Jacobian Calculations for nls()

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Contents

ISSUES

- ExDerivs.R file causes a number of failures in the ORIGINAL numericDeriv.
- Need to verify nlsalt:: version of numericDeriv() matches all cases of nlspkg:: version
- Do we need to get a model frame? How? and How to use it?

TODOS (mostly from nlsr vignette nlsr-devdoc.Rmd)

- how to insert numerical derivatives when Deriv unable to get result
- approximations for jacfn beyond fwd approximation. How to specify??

- how to force numerical approximations in nlfb() in a manner consistent with that used in `optimx::optimr()`, that is, to surround the name of `jacfn` with quotes if it is a numerical approximation, or to provide a logical control to `nlxb()` for this purpose.

Jacobians in nls()

This document source is in file **DerivsNLS.Rmd**.

`nls()` needs Jacobians calculated at the current set of trial nonlinear model parameters to set up the Gauss-Newton equations. Unfortunately, `nls()` calls the Jacobian the "gradient," and uses function `numericDerivs()` to compute them. This document is an attempt to describe different ways to compute the Jacobian for use in nls() and related software, and to evaluate the performance of these approaches.

In evaluating performance, we need to know the conditions under which the evaluation was conducted. Thus the computations included in this document, which is built using `Rmarkdown`, are specific to the computer in which the document is processed. We will add tables that give the results for different computing environments at the bottom.

An example problem

We will use the Hobbs weed infestation problem (Nash (1979), page 120).

```
# Data for Hobbs problem
ydat  <-  c(5.308, 7.24, 9.638, 12.866, 17.069, 23.192, 31.443,
            38.558, 50.156, 62.948, 75.995, 91.972) # for testing
tdat  <-  seq_along(ydat) # for testing

# A simple starting vector -- must have named parameters for nlxb, nls, wrapnlsr.
start1  <-  c(b1=1, b2=1, b3=1)
eunsc  <-   y ~ b1/(1+b2*exp(-b3*tt))
str(eunsc)
```

```
## Class 'formula'  language y ~ b1/(1 + b2 * exp(-b3 * tt))
##   ..- attr(*, ".Environment")=<environment: R_GlobalEnv>
```

```
# Can we convert a string form of this "model" to a formula
ceunsc <- " y ~ b1/(1+b2*exp(-b3*tt))"
str(ceunsc)
```

```
##  chr " y ~ b1/(1+b2*exp(-b3*tt))"
```

```
# Will be TRUE if we have made the conversion
print(as.formula(ceunsc)==eunsc)
```

```
## [1] TRUE
```

```
## LOCAL DATA IN DATA FRAMES
weeddata1  <-  data.frame(y=ydat, tt=tdat)

## Put data in an Environment
weedenv <- list2env(weeddata1)
weedenv$b1 <- start1[[1]]
weedenv$b2 <- start1[[2]]
weedenv$b3 <- start1[[3]]
# Display content of the Environment
## Note that may need to do further commands to get everything
ls.str(weedenv)
```

```
## b1 :  num 1
## b2 :  num 1
## b3 :  num 1
## tt :  int [1:12] 1 2 3 4 5 6 7 8 9 10 ...
## y :  num [1:12] 5.31 7.24 9.64 12.87 17.07 ...
```

```
# Generate the residual "call"
rexpr<-call("-",eunsc[[3]], eunsc[[2]])
# Get the residuals
r0<-eval(rexpr, weedenv)
print(r0)
```

```
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
```

```
cat("Sumsquares at 1,1,1 is ",sum(r0^2),"\n")
```

```
## Sumsquares at 1,1,1 is  23520.58
```

```
## Another way
ldata<-list2env(as.list(start1),envir=weedenv)
ldata
```

```
## <environment: 0x560b412576d8>
```

```
ls.str(ldata)
```

```
## b1 :  num 1
## b2 :  num 1
## b3 :  num 1
## tt :  int [1:12] 1 2 3 4 5 6 7 8 9 10 ...
## y :  num [1:12] 5.31 7.24 9.64 12.87 17.07 ...
```

```
eval(rexpr,envir=ldata)
```

```
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
```

```
## Do we need to get a model frame? How? and How to use it?

## Now ready to try things out.
```

Tools for Jacobians

numericDeriv() original version

`numericDeriv` is the R function used by `nls()` to evaluate Jacobians for its Gauss-Newton equations. The R source code is in the file `nls.R`. It calls a C function numeric_deriv in `nls.c`.

```
## Seems to work -- BUT note file ExDerivs.R has many "failures"??
theta <- c("b1", "b2", "b3")
ndeunsc<-nlspkg::numericDeriv(rexpr, theta, rho=weedenv)
```

```
## Registered S3 methods overwritten by 'nlspkg':
##   method            from
##   anova.nls         stats
##   coef.nls          stats
##   confint.nls       stats
##   deviance.nls      stats
```

```
##   df.residual.nls   stats
##   fitted.nls        stats
##   formula.nls       stats
##   logLik.nls        stats
##   nobs.nls          stats
##   plot.profile.nls  stats
##   predict.nls       stats
##   print.nls         stats
##   print.summary.nls stats
##   profile.nls       stats
##   residuals.nls     stats
##   summary.nls       stats
##   vcov.nls          stats
##   weights.nls       stats
```

```
print(ndeunsc)
```

```
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##            [,1]          [,2]         [,3]
##  [1,] 0.7310585 -1.966119e-01 0.1966118813
##  [2,] 0.8807971 -1.049936e-01 0.2099871635
##  [3,] 0.9525741 -4.517674e-02 0.1355299950
##  [4,] 0.9820137 -1.766276e-02 0.0706508160
##  [5,] 0.9933071 -6.648064e-03 0.0332403183
##  [6,] 0.9975274 -2.466440e-03 0.0147991180
##  [7,] 0.9990890 -9.102821e-04 0.0063714981
##  [8,] 0.9996643 -3.356934e-04 0.0026817322
##  [9,] 0.9998765 -1.235008e-04 0.0011105537
## [10,] 0.9999547 -4.529953e-05 0.0004539490
## [11,] 0.9999828 -1.716614e-05 0.0001831055
## [12,] 0.9999943 -5.722046e-06 0.0000743866
```

```
print(sum(ndeunsc^2))
```

```
## [1] 23520.58
```

```
tndeunsc<-microbenchmark(ndeunsc<-nlspkg::numericDeriv(rexpr, theta, rho=weedenv))
print(tndeunsc)
```

```
## Unit: microseconds
##                                                        expr    min     lq
##  ndeunsc <- nlspkg::numericDeriv(rexpr, theta, rho = weedenv) 10.059 10.284
##      mean median     uq    max neval
##  10.77702 10.392 10.572 38.831   100
```

```
## numericDeriv also has central difference option, as well as choice of eps parameter
## Central diff
ndeunsc2<-nlspkg::numericDeriv(rexpr, theta, rho=weedenv, central=TRUE)
print(ndeunsc2)
```

```
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##            [,1]          [,2]         [,3]
##  [1,] 0.7310586 -1.966119e-01 1.966119e-01
```

```
##  [2,] 0.8807971 -1.049936e-01 2.099872e-01
##  [3,] 0.9525741 -4.517666e-02 1.355300e-01
##  [4,] 0.9820138 -1.766271e-02 7.065082e-02
##  [5,] 0.9933071 -6.648057e-03 3.324028e-02
##  [6,] 0.9975274 -2.466509e-03 1.479906e-02
##  [7,] 0.9990889 -9.102211e-04 6.371548e-03
##  [8,] 0.9996647 -3.352378e-04 2.681902e-03
##  [9,] 0.9998766 -1.233799e-04 1.110414e-03
## [10,] 0.9999546 -4.539623e-05 4.539581e-04
## [11,] 0.9999833 -1.670090e-05 1.837134e-04
## [12,] 0.9999939 -6.143885e-06 7.372897e-05
```

```
print(sum(ndeunsc2^2))
```

```
## [1] 23520.58
```

```
tndeunsc2<-microbenchmark(ndeunsc2<-nlspkg::numericDeriv(rexpr, theta, rho=weedenv, central=TRUE))
print(tndeunsc2)
```

```
## Unit: microseconds
##                                                                          expr
##  ndeunsc2 <- nlspkg::numericDeriv(rexpr, theta, rho = weedenv,      central = TRUE)
##   min     lq     mean  median      uq    max neval
##  12.6 12.958 13.81619 13.1975 13.5545 64.875   100
```

```
## Forward diff with smaller eps
ndeunscx<-nlspkg::numericDeriv(rexpr, theta, rho=weedenv, eps=1e-10)
print(ndeunscx)
```

```
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##            [,1]          [,2]         [,3]
##  [1,] 0.7310597 -0.1966160568 0.1966071750
##  [2,] 0.8807977 -0.1049915710 0.2099920238
##  [3,] 0.9525714 -0.0451905180 0.1355182633
##  [4,] 0.9820056 -0.0176747506 0.0706457115
##  [5,] 0.9933032 -0.0066435746 0.0332534000
##  [6,] 0.9975309 -0.0024513724 0.0148148160
##  [7,] 0.9990941 -0.0009237056 0.0063593575
##  [8,] 0.9996626 -0.0003552714 0.0026290081
##  [9,] 0.9998757 -0.0001421085 0.0011368684
## [10,] 0.9999468  0.0000000000 0.0004973799
## [11,] 0.9998757 -0.0001421085 0.0001421085
## [12,] 1.0000178  0.0000000000 0.0001421085
```

```
print(sum(ndeunscx^2))
```

```
## [1] 23520.58
```

```
tndeunscx<-microbenchmark(ndeunscx2<-nlspkg::numericDeriv(rexpr, theta, rho=weedenv, eps=1e-10))
print(tndeunscx)
```

```
## Unit: microseconds
##                                                                        expr
##  ndeunscx2 <- nlspkg::numericDeriv(rexpr, theta, rho = weedenv,      eps = 1e-10)
##    min     lq    mean median    uq    max neval
##  8.972 9.2485 9.82862 9.3615 9.544 36.814   100
```

```
## Central diff with smaller eps
ndeunscx2<-nlspkg::numericDeriv(rexpr, theta, rho=weedenv, central=TRUE, eps=1e-10)
print(ndeunscx2)
```

```
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##            [,1]          [,2]         [,3]
##  [1,] 0.7310597 -1.966116e-01 1.966116e-01
##  [2,] 0.8807977 -1.049916e-01 2.099876e-01
##  [3,] 0.9525714 -4.518164e-02 1.355271e-01
##  [4,] 0.9820145 -1.766587e-02 7.065459e-02
##  [5,] 0.9933032 -6.643575e-03 3.325340e-02
##  [6,] 0.9975309 -2.451372e-03 1.481482e-02
##  [7,] 0.9990941 -9.059420e-04 6.359357e-03
##  [8,] 0.9996981 -3.197442e-04 2.664535e-03
##  [9,] 0.9998757 -1.421085e-04 1.136868e-03
## [10,] 0.9999468 -3.552714e-05 4.618528e-04
## [11,] 0.9999468 -7.105427e-05 2.131628e-04
## [12,] 1.0000178  0.000000e+00 7.105427e-05
```

```
print(sum(ndeunscx2^2))
```

```
## [1] 23520.58
```

```
tndeunscx2<-microbenchmark(ndeunscx2<-nlspkg::numericDeriv(rexpr, theta, rho=weedenv, central=TRUE, eps=
print(tndeunscx2)
```

```
## Unit: microseconds
##                                                                                                  expr
##  ndeunscx2 <- nlspkg::numericDeriv(rexpr, theta, rho = weedenv,      central = TRUE, eps = 1e-10)
##     min      lq     mean  median     uq    max neval
##  11.869 12.0625 12.83785 12.2815 12.637 41.962   100
```

```
## Add dir parameter
## to forward diff
ndeunsc<-nlspkg::numericDeriv(rexpr, theta, rho=weedenv)
print(ndeunsc)
```

```
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##            [,1]          [,2]         [,3]
##  [1,] 0.7310585 -1.966119e-01 0.1966118813
##  [2,] 0.8807971 -1.049936e-01 0.2099871635
##  [3,] 0.9525741 -4.517674e-02 0.1355299950
##  [4,] 0.9820137 -1.766276e-02 0.0706508160
##  [5,] 0.9933071 -6.648064e-03 0.0332403183
##  [6,] 0.9975274 -2.466440e-03 0.0147991180
##  [7,] 0.9990890 -9.102821e-04 0.0063714981
##  [8,] 0.9996643 -3.356934e-04 0.0026817322
##  [9,] 0.9998765 -1.235008e-04 0.0011105537
## [10,] 0.9999547 -4.529953e-05 0.0004539490
## [11,] 0.9999828 -1.716614e-05 0.0001831055
## [12,] 0.9999943 -5.722046e-06 0.0000743866
```

```
ndeunscd<-nlspkg::numericDeriv(rexpr, theta, rho=weedenv, dir=-1)
print(ndeunscd)
```

```
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##            [,1]          [,2]         [,3]
##  [1,] 0.7310586 -1.966119e-01 1.966119e-01
##  [2,] 0.8807971 -1.049936e-01 2.099872e-01
##  [3,] 0.9525741 -4.517663e-02 1.355300e-01
##  [4,] 0.9820138 -1.766264e-02 7.065094e-02
##  [5,] 0.9933071 -6.648064e-03 3.324032e-02
##  [6,] 0.9975274 -2.466440e-03 1.479912e-02
##  [7,] 0.9990890 -9.102821e-04 6.371498e-03
##  [8,] 0.9996648 -3.352165e-04 2.682209e-03
##  [9,] 0.9998765 -1.235008e-04 1.110554e-03
## [10,] 0.9999547 -4.529953e-05 4.539490e-04
## [11,] 0.9999838 -1.621246e-05 1.840591e-04
## [12,] 0.9999933 -6.675720e-06 7.343292e-05
```

```
ndeunscc<-nlspkg::numericDeriv(rexpr, theta, rho=weedenv, central=TRUE)
print(ndeunscc)
```

```
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##            [,1]          [,2]         [,3]
##  [1,] 0.7310586 -1.966119e-01 1.966119e-01
##  [2,] 0.8807971 -1.049936e-01 2.099872e-01
##  [3,] 0.9525741 -4.517666e-02 1.355300e-01
##  [4,] 0.9820138 -1.766271e-02 7.065082e-02
##  [5,] 0.9933071 -6.648057e-03 3.324028e-02
##  [6,] 0.9975274 -2.466509e-03 1.479906e-02
##  [7,] 0.9990889 -9.102211e-04 6.371548e-03
##  [8,] 0.9996647 -3.352378e-04 2.681902e-03
##  [9,] 0.9998766 -1.233799e-04 1.110414e-03
## [10,] 0.9999546 -4.539623e-05 4.539581e-04
## [11,] 0.9999833 -1.670090e-05 1.837134e-04
## [12,] 0.9999939 -6.143885e-06 7.372897e-05
```

```
ndeunsccd<-nlspkg::numericDeriv(rexpr, theta, rho=weedenv, central=TRUE, dir=-1)
print(ndeunsccd)
```

```
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##            [,1]          [,2]         [,3]
##  [1,] 0.7310586 -1.966119e-01 1.966119e-01
##  [2,] 0.8807971 -1.049936e-01 2.099872e-01
##  [3,] 0.9525741 -4.517666e-02 1.355300e-01
##  [4,] 0.9820138 -1.766271e-02 7.065082e-02
##  [5,] 0.9933071 -6.648057e-03 3.324028e-02
##  [6,] 0.9975274 -2.466509e-03 1.479906e-02
##  [7,] 0.9990889 -9.102211e-04 6.371548e-03
##  [8,] 0.9996647 -3.352378e-04 2.681902e-03
```

```
##  [9,] 0.9998766 -1.233799e-04 1.110414e-03
## [10,] 0.9999546 -4.539623e-05 4.539581e-04
## [11,] 0.9999833 -1.670090e-05 1.837134e-04
## [12,] 0.9999939 -6.143885e-06 7.372897e-05
```

numericDeriv() alternative pure-R version

This version (see Appendix 2) has C code replaced with R equivalents.

```
## Try ExDerivs.R ??
andeunsc<-nlsalt::numericDeriv(rexpr, theta, rho=weedenv)
```

```
## Registered S3 methods overwritten by 'nlsalt':
##   method            from
##   anova.nls         nlspkg
##   coef.nls          nlspkg
##   confint.nls       MASS
##   deviance.nls      nlspkg
##   df.residual.nls   nlspkg
##   fitted.nls        nlspkg
##   formula.nls       nlspkg
##   logLik.nls        nlspkg
##   nobs.nls          nlspkg
##   plot.profile.nls  nlspkg
##   predict.nls       nlspkg
##   print.nls         nlspkg
##   print.summary.nls nlspkg
##   profile.nls       nlspkg
##   residuals.nls     nlspkg
##   summary.nls       nlspkg
##   vcov.nls          nlspkg
##   weights.nls       nlspkg
```

```
## par:1  1  1
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##            [,1]          [,2]         [,3]
##  [1,] 0.7310585 -1.966119e-01 0.1966118813
##  [2,] 0.8807971 -1.049936e-01 0.2099871635
##  [3,] 0.9525741 -4.517674e-02 0.1355299950
##  [4,] 0.9820137 -1.766276e-02 0.0706508160
##  [5,] 0.9933071 -6.648064e-03 0.0332403183
##  [6,] 0.9975274 -2.466440e-03 0.0147991180
##  [7,] 0.9990890 -9.102821e-04 0.0063714981
##  [8,] 0.9996643 -3.356934e-04 0.0026817322
##  [9,] 0.9998765 -1.235008e-04 0.0011105537
## [10,] 0.9999547 -4.529953e-05 0.0004539490
## [11,] 0.9999828 -1.716614e-05 0.0001831055
## [12,] 0.9999943 -5.722046e-06 0.0000743866
```

```
print(andeunsc)
```

```
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
```

```
##            [,1]          [,2]         [,3]
##  [1,] 0.7310585 -1.966119e-01 0.1966118813
##  [2,] 0.8807971 -1.049936e-01 0.2099871635
##  [3,] 0.9525741 -4.517674e-02 0.1355299950
##  [4,] 0.9820137 -1.766276e-02 0.0706508160
##  [5,] 0.9933071 -6.648064e-03 0.0332403183
##  [6,] 0.9975274 -2.466440e-03 0.0147991180
##  [7,] 0.9990890 -9.102821e-04 0.0063714981
##  [8,] 0.9996643 -3.356934e-04 0.0026817322
##  [9,] 0.9998765 -1.235008e-04 0.0011105537
## [10,] 0.9999547 -4.529953e-05 0.0004539490
## [11,] 0.9999828 -1.716614e-05 0.0001831055
## [12,] 0.9999943 -5.722046e-06 0.0000743866
```

```
print(sum(andeunsc^2))
```

```
## [1] 23520.58
```

```
tandeunsc<-microbenchmark(andeunsc<-nlsalt::numericDeriv(rexpr, theta, rho=weedenv))
```

```
## par:1  1  1
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##            [,1]          [,2]         [,3]
##  [1,] 0.7310585 -1.966119e-01 0.1966118813
##  [2,] 0.8807971 -1.049936e-01 0.2099871635
##  [3,] 0.9525741 -4.517674e-02 0.1355299950
##  [4,] 0.9820137 -1.766276e-02 0.0706508160
##  [5,] 0.9933071 -6.648064e-03 0.0332403183
##  [6,] 0.9975274 -2.466440e-03 0.0147991180
##  [7,] 0.9990890 -9.102821e-04 0.0063714981
##  [8,] 0.9996643 -3.356934e-04 0.0026817322
##  [9,] 0.9998765 -1.235008e-04 0.0011105537
## [10,] 0.9999547 -4.529953e-05 0.0004539490
## [11,] 0.9999828 -1.716614e-05 0.0001831055
## [12,] 0.9999943 -5.722046e-06 0.0000743866
## par:1  1  1
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
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##  [3,] 0.9525741 -4.517674e-02 0.1355299950
##  [4,] 0.9820137 -1.766276e-02 0.0706508160
##  [5,] 0.9933071 -6.648064e-03 0.0332403183
##  [6,] 0.9975274 -2.466440e-03 0.0147991180
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## [11,] 0.9999828 -1.716614e-05 0.0001831055
## [12,] 0.9999943 -5.722046e-06 0.0000743866
## par:1  1  1
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
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##  [5,] 0.9933071 -6.648064e-03 0.0332403183
##  [6,] 0.9975274 -2.466440e-03 0.0147991180
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##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
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##  [5,] 0.9933071 -6.648064e-03 0.0332403183
##  [6,] 0.9975274 -2.466440e-03 0.0147991180
##  [7,] 0.9990890 -9.102821e-04 0.0063714981
##  [8,] 0.9996643 -3.356934e-04 0.0026817322
##  [9,] 0.9998765 -1.235008e-04 0.0011105537
## [10,] 0.9999547 -4.529953e-05 0.0004539490
## [11,] 0.9999828 -1.716614e-05 0.0001831055
## [12,] 0.9999943 -5.722046e-06 0.0000743866
## par:1  1  1
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##            [,1]          [,2]         [,3]
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##  [6,] 0.9975274 -2.466440e-03 0.0147991180
##  [7,] 0.9990890 -9.102821e-04 0.0063714981
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##  [9,] 0.9998765 -1.235008e-04 0.0011105537
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## [11,] 0.9999828 -1.716614e-05 0.0001831055
## [12,] 0.9999943 -5.722046e-06 0.0000743866
## par:1  1  1
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
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##  [2,] 0.8807971 -1.049936e-01 0.2099871635
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##  [7,] 0.9990890 -9.102821e-04 0.0063714981
##  [8,] 0.9996643 -3.356934e-04 0.0026817322
##  [9,] 0.9998765 -1.235008e-04 0.0011105537
## [10,] 0.9999547 -4.529953e-05 0.0004539490
## [11,] 0.9999828 -1.716614e-05 0.0001831055
## [12,] 0.9999943 -5.722046e-06 0.0000743866
```

```
## par:1  1  1
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##            [,1]          [,2]         [,3]
##  [1,] 0.7310585 -1.966119e-01 0.1966118813
##  [2,] 0.8807971 -1.049936e-01 0.2099871635
##  [3,] 0.9525741 -4.517674e-02 0.1355299950
##  [4,] 0.9820137 -1.766276e-02 0.0706508160
##  [5,] 0.9933071 -6.648064e-03 0.0332403183
##  [6,] 0.9975274 -2.466440e-03 0.0147991180
##  [7,] 0.9990890 -9.102821e-04 0.0063714981
##  [8,] 0.9996643 -3.356934e-04 0.0026817322
##  [9,] 0.9998765 -1.235008e-04 0.0011105537
## [10,] 0.9999547 -4.529953e-05 0.0004539490
## [11,] 0.9999828 -1.716614e-05 0.0001831055
## [12,] 0.9999943 -5.722046e-06 0.0000743866
## par:1  1  1
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##            [,1]          [,2]         [,3]
##  [1,] 0.7310585 -1.966119e-01 0.1966118813
##  [2,] 0.8807971 -1.049936e-01 0.2099871635
##  [3,] 0.9525741 -4.517674e-02 0.1355299950
##  [4,] 0.9820137 -1.766276e-02 0.0706508160
##  [5,] 0.9933071 -6.648064e-03 0.0332403183
##  [6,] 0.9975274 -2.466440e-03 0.0147991180
##  [7,] 0.9990890 -9.102821e-04 0.0063714981
##  [8,] 0.9996643 -3.356934e-04 0.0026817322
##  [9,] 0.9998765 -1.235008e-04 0.0011105537
## [10,] 0.9999547 -4.529953e-05 0.0004539490
## [11,] 0.9999828 -1.716614e-05 0.0001831055
## [12,] 0.9999943 -5.722046e-06 0.0000743866
## par:1  1  1
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##            [,1]          [,2]         [,3]
##  [1,] 0.7310585 -1.966119e-01 0.1966118813
##  [2,] 0.8807971 -1.049936e-01 0.2099871635
##  [3,] 0.9525741 -4.517674e-02 0.1355299950
##  [4,] 0.9820137 -1.766276e-02 0.0706508160
##  [5,] 0.9933071 -6.648064e-03 0.0332403183
##  [6,] 0.9975274 -2.466440e-03 0.0147991180
##  [7,] 0.9990890 -9.102821e-04 0.0063714981
##  [8,] 0.9996643 -3.356934e-04 0.0026817322
##  [9,] 0.9998765 -1.235008e-04 0.0011105537
## [10,] 0.9999547 -4.529953e-05 0.0004539490
## [11,] 0.9999828 -1.716614e-05 0.0001831055
## [12,] 0.9999943 -5.722046e-06 0.0000743866
## par:1  1  1
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
```

```
## attr(,"gradient")
##            [,1]          [,2]         [,3]
##  [1,] 0.7310585 -1.966119e-01 0.1966118813
##  [2,] 0.8807971 -1.049936e-01 0.2099871635
##  [3,] 0.9525741 -4.517674e-02 0.1355299950
##  [4,] 0.9820137 -1.766276e-02 0.0706508160
##  [5,] 0.9933071 -6.648064e-03 0.0332403183
##  [6,] 0.9975274 -2.466440e-03 0.0147991180
##  [7,] 0.9990890 -9.102821e-04 0.0063714981
##  [8,] 0.9996643 -3.356934e-04 0.0026817322
##  [9,] 0.9998765 -1.235008e-04 0.0011105537
## [10,] 0.9999547 -4.529953e-05 0.0004539490
## [11,] 0.9999828 -1.716614e-05 0.0001831055
## [12,] 0.9999943 -5.722046e-06 0.0000743866
## par:1  1  1
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##            [,1]          [,2]         [,3]
##  [1,] 0.7310585 -1.966119e-01 0.1966118813
##  [2,] 0.8807971 -1.049936e-01 0.2099871635
##  [3,] 0.9525741 -4.517674e-02 0.1355299950
##  [4,] 0.9820137 -1.766276e-02 0.0706508160
##  [5,] 0.9933071 -6.648064e-03 0.0332403183
##  [6,] 0.9975274 -2.466440e-03 0.0147991180
##  [7,] 0.9990890 -9.102821e-04 0.0063714981
##  [8,] 0.9996643 -3.356934e-04 0.0026817322
##  [9,] 0.9998765 -1.235008e-04 0.0011105537
## [10,] 0.9999547 -4.529953e-05 0.0004539490
## [11,] 0.9999828 -1.716614e-05 0.0001831055
## [12,] 0.9999943 -5.722046e-06 0.0000743866
## par:1  1  1
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##            [,1]          [,2]         [,3]
##  [1,] 0.7310585 -1.966119e-01 0.1966118813
##  [2,] 0.8807971 -1.049936e-01 0.2099871635
##  [3,] 0.9525741 -4.517674e-02 0.1355299950
##  [4,] 0.9820137 -1.766276e-02 0.0706508160
##  [5,] 0.9933071 -6.648064e-03 0.0332403183
##  [6,] 0.9975274 -2.466440e-03 0.0147991180
##  [7,] 0.9990890 -9.102821e-04 0.0063714981
##  [8,] 0.9996643 -3.356934e-04 0.0026817322
##  [9,] 0.9998765 -1.235008e-04 0.0011105537
## [10,] 0.9999547 -4.529953e-05 0.0004539490
## [11,] 0.9999828 -1.716614e-05 0.0001831055
## [12,] 0.9999943 -5.722046e-06 0.0000743866
## par:1  1  1
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##            [,1]          [,2]         [,3]
##  [1,] 0.7310585 -1.966119e-01 0.1966118813
```

```
##  [2,] 0.8807971 -1.049936e-01 0.2099871635
##  [3,] 0.9525741 -4.517674e-02 0.1355299950
##  [4,] 0.9820137 -1.766276e-02 0.0706508160
##  [5,] 0.9933071 -6.648064e-03 0.0332403183
##  [6,] 0.9975274 -2.466440e-03 0.0147991180
##  [7,] 0.9990890 -9.102821e-04 0.0063714981
##  [8,] 0.9996643 -3.356934e-04 0.0026817322
##  [9,] 0.9998765 -1.235008e-04 0.0011105537
## [10,] 0.9999547 -4.529953e-05 0.0004539490
## [11,] 0.9999828 -1.716614e-05 0.0001831055
## [12,] 0.9999943 -5.722046e-06 0.0000743866
## par:1  1  1
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##           [,1]          [,2]         [,3]
##  [1,] 0.7310585 -1.966119e-01 0.1966118813
##  [2,] 0.8807971 -1.049936e-01 0.2099871635
##  [3,] 0.9525741 -4.517674e-02 0.1355299950
##  [4,] 0.9820137 -1.766276e-02 0.0706508160
##  [5,] 0.9933071 -6.648064e-03 0.0332403183
##  [6,] 0.9975274 -2.466440e-03 0.0147991180
##  [7,] 0.9990890 -9.102821e-04 0.0063714981
##  [8,] 0.9996643 -3.356934e-04 0.0026817322
##  [9,] 0.9998765 -1.235008e-04 0.0011105537
## [10,] 0.9999547 -4.529953e-05 0.0004539490
## [11,] 0.9999828 -1.716614e-05 0.0001831055
## [12,] 0.9999943 -5.722046e-06 0.0000743866
## par:1  1  1
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##           [,1]          [,2]         [,3]
##  [1,] 0.7310585 -1.966119e-01 0.1966118813
##  [2,] 0.8807971 -1.049936e-01 0.2099871635
##  [3,] 0.9525741 -4.517674e-02 0.1355299950
##  [4,] 0.9820137 -1.766276e-02 0.0706508160
##  [5,] 0.9933071 -6.648064e-03 0.0332403183
##  [6,] 0.9975274 -2.466440e-03 0.0147991180
##  [7,] 0.9990890 -9.102821e-04 0.0063714981
##  [8,] 0.9996643 -3.356934e-04 0.0026817322
##  [9,] 0.9998765 -1.235008e-04 0.0011105537
## [10,] 0.9999547 -4.529953e-05 0.0004539490
## [11,] 0.9999828 -1.716614e-05 0.0001831055
## [12,] 0.9999943 -5.722046e-06 0.0000743866
```

```
print(tandeunsc)
```

```
## Unit: microseconds
##                                                          expr     min       lq
##  andeunsc <- nlsalt::numericDeriv(rexpr, theta, rho = weedenv) 121.267 145.5875
##      mean  median       uq    max neval
##  174.2673 174.134 202.5205 278.38   100
```

```
## numericDeriv also has central difference option, as well as choice of eps parameter
## Central diff
andeunsc2<-nlsalt::numericDeriv(rexpr, theta, rho=weedenv, central=TRUE)
```

```
## par:1  1  1
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##             [,1]          [,2]         [,3]
##  [1,] 0.7310586 -1.966119e-01 1.966119e-01
##  [2,] 0.8807971 -1.049936e-01 2.099872e-01
##  [3,] 0.9525741 -4.517666e-02 1.355300e-01
##  [4,] 0.9820138 -1.766271e-02 7.065082e-02
##  [5,] 0.9933071 -6.648057e-03 3.324028e-02
##  [6,] 0.9975274 -2.466509e-03 1.479906e-02
##  [7,] 0.9990889 -9.102211e-04 6.371548e-03
##  [8,] 0.9996647 -3.352378e-04 2.681902e-03
##  [9,] 0.9998766 -1.233799e-04 1.110414e-03
## [10,] 0.9999546 -4.539623e-05 4.539581e-04
## [11,] 0.9999833 -1.670090e-05 1.837134e-04
## [12,] 0.9999939 -6.143885e-06 7.372897e-05
```

```
print(andeunsc2)
```

```
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##             [,1]          [,2]         [,3]
##  [1,] 0.7310586 -1.966119e-01 1.966119e-01
##  [2,] 0.8807971 -1.049936e-01 2.099872e-01
##  [3,] 0.9525741 -4.517666e-02 1.355300e-01
##  [4,] 0.9820138 -1.766271e-02 7.065082e-02
##  [5,] 0.9933071 -6.648057e-03 3.324028e-02
##  [6,] 0.9975274 -2.466509e-03 1.479906e-02
##  [7,] 0.9990889 -9.102211e-04 6.371548e-03
##  [8,] 0.9996647 -3.352378e-04 2.681902e-03
##  [9,] 0.9998766 -1.233799e-04 1.110414e-03
## [10,] 0.9999546 -4.539623e-05 4.539581e-04
## [11,] 0.9999833 -1.670090e-05 1.837134e-04
## [12,] 0.9999939 -6.143885e-06 7.372897e-05
```

```
print(sum(andeunsc2^2))
```

```
## [1] 23520.58
```

```
tandeunsc2<-microbenchmark(andeunsc2<-nlsalt::numericDeriv(rexpr, theta, rho=weedenv, central=TRUE))
```

```
## par:1  1  1
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##             [,1]          [,2]         [,3]
##  [1,] 0.7310586 -1.966119e-01 1.966119e-01
##  [2,] 0.8807971 -1.049936e-01 2.099872e-01
##  [3,] 0.9525741 -4.517666e-02 1.355300e-01
##  [4,] 0.9820138 -1.766271e-02 7.065082e-02
```

```
##  [5,] 0.9933071 -6.648057e-03 3.324028e-02
##  [6,] 0.9975274 -2.466509e-03 1.479906e-02
##  [7,] 0.9990889 -9.102211e-04 6.371548e-03
##  [8,] 0.9996647 -3.352378e-04 2.681902e-03
##  [9,] 0.9998766 -1.233799e-04 1.110414e-03
## [10,] 0.9999546 -4.539623e-05 4.539581e-04
## [11,] 0.9999833 -1.670090e-05 1.837134e-04
## [12,] 0.9999939 -6.143885e-06 7.372897e-05
## par:1  1  1
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##            [,1]          [,2]         [,3]
##  [1,] 0.7310586 -1.966119e-01 1.966119e-01
##  [2,] 0.8807971 -1.049936e-01 2.099872e-01
##  [3,] 0.9525741 -4.517666e-02 1.355300e-01
##  [4,] 0.9820138 -1.766271e-02 7.065082e-02
##  [5,] 0.9933071 -6.648057e-03 3.324028e-02
##  [6,] 0.9975274 -2.466509e-03 1.479906e-02
##  [7,] 0.9990889 -9.102211e-04 6.371548e-03
##  [8,] 0.9996647 -3.352378e-04 2.681902e-03
##  [9,] 0.9998766 -1.233799e-04 1.110414e-03
## [10,] 0.9999546 -4.539623e-05 4.539581e-04
## [11,] 0.9999833 -1.670090e-05 1.837134e-04
## [12,] 0.9999939 -6.143885e-06 7.372897e-05
## par:1  1  1
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##            [,1]          [,2]         [,3]
##  [1,] 0.7310586 -1.966119e-01 1.966119e-01
##  [2,] 0.8807971 -1.049936e-01 2.099872e-01
##  [3,] 0.9525741 -4.517666e-02 1.355300e-01
##  [4,] 0.9820138 -1.766271e-02 7.065082e-02
##  [5,] 0.9933071 -6.648057e-03 3.324028e-02
##  [6,] 0.9975274 -2.466509e-03 1.479906e-02
##  [7,] 0.9990889 -9.102211e-04 6.371548e-03
##  [8,] 0.9996647 -3.352378e-04 2.681902e-03
##  [9,] 0.9998766 -1.233799e-04 1.110414e-03
## [10,] 0.9999546 -4.539623e-05 4.539581e-04
## [11,] 0.9999833 -1.670090e-05 1.837134e-04
## [12,] 0.9999939 -6.143885e-06 7.372897e-05
## par:1  1  1
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##            [,1]          [,2]         [,3]
##  [1,] 0.7310586 -1.966119e-01 1.966119e-01
##  [2,] 0.8807971 -1.049936e-01 2.099872e-01
##  [3,] 0.9525741 -4.517666e-02 1.355300e-01
##  [4,] 0.9820138 -1.766271e-02 7.065082e-02
##  [5,] 0.9933071 -6.648057e-03 3.324028e-02
##  [6,] 0.9975274 -2.466509e-03 1.479906e-02
##  [7,] 0.9990889 -9.102211e-04 6.371548e-03
```

```
##  [8,] 0.9996647 -3.352378e-04 2.681902e-03
##  [9,] 0.9998766 -1.233799e-04 1.110414e-03
## [10,] 0.9999546 -4.539623e-05 4.539581e-04
## [11,] 0.9999833 -1.670090e-05 1.837134e-04
## [12,] 0.9999939 -6.143885e-06 7.372897e-05
## par:1  1  1
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##            [,1]          [,2]         [,3]
##  [1,] 0.7310586 -1.966119e-01 1.966119e-01
##  [2,] 0.8807971 -1.049936e-01 2.099872e-01
##  [3,] 0.9525741 -4.517666e-02 1.355300e-01
##  [4,] 0.9820138 -1.766271e-02 7.065082e-02
##  [5,] 0.9933071 -6.648057e-03 3.324028e-02
##  [6,] 0.9975274 -2.466509e-03 1.479906e-02
##  [7,] 0.9990889 -9.102211e-04 6.371548e-03
##  [8,] 0.9996647 -3.352378e-04 2.681902e-03
##  [9,] 0.9998766 -1.233799e-04 1.110414e-03
## [10,] 0.9999546 -4.539623e-05 4.539581e-04
## [11,] 0.9999833 -1.670090e-05 1.837134e-04
## [12,] 0.9999939 -6.143885e-06 7.372897e-05
## par:1  1  1
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##            [,1]          [,2]         [,3]
##  [1,] 0.7310586 -1.966119e-01 1.966119e-01
##  [2,] 0.8807971 -1.049936e-01 2.099872e-01
##  [3,] 0.9525741 -4.517666e-02 1.355300e-01
##  [4,] 0.9820138 -1.766271e-02 7.065082e-02
##  [5,] 0.9933071 -6.648057e-03 3.324028e-02
##  [6,] 0.9975274 -2.466509e-03 1.479906e-02
##  [7,] 0.9990889 -9.102211e-04 6.371548e-03
##  [8,] 0.9996647 -3.352378e-04 2.681902e-03
##  [9,] 0.9998766 -1.233799e-04 1.110414e-03
## [10,] 0.9999546 -4.539623e-05 4.539581e-04
## [11,] 0.9999833 -1.670090e-05 1.837134e-04
## [12,] 0.9999939 -6.143885e-06 7.372897e-05
## par:1  1  1
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##            [,1]          [,2]         [,3]
##  [1,] 0.7310586 -1.966119e-01 1.966119e-01
##  [2,] 0.8807971 -1.049936e-01 2.099872e-01
##  [3,] 0.9525741 -4.517666e-02 1.355300e-01
##  [4,] 0.9820138 -1.766271e-02 7.065082e-02
##  [5,] 0.9933071 -6.648057e-03 3.324028e-02
##  [6,] 0.9975274 -2.466509e-03 1.479906e-02
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##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
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##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##            [,1]          [,2]         [,3]
##  [1,] 0.7310586 -1.966119e-01 1.966119e-01
##  [2,] 0.8807971 -1.049936e-01 2.099872e-01
##  [3,] 0.9525741 -4.517666e-02 1.355300e-01
##  [4,] 0.9820138 -1.766271e-02 7.065082e-02
##  [5,] 0.9933071 -6.648057e-03 3.324028e-02
##  [6,] 0.9975274 -2.466509e-03 1.479906e-02
##  [7,] 0.9990889 -9.102211e-04 6.371548e-03
##  [8,] 0.9996647 -3.352378e-04 2.681902e-03
##  [9,] 0.9998766 -1.233799e-04 1.110414e-03
## [10,] 0.9999546 -4.539623e-05 4.539581e-04
## [11,] 0.9999833 -1.670090e-05 1.837134e-04
## [12,] 0.9999939 -6.143885e-06 7.372897e-05
## par:1  1  1
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##            [,1]          [,2]         [,3]
##  [1,] 0.7310586 -1.966119e-01 1.966119e-01
##  [2,] 0.8807971 -1.049936e-01 2.099872e-01
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##  [7,] 0.9990889 -9.102211e-04 6.371548e-03
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##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
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## [10,] 0.9999546 -4.539623e-05 4.539581e-04
## [11,] 0.9999833 -1.670090e-05 1.837134e-04
```

```
## [12,] 0.9999939 -6.143885e-06 7.372897e-05
## par:1  1  1
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
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##  [7,] 0.9990889 -9.102211e-04 6.371548e-03
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##  [5,] 0.9933071 -6.648057e-03 3.324028e-02
##  [6,] 0.9975274 -2.466509e-03 1.479906e-02
##  [7,] 0.9990889 -9.102211e-04 6.371548e-03
##  [8,] 0.9996647 -3.352378e-04 2.681902e-03
##  [9,] 0.9998766 -1.233799e-04 1.110414e-03
## [10,] 0.9999546 -4.539623e-05 4.539581e-04
## [11,] 0.9999833 -1.670090e-05 1.837134e-04
## [12,] 0.9999939 -6.143885e-06 7.372897e-05
## par:1  1  1
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##            [,1]          [,2]         [,3]
##  [1,] 0.7310586 -1.966119e-01 1.966119e-01
##  [2,] 0.8807971 -1.049936e-01 2.099872e-01
##  [3,] 0.9525741 -4.517666e-02 1.355300e-01
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```
## [10,] 0.9999546 -4.539623e-05 4.539581e-04
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##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
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## [11,] 0.9999833 -1.670090e-05 1.837134e-04
## [12,] 0.9999939 -6.143885e-06 7.372897e-05
## par:1  1  1
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##            [,1]          [,2]         [,3]
##  [1,] 0.7310586 -1.966119e-01 1.966119e-01
##  [2,] 0.8807971 -1.049936e-01 2.099872e-01
##  [3,] 0.9525741 -4.517666e-02 1.355300e-01
##  [4,] 0.9820138 -1.766271e-02 7.065082e-02
##  [5,] 0.9933071 -6.648057e-03 3.324028e-02
##  [6,] 0.9975274 -2.466509e-03 1.479906e-02
##  [7,] 0.9990889 -9.102211e-04 6.371548e-03
##  [8,] 0.9996647 -3.352378e-04 2.681902e-03
##  [9,] 0.9998766 -1.233799e-04 1.110414e-03
## [10,] 0.9999546 -4.539623e-05 4.539581e-04
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## par:1  1  1
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##            [,1]          [,2]         [,3]
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##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
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## [10,] 0.9999546 -4.539623e-05 4.539581e-04
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## par:1  1  1
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##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
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##  [1,] 0.7310586 -1.966119e-01 1.966119e-01
##  [2,] 0.8807971 -1.049936e-01 2.099872e-01
##  [3,] 0.9525741 -4.517666e-02 1.355300e-01
##  [4,] 0.9820138 -1.766271e-02 7.065082e-02
##  [5,] 0.9933071 -6.648057e-03 3.324028e-02
##  [6,] 0.9975274 -2.466509e-03 1.479906e-02
##  [7,] 0.9990889 -9.102211e-04 6.371548e-03
##  [8,] 0.9996647 -3.352378e-04 2.681902e-03
##  [9,] 0.9998766 -1.233799e-04 1.110414e-03
## [10,] 0.9999546 -4.539623e-05 4.539581e-04
## [11,] 0.9999833 -1.670090e-05 1.837134e-04
## [12,] 0.9999939 -6.143885e-06 7.372897e-05
## par:1  1  1
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##            [,1]          [,2]         [,3]
##  [1,] 0.7310586 -1.966119e-01 1.966119e-01
##  [2,] 0.8807971 -1.049936e-01 2.099872e-01
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##  [8,] 0.9996647 -3.352378e-04 2.681902e-03
```

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##  [9,] 0.9998766 -1.233799e-04 1.110414e-03
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```
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```
##  [7,] 0.9990889 -9.102211e-04 6.371548e-03
##  [8,] 0.9996647 -3.352378e-04 2.681902e-03
##  [9,] 0.9998766 -1.233799e-04 1.110414e-03
## [10,] 0.9999546 -4.539623e-05 4.539581e-04
## [11,] 0.9999833 -1.670090e-05 1.837134e-04
## [12,] 0.9999939 -6.143885e-06 7.372897e-05
## par:1  1  1
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##            [,1]          [,2]         [,3]
##  [1,] 0.7310586 -1.966119e-01 1.966119e-01
##  [2,] 0.8807971 -1.049936e-01 2.099872e-01
##  [3,] 0.9525741 -4.517666e-02 1.355300e-01
##  [4,] 0.9820138 -1.766271e-02 7.065082e-02
##  [5,] 0.9933071 -6.648057e-03 3.324028e-02
##  [6,] 0.9975274 -2.466509e-03 1.479906e-02
##  [7,] 0.9990889 -9.102211e-04 6.371548e-03
##  [8,] 0.9996647 -3.352378e-04 2.681902e-03
##  [9,] 0.9998766 -1.233799e-04 1.110414e-03
## [10,] 0.9999546 -4.539623e-05 4.539581e-04
## [11,] 0.9999833 -1.670090e-05 1.837134e-04
## [12,] 0.9999939 -6.143885e-06 7.372897e-05
## par:1  1  1
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##            [,1]          [,2]         [,3]
##  [1,] 0.7310586 -1.966119e-01 1.966119e-01
##  [2,] 0.8807971 -1.049936e-01 2.099872e-01
##  [3,] 0.9525741 -4.517666e-02 1.355300e-01
##  [4,] 0.9820138 -1.766271e-02 7.065082e-02
##  [5,] 0.9933071 -6.648057e-03 3.324028e-02
##  [6,] 0.9975274 -2.466509e-03 1.479906e-02
##  [7,] 0.9990889 -9.102211e-04 6.371548e-03
##  [8,] 0.9996647 -3.352378e-04 2.681902e-03
##  [9,] 0.9998766 -1.233799e-04 1.110414e-03
## [10,] 0.9999546 -4.539623e-05 4.539581e-04
## [11,] 0.9999833 -1.670090e-05 1.837134e-04
## [12,] 0.9999939 -6.143885e-06 7.372897e-05
## par:1  1  1
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##            [,1]          [,2]         [,3]
##  [1,] 0.7310586 -1.966119e-01 1.966119e-01
##  [2,] 0.8807971 -1.049936e-01 2.099872e-01
##  [3,] 0.9525741 -4.517666e-02 1.355300e-01
##  [4,] 0.9820138 -1.766271e-02 7.065082e-02
##  [5,] 0.9933071 -6.648057e-03 3.324028e-02
##  [6,] 0.9975274 -2.466509e-03 1.479906e-02
##  [7,] 0.9990889 -9.102211e-04 6.371548e-03
##  [8,] 0.9996647 -3.352378e-04 2.681902e-03
##  [9,] 0.9998766 -1.233799e-04 1.110414e-03
```

```
## [10,] 0.9999546 -4.539623e-05 4.539581e-04
## [11,] 0.9999833 -1.670090e-05 1.837134e-04
## [12,] 0.9999939 -6.143885e-06 7.372897e-05
## par:1  1  1
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##             [,1]          [,2]         [,3]
##  [1,] 0.7310586 -1.966119e-01 1.966119e-01
##  [2,] 0.8807971 -1.049936e-01 2.099872e-01
##  [3,] 0.9525741 -4.517666e-02 1.355300e-01
##  [4,] 0.9820138 -1.766271e-02 7.065082e-02
##  [5,] 0.9933071 -6.648057e-03 3.324028e-02
##  [6,] 0.9975274 -2.466509e-03 1.479906e-02
##  [7,] 0.9990889 -9.102211e-04 6.371548e-03
##  [8,] 0.9996647 -3.352378e-04 2.681902e-03
##  [9,] 0.9998766 -1.233799e-04 1.110414e-03
## [10,] 0.9999546 -4.539623e-05 4.539581e-04
## [11,] 0.9999833 -1.670090e-05 1.837134e-04
## [12,] 0.9999939 -6.143885e-06 7.372897e-05
## par:1  1  1
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##             [,1]          [,2]         [,3]
##  [1,] 0.7310586 -1.966119e-01 1.966119e-01
##  [2,] 0.8807971 -1.049936e-01 2.099872e-01
##  [3,] 0.9525741 -4.517666e-02 1.355300e-01
##  [4,] 0.9820138 -1.766271e-02 7.065082e-02
##  [5,] 0.9933071 -6.648057e-03 3.324028e-02
##  [6,] 0.9975274 -2.466509e-03 1.479906e-02
##  [7,] 0.9990889 -9.102211e-04 6.371548e-03
##  [8,] 0.9996647 -3.352378e-04 2.681902e-03
##  [9,] 0.9998766 -1.233799e-04 1.110414e-03
## [10,] 0.9999546 -4.539623e-05 4.539581e-04
## [11,] 0.9999833 -1.670090e-05 1.837134e-04
## [12,] 0.9999939 -6.143885e-06 7.372897e-05
## par:1  1  1
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##             [,1]          [,2]         [,3]
##  [1,] 0.7310586 -1.966119e-01 1.966119e-01
##  [2,] 0.8807971 -1.049936e-01 2.099872e-01
##  [3,] 0.9525741 -4.517666e-02 1.355300e-01
##  [4,] 0.9820138 -1.766271e-02 7.065082e-02
##  [5,] 0.9933071 -6.648057e-03 3.324028e-02
##  [6,] 0.9975274 -2.466509e-03 1.479906e-02
##  [7,] 0.9990889 -9.102211e-04 6.371548e-03
##  [8,] 0.9996647 -3.352378e-04 2.681902e-03
##  [9,] 0.9998766 -1.233799e-04 1.110414e-03
## [10,] 0.9999546 -4.539623e-05 4.539581e-04
## [11,] 0.9999833 -1.670090e-05 1.837134e-04
## [12,] 0.9999939 -6.143885e-06 7.372897e-05
```

```
## par:1  1  1
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##            [,1]          [,2]         [,3]
##  [1,] 0.7310586 -1.966119e-01 1.966119e-01
##  [2,] 0.8807971 -1.049936e-01 2.099872e-01
##  [3,] 0.9525741 -4.517666e-02 1.355300e-01
##  [4,] 0.9820138 -1.766271e-02 7.065082e-02
##  [5,] 0.9933071 -6.648057e-03 3.324028e-02
##  [6,] 0.9975274 -2.466509e-03 1.479906e-02
##  [7,] 0.9990889 -9.102211e-04 6.371548e-03
##  [8,] 0.9996647 -3.352378e-04 2.681902e-03
##  [9,] 0.9998766 -1.233799e-04 1.110414e-03
## [10,] 0.9999546 -4.539623e-05 4.539581e-04
## [11,] 0.9999833 -1.670090e-05 1.837134e-04
## [12,] 0.9999939 -6.143885e-06 7.372897e-05
```

```
print(tandeunsc2)
```

```
## Unit: microseconds
##                                                                          expr
##  andeunsc2 <- nlsalt::numericDeriv(rexpr, theta, rho = weedenv,      central = TRUE)
##      min       lq     mean   median       uq     max neval
##  134.517 157.6255 185.7384 186.9365 215.466 258.006   100
```

```
## Forward diff with smaller eps
andeunscx<-nlsalt::numericDeriv(rexpr, theta, rho=weedenv, eps=1e-10)
```

```
## par:1  1  1
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##            [,1]          [,2]         [,3]
##  [1,] 0.7310597 -0.1966160568 0.1966071750
##  [2,] 0.8807977 -0.1049915710 0.2099920238
##  [3,] 0.9525714 -0.0451905180 0.1355182633
##  [4,] 0.9820056 -0.0176747506 0.0706457115
##  [5,] 0.9933032 -0.0066435746 0.0332534000
##  [6,] 0.9975309 -0.0024513724 0.0148148160
##  [7,] 0.9990941 -0.0009237056 0.0063593575
##  [8,] 0.9996626 -0.0003552714 0.0026290081
##  [9,] 0.9998757 -0.0001421085 0.0011368684
## [10,] 0.9999468  0.0000000000 0.0004973799
## [11,] 0.9998757 -0.0001421085 0.0001421085
## [12,] 1.0000178  0.0000000000 0.0001421085
```

```
print(andeunscx)
```

```
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##            [,1]          [,2]         [,3]
##  [1,] 0.7310597 -0.1966160568 0.1966071750
##  [2,] 0.8807977 -0.1049915710 0.2099920238
##  [3,] 0.9525714 -0.0451905180 0.1355182633
```

```
##  [4,] 0.9820056 -0.0176747506 0.0706457115
##  [5,] 0.9933032 -0.0066435746 0.0332534000
##  [6,] 0.9975309 -0.0024513724 0.0148148160
##  [7,] 0.9990941 -0.0009237056 0.0063593575
##  [8,] 0.9996626 -0.0003552714 0.0026290081
##  [9,] 0.9998757 -0.0001421085 0.0011368684
## [10,] 0.9999468  0.0000000000 0.0004973799
## [11,] 0.9998757 -0.0001421085 0.0001421085
## [12,] 1.0000178  0.0000000000 0.0001421085
```

```
print(sum(andeunscx^2))
```

```
## [1] 23520.58
```

```
tandeunscx<-microbenchmark(andeunscx2<-nlsalt::numericDeriv(rexpr, theta, rho=weedenv, eps=1e-10))
```

```
## par:1  1  1
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##            [,1]          [,2]         [,3]
##  [1,] 0.7310597 -0.1966160568 0.1966071750
##  [2,] 0.8807977 -0.1049915710 0.2099920238
##  [3,] 0.9525714 -0.0451905180 0.1355182633
##  [4,] 0.9820056 -0.0176747506 0.0706457115
##  [5,] 0.9933032 -0.0066435746 0.0332534000
##  [6,] 0.9975309 -0.0024513724 0.0148148160
##  [7,] 0.9990941 -0.0009237056 0.0063593575
##  [8,] 0.9996626 -0.0003552714 0.0026290081
##  [9,] 0.9998757 -0.0001421085 0.0011368684
## [10,] 0.9999468  0.0000000000 0.0004973799
## [11,] 0.9998757 -0.0001421085 0.0001421085
## [12,] 1.0000178  0.0000000000 0.0001421085
## par:1  1  1
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##            [,1]          [,2]         [,3]
##  [1,] 0.7310597 -0.1966160568 0.1966071750
##  [2,] 0.8807977 -0.1049915710 0.2099920238
##  [3,] 0.9525714 -0.0451905180 0.1355182633
##  [4,] 0.9820056 -0.0176747506 0.0706457115
##  [5,] 0.9933032 -0.0066435746 0.0332534000
##  [6,] 0.9975309 -0.0024513724 0.0148148160
##  [7,] 0.9990941 -0.0009237056 0.0063593575
##  [8,] 0.9996626 -0.0003552714 0.0026290081
##  [9,] 0.9998757 -0.0001421085 0.0011368684
## [10,] 0.9999468  0.0000000000 0.0004973799
## [11,] 0.9998757 -0.0001421085 0.0001421085
## [12,] 1.0000178  0.0000000000 0.0001421085
## par:1  1  1
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##            [,1]          [,2]         [,3]
```

```
##  [1,] 0.7310597 -0.1966160568 0.1966071750
##  [2,] 0.8807977 -0.1049915710 0.2099920238
##  [3,] 0.9525714 -0.0451905180 0.1355182633
##  [4,] 0.9820056 -0.0176747506 0.0706457115
##  [5,] 0.9933032 -0.0066435746 0.0332534000
##  [6,] 0.9975309 -0.0024513724 0.0148148160
##  [7,] 0.9990941 -0.0009237056 0.0063593575
##  [8,] 0.9996626 -0.0003552714 0.0026290081
##  [9,] 0.9998757 -0.0001421085 0.0011368684
## [10,] 0.9999468  0.0000000000 0.0004973799
## [11,] 0.9998757 -0.0001421085 0.0001421085
## [12,] 1.0000178  0.0000000000 0.0001421085
## par:1  1  1
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##            [,1]          [,2]         [,3]
##  [1,] 0.7310597 -0.1966160568 0.1966071750
##  [2,] 0.8807977 -0.1049915710 0.2099920238
##  [3,] 0.9525714 -0.0451905180 0.1355182633
##  [4,] 0.9820056 -0.0176747506 0.0706457115
##  [5,] 0.9933032 -0.0066435746 0.0332534000
##  [6,] 0.9975309 -0.0024513724 0.0148148160
##  [7,] 0.9990941 -0.0009237056 0.0063593575
##  [8,] 0.9996626 -0.0003552714 0.0026290081
##  [9,] 0.9998757 -0.0001421085 0.0011368684
## [10,] 0.9999468  0.0000000000 0.0004973799
## [11,] 0.9998757 -0.0001421085 0.0001421085
## [12,] 1.0000178  0.0000000000 0.0001421085
## par:1  1  1
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##            [,1]          [,2]         [,3]
##  [1,] 0.7310597 -0.1966160568 0.1966071750
##  [2,] 0.8807977 -0.1049915710 0.2099920238
##  [3,] 0.9525714 -0.0451905180 0.1355182633
##  [4,] 0.9820056 -0.0176747506 0.0706457115
##  [5,] 0.9933032 -0.0066435746 0.0332534000
##  [6,] 0.9975309 -0.0024513724 0.0148148160
##  [7,] 0.9990941 -0.0009237056 0.0063593575
##  [8,] 0.9996626 -0.0003552714 0.0026290081
##  [9,] 0.9998757 -0.0001421085 0.0011368684
## [10,] 0.9999468  0.0000000000 0.0004973799
## [11,] 0.9998757 -0.0001421085 0.0001421085
## [12,] 1.0000178  0.0000000000 0.0001421085
## par:1  1  1
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##            [,1]          [,2]         [,3]
##  [1,] 0.7310597 -0.1966160568 0.1966071750
##  [2,] 0.8807977 -0.1049915710 0.2099920238
##  [3,] 0.9525714 -0.0451905180 0.1355182633
```

```
## [4,] 0.9820056 -0.0176747506 0.0706457115
## [5,] 0.9933032 -0.0066435746 0.0332534000
## [6,] 0.9975309 -0.0024513724 0.0148148160
## [7,] 0.9990941 -0.0009237056 0.0063593575
## [8,] 0.9996626 -0.0003552714 0.0026290081
## [9,] 0.9998757 -0.0001421085 0.0011368684
## [10,] 0.9999468  0.0000000000 0.0004973799
## [11,] 0.9998757 -0.0001421085 0.0001421085
## [12,] 1.0000178  0.0000000000 0.0001421085
## par:1  1  1
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##            [,1]          [,2]         [,3]
##  [1,] 0.7310597 -0.1966160568 0.1966071750
##  [2,] 0.8807977 -0.1049915710 0.2099920238
##  [3,] 0.9525714 -0.0451905180 0.1355182633
##  [4,] 0.9820056 -0.0176747506 0.0706457115
##  [5,] 0.9933032 -0.0066435746 0.0332534000
##  [6,] 0.9975309 -0.0024513724 0.0148148160
##  [7,] 0.9990941 -0.0009237056 0.0063593575
##  [8,] 0.9996626 -0.0003552714 0.0026290081
##  [9,] 0.9998757 -0.0001421085 0.0011368684
## [10,] 0.9999468  0.0000000000 0.0004973799
## [11,] 0.9998757 -0.0001421085 0.0001421085
## [12,] 1.0000178  0.0000000000 0.0001421085
## par:1  1  1
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##            [,1]          [,2]         [,3]
##  [1,] 0.7310597 -0.1966160568 0.1966071750
##  [2,] 0.8807977 -0.1049915710 0.2099920238
##  [3,] 0.9525714 -0.0451905180 0.1355182633
##  [4,] 0.9820056 -0.0176747506 0.0706457115
##  [5,] 0.9933032 -0.0066435746 0.0332534000
##  [6,] 0.9975309 -0.0024513724 0.0148148160
##  [7,] 0.9990941 -0.0009237056 0.0063593575
##  [8,] 0.9996626 -0.0003552714 0.0026290081
##  [9,] 0.9998757 -0.0001421085 0.0011368684
## [10,] 0.9999468  0.0000000000 0.0004973799
## [11,] 0.9998757 -0.0001421085 0.0001421085
## [12,] 1.0000178  0.0000000000 0.0001421085
## par:1  1  1
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##            [,1]          [,2]         [,3]
##  [1,] 0.7310597 -0.1966160568 0.1966071750
##  [2,] 0.8807977 -0.1049915710 0.2099920238
##  [3,] 0.9525714 -0.0451905180 0.1355182633
##  [4,] 0.9820056 -0.0176747506 0.0706457115
##  [5,] 0.9933032 -0.0066435746 0.0332534000
##  [6,] 0.9975309 -0.0024513724 0.0148148160
```

```
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## [11,] 0.9998757 -0.0001421085 0.0001421085
## [12,] 1.0000178  0.0000000000 0.0001421085
## par:1  1  1
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##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
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## [12,] 1.0000178  0.0000000000 0.0001421085
## par:1  1  1
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
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##            [,1]          [,2]         [,3]
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##  [2,] 0.8807977 -0.1049915710 0.2099920238
##  [3,] 0.9525714 -0.0451905180 0.1355182633
##  [4,] 0.9820056 -0.0176747506 0.0706457115
##  [5,] 0.9933032 -0.0066435746 0.0332534000
##  [6,] 0.9975309 -0.0024513724 0.0148148160
##  [7,] 0.9990941 -0.0009237056 0.0063593575
##  [8,] 0.9996626 -0.0003552714 0.0026290081
##  [9,] 0.9998757 -0.0001421085 0.0011368684
## [10,] 0.9999468  0.0000000000 0.0004973799
## [11,] 0.9998757 -0.0001421085 0.0001421085
## [12,] 1.0000178  0.0000000000 0.0001421085
## par:1  1  1
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##            [,1]          [,2]         [,3]
##  [1,] 0.7310597 -0.1966160568 0.1966071750
##  [2,] 0.8807977 -0.1049915710 0.2099920238
##  [3,] 0.9525714 -0.0451905180 0.1355182633
##  [4,] 0.9820056 -0.0176747506 0.0706457115
##  [5,] 0.9933032 -0.0066435746 0.0332534000
##  [6,] 0.9975309 -0.0024513724 0.0148148160
##  [7,] 0.9990941 -0.0009237056 0.0063593575
##  [8,] 0.9996626 -0.0003552714 0.0026290081
##  [9,] 0.9998757 -0.0001421085 0.0011368684
## [10,] 0.9999468  0.0000000000 0.0004973799
## [11,] 0.9998757 -0.0001421085 0.0001421085
## [12,] 1.0000178  0.0000000000 0.0001421085
## par:1  1  1
```

```
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##            [,1]          [,2]         [,3]
##  [1,] 0.7310597 -0.1966160568 0.1966071750
##  [2,] 0.8807977 -0.1049915710 0.2099920238
##  [3,] 0.9525714 -0.0451905180 0.1355182633
##  [4,] 0.9820056 -0.0176747506 0.0706457115
##  [5,] 0.9933032 -0.0066435746 0.0332534000
##  [6,] 0.9975309 -0.0024513724 0.0148148160
##  [7,] 0.9990941 -0.0009237056 0.0063593575
##  [8,] 0.9996626 -0.0003552714 0.0026290081
##  [9,] 0.9998757 -0.0001421085 0.0011368684
## [10,] 0.9999468  0.0000000000 0.0004973799
## [11,] 0.9998757 -0.0001421085 0.0001421085
## [12,] 1.0000178  0.0000000000 0.0001421085
## par:1  1  1
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##            [,1]          [,2]         [,3]
##  [1,] 0.7310597 -0.1966160568 0.1966071750
##  [2,] 0.8807977 -0.1049915710 0.2099920238
##  [3,] 0.9525714 -0.0451905180 0.1355182633
##  [4,] 0.9820056 -0.0176747506 0.0706457115
##  [5,] 0.9933032 -0.0066435746 0.0332534000
##  [6,] 0.9975309 -0.0024513724 0.0148148160
##  [7,] 0.9990941 -0.0009237056 0.0063593575
##  [8,] 0.9996626 -0.0003552714 0.0026290081
##  [9,] 0.9998757 -0.0001421085 0.0011368684
## [10,] 0.9999468  0.0000000000 0.0004973799
## [11,] 0.9998757 -0.0001421085 0.0001421085
## [12,] 1.0000178  0.0000000000 0.0001421085
## par:1  1  1
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##            [,1]          [,2]         [,3]
##  [1,] 0.7310597 -0.1966160568 0.1966071750
##  [2,] 0.8807977 -0.1049915710 0.2099920238
##  [3,] 0.9525714 -0.0451905180 0.1355182633
##  [4,] 0.9820056 -0.0176747506 0.0706457115
##  [5,] 0.9933032 -0.0066435746 0.0332534000
##  [6,] 0.9975309 -0.0024513724 0.0148148160
##  [7,] 0.9990941 -0.0009237056 0.0063593575
##  [8,] 0.9996626 -0.0003552714 0.0026290081
##  [9,] 0.9998757 -0.0001421085 0.0011368684
## [10,] 0.9999468  0.0000000000 0.0004973799
## [11,] 0.9998757 -0.0001421085 0.0001421085
## [12,] 1.0000178  0.0000000000 0.0001421085
## par:1  1  1
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
```

```
##              [,1]          [,2]         [,3]
##  [1,] 0.7310597 -0.1966160568 0.1966071750
##  [2,] 0.8807977 -0.1049915710 0.2099920238
##  [3,] 0.9525714 -0.0451905180 0.1355182633
##  [4,] 0.9820056 -0.0176747506 0.0706457115
##  [5,] 0.9933032 -0.0066435746 0.0332534000
##  [6,] 0.9975309 -0.0024513724 0.0148148160
##  [7,] 0.9990941 -0.0009237056 0.0063593575
##  [8,] 0.9996626 -0.0003552714 0.0026290081
##  [9,] 0.9998757 -0.0001421085 0.0011368684
## [10,] 0.9999468  0.0000000000 0.0004973799
## [11,] 0.9998757 -0.0001421085 0.0001421085
## [12,] 1.0000178  0.0000000000 0.0001421085
## par:1  1  1
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##              [,1]          [,2]         [,3]
##  [1,] 0.7310597 -0.1966160568 0.1966071750
##  [2,] 0.8807977 -0.1049915710 0.2099920238
##  [3,] 0.9525714 -0.0451905180 0.1355182633
##  [4,] 0.9820056 -0.0176747506 0.0706457115
##  [5,] 0.9933032 -0.0066435746 0.0332534000
##  [6,] 0.9975309 -0.0024513724 0.0148148160
##  [7,] 0.9990941 -0.0009237056 0.0063593575
##  [8,] 0.9996626 -0.0003552714 0.0026290081
##  [9,] 0.9998757 -0.0001421085 0.0011368684
## [10,] 0.9999468  0.0000000000 0.0004973799
## [11,] 0.9998757 -0.0001421085 0.0001421085
## [12,] 1.0000178  0.0000000000 0.0001421085
## par:1  1  1
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##              [,1]          [,2]         [,3]
##  [1,] 0.7310597 -0.1966160568 0.1966071750
##  [2,] 0.8807977 -0.1049915710 0.2099920238
##  [3,] 0.9525714 -0.0451905180 0.1355182633
##  [4,] 0.9820056 -0.0176747506 0.0706457115
##  [5,] 0.9933032 -0.0066435746 0.0332534000
##  [6,] 0.9975309 -0.0024513724 0.0148148160
##  [7,] 0.9990941 -0.0009237056 0.0063593575
##  [8,] 0.9996626 -0.0003552714 0.0026290081
##  [9,] 0.9998757 -0.0001421085 0.0011368684
## [10,] 0.9999468  0.0000000000 0.0004973799
## [11,] 0.9998757 -0.0001421085 0.0001421085
## [12,] 1.0000178  0.0000000000 0.0001421085
## par:1  1  1
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##              [,1]          [,2]         [,3]
##  [1,] 0.7310597 -0.1966160568 0.1966071750
##  [2,] 0.8807977 -0.1049915710 0.2099920238
```

```
##  [3,] 0.9525714 -0.0451905180 0.1355182633
##  [4,] 0.9820056 -0.0176747506 0.0706457115
##  [5,] 0.9933032 -0.0066435746 0.0332534000
##  [6,] 0.9975309 -0.0024513724 0.0148148160
##  [7,] 0.9990941 -0.0009237056 0.0063593575
##  [8,] 0.9996626 -0.0003552714 0.0026290081
##  [9,] 0.9998757 -0.0001421085 0.0011368684
## [10,] 0.9999468  0.0000000000 0.0004973799
## [11,] 0.9998757 -0.0001421085 0.0001421085
## [12,] 1.0000178  0.0000000000 0.0001421085
## par:1  1  1
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##            [,1]          [,2]         [,3]
##  [1,] 0.7310597 -0.1966160568 0.1966071750
##  [2,] 0.8807977 -0.1049915710 0.2099920238
##  [3,] 0.9525714 -0.0451905180 0.1355182633
##  [4,] 0.9820056 -0.0176747506 0.0706457115
##  [5,] 0.9933032 -0.0066435746 0.0332534000
##  [6,] 0.9975309 -0.0024513724 0.0148148160
##  [7,] 0.9990941 -0.0009237056 0.0063593575
##  [8,] 0.9996626 -0.0003552714 0.0026290081
##  [9,] 0.9998757 -0.0001421085 0.0011368684
## [10,] 0.9999468  0.0000000000 0.0004973799
## [11,] 0.9998757 -0.0001421085 0.0001421085
## [12,] 1.0000178  0.0000000000 0.0001421085
## par:1  1  1
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##            [,1]          [,2]         [,3]
##  [1,] 0.7310597 -0.1966160568 0.1966071750
##  [2,] 0.8807977 -0.1049915710 0.2099920238
##  [3,] 0.9525714 -0.0451905180 0.1355182633
##  [4,] 0.9820056 -0.0176747506 0.0706457115
##  [5,] 0.9933032 -0.0066435746 0.0332534000
##  [6,] 0.9975309 -0.0024513724 0.0148148160
##  [7,] 0.9990941 -0.0009237056 0.0063593575
##  [8,] 0.9996626 -0.0003552714 0.0026290081
##  [9,] 0.9998757 -0.0001421085 0.0011368684
## [10,] 0.9999468  0.0000000000 0.0004973799
## [11,] 0.9998757 -0.0001421085 0.0001421085
## [12,] 1.0000178  0.0000000000 0.0001421085
## par:1  1  1
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##            [,1]          [,2]         [,3]
##  [1,] 0.7310597 -0.1966160568 0.1966071750
##  [2,] 0.8807977 -0.1049915710 0.2099920238
##  [3,] 0.9525714 -0.0451905180 0.1355182633
##  [4,] 0.9820056 -0.0176747506 0.0706457115
##  [5,] 0.9933032 -0.0066435746 0.0332534000
```

```
##  [6,] 0.9975309 -0.0024513724 0.0148148160
##  [7,] 0.9990941 -0.0009237056 0.0063593575
##  [8,] 0.9996626 -0.0003552714 0.0026290081
##  [9,] 0.9998757 -0.0001421085 0.0011368684
## [10,] 0.9999468  0.0000000000 0.0004973799
## [11,] 0.9998757 -0.0001421085 0.0001421085
## [12,] 1.0000178  0.0000000000 0.0001421085
## par:1  1  1
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##           [,1]          [,2]         [,3]
##  [1,] 0.7310597 -0.1966160568 0.1966071750
##  [2,] 0.8807977 -0.1049915710 0.2099920238
##  [3,] 0.9525714 -0.0451905180 0.1355182633
##  [4,] 0.9820056 -0.0176747506 0.0706457115
##  [5,] 0.9933032 -0.0066435746 0.0332534000
##  [6,] 0.9975309 -0.0024513724 0.0148148160
##  [7,] 0.9990941 -0.0009237056 0.0063593575
##  [8,] 0.9996626 -0.0003552714 0.0026290081
##  [9,] 0.9998757 -0.0001421085 0.0011368684
## [10,] 0.9999468  0.0000000000 0.0004973799
## [11,] 0.9998757 -0.0001421085 0.0001421085
## [12,] 1.0000178  0.0000000000 0.0001421085
## par:1  1  1
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##           [,1]          [,2]         [,3]
##  [1,] 0.7310597 -0.1966160568 0.1966071750
##  [2,] 0.8807977 -0.1049915710 0.2099920238
##  [3,] 0.9525714 -0.0451905180 0.1355182633
##  [4,] 0.9820056 -0.0176747506 0.0706457115
##  [5,] 0.9933032 -0.0066435746 0.0332534000
##  [6,] 0.9975309 -0.0024513724 0.0148148160
##  [7,] 0.9990941 -0.0009237056 0.0063593575
##  [8,] 0.9996626 -0.0003552714 0.0026290081
##  [9,] 0.9998757 -0.0001421085 0.0011368684
## [10,] 0.9999468  0.0000000000 0.0004973799
## [11,] 0.9998757 -0.0001421085 0.0001421085
## [12,] 1.0000178  0.0000000000 0.0001421085
```

```
print(tandeunscx)
```

```
## Unit: microseconds
##                                                                                 expr
##  andeunscx2 <- nlsalt::numericDeriv(rexpr, theta, rho = weedenv,      eps = 1e-10)
##      min       lq     mean  median      uq     max neval
##  127.101 152.5045 180.4861 180.306 207.431 236.271   100
```

```
## Central diff with smaller eps
andeunscx2<-nlsalt::numericDeriv(rexpr, theta, rho=weedenv, central=TRUE, eps=1e-10)
```

```
## par:1  1  1
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
```

```
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##            [,1]          [,2]         [,3]
##  [1,] 0.7310597 -1.966116e-01 1.966116e-01
##  [2,] 0.8807977 -1.049916e-01 2.099876e-01
##  [3,] 0.9525714 -4.518164e-02 1.355271e-01
##  [4,] 0.9820145 -1.766587e-02 7.065459e-02
##  [5,] 0.9933032 -6.643575e-03 3.325340e-02
##  [6,] 0.9975309 -2.451372e-03 1.481482e-02
##  [7,] 0.9990941 -9.059420e-04 6.359357e-03
##  [8,] 0.9996981 -3.197442e-04 2.664535e-03
##  [9,] 0.9998757 -1.421085e-04 1.136868e-03
## [10,] 0.9999468 -3.552714e-05 4.618528e-04
## [11,] 0.9999468 -7.105427e-05 2.131628e-04
## [12,] 1.0000178  0.000000e+00 7.105427e-05
```

```
print(andeunscx2)
```

```
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##            [,1]          [,2]         [,3]
##  [1,] 0.7310597 -1.966116e-01 1.966116e-01
##  [2,] 0.8807977 -1.049916e-01 2.099876e-01
##  [3,] 0.9525714 -4.518164e-02 1.355271e-01
##  [4,] 0.9820145 -1.766587e-02 7.065459e-02
##  [5,] 0.9933032 -6.643575e-03 3.325340e-02
##  [6,] 0.9975309 -2.451372e-03 1.481482e-02
##  [7,] 0.9990941 -9.059420e-04 6.359357e-03
##  [8,] 0.9996981 -3.197442e-04 2.664535e-03
##  [9,] 0.9998757 -1.421085e-04 1.136868e-03
## [10,] 0.9999468 -3.552714e-05 4.618528e-04
## [11,] 0.9999468 -7.105427e-05 2.131628e-04
## [12,] 1.0000178  0.000000e+00 7.105427e-05
```

```
print(sum(andeunscx2^2))
```

```
## [1] 23520.58
```

```
tandeunscx2<-microbenchmark(andeunscx2<-nlsalt::numericDeriv(rexpr, theta, rho=weedenv, central=TRUE, ep
```

```
## par:1  1  1
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##            [,1]          [,2]         [,3]
##  [1,] 0.7310597 -1.966116e-01 1.966116e-01
##  [2,] 0.8807977 -1.049916e-01 2.099876e-01
##  [3,] 0.9525714 -4.518164e-02 1.355271e-01
##  [4,] 0.9820145 -1.766587e-02 7.065459e-02
##  [5,] 0.9933032 -6.643575e-03 3.325340e-02
##  [6,] 0.9975309 -2.451372e-03 1.481482e-02
##  [7,] 0.9990941 -9.059420e-04 6.359357e-03
##  [8,] 0.9996981 -3.197442e-04 2.664535e-03
##  [9,] 0.9998757 -1.421085e-04 1.136868e-03
## [10,] 0.9999468 -3.552714e-05 4.618528e-04
```

```
## [11,] 0.9999468 -7.105427e-05 2.131628e-04
## [12,] 1.0000178  0.000000e+00 7.105427e-05
## par:1  1  1
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##            [,1]          [,2]         [,3]
##  [1,] 0.7310597 -1.966116e-01 1.966116e-01
##  [2,] 0.8807977 -1.049916e-01 2.099876e-01
##  [3,] 0.9525714 -4.518164e-02 1.355271e-01
##  [4,] 0.9820145 -1.766587e-02 7.065459e-02
##  [5,] 0.9933032 -6.643575e-03 3.325340e-02
##  [6,] 0.9975309 -2.451372e-03 1.481482e-02
##  [7,] 0.9990941 -9.059420e-04 6.359357e-03
##  [8,] 0.9996981 -3.197442e-04 2.664535e-03
##  [9,] 0.9998757 -1.421085e-04 1.136868e-03
## [10,] 0.9999468 -3.552714e-05 4.618528e-04
## [11,] 0.9999468 -7.105427e-05 2.131628e-04
## [12,] 1.0000178  0.000000e+00 7.105427e-05
## par:1  1  1
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##            [,1]          [,2]         [,3]
##  [1,] 0.7310597 -1.966116e-01 1.966116e-01
##  [2,] 0.8807977 -1.049916e-01 2.099876e-01
##  [3,] 0.9525714 -4.518164e-02 1.355271e-01
##  [4,] 0.9820145 -1.766587e-02 7.065459e-02
##  [5,] 0.9933032 -6.643575e-03 3.325340e-02
##  [6,] 0.9975309 -2.451372e-03 1.481482e-02
##  [7,] 0.9990941 -9.059420e-04 6.359357e-03
##  [8,] 0.9996981 -3.197442e-04 2.664535e-03
##  [9,] 0.9998757 -1.421085e-04 1.136868e-03
## [10,] 0.9999468 -3.552714e-05 4.618528e-04
## [11,] 0.9999468 -7.105427e-05 2.131628e-04
## [12,] 1.0000178  0.000000e+00 7.105427e-05
## par:1  1  1
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##            [,1]          [,2]         [,3]
##  [1,] 0.7310597 -1.966116e-01 1.966116e-01
##  [2,] 0.8807977 -1.049916e-01 2.099876e-01
##  [3,] 0.9525714 -4.518164e-02 1.355271e-01
##  [4,] 0.9820145 -1.766587e-02 7.065459e-02
##  [5,] 0.9933032 -6.643575e-03 3.325340e-02
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##  [8,] 0.9996981 -3.197442e-04 2.664535e-03
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## [10,] 0.9999468 -3.552714e-05 4.618528e-04
## [11,] 0.9999468 -7.105427e-05 2.131628e-04
## [12,] 1.0000178  0.000000e+00 7.105427e-05
## par:1  1  1
```

```
## [1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
## [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##           [,1]          [,2]         [,3]
##  [1,] 0.7310597 -1.966116e-01 1.966116e-01
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##  [7,] 0.9990941 -9.059420e-04 6.359357e-03
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## par:1  1  1
## [1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
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##  [3,] 0.9525714 -4.518164e-02 1.355271e-01
##  [4,] 0.9820145 -1.766587e-02 7.065459e-02
##  [5,] 0.9933032 -6.643575e-03 3.325340e-02
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##           [,1]          [,2]         [,3]
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##  [7,] 0.9990941 -9.059420e-04 6.359357e-03
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##             [,1]          [,2]         [,3]
##  [1,] 0.7310597 -1.966116e-01 1.966116e-01
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##             [,1]          [,2]         [,3]
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```

```
##  [3,] 0.9525714 -4.518164e-02 1.355271e-01
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##  [5,] 0.9933032 -6.643575e-03 3.325340e-02
##  [6,] 0.9975309 -2.451372e-03 1.481482e-02
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##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
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##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##            [,1]          [,2]         [,3]
##  [1,] 0.7310597 -1.966116e-01 1.966116e-01
##  [2,] 0.8807977 -1.049916e-01 2.099876e-01
##  [3,] 0.9525714 -4.518164e-02 1.355271e-01
##  [4,] 0.9820145 -1.766587e-02 7.065459e-02
##  [5,] 0.9933032 -6.643575e-03 3.325340e-02
##  [6,] 0.9975309 -2.451372e-03 1.481482e-02
##  [7,] 0.9990941 -9.059420e-04 6.359357e-03
##  [8,] 0.9996981 -3.197442e-04 2.664535e-03
##  [9,] 0.9998757 -1.421085e-04 1.136868e-03
## [10,] 0.9999468 -3.552714e-05 4.618528e-04
## [11,] 0.9999468 -7.105427e-05 2.131628e-04
## [12,] 1.0000178  0.000000e+00 7.105427e-05
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##  [8,] 0.9996981 -3.197442e-04 2.664535e-03
##  [9,] 0.9998757 -1.421085e-04 1.136868e-03
## [10,] 0.9999468 -3.552714e-05 4.618528e-04
## [11,] 0.9999468 -7.105427e-05 2.131628e-04
## [12,] 1.0000178  0.000000e+00 7.105427e-05
## par:1  1  1
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##            [,1]          [,2]         [,3]
##  [1,] 0.7310597 -1.966116e-01 1.966116e-01
##  [2,] 0.8807977 -1.049916e-01 2.099876e-01
##  [3,] 0.9525714 -4.518164e-02 1.355271e-01
##  [4,] 0.9820145 -1.766587e-02 7.065459e-02
##  [5,] 0.9933032 -6.643575e-03 3.325340e-02
##  [6,] 0.9975309 -2.451372e-03 1.481482e-02
##  [7,] 0.9990941 -9.059420e-04 6.359357e-03
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## [12,] 1.0000178  0.000000e+00 7.105427e-05
```

```
## par:1  1  1
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
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##            [,1]          [,2]         [,3]
##  [1,] 0.7310597 -1.966116e-01 1.966116e-01
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##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
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## attr(,"gradient")
##            [,1]          [,2]         [,3]
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## attr(,"gradient")
##              [,1]          [,2]         [,3]
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## attr(,"gradient")
##              [,1]          [,2]         [,3]
##  [1,] 0.7310597 -1.966116e-01 1.966116e-01
```

```
##  [2,] 0.8807977 -1.049916e-01 2.099876e-01
##  [3,] 0.9525714 -4.518164e-02 1.355271e-01
##  [4,] 0.9820145 -1.766587e-02 7.065459e-02
##  [5,] 0.9933032 -6.643575e-03 3.325340e-02
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```
##  [5,] 0.9933032 -6.643575e-03 3.325340e-02
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##            [,1]          [,2]         [,3]
##  [1,] 0.7310597 -1.966116e-01 1.966116e-01
##  [2,] 0.8807977 -1.049916e-01 2.099876e-01
##  [3,] 0.9525714 -4.518164e-02 1.355271e-01
##  [4,] 0.9820145 -1.766587e-02 7.065459e-02
##  [5,] 0.9933032 -6.643575e-03 3.325340e-02
##  [6,] 0.9975309 -2.451372e-03 1.481482e-02
##  [7,] 0.9990941 -9.059420e-04 6.359357e-03
##  [8,] 0.9996981 -3.197442e-04 2.664535e-03
##  [9,] 0.9998757 -1.421085e-04 1.136868e-03
## [10,] 0.9999468 -3.552714e-05 4.618528e-04
## [11,] 0.9999468 -7.105427e-05 2.131628e-04
## [12,] 1.0000178  0.000000e+00 7.105427e-05
## par:1  1  1
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##            [,1]          [,2]         [,3]
##  [1,] 0.7310597 -1.966116e-01 1.966116e-01
##  [2,] 0.8807977 -1.049916e-01 2.099876e-01
##  [3,] 0.9525714 -4.518164e-02 1.355271e-01
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##  [7,] 0.9990941 -9.059420e-04 6.359357e-03
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## [10,] 0.9999468 -3.552714e-05 4.618528e-04
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## par:1  1  1
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##            [,1]          [,2]         [,3]
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## [12,] 1.0000178  0.000000e+00 7.105427e-05
## par:1  1  1
```

```
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
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##  [1,] 0.7310597 -1.966116e-01 1.966116e-01
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##  [3,] 0.9525714 -4.518164e-02 1.355271e-01
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```

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## [10,] 0.9999468 -3.552714e-05 4.618528e-04
## [11,] 0.9999468 -7.105427e-05 2.131628e-04
## [12,] 1.0000178  0.000000e+00 7.105427e-05
## par:1  1  1
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##            [,1]          [,2]         [,3]
##  [1,] 0.7310597 -1.966116e-01 1.966116e-01
##  [2,] 0.8807977 -1.049916e-01 2.099876e-01
##  [3,] 0.9525714 -4.518164e-02 1.355271e-01
##  [4,] 0.9820145 -1.766587e-02 7.065459e-02
##  [5,] 0.9933032 -6.643575e-03 3.325340e-02
##  [6,] 0.9975309 -2.451372e-03 1.481482e-02
##  [7,] 0.9990941 -9.059420e-04 6.359357e-03
##  [8,] 0.9996981 -3.197442e-04 2.664535e-03
##  [9,] 0.9998757 -1.421085e-04 1.136868e-03
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```

```
## [12,] 1.0000178  0.000000e+00 7.105427e-05
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##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##            [,1]          [,2]         [,3]
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##  [2,] 0.8807977 -1.049916e-01 2.099876e-01
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##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
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##            [,1]          [,2]         [,3]
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##  [2,] 0.8807977 -1.049916e-01 2.099876e-01
##  [3,] 0.9525714 -4.518164e-02 1.355271e-01
##  [4,] 0.9820145 -1.766587e-02 7.065459e-02
##  [5,] 0.9933032 -6.643575e-03 3.325340e-02
##  [6,] 0.9975309 -2.451372e-03 1.481482e-02
##  [7,] 0.9990941 -9.059420e-04 6.359357e-03
##  [8,] 0.9996981 -3.197442e-04 2.664535e-03
##  [9,] 0.9998757 -1.421085e-04 1.136868e-03
## [10,] 0.9999468 -3.552714e-05 4.618528e-04
## [11,] 0.9999468 -7.105427e-05 2.131628e-04
## [12,] 1.0000178  0.000000e+00 7.105427e-05
## par:1  1  1
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##            [,1]          [,2]         [,3]
##  [1,] 0.7310597 -1.966116e-01 1.966116e-01
##  [2,] 0.8807977 -1.049916e-01 2.099876e-01
##  [3,] 0.9525714 -4.518164e-02 1.355271e-01
##  [4,] 0.9820145 -1.766587e-02 7.065459e-02
##  [5,] 0.9933032 -6.643575e-03 3.325340e-02
##  [6,] 0.9975309 -2.451372e-03 1.481482e-02
##  [7,] 0.9990941 -9.059420e-04 6.359357e-03
##  [8,] 0.9996981 -3.197442e-04 2.664535e-03
##  [9,] 0.9998757 -1.421085e-04 1.136868e-03
## [10,] 0.9999468 -3.552714e-05 4.618528e-04
## [11,] 0.9999468 -7.105427e-05 2.131628e-04
## [12,] 1.0000178  0.000000e+00 7.105427e-05
## par:1  1  1
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##            [,1]          [,2]         [,3]
##  [1,] 0.7310597 -1.966116e-01 1.966116e-01
##  [2,] 0.8807977 -1.049916e-01 2.099876e-01
##  [3,] 0.9525714 -4.518164e-02 1.355271e-01
##  [4,] 0.9820145 -1.766587e-02 7.065459e-02
##  [5,] 0.9933032 -6.643575e-03 3.325340e-02
##  [6,] 0.9975309 -2.451372e-03 1.481482e-02
##  [7,] 0.9990941 -9.059420e-04 6.359357e-03
##  [8,] 0.9996981 -3.197442e-04 2.664535e-03
##  [9,] 0.9998757 -1.421085e-04 1.136868e-03
## [10,] 0.9999468 -3.552714e-05 4.618528e-04
## [11,] 0.9999468 -7.105427e-05 2.131628e-04
## [12,] 1.0000178  0.000000e+00 7.105427e-05
```

```
## par:1  1  1
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##             [,1]          [,2]         [,3]
##  [1,] 0.7310597 -1.966116e-01 1.966116e-01
##  [2,] 0.8807977 -1.049916e-01 2.099876e-01
##  [3,] 0.9525714 -4.518164e-02 1.355271e-01
##  [4,] 0.9820145 -1.766587e-02 7.065459e-02
##  [5,] 0.9933032 -6.643575e-03 3.325340e-02
##  [6,] 0.9975309 -2.451372e-03 1.481482e-02
##  [7,] 0.9990941 -9.059420e-04 6.359357e-03
##  [8,] 0.9996981 -3.197442e-04 2.664535e-03
##  [9,] 0.9998757 -1.421085e-04 1.136868e-03
## [10,] 0.9999468 -3.552714e-05 4.618528e-04
## [11,] 0.9999468 -7.105427e-05 2.131628e-04
## [12,] 1.0000178  0.000000e+00 7.105427e-05
## par:1  1  1
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##             [,1]          [,2]         [,3]
##  [1,] 0.7310597 -1.966116e-01 1.966116e-01
##  [2,] 0.8807977 -1.049916e-01 2.099876e-01
##  [3,] 0.9525714 -4.518164e-02 1.355271e-01
##  [4,] 0.9820145 -1.766587e-02 7.065459e-02
##  [5,] 0.9933032 -6.643575e-03 3.325340e-02
##  [6,] 0.9975309 -2.451372e-03 1.481482e-02
##  [7,] 0.9990941 -9.059420e-04 6.359357e-03
##  [8,] 0.9996981 -3.197442e-04 2.664535e-03
##  [9,] 0.9998757 -1.421085e-04 1.136868e-03
## [10,] 0.9999468 -3.552714e-05 4.618528e-04
## [11,] 0.9999468 -7.105427e-05 2.131628e-04
## [12,] 1.0000178  0.000000e+00 7.105427e-05
## par:1  1  1
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##             [,1]          [,2]         [,3]
##  [1,] 0.7310597 -1.966116e-01 1.966116e-01
##  [2,] 0.8807977 -1.049916e-01 2.099876e-01
##  [3,] 0.9525714 -4.518164e-02 1.355271e-01
##  [4,] 0.9820145 -1.766587e-02 7.065459e-02
##  [5,] 0.9933032 -6.643575e-03 3.325340e-02
##  [6,] 0.9975309 -2.451372e-03 1.481482e-02
##  [7,] 0.9990941 -9.059420e-04 6.359357e-03
##  [8,] 0.9996981 -3.197442e-04 2.664535e-03
##  [9,] 0.9998757 -1.421085e-04 1.136868e-03
## [10,] 0.9999468 -3.552714e-05 4.618528e-04
## [11,] 0.9999468 -7.105427e-05 2.131628e-04
## [12,] 1.0000178  0.000000e+00 7.105427e-05
## par:1  1  1
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
```

```
## attr(,"gradient")
##            [,1]          [,2]         [,3]
##  [1,] 0.7310597 -1.966116e-01 1.966116e-01
##  [2,] 0.8807977 -1.049916e-01 2.099876e-01
##  [3,] 0.9525714 -4.518164e-02 1.355271e-01
##  [4,] 0.9820145 -1.766587e-02 7.065459e-02
##  [5,] 0.9933032 -6.643575e-03 3.325340e-02
##  [6,] 0.9975309 -2.451372e-03 1.481482e-02
##  [7,] 0.9990941 -9.059420e-04 6.359357e-03
##  [8,] 0.9996981 -3.197442e-04 2.664535e-03
##  [9,] 0.9998757 -1.421085e-04 1.136868e-03
## [10,] 0.9999468 -3.552714e-05 4.618528e-04
## [11,] 0.9999468 -7.105427e-05 2.131628e-04
## [12,] 1.0000178  0.000000e+00 7.105427e-05
## par:1  1  1
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##            [,1]          [,2]         [,3]
##  [1,] 0.7310597 -1.966116e-01 1.966116e-01
##  [2,] 0.8807977 -1.049916e-01 2.099876e-01
##  [3,] 0.9525714 -4.518164e-02 1.355271e-01
##  [4,] 0.9820145 -1.766587e-02 7.065459e-02
##  [5,] 0.9933032 -6.643575e-03 3.325340e-02
##  [6,] 0.9975309 -2.451372e-03 1.481482e-02
##  [7,] 0.9990941 -9.059420e-04 6.359357e-03
##  [8,] 0.9996981 -3.197442e-04 2.664535e-03
##  [9,] 0.9998757 -1.421085e-04 1.136868e-03
## [10,] 0.9999468 -3.552714e-05 4.618528e-04
## [11,] 0.9999468 -7.105427e-05 2.131628e-04
## [12,] 1.0000178  0.000000e+00 7.105427e-05
## par:1  1  1
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##            [,1]          [,2]         [,3]
##  [1,] 0.7310597 -1.966116e-01 1.966116e-01
##  [2,] 0.8807977 -1.049916e-01 2.099876e-01
##  [3,] 0.9525714 -4.518164e-02 1.355271e-01
##  [4,] 0.9820145 -1.766587e-02 7.065459e-02
##  [5,] 0.9933032 -6.643575e-03 3.325340e-02
##  [6,] 0.9975309 -2.451372e-03 1.481482e-02
##  [7,] 0.9990941 -9.059420e-04 6.359357e-03
##  [8,] 0.9996981 -3.197442e-04 2.664535e-03
##  [9,] 0.9998757 -1.421085e-04 1.136868e-03
## [10,] 0.9999468 -3.552714e-05 4.618528e-04
## [11,] 0.9999468 -7.105427e-05 2.131628e-04
## [12,] 1.0000178  0.000000e+00 7.105427e-05
## par:1  1  1
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##            [,1]          [,2]         [,3]
##  [1,] 0.7310597 -1.966116e-01 1.966116e-01
```

```
##  [2,] 0.8807977 -1.049916e-01 2.099876e-01
##  [3,] 0.9525714 -4.518164e-02 1.355271e-01
##  [4,] 0.9820145 -1.766587e-02 7.065459e-02
##  [5,] 0.9933032 -6.643575e-03 3.325340e-02
##  [6,] 0.9975309 -2.451372e-03 1.481482e-02
##  [7,] 0.9990941 -9.059420e-04 6.359357e-03
##  [8,] 0.9996981 -3.197442e-04 2.664535e-03
##  [9,] 0.9998757 -1.421085e-04 1.136868e-03
## [10,] 0.9999468 -3.552714e-05 4.618528e-04
## [11,] 0.9999468 -7.105427e-05 2.131628e-04
## [12,] 1.0000178  0.000000e+00 7.105427e-05
```

```
print(tandeunscx2)
```

```
## Unit: microseconds
##                                                                                                 expr
##  andeunscx2 <- nlsalt::numericDeriv(rexpr, theta, rho = weedenv,      central = TRUE, eps = 1e-10)
##      min       lq     mean   median       uq     max neval
##  128.038 153.9095 181.7733 184.6065 210.465 235.228   100
```

The `dir` parameter allows us to use a backward difference for the derivative. This appears in `nlsModel()` for the case where a parameter is on an upper bound for the case `algorithm="port"`. It does not check for nearness to the bound, and for the lower bound assumes that we are stepping AWAY from the bound in the default direction (`dir=+1`). None of the code addresses the issue where bounds are closer together than the step used for the finite difference, so there are situations where we could crash the code. Nor does the code check if the central difference is specified when near a bound.

- In the case of lower bounds, a central difference can overstep the bound when a parameter is "close" or on the bound.
- In the case of an upper bound, changing the `dir` will not change the derivative expression and steps in both forward and backward directions of the parameter are taken.

Symbolic methods from `nlsr`

The package `nlsr` has a function `model2rjfun()` that converts an expression describing how the residual functions are computed into an R function that computes the residuals at a particular set of parameters and sets the **attribute** "gradient" of the vector of residual values to the Jacobian at the particular set of parameters.

```
# nlsr has function model2rjfun. We can evaluate just the residuals
res0<-model2rjfun(eunsc, start1, data=weeddata1, jacobian=FALSE)
res0(start1)
```

```
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
```

```
# or the residuals and jacobian
## nlsr::model2rjfun forms a function with gradient (jacobian) attribute
funsc <- model2rjfun(eunsc, start1, data=weeddata1) # from nlsr: creates a function
tmodel2rjfun <- microbenchmark(model2rjfun(eunsc, start1, data=weeddata1))
print(tmodel2rjfun)
```

```
## Unit: microseconds
##                                         expr    min      lq     mean  median
##  model2rjfun(eunsc, start1, data = weeddata1) 82.015 84.0035 90.04153 85.0975
##       uq     max neval
##  87.1425 197.034   100
```

```
print(funsc)
```

```
## function(prm) {
##         if (is.null(names(prm)))
##      names(prm) <- names(pvec)
##    localdata <- list2env(as.list(prm), parent = data)
##    eval(residexpr, envir = localdata)
##         # Saves Jacobian matrix as "gradient" attribute (consistent with deriv())
##     }
## <bytecode: 0x560b45e740b0>
## <environment: 0x560b45603670>
```

```
print(funsc(start1))
```

```
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##              b1            b2           b3
##  [1,] 0.7310586 -1.966119e-01 1.966119e-01
##  [2,] 0.8807971 -1.049936e-01 2.099872e-01
##  [3,] 0.9525741 -4.517666e-02 1.355300e-01
##  [4,] 0.9820138 -1.766271e-02 7.065082e-02
##  [5,] 0.9933071 -6.648057e-03 3.324028e-02
##  [6,] 0.9975274 -2.466509e-03 1.479906e-02
##  [7,] 0.9990889 -9.102212e-04 6.371548e-03
##  [8,] 0.9996646 -3.352377e-04 2.681901e-03
##  [9,] 0.9998766 -1.233793e-04 1.110414e-03
## [10,] 0.9999546 -4.539581e-05 4.539581e-04
## [11,] 0.9999833 -1.670114e-05 1.837126e-04
## [12,] 0.9999939 -6.144137e-06 7.372964e-05
```

```
print(environment(funsc))
```

```
## <environment: 0x560b45603670>
```

```
print(ls.str(environment(funsc)))
```

```
## data : <environment: 0x560b455f6870>
## jacobian :  logi TRUE
## modelformula : Class 'formula'  language y ~ b1/(1 + b2 * exp(-b3 * tt))
## pvec :  Named num [1:3] 1 1 1
## residexpr :   expression({  .expr3 <- exp(-b3 * tt)  .expr5 <- 1 + b2 * .expr3  .expr10 <- .expr5^2
## rjfun : function (prm)
## testresult :  logi TRUE
```

```
print(ls(environment(funsc)$data))
```

```
## [1] "tt" "y"
```

```
eval(eunsc, environment(funsc))
```

```
## y ~ b1/(1 + b2 * exp(-b3 * tt))
```

```
vfunsc<-funsc(start1)
print(vfunsc)
```

```
##  [1]  -4.576941  -6.359203  -8.685426 -11.883986 -16.075693 -22.194473
##  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
```

```
## attr(,"gradient")
##              b1            b2           b3
##  [1,] 0.7310586 -1.966119e-01 1.966119e-01
##  [2,] 0.8807971 -1.049936e-01 2.099872e-01
##  [3,] 0.9525741 -4.517666e-02 1.355300e-01
##  [4,] 0.9820138 -1.766271e-02 7.065082e-02
##  [5,] 0.9933071 -6.648057e-03 3.324028e-02
##  [6,] 0.9975274 -2.466509e-03 1.479906e-02
##  [7,] 0.9990889 -9.102212e-04 6.371548e-03
##  [8,] 0.9996646 -3.352377e-04 2.681901e-03
##  [9,] 0.9998766 -1.233793e-04 1.110414e-03
## [10,] 0.9999546 -4.539581e-05 4.539581e-04
## [11,] 0.9999833 -1.670114e-05 1.837126e-04
## [12,] 0.9999939 -6.144137e-06 7.372964e-05
```

```
tfunsc<-microbenchmark(funsc(start1))
print(tfunsc)
```

```
## Unit: microseconds
##           expr   min      lq     mean  median     uq   max neval
##  funsc(start1) 12.87 13.3315 14.36131 13.4885 13.768 53.95   100
```

numDeriv package

The package `numDeriv` includes a function `jacobian()` that acts on a user function `resid()` to produce the Jacobian at a set of parameters by several choices of approximation.

```
# We use the residual function (without gradient attribute) from nlsr
jeunsc<-jacobian(res0, start1)
jeunsc
```

```
##            [,1]          [,2]         [,3]
##  [1,] 0.7310586 -1.966119e-01 1.966119e-01
##  [2,] 0.8807971 -1.049936e-01 2.099872e-01
##  [3,] 0.9525741 -4.517666e-02 1.355300e-01
##  [4,] 0.9820138 -1.766271e-02 7.065082e-02
##  [5,] 0.9933071 -6.648057e-03 3.324028e-02
##  [6,] 0.9975274 -2.466509e-03 1.479906e-02
##  [7,] 0.9990889 -9.102212e-04 6.371548e-03
##  [8,] 0.9996647 -3.352378e-04 2.681902e-03
##  [9,] 0.9998766 -1.233791e-04 1.110414e-03
## [10,] 0.9999546 -4.539572e-05 4.539580e-04
## [11,] 0.9999833 -1.670116e-05 1.837129e-04
## [12,] 0.9999939 -6.144205e-06 7.373002e-05
```

```
# Timings of the analytic jacobian calculations
tjeunsc<-microbenchmark(jeunsc<-jacobian(res0, start1))
print(tjeunsc)
```

```
## Unit: microseconds
##                              expr     min       lq    mean  median      uq
##  jeunsc <- jacobian(res0, start1) 348.935 355.4015 368.019 362.523 372.638
##     max neval
##  453.63   100
```

Note that the manual pages for `numDeriv` offer many options for the functions in the package. At 2021-5-27

we have yet to explore these.

Comparisons

In the following, we are comparing to `vfunsc`, which is the evaluated residual vector at `start1=c(1,1,1)` with "gradient" attribute (jacobian) included, as developed using package `nlsr`. This is taken as the "correct" result, even though it is possible that the generated order of calculations may introduce inaccuracies in the supposedly analytic derivatives.

`numericDeriv` computes a similar structure (residuals with "gradient" attribute): `ndeunsc`: the forward difference result with default `eps` (1e-07 according to manual) `ndeunsc2`: Central difference with default `eps` `ndeunscx`: Forward difference with smaller eps=1e-10 `ndeunscx2`: Central difference with smaller eps=1e-10

`jeunsc`: numDeriv::jacobian() result with default settings.

```
## Matrix comparisons
attr(ndeunsc, "gradient")-attr(vfunsc,"gradient")
```

```
##                   b1            b2            b3
##  [1,] -4.066995e-08 -7.619266e-09 -5.198538e-08
##  [2,]  1.016833e-08  3.631656e-09 -7.263312e-09
##  [3,]  7.050552e-09 -8.473015e-08  1.577186e-08
##  [4,] -8.764533e-08 -5.738229e-08 -8.889419e-09
##  [5,] -3.542825e-08 -6.988878e-09  3.494439e-08
##  [6,] -1.592723e-08  6.909055e-08  6.229383e-08
##  [7,]  5.380365e-08 -6.095489e-08 -5.015294e-08
##  [8,] -3.432289e-07 -4.556886e-07 -1.691883e-07
##  [9,] -1.062480e-07 -1.214742e-07  1.395936e-07
## [10,]  9.833867e-08  9.627771e-08 -9.102750e-09
## [11,] -4.647158e-07 -4.649948e-07 -6.071033e-07
## [12,]  4.221287e-07  4.220910e-07  6.569545e-07
```

```
attr(ndeunsc2, "gradient")-attr(vfunsc,"gradient")
```

```
##                   b1            b2            b3
##  [1,] -5.513268e-11  1.371020e-11  5.962686e-11
##  [2,]  6.850076e-13 -3.831403e-11  3.291006e-12
##  [3,] -2.144829e-11 -1.208666e-10  6.925160e-11
##  [4,] -2.665634e-11  4.123477e-11 -1.826495e-11
##  [5,]  2.175706e-10 -7.825720e-11  9.793778e-11
##  [6,] -2.416068e-10 -1.666460e-10 -1.735172e-10
##  [7,] -8.350343e-11  5.415257e-11 -8.571976e-11
##  [8,]  3.255913e-10 -9.864836e-11  2.024904e-10
##  [9,]  1.379652e-11 -5.685668e-10 -1.631673e-10
## [10,] -9.296786e-11 -4.173983e-10  6.710748e-11
## [11,] -4.266865e-11  2.415372e-10  8.632699e-10
## [12,] -2.738492e-10  2.515432e-10 -6.717320e-10
```

```
attr(ndeunscx, "gradient")-attr(vfunsc,"gradient")
```

```
##                   b1            b2            b3
##  [1,]  1.078625e-06 -4.123528e-06 -4.758257e-06
##  [2,]  5.790545e-07  2.014411e-06  4.852963e-06
##  [3,] -2.771694e-06 -1.385826e-05 -1.171591e-05
##  [4,] -8.202081e-06 -1.204434e-05 -5.113350e-06
##  [5,] -3.931620e-06  4.482091e-06  1.311668e-05
##  [6,]  3.569890e-06  1.513685e-05  1.576029e-05
```

```
##  [7,]  5.191947e-06 -1.348438e-05 -1.219078e-05
##  [8,] -2.074929e-06 -2.003370e-05 -5.289324e-05
##  [9,] -8.676624e-07 -1.872920e-05  2.645423e-05
## [10,] -7.810096e-06  4.539581e-05  4.342184e-05
## [11,] -1.075608e-04 -1.254074e-04 -4.160402e-05
## [12,]  2.399048e-05  6.144137e-06  6.837890e-05
```

```
attr(ndeunscx2, "gradient")-attr(vfunsc,"gradient")
```

```
##                  b1            b2            b3
##  [1,]  1.078625e-06  3.173646e-07 -3.173646e-07
##  [2,]  5.790545e-07  2.014411e-06  4.120706e-07
##  [3,] -2.771694e-06 -4.976479e-06 -2.834131e-06
##  [4,]  6.797035e-07 -3.162555e-06  3.768434e-06
##  [5,] -3.931620e-06  4.482091e-06  1.311668e-05
##  [6,]  3.569890e-06  1.513685e-05  1.576029e-05
##  [7,]  5.191947e-06  4.279192e-06 -1.219078e-05
##  [8,]  3.345221e-05  1.549344e-05 -1.736611e-05
##  [9,] -8.676624e-07 -1.872920e-05  2.645423e-05
## [10,] -7.810096e-06  9.868671e-06  7.894701e-06
## [11,] -3.650654e-05 -5.435313e-05  2.945025e-05
## [12,]  2.399048e-05  6.144137e-06 -2.675369e-06
```

```
jeunsc-attr(vfunsc,"gradient")
```

```
##                  b1            b2            b3
##  [1,] -2.239464e-11  7.806283e-12 -1.156686e-12
##  [2,] -2.267631e-12  2.974312e-11  2.957756e-11
##  [3,] -8.948509e-12  3.630193e-11 -1.256267e-11
##  [4,] -1.649125e-12  1.182179e-13  6.369841e-11
##  [5,] -4.272493e-11 -1.109757e-11  3.501116e-11
##  [6,]  1.867381e-10  1.793287e-11  3.319552e-11
##  [7,]  1.090728e-11  1.840947e-11  9.946462e-12
##  [8,]  2.035664e-10 -1.520996e-10  1.911593e-10
##  [9,] -3.582228e-10  2.039028e-10 -1.905254e-10
## [10,]  3.202474e-10  8.291927e-11 -6.263366e-11
## [11,]  5.931922e-12 -1.779933e-11  3.682277e-10
## [12,]  4.132902e-10 -6.831839e-11  3.817154e-10
```

```
## Summary comparisons
max(abs(attr(ndeunsc, "gradient")-attr(vfunsc,"gradient")))
```

```
## [1] 6.569545e-07
```

```
max(abs(attr(ndeunsc2, "gradient")-attr(vfunsc,"gradient")))
```

```
## [1] 8.632699e-10
```

```
max(abs(attr(ndeunscx, "gradient")-attr(vfunsc,"gradient")))
```

```
## [1] 0.0001254074
```

```
max(abs(attr(ndeunscx2, "gradient")-attr(vfunsc,"gradient")))
```

```
## [1] 5.435313e-05
```

```
max(abs(jeunsc-attr(vfunsc,"gradient")))
```

```
## [1] 4.132902e-10
```

Performance results for different computing environments

Here we present tables of the results, preceded by identified descriptions of the machines we used. We use ideas and functions from the document `MachineSummary` to provide a characterization and identity for each machine used.

M21-LM20.1

?? still to be run

?? What machines provide a range of possibilities.

Discussion of derivative computation for nonlinear least squares

In no particular order, we comment on some issues relating to the Jacobian calculations in nonlinear least squares.

Nomenclature

R is not in step with many other areas of numerical computation when labelling different objects in the nonlinear least squares problem. In particular, R uses the term "gradient" when the object of interest is the Jacobian matrix. In that it is useful in performing iterations of the Gauss-Newton or related equations to have the Jacobian associated with the residuals, and the rows of the Jacobian matrix are "gradients" of the respective residuals, we can accept the attribute name "gradient" to select the required information. Moreover, as in package `nlsr` it is very useful to have the Jacobian matrix as an attribute of the residual vector, since the main solver function, in this case `nlsr::nlfb()`, can be called with the same input for the arguments `res` and `jac`. These are the functions required to compute the residual and the Jacobian, and using the same function for both is very convenient, but needs some way to return both the residual vector and Jacobian matrix in a coherent fashion.

Numerical approximation near constraints

As far as we are aware there is no software that implements a fully safeguarded system to compute numerical approximation of the Jacobian (or gradients in general optimization) near constraints. The same statement applies even in the case of the much simpler bounds constraints. Users have a perverse tendency to devise ways to foil our best efforts. For example, they may decide that a good way to specify fixed (i.e., masked) parameters that they do not want to vary during a particular calculation is to specify the lower and upper bound of a parameter at the same value. Later runs may want the parameter constraints relaxed.

In `nlsr::nlxb()`, users may, in fact, specify masked parameters this way. This is a case of "if you can't beat them, join them," but it does provide an easily understood way for users to fix values.

More tricky is dealing with constraints that are close together. Note that these may arise from, for example, two linear (planar) constraints that approach at a narrow angle. In the apex where these constraints intersect, we will have tight bounds on parameters. If the constraint is not one that is imposed by the nature of the residual or objective function, for example, a log() or square root near zero, then we can generally proceed and allow the derivative approximator to evaluate outside the constraints. Things are decidedly nastier if we do have inadmissible values.

The issue of constraints and the need for a step in parameter values for derivative approximations was one of the motivations for trying to find analytic derivatives in package `nlsr` and the continuing effort to bring them into other R tools.

Appendix 1: Base R numericDeriv code

This code is in two files, nls.R and nls.c and is extracted here.

From nls.R

```
numericDeriv <- function(expr, theta, rho = parent.frame(), dir = 1,
                 eps = .Machine$double.eps ^ (1/if(central) 3 else 2), central = FALSE)
## Note: this expr must be set up as a call to work properly according to JN??
## ?? we set eps conditional on central. But central set AFTER eps. Is this OK.
{    cat("numericDeriv-Alt\n")
    dir <- rep_len(dir, length(theta))
    stopifnot(is.finite(eps), eps > 0)
    rho1 <- new.env(FALSE, rho, 0)
    if (!is.character(theta) ) {stop("'theta' should be of type character")}
    if (is.null(rho)) {
            stop("use of NULL environment is defunct")
            #        rho <- R_BaseEnv;
    } else {
          if(! is.environment(rho)) {stop("'rho' should be an environment")}
          #    int nprot = 3;
    }
    if( ! ((length(dir) == length(theta) ) & (is.numeric(dir) ) ) )
              {stop("'dir' is not a numeric vector of the correct length") }
    if(is.na(central)) { stop("'central' is NA, but must be TRUE or FALSE") }
    res0 <- eval(expr, rho) # the base residuals. ?? C has a check for REAL ANS=res0
    if (any(is.infinite(res0)) ) {stop("residuals cannot be evaluated at base point")}
    ##  CHECK_FN_VAL(res, ans);  ?? how to do this. Is it necessary?
    nt <- length(theta) # number of parameters
    mr <- length(res0) # number of residuals
    JJ <- matrix(NA, nrow=mr, ncol=nt) # Initialize the Jacobian
    for (j in 1:nt){
       origPar<-get(theta[j],rho)
       xx <- abs(origPar)
       delta <- if (xx == 0.0) {eps} else { xx*eps }
       ## JN: I prefer eps*(xx + eps)  which is simpler ?? Should we suggest / use a control switch
       prmx<-origPar+delta*dir[j]
       assign(theta[j],prmx,rho)
       res1 <- eval(expr, rho) # new residuals (forward step)
       if (central) { # compute backward step resids for central diff
          prmb <- origPar - dir[j]*delta
          assign(theta[j], prmb, envir=rho) # may be able to make more efficient later??
          resb <- eval(expr, rho)
          JJ[, j] <- dir[j]*(res1-resb)/(2*delta) # vectorized
       } else { ## forward diff
          JJ[,j] <- dir[j]*(res1-res0)/delta
       }  # end forward diff
    } # end loop over the parameters
    attr(res0, "gradient") <- JJ
    return(res0)
}
```

From nls.c

```
#include <stdlib.h>
#include <string.h>
#include <math.h>
#include <float.h>
#include <R.h>
```

```
#include <Rinternals.h>
#include "nls.h"
#include "internals.h"

#ifndef MIN
#define MIN(a,b) (((a)<(b))?(a):(b))
#endif

/*
 *  call to numeric_deriv from R -
 *  .Call("numeric_deriv", expr, theta, rho, dir = 1., eps = .Machine$double.eps, central=FALSE)
 *  Returns: ans
 */
SEXP
numeric_deriv(SEXP expr, SEXP theta, SEXP rho, SEXP dir, SEXP eps_, SEXP centr,
              SEXP rho1)
{
    if(!isString(theta))
    error(_("'theta' should be of type character"));
    if (isNull(rho)) {
    error(_("use of NULL environment is defunct"));
    rho = R_BaseEnv;
    } else
    if(!isEnvironment(rho))
        error(_("'rho' should be an environment"));
    int nprot = 3;
    if(TYPEOF(dir) != REALSXP) {
    PROTECT(dir = coerceVector(dir, REALSXP)); nprot++;
    }
    if(LENGTH(dir) != LENGTH(theta))
    error(_("'dir' is not a numeric vector of the correct length"));
    Rboolean central = asLogical(centr);
    if(central == NA_LOGICAL)
    error(_("'central' is NA, but must be TRUE or FALSE"));
//    SEXP rho1 = PROTECT(R_NewEnv(rho, FALSE, 0));
//    nprot++;
    SEXP
    pars = PROTECT(allocVector(VECSXP, LENGTH(theta))),
        ans  = PROTECT(duplicate(eval(expr, rho1)));
    double *rDir = REAL(dir),  *res = NULL; // -Wall
#define CHECK_FN_VAL(_r_, _ANS_) do {                   \
    if(!isReal(_ANS_)) {                        \
    SEXP temp = coerceVector(_ANS_, REALSXP);           \
    UNPROTECT(1);/*: _ANS_ *must* have been the last PROTECT() ! */ \
    PROTECT(_ANS_ = temp);                      \
    }                                   \
    _r_ = REAL(_ANS_);                          \
    for(int i = 0; i < LENGTH(_ANS_); i++) {                \
    if (!R_FINITE(_r_[i]))                      \
        error(_("Missing value or an infinity produced when evaluating the model")); \
    }                                   \
} while(0)

    CHECK_FN_VAL(res, ans);
```

```
    const void *vmax = vmaxget();
    int lengthTheta = 0;
    for(int i = 0; i < LENGTH(theta); i++) {
    const char *name = translateChar(STRING_ELT(theta, i));
    SEXP s_name = install(name);
    SEXP temp = findVar(s_name, rho1);
    if(isInteger(temp))
        error(_("variable '%s' is integer, not numeric"), name);
    if(!isReal(temp))
        error(_("variable '%s' is not numeric"), name);
    // We'll be modifying the variable, so need to make a copy PR#15849
    defineVar(s_name, temp = duplicate(temp), rho1);
    MARK_NOT_MUTABLE(temp);
    SET_VECTOR_ELT(pars, i, temp);
    lengthTheta += LENGTH(VECTOR_ELT(pars, i));
    }
    vmaxset(vmax);
    SEXP gradient = PROTECT(allocMatrix(REALSXP, LENGTH(ans), lengthTheta));
    double *grad = REAL(gradient);
    double eps = asReal(eps_); // was hardcoded sqrt(DOUBLE_EPS) { ~= 1.49e-08, typically}
    for(int start = 0, i = 0; i < LENGTH(theta); i++) {
    double *pars_i = REAL(VECTOR_ELT(pars, i));
    for(int j = 0; j < LENGTH(VECTOR_ELT(pars, i)); j++, start += LENGTH(ans)) {
        double
        origPar = pars_i[j],
        xx = fabs(origPar),
        delta = (xx == 0) ? eps : xx*eps;
        pars_i[j] += rDir[i] * delta;
        SEXP ans_del = PROTECT(eval(expr, rho1));
        double *rDel = NULL;
        CHECK_FN_VAL(rDel, ans_del);
        if(central) {
        pars_i[j] = origPar - rDir[i] * delta;
        SEXP ans_de2 = PROTECT(eval(expr, rho1));
        double *rD2 = NULL;
        CHECK_FN_VAL(rD2, ans_de2);
        for(int k = 0; k < LENGTH(ans); k++) {
            grad[start + k] = rDir[i] * (rDel[k] - rD2[k])/(2 * delta);
        }
        } else { // forward difference  (previously hardwired):
        for(int k = 0; k < LENGTH(ans); k++) {
            grad[start + k] = rDir[i] * (rDel[k] - res[k])/delta;
        }
        }
        UNPROTECT(central ? 2 : 1); // ansDel & possibly ans
        pars_i[j] = origPar;
    }
    }
    setAttrib(ans, install("gradient"), gradient);
    UNPROTECT(nprot);
    return ans;
}
```

Appendix 2: numericDeriv() from nlsalt package (all in R)

```
#  File src/library/stats/R/nlsnd.R
#  Part of the modified R package, https://www.R-project.org
#
#  Copyright (C) 2000-2020 The R Core Team
#  Copyright (C) 1999-1999 Saikat DebRoy, Douglas M. Bates, Jose C. Pinheiro
#  J C Nash 2021
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#  https://www.R-project.org/Licenses/

###
###            numeric Jacobian for Nonlinear least squares for R
###

numericDeriv <- function(expr, theta, rho = parent.frame(), dir = 1,
                 eps = .Machine$double.eps ^ (1/if(central) 3 else 2), central = FALSE)
## Note: this expr must be set up as a call to work properly according to JN??
## ?? we set eps conditional on central. But central set AFTER eps. Is this OK.
{   ndtrace<-TRUE
#    if(ndtrace) cat("numericDeriv-Alt\n")
    dir <- rep_len(dir, length(theta))
    stopifnot(is.finite(eps), eps > 0)
    rho1 <- new.env(FALSE, rho, 0)
    if (!is.character(theta) ) {stop("'theta' should be of type character")}
    if (is.null(rho)) {
            stop("use of NULL environment is defunct")
            #        rho <- R_BaseEnv;
    } else {
          if(! is.environment(rho)) {stop("'rho' should be an environment")}
          #    int nprot = 3;
    }
    if( ! ((length(dir) == length(theta) ) & (is.numeric(dir) ) ) )
              {stop("'dir' is not a numeric vector of the correct length") }
    if(is.na(central)) { stop("'central' is NA, but must be TRUE or FALSE") }
    res0 <- eval(expr, rho) # the base residuals. ?? C has a check for REAL ANS=res0
    if (any(is.infinite(res0)) ) {stop("residuals cannot be evaluated at base point")}
    ##  CHECK_FN_VAL(res, ans);  ?? how to do this. Is it necessary?
    nt <- length(theta) # number of parameters
    mr <- length(res0) # number of residuals
    JJ <- matrix(NA, nrow=mr, ncol=nt) # Initialize the Jacobian
    for (j in 1:nt){
       origPar<-get(theta[j],rho)
```

```
      xx <- abs(origPar)
      delta <- if (xx == 0.0) {eps} else { xx*eps }
      ## JN: I prefer eps*(xx + eps)  which is simpler ?? Should we suggest / use a control switch
      prmx<-origPar+delta*dir[j]
      assign(theta[j],prmx,rho)
      res1 <- eval(expr, rho) # new residuals (forward step)
      if (central) { # compute backward step resids for central diff
         prmb <- origPar - dir[j]*delta
         assign(theta[j], prmb, envir=rho) # may be able to make more efficient later??
         resb <- eval(expr, rho)
         JJ[, j] <- dir[j]*(res1-resb)/(2*delta) # vectorized
      } else { ## forward diff
         JJ[,j] <- dir[j]*(res1-res0)/delta
      }  # end forward diff
      assign(theta[j],origPar,rho) # restore the parameter value !! IMPORTANT
   } # end loop over the parameters
   attr(res0, "gradient") <- JJ
   if (ndtrace){
      cat("par:")
      for (j in 1:nt){ cat(get(theta[j],rho)," ") }
      cat("\n")
      print(res0)
   }
   return(res0)
}
```

Nash, John C. 1979. *Compact Numerical Methods for Computers : Linear Algebra and Function Minimisation.* Book. Hilger: Bristol.