##  [9,] -8.676624e-07 -1.872920e-05  2.645423e-05
## [10,] -7.810096e-06  4.539581e-05  4.342184e-05
## [11,] -1.075608e-04 -1.254074e-04 -4.160402e-05
## [12,]  2.399048e-05  6.144137e-06  6.837890e-05
```

```
attr(ndeunscx2, "gradient")-attr(vfunsc,"gradient")
```

```
##                 b1            b2            b3
##  [1,]  1.078625e-06  3.173646e-07 -3.173646e-07
##  [2,]  5.790545e-07  2.014411e-06  4.120706e-07
##  [3,] -2.771694e-06 -4.976479e-06 -2.834131e-06
##  [4,]  6.797035e-07 -3.162555e-06  3.768434e-06
##  [5,] -3.931620e-06  4.482091e-06  1.311668e-05
##  [6,]  3.569890e-06  1.513685e-05  1.576029e-05
##  [7,]  5.191947e-06  4.279192e-06 -1.219078e-05
##  [8,]  3.345221e-05  1.549344e-05 -1.736611e-05
##  [9,] -8.676624e-07 -1.872920e-05  2.645423e-05
## [10,] -7.810096e-06  9.868671e-06  7.894701e-06
## [11,] -3.650654e-05 -5.435313e-05  2.945025e-05
## [12,]  2.399048e-05  6.144137e-06 -2.675369e-06
```

```
jeunsc-attr(vfunsc,"gradient")
```

```
##                 b1            b2            b3
##  [1,] -2.239464e-11  7.806283e-12 -1.156686e-12
##  [2,] -2.267631e-12  2.974312e-11  2.957756e-11
##  [3,] -8.948509e-12  3.630193e-11 -1.256267e-11
##  [4,] -1.649125e-12  1.182179e-13  6.369841e-11
##  [5,] -4.272493e-11 -1.109757e-11  3.501116e-11
##  [6,]  1.867381e-10  1.793287e-11  3.319552e-11
##  [7,]  1.090728e-11  1.840947e-11  9.946462e-12
##  [8,]  2.035664e-10 -1.520996e-10  1.911593e-10
##  [9,] -3.582228e-10  2.039028e-10 -1.905254e-10
## [10,]  3.202474e-10  8.291927e-11 -6.263366e-11
## [11,]  5.931922e-12 -1.779933e-11  3.682277e-10
## [12,]  4.132902e-10 -6.831839e-11  3.817154e-10
```

```
## Summary comparisons
max(abs(attr(ndeunsc, "gradient")-attr(vfunsc,"gradient")))
```

```
## [1] 6.569545e-07
```

```
max(abs(attr(ndeunsc2, "gradient")-attr(vfunsc,"gradient")))
```

```
## [1] 8.632699e-10
```

```
max(abs(attr(ndeunscx, "gradient")-attr(vfunsc,"gradient")))
```

```
## [1] 0.0001254074
```

```
max(abs(attr(ndeunscx2, "gradient")-attr(vfunsc,"gradient")))
```

```
## [1] 5.435313e-05
```

```
max(abs(jeunsc-attr(vfunsc,"gradient")))
```

```
## [1] 4.132902e-10
```

## Performance results for different computing environments

Here we present tables of the results, preceded by identified descriptions of the machines we used.

M21-LM20.1

```
sessionInfo()
```

```
## R version 4.1.0 (2021-05-18)
## Platform: x86_64-pc-linux-gnu (64-bit)
## Running under: Linux Mint 20.1
##
## Matrix products: default
## BLAS:   /usr/lib/x86_64-linux-gnu/openblas-pthread/libblas.so.3
## LAPACK: /usr/lib/x86_64-linux-gnu/openblas-pthread/liblapack.so.3
##
## locale:
##  [1] LC_CTYPE=en_CA.UTF-8       LC_NUMERIC=C
##  [3] LC_TIME=en_CA.UTF-8        LC_COLLATE=en_CA.UTF-8
##  [5] LC_MONETARY=en_CA.UTF-8    LC_MESSAGES=en_CA.UTF-8
##  [7] LC_PAPER=en_CA.UTF-8       LC_NAME=C
##  [9] LC_ADDRESS=C               LC_TELEPHONE=C
## [11] LC_MEASUREMENT=en_CA.UTF-8 LC_IDENTIFICATION=C
##
## attached base packages:
## [1] stats     graphics  grDevices utils     datasets  methods   base
##
## other attached packages:
## [1] microbenchmark_1.4-7 numDeriv_2016.8-1.1  nlsr_2019.9.7
##
## loaded via a namespace (and not attached):
##  [1] compiler_4.1.0    magrittr_2.0.1    tools_4.1.0       htmltools_0.5.1.1
##  [5] yaml_2.2.1        stringi_1.6.2     rmarkdown_2.8     knitr_1.33
##  [9] stringr_1.4.0     xfun_0.23         digest_0.6.27     rlang_0.4.11
## [13] evaluate_0.14
```

To get a good picture of the physical and logical machine that is M21-LM20.1, we can run

```
inxi -F >tlinux.txt
```

in a command line terminal in the host machine.

While it may be tempting to run either

```
system('inxi -F >../t.txt')
```

or

```
inxi -F >../t2.txt
```

it turns out that the encoding of the files is different. Indeed the files are different sizes!

```
ls -al ../t*.txt
```

```
## -rw-rw-r-- 1 john john 2408 Jun  3 09:07 ../t2.txt
## -rw-rw-r-- 1 john john  543 Jan  8 10:13 ../test1data.txt
## -rw-rw-r-- 1 john john 2413 May 27 10:17 ../tlinux2.txt
## -rw-rw-r-- 1 john john 1917 May 27 09:48 ../tlinuxOS.txt
## -rw-rw-r-- 1 john john 1914 Jun  3 08:46 ../tl.txt
## -rw-rw-r-- 1 john john 2051 Jun  3 08:57 ../tlx-inxirunFromBash.txt
## -rw-rw-r-- 1 john john 1920 Jun  3 08:59 ../tlx-inxirunFromXterm.txt
## -rw-rw-r-- 1 john john 1926 Jun  3 09:02 ../tlx-konsole-e-prog.txt
## -rw-rw-r-- 1 john john 1923 Jun  3 09:06 ../tlx.txt
## -rw-rw-r-- 1 john john 1923 Jun  3 09:05 ../tlx-xterm-e-prog.txt
## -rw-rw-r-- 1 john john 2400 Jun  3 09:07 ../t.txt
```

Here is the result from running the `inxi` command in the native Linux terminal. (?? note that we need each "user to do following.) Note that the file tlinuxOS.txt needs to be created via the command

```
inxi -F > tlinuxOS.txt
```

in the directory immediately ABOVE the `improvenls` local git repository where the present docoument is stored i.e., outside the version control.

```r
cat(readLines('../tlinuxOS.txt'), sep = '\n')
```

```
## System:
##   Host: M21 Kernel: 5.4.0-73-generic x86_64 bits: 64 Desktop: MATE 1.24.0
##   Distro: Linux Mint 20.1 Ulyssa
## Machine:
##   Type: Desktop System: ASUS product: N/A v: N/A
##   serial: <superuser/root required>
##   Mobo: ASUSTeK model: PRIME Z490-A v: Rev 1.xx
##   serial: <superuser/root required> UEFI: American Megatrends v: 0607
##   date: 05/29/2020
## CPU:
##   Topology: 6-Core model: Intel Core i5-10400 bits: 64 type: MT MCP
##   L2 cache: 12.0 MiB
##   Speed: 800 MHz min/max: 800/4300 MHz Core speeds (MHz): 1: 800 2: 800
##   3: 800 4: 800 5: 800 6: 800 7: 800 8: 800 9: 800 10: 800 11: 800 12: 800
## Graphics:
##   Device-1: Intel driver: i915 v: kernel
##   Display: x11 server: X.Org 1.20.9 driver: modesetting unloaded: fbdev,vesa
##   resolution: 1920x1080~60Hz, 1920x1080~60Hz
##   OpenGL: renderer: Mesa Intel UHD Graphics 630 (CML GT2) v: 4.6 Mesa 20.2.6
## Audio:
##   Device-1: Intel Comet Lake PCH cAVS driver: snd_hda_intel
##   Sound Server: ALSA v: k5.4.0-73-generic
## Network:
##   Device-1: Intel driver: igc
##   IF: enp3s0 state: down mac: 3c:7c:3f:2d:e8:8b
##   Device-2: ASIX AX88179 Gigabit Ethernet type: USB driver: ax88179_178a
##   IF: enx00249b1580ea state: up speed: 100 Mbps duplex: full
##   mac: 00:24:9b:15:80:ea
## Drives:
##   Local Storage: total: 4.55 TiB used: 1.09 TiB (24.0%)
##   ID-1: /dev/nvme0n1 vendor: Western Digital model: WDS100T3X0C-00SJG0
##   size: 931.51 GiB
##   ID-2: /dev/sda vendor: Seagate model: ST2000DM008-2FR102 size: 1.82 TiB
##   ID-3: /dev/sdb vendor: Seagate model: ST2000DM008-2FR102 size: 1.82 TiB
## RAID:
##   Device-1: m21z type: zfs status: ONLINE size: 1.81 TiB free: 950.00 GiB
##   Components: online: sda sdb
## Partition:
##   ID-1: / size: 915.40 GiB used: 212.99 GiB (23.3%) fs: ext4
##   dev: /dev/nvme0n1p2
## Sensors:
##   System Temperatures: cpu: 27.8 C mobo: N/A
##   Fan Speeds (RPM): N/A
## Info:
##   Processes: 408 Uptime: 1h 25m Memory: 31.19 GiB used: 4.54 GiB (14.6%)
##   Shell: bash inxi: 3.0.38
```

```
#!/bin/sh
inxi -F >../tlx.txt
echo "done!"
```

```r
system("mate-terminal -e ./inxirun.sh")
cat(readLines('../tlx.txt'), sep = '\n')
```

```
## System:
##   Host: M21 Kernel: 5.4.0-73-generic x86_64 bits: 64 Desktop: MATE 1.24.0
##   Distro: Linux Mint 20.1 Ulyssa
## Machine:
##   Type: Desktop System: ASUS product: N/A v: N/A
##   serial: <superuser/root required>
##   Mobo: ASUSTeK model: PRIME Z490-A v: Rev 1.xx
##   serial: <superuser/root required> UEFI: American Megatrends v: 0607
##   date: 05/29/2020
## CPU:
##   Topology: 6-Core model: Intel Core i5-10400 bits: 64 type: MT MCP
##   L2 cache: 12.0 MiB
##   Speed: 800 MHz min/max: 800/4300 MHz Core speeds (MHz): 1: 800 2: 800
##   3: 800 4: 800 5: 800 6: 800 7: 800 8: 800 9: 800 10: 800 11: 800 12: 801
## Graphics:
##   Device-1: Intel driver: i915 v: kernel
##   Display: x11 server: X.Org 1.20.9 driver: modesetting unloaded: fbdev,vesa
##   resolution: 1920x1080~60Hz, 1920x1080~60Hz
##   OpenGL: renderer: Mesa Intel UHD Graphics 630 (CML GT2) v: 4.6 Mesa 20.2.6
## Audio:
##   Device-1: Intel Comet Lake PCH cAVS driver: snd_hda_intel
##   Sound Server: ALSA v: k5.4.0-73-generic
## Network:
##   Device-1: Intel driver: igc
##   IF: enp3s0 state: down mac: 3c:7c:3f:2d:e8:8b
##   Device-2: ASIX AX88179 Gigabit Ethernet type: USB driver: ax88179_178a
##   IF: enx00249b1580ea state: up speed: 100 Mbps duplex: full
##   mac: 00:24:9b:15:80:ea
## Drives:
##   Local Storage: total: 4.55 TiB used: 957.37 GiB (20.6%)
##   ID-1: /dev/nvme0n1 vendor: Western Digital model: WDS100T3X0C-00SJG0
##   size: 931.51 GiB
##   ID-2: /dev/sda vendor: Seagate model: ST2000DM008-2FR102 size: 1.82 TiB
##   ID-3: /dev/sdb vendor: Seagate model: ST2000DM008-2FR102 size: 1.82 TiB
## RAID:
##   Device-1: m21z type: zfs status: ONLINE size: 1.81 TiB free: 1.08 TiB
##   Components: online: sda sdb
## Partition:
##   ID-1: / size: 915.40 GiB used: 209.87 GiB (22.9%) fs: ext4
##   dev: /dev/nvme0n1p2
## Sensors:
##   System Temperatures: cpu: 27.8 C mobo: N/A
##   Fan Speeds (RPM): N/A
## Info:
##   Processes: 431 Uptime: 1h 07m Memory: 31.19 GiB used: 4.55 GiB (14.6%)
##   Shell: inxirun.sh inxi: 3.0.38
```