Jacobian Calculations for nls()

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ISSUES

- ExDerivs.R file causes a number of failures in the ORIGINAL numericDeriv.
- Need to verify nlsalt:: version matches all cases of nlspkg:: version

TODOS (mostly from nlsr vignette nlsr-devdoc.Rmd)

- explain JN approach to relative offset convergence criterion
- how to insert numerical derivatives when Deriv unable to get result
- approximations for jacfn beyond fwd approximation. How to specify??
- how to force numerical approximations in nlfb() in a manner consistent with that used in optimx::optimr(), that is, to surround the name of jacfn with quotes if it is a numerical approximation, or to provide a logical control to nlxb() for this purpose.

Jacobians in nls()

nls() needs Jacobians calculated at the current set of trial nonlinear model parameters to set up the Gauss-Newton equations. Unfortunately, nls() calls the Jacobian the "gradient," and uses function numericDerivs() to compute them. This document is an attempt to describe different ways to compute the Jacobian for use in nls() and related software, and to evaluate the performance of these approaches.

In evaluating performance, we need to know the conditions under which the evaluation was conducted. Thus the computations included in this document, which is built using Rmarkdown, are specific to the computer in which the document is processed. We will add tables that give the results for different computing environments at the bottom.

An example problem

We will use the Hobbs weed infestation problem (Nash (1979), page 120).

```
## Class 'formula' language y \sim b1/(1 + b2 * exp(-b3 * tt))
## ..- attr(*, ".Environment")=<environment: R_GlobalEnv>
# Can we convert a string form of this "model" to a formula
ceunsc <- " y ~ b1/(1+b2*exp(-b3*tt))"
str(ceunsc)
## chr " y ~ b1/(1+b2*exp(-b3*tt))"
# Will be TRUE if we have made the conversion
print(as.formula(ceunsc)==eunsc)
## [1] TRUE
## LOCAL DATA IN DATA FRAMES
weeddata1 <- data.frame(y=ydat, tt=tdat)</pre>
## Put data in an Environment
weedenv <- list2env(weeddata1)</pre>
weedenv$b1 <- start1[[1]]</pre>
weedenv$b2 <- start1[[2]]</pre>
weedenv$b3 <- start1[[3]]</pre>
# Display content of the Environment
## Note that may need to do further commands to get everything
ls.str(weedenv)
## b1 : num 1
## b2 : num 1
## b3 : num 1
## tt : int [1:12] 1 2 3 4 5 6 7 8 9 10 ...
## y : num [1:12] 5.31 7.24 9.64 12.87 17.07 ...
# Generate the residual "call"
rexpr<-call("-",eunsc[[3]], eunsc[[2]])
# Get the residuals
r0<-eval(rexpr, weedenv)
print(r0)
## [1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
## [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
cat("Sumsquares at 1,1,1 is ",sum(r0^2),"\n")
## Sumsquares at 1,1,1 is 23520.58
## Another way
ldata<-list2env(as.list(start1),envir=weedenv)</pre>
## <environment: 0x5634eec2d990>
ls.str(ldata)
## b1 : num 1
## b2 : num 1
## b3 : num 1
## tt : int [1:12] 1 2 3 4 5 6 7 8 9 10 ...
## y : num [1:12] 5.31 7.24 9.64 12.87 17.07 ...
```

```
eval(rexpr,envir=ldata)
   [1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## Do we need to get a model frame? How? and How to use it?
## Now ready to try things out.
```

Tools for Jacobians

numericDeriv() original version

numericDeriv is the R function used by nls() to evaluate Jacobians for its Gauss-Newton equations. The

```
R source code is in the file nls.R. It calls a C function numeric_deriv in nls.c.
## Seems to work -- BUT note file ExDerivs.R has many "failures"??
theta <- c("b1", "b2", "b3")
ndeunsc<-nlspkg::numericDeriv(rexpr, theta, rho=weedenv)
## Registered S3 methods overwritten by 'nlspkg':
     method
##
##
     anova.nls
                       stats
##
     coef.nls
                       stats
##
     confint.nls
                       stats
##
     deviance.nls
                       stats
##
     df.residual.nls
                       stats
##
     fitted.nls
                       stats
##
     formula.nls
                       stats
##
     logLik.nls
                       stats
##
     nobs.nls
                       stats
     plot.profile.nls
##
                       stats
##
     predict.nls
                       stats
##
     print.nls
                       stats
     print.summary.nls stats
##
##
     profile.nls
                       stats
##
     residuals.nls
                       stats
##
     summary.nls
                       stats
##
     vcov.nls
                       stats
##
     weights.nls
                       stats
print(ndeunsc)
##
    \begin{bmatrix} 1 \end{bmatrix} -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
    attr(, "gradient")
##
##
              [,1]
                                         [,3]
                            [,2]
##
   [1,] 0.7310585 -1.966119e-01 0.1966118813
##
   [2,] 0.8807971 -1.049936e-01 0.2099871635
##
    [3,] 0.9525741 -4.517674e-02 0.1355299950
##
  [4,] 0.9820137 -1.766276e-02 0.0706508160
## [5,] 0.9933071 -6.648064e-03 0.0332403183
## [6,] 0.9975274 -2.466440e-03 0.0147991180
##
   [7,] 0.9990890 -9.102821e-04 0.0063714981
```

[8,] 0.9996643 -3.356934e-04 0.0026817322

```
## [9,] 0.9998765 -1.235008e-04 0.0011105537
## [10,] 0.9999547 -4.529953e-05 0.0004539490
## [11,] 0.9999828 -1.716614e-05 0.0001831055
## [12,] 0.9999943 -5.722046e-06 0.0000743866
print(sum(ndeunsc^2))
## [1] 23520.58
tndeunsc<-microbenchmark(ndeunsc<-nlspkg::numericDeriv(rexpr, theta, rho=weedenv))</pre>
print(tndeunsc)
## Unit: microseconds
##
                                                                             lq
                                                            expr
                                                                    min
##
   ndeunsc <- nlspkg::numericDeriv(rexpr, theta, rho = weedenv) 10.236 10.5565
        mean median
                          uq
                                max neval
## 11.66487 10.7465 10.9635 53.377
## numericDeriv also has central difference option, as well as choice of eps parameter
## Central diff
ndeunsc2<-nlspkg::numericDeriv(rexpr, theta, rho=weedenv, central=TRUE)
print(ndeunsc2)
   [1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
    [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##
              [,1]
                            [,2]
                                         [,3]
##
  [1,] 0.7310586 -1.966119e-01 1.966119e-01
## [2,] 0.8807971 -1.049936e-01 2.099872e-01
## [3,] 0.9525741 -4.517666e-02 1.355300e-01
## [4,] 0.9820138 -1.766271e-02 7.065082e-02
## [5,] 0.9933071 -6.648057e-03 3.324028e-02
## [6,] 0.9975274 -2.466509e-03 1.479906e-02
## [7,] 0.9990889 -9.102211e-04 6.371548e-03
## [8,] 0.9996647 -3.352378e-04 2.681902e-03
## [9,] 0.9998766 -1.233799e-04 1.110414e-03
## [10,] 0.9999546 -4.539623e-05 4.539581e-04
## [11,] 0.9999833 -1.670090e-05 1.837134e-04
## [12,] 0.9999939 -6.143885e-06 7.372897e-05
print(sum(ndeunsc2^2))
## [1] 23520.58
tndeunsc2<-microbenchmark(ndeunsc2<-nlspkg::numericDeriv(rexpr, theta, rho=weedenv, central=TRUE))
print(tndeunsc2)
## Unit: microseconds
##
                                                                                   expr
##
   ndeunsc2 <- nlspkg::numericDeriv(rexpr, theta, rho = weedenv,</pre>
                                                                        central = TRUE)
##
              lq mean median
                                      uq
                                            max neval
   12.837 13.239 13.829 13.4195 13.6435 43.834
## Forward diff with smaller eps
ndeunscx<-nlspkg::numericDeriv(rexpr, theta, rho=weedenv, eps=1e-10)
print(ndeunscx)
```

[1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473

```
## [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##
              [,1]
                            [,2]
##
  [1,] 0.7310597 -0.1966160568 0.1966071750
##
   [2,] 0.8807977 -0.1049915710 0.2099920238
## [3,] 0.9525714 -0.0451905180 0.1355182633
## [4,] 0.9820056 -0.0176747506 0.0706457115
## [5,] 0.9933032 -0.0066435746 0.0332534000
## [6,] 0.9975309 -0.0024513724 0.0148148160
## [7,] 0.9990941 -0.0009237056 0.0063593575
## [8,] 0.9996626 -0.0003552714 0.0026290081
## [9,] 0.9998757 -0.0001421085 0.0011368684
## [10,] 0.9999468 0.000000000 0.0004973799
## [11,] 0.9998757 -0.0001421085 0.0001421085
## [12,] 1.0000178 0.000000000 0.0001421085
print(sum(ndeunscx^2))
## [1] 23520.58
tndeunscx<-microbenchmark(ndeunscx2<-nlspkg::numericDeriv(rexpr, theta, rho=weedenv, eps=1e-10))
print(tndeunscx)
## Unit: microseconds
                                                                                expr
  ndeunscx2 <- nlspkg::numericDeriv(rexpr, theta, rho = weedenv,</pre>
##
                                                                      eps = 1e-10)
              lq mean median
                                           max neval
                                     uq
  9.365 10.5875 11.3392 10.959 11.3125 37.587
## Central diff with smaller eps
ndeunscx2<-nlspkg::numericDeriv(rexpr, theta, rho-weedenv, central=TRUE, eps=1e-10)
print(ndeunscx2)
   [1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
## [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##
              [,1]
                            [,2]
                                         [.3]
## [1,] 0.7310597 -1.966116e-01 1.966116e-01
## [2,] 0.8807977 -1.049916e-01 2.099876e-01
## [3,] 0.9525714 -4.518164e-02 1.355271e-01
## [4,] 0.9820145 -1.766587e-02 7.065459e-02
## [5,] 0.9933032 -6.643575e-03 3.325340e-02
   [6,] 0.9975309 -2.451372e-03 1.481482e-02
## [7,] 0.9990941 -9.059420e-04 6.359357e-03
## [8,] 0.9996981 -3.197442e-04 2.664535e-03
## [9,] 0.9998757 -1.421085e-04 1.136868e-03
## [10,] 0.9999468 -3.552714e-05 4.618528e-04
## [11,] 0.9999468 -7.105427e-05 2.131628e-04
## [12,] 1.0000178 0.000000e+00 7.105427e-05
print(sum(ndeunscx2^2))
## [1] 23520.58
tndeunscx2<-microbenchmark(ndeunscx2<-nlspkg::numericDeriv(rexpr, theta, rho=weedenv, central=TRUE, eps
print(tndeunscx2)
```

Unit: microseconds

```
##
                                                                                                 expr
## ndeunscx2 <- nlspkg::numericDeriv(rexpr, theta, rho = weedenv, central = TRUE, eps = 1e-10)
                     mean median
                lq
                                       uq
                                             max neval
## 11.941 12.2755 13.01076 12.536 12.765 39.704
## Add dir parameter
## to forward diff
ndeunsc<-nlspkg::numericDeriv(rexpr, theta, rho=weedenv)
print(ndeunsc)
    [1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
    [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##
              [,1]
                            [,2]
  [1,] 0.7310585 -1.966119e-01 0.1966118813
##
   [2,] 0.8807971 -1.049936e-01 0.2099871635
  [3,] 0.9525741 -4.517674e-02 0.1355299950
## [4,] 0.9820137 -1.766276e-02 0.0706508160
## [5,] 0.9933071 -6.648064e-03 0.0332403183
## [6,] 0.9975274 -2.466440e-03 0.0147991180
## [7,] 0.9990890 -9.102821e-04 0.0063714981
## [8,] 0.9996643 -3.356934e-04 0.0026817322
## [9,] 0.9998765 -1.235008e-04 0.0011105537
## [10,] 0.9999547 -4.529953e-05 0.0004539490
## [11,] 0.9999828 -1.716614e-05 0.0001831055
## [12,] 0.9999943 -5.722046e-06 0.0000743866
ndeunscd<-nlspkg::numericDeriv(rexpr, theta, rho=weedenv, dir=-1)
print(ndeunscd)
    \begin{bmatrix} 1 \end{bmatrix} -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
   [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##
              [,1]
                            [,2]
                                         [,3]
##
  [1,] 0.7310586 -1.966119e-01 1.966119e-01
## [2,] 0.8807971 -1.049936e-01 2.099872e-01
   [3,] 0.9525741 -4.517663e-02 1.355300e-01
## [4,] 0.9820138 -1.766264e-02 7.065094e-02
## [5,] 0.9933071 -6.648064e-03 3.324032e-02
## [6,] 0.9975274 -2.466440e-03 1.479912e-02
## [7,] 0.9990890 -9.102821e-04 6.371498e-03
## [8,] 0.9996648 -3.352165e-04 2.682209e-03
## [9,] 0.9998765 -1.235008e-04 1.110554e-03
## [10,] 0.9999547 -4.529953e-05 4.539490e-04
## [11,] 0.9999838 -1.621246e-05 1.840591e-04
## [12,] 0.9999933 -6.675720e-06 7.343292e-05
ndeunscc<-nlspkg::numericDeriv(rexpr, theta, rho=weedenv, central=TRUE)
print(ndeunscc)
   [1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
## [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##
              [,1]
                            [,2]
                                         [,3]
  [1,] 0.7310586 -1.966119e-01 1.966119e-01
## [2,] 0.8807971 -1.049936e-01 2.099872e-01
```

```
[3,] 0.9525741 -4.517666e-02 1.355300e-01
##
   [4,] 0.9820138 -1.766271e-02 7.065082e-02
  [5,] 0.9933071 -6.648057e-03 3.324028e-02
## [6,] 0.9975274 -2.466509e-03 1.479906e-02
   [7,] 0.9990889 -9.102211e-04 6.371548e-03
## [8,] 0.9996647 -3.352378e-04 2.681902e-03
## [9,] 0.9998766 -1.233799e-04 1.110414e-03
## [10,] 0.9999546 -4.539623e-05 4.539581e-04
## [11,] 0.9999833 -1.670090e-05 1.837134e-04
## [12,] 0.9999939 -6.143885e-06 7.372897e-05
ndeunsccd<-nlspkg::numericDeriv(rexpr, theta, rho=weedenv, central=TRUE, dir=-1)
print(ndeunsccd)
    \begin{bmatrix} 1 \end{bmatrix} -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
##
    [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
  attr(, "gradient")
##
              [,1]
                             [,2]
                                          [,3]
   [1,] 0.7310586 -1.966119e-01 1.966119e-01
##
##
   [2,] 0.8807971 -1.049936e-01 2.099872e-01
   [3,] 0.9525741 -4.517666e-02 1.355300e-01
##
   [4,] 0.9820138 -1.766271e-02 7.065082e-02
   [5,] 0.9933071 -6.648057e-03 3.324028e-02
  [6,] 0.9975274 -2.466509e-03 1.479906e-02
  [7,] 0.9990889 -9.102211e-04 6.371548e-03
## [8,] 0.9996647 -3.352378e-04 2.681902e-03
## [9,] 0.9998766 -1.233799e-04 1.110414e-03
## [10,] 0.9999546 -4.539623e-05 4.539581e-04
## [11,] 0.9999833 -1.670090e-05 1.837134e-04
## [12,] 0.9999939 -6.143885e-06 7.372897e-05
```

numericDeriv() alternative pure-R version

This version (see Appendix 2) has C code replaced with R equivalents.

```
## Try ExDerivs.R ??
andeunsc<-nlsalt::numericDeriv(rexpr, theta, rho=weedenv)

## Registered S3 methods overwritten by 'nlsalt':</pre>
```

```
##
     method
                        from
##
     anova.nls
                        nlspkg
##
     coef.nls
                        nlspkg
##
     confint.nls
                        MASS
##
     deviance.nls
                        nlspkg
##
     df.residual.nls
                        nlspkg
##
     fitted.nls
                        nlspkg
##
     formula.nls
                        nlspkg
##
     logLik.nls
                        nlspkg
##
     nobs.nls
                        nlspkg
##
     plot.profile.nls
                        nlspkg
##
     predict.nls
                        nlspkg
##
     print.nls
                        nlspkg
##
     print.summary.nls nlspkg
##
     profile.nls
                        nlspkg
##
     residuals.nls
                        nlspkg
##
     summary.nls
                        nlspkg
```

```
##
     vcov.nls
                       nlspkg
##
     weights.nls
                       nlspkg
## par:1 1 1
    \begin{bmatrix} 1 \end{bmatrix} -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
    [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
  attr(, "gradient")
##
              [,1]
                             [,2]
                                          [,3]
##
    [1,] 0.7310585 -1.966119e-01 0.1966118813
   [2,] 0.8807971 -1.049936e-01 0.2099871635
##
  [3,] 0.9525741 -4.517674e-02 0.1355299950
## [4,] 0.9820137 -1.766276e-02 0.0706508160
   [5,] 0.9933071 -6.648064e-03 0.0332403183
  [6,] 0.9975274 -2.466440e-03 0.0147991180
  [7,] 0.9990890 -9.102821e-04 0.0063714981
  [8,] 0.9996643 -3.356934e-04 0.0026817322
## [9,] 0.9998765 -1.235008e-04 0.0011105537
## [10,] 0.9999547 -4.529953e-05 0.0004539490
## [11,] 0.9999828 -1.716614e-05 0.0001831055
## [12,] 0.9999943 -5.722046e-06 0.0000743866
print(andeunsc)
    \begin{bmatrix} 1 \end{bmatrix} -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
   [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##
              [,1]
                             [,2]
                                          [,3]
##
   [1,] 0.7310585 -1.966119e-01 0.1966118813
  [2,] 0.8807971 -1.049936e-01 0.2099871635
  [3,] 0.9525741 -4.517674e-02 0.1355299950
  [4,] 0.9820137 -1.766276e-02 0.0706508160
## [5,] 0.9933071 -6.648064e-03 0.0332403183
## [6,] 0.9975274 -2.466440e-03 0.0147991180
## [7,] 0.9990890 -9.102821e-04 0.0063714981
## [8,] 0.9996643 -3.356934e-04 0.0026817322
## [9,] 0.9998765 -1.235008e-04 0.0011105537
## [10,] 0.9999547 -4.529953e-05 0.0004539490
## [11,] 0.9999828 -1.716614e-05 0.0001831055
## [12,] 0.9999943 -5.722046e-06 0.0000743866
print(sum(andeunsc^2))
## [1] 23520.58
tandeunsc<-microbenchmark(andeunsc<-nlsalt::numericDeriv(rexpr, theta, rho=weedenv))
## par:1 1 1
   [1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
   [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##
              [,1]
                             [,2]
   [1,] 0.7310585 -1.966119e-01 0.1966118813
##
## [2,] 0.8807971 -1.049936e-01 0.2099871635
## [3,] 0.9525741 -4.517674e-02 0.1355299950
## [4,] 0.9820137 -1.766276e-02 0.0706508160
## [5,] 0.9933071 -6.648064e-03 0.0332403183
## [6,] 0.9975274 -2.466440e-03 0.0147991180
```

```
[7,] 0.9990890 -9.102821e-04 0.0063714981
## [8,] 0.9996643 -3.356934e-04 0.0026817322
## [9,] 0.9998765 -1.235008e-04 0.0011105537
## [10,] 0.9999547 -4.529953e-05 0.0004539490
## [11,] 0.9999828 -1.716614e-05 0.0001831055
## [12,] 0.9999943 -5.722046e-06 0.0000743866
## par:1 1 1
    [1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
    [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
  attr(,"gradient")
              [,1]
                            [,2]
   [1,] 0.7310585 -1.966119e-01 0.1966118813
##
   [2,] 0.8807971 -1.049936e-01 0.2099871635
  [3,] 0.9525741 -4.517674e-02 0.1355299950
## [4,] 0.9820137 -1.766276e-02 0.0706508160
##
   [5,] 0.9933071 -6.648064e-03 0.0332403183
##
  [6,] 0.9975274 -2.466440e-03 0.0147991180
## [7,] 0.9990890 -9.102821e-04 0.0063714981
## [8,] 0.9996643 -3.356934e-04 0.0026817322
   [9,] 0.9998765 -1.235008e-04 0.0011105537
## [10,] 0.9999547 -4.529953e-05 0.0004539490
## [11,] 0.9999828 -1.716614e-05 0.0001831055
## [12,] 0.9999943 -5.722046e-06 0.0000743866
## par:1 1 1
   [1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
   [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##
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                            [,2]
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```
## attr(,"gradient")
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              [,1]
##
                                          [.3]
    [1,] 0.7310585 -1.966119e-01 0.1966118813
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                            [,2]
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   [3,] 0.9525741 -4.517674e-02 0.1355299950
## [4,] 0.9820137 -1.766276e-02 0.0706508160
## [5,] 0.9933071 -6.648064e-03 0.0332403183
## [6,] 0.9975274 -2.466440e-03 0.0147991180
## [7,] 0.9990890 -9.102821e-04 0.0063714981
```

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## [8,] 0.9996643 -3.356934e-04 0.0026817322
## [9,] 0.9998765 -1.235008e-04 0.0011105537
## [10,] 0.9999547 -4.529953e-05 0.0004539490
## [11,] 0.9999828 -1.716614e-05 0.0001831055
## [12,] 0.9999943 -5.722046e-06 0.0000743866
## par:1 1 1
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   [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##
              [,1]
                            [,2]
                                         [,3]
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## [7,] 0.9990890 -9.102821e-04 0.0063714981
## [8,] 0.9996643 -3.356934e-04 0.0026817322
## [9,] 0.9998765 -1.235008e-04 0.0011105537
## [10,] 0.9999547 -4.529953e-05 0.0004539490
## [11,] 0.9999828 -1.716614e-05 0.0001831055
## [12,] 0.9999943 -5.722046e-06 0.0000743866
## par:1 1 1
##
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   [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##
              [,1]
                             [,2]
##
   [1,] 0.7310585 -1.966119e-01 0.1966118813
   [2,] 0.8807971 -1.049936e-01 0.2099871635
   [3,] 0.9525741 -4.517674e-02 0.1355299950
##
   [4,] 0.9820137 -1.766276e-02 0.0706508160
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  [5,] 0.9933071 -6.648064e-03 0.0332403183
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## [9,] 0.9998765 -1.235008e-04 0.0011105537
## [10,] 0.9999547 -4.529953e-05 0.0004539490
## [11,] 0.9999828 -1.716614e-05 0.0001831055
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## attr(,"gradient")
##
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                                         [.3]
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## [12,] 0.9999943 -5.722046e-06 0.0000743866
## par:1 1 1
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## par:1 1 1
   [1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
   [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##
              [,1]
                            [,2]
                                         [,3]
##
   [1,] 0.7310585 -1.966119e-01 0.1966118813
  [2,] 0.8807971 -1.049936e-01 0.2099871635
   [3,] 0.9525741 -4.517674e-02 0.1355299950
  [4,] 0.9820137 -1.766276e-02 0.0706508160
  [5,] 0.9933071 -6.648064e-03 0.0332403183
## [6,] 0.9975274 -2.466440e-03 0.0147991180
   [7,] 0.9990890 -9.102821e-04 0.0063714981
## [8,] 0.9996643 -3.356934e-04 0.0026817322
## [9,] 0.9998765 -1.235008e-04 0.0011105537
## [10,] 0.9999547 -4.529953e-05 0.0004539490
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   [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##
              [,1]
                            [,2]
                                         [,3]
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##
   [2,] 0.8807971 -1.049936e-01 0.2099871635
   [3,] 0.9525741 -4.517674e-02 0.1355299950
  [4,] 0.9820137 -1.766276e-02 0.0706508160
## [5,] 0.9933071 -6.648064e-03 0.0332403183
   [6,] 0.9975274 -2.466440e-03 0.0147991180
## [7,] 0.9990890 -9.102821e-04 0.0063714981
## [8,] 0.9996643 -3.356934e-04 0.0026817322
## [9,] 0.9998765 -1.235008e-04 0.0011105537
## [10,] 0.9999547 -4.529953e-05 0.0004539490
## [11,] 0.9999828 -1.716614e-05 0.0001831055
## [12,] 0.9999943 -5.722046e-06 0.0000743866
## par:1 1 1
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   [1,] 0.7310585 -1.966119e-01 0.1966118813
   [2,] 0.8807971 -1.049936e-01 0.2099871635
   [3,] 0.9525741 -4.517674e-02 0.1355299950
##
   [4,] 0.9820137 -1.766276e-02 0.0706508160
##
  [5,] 0.9933071 -6.648064e-03 0.0332403183
## [6,] 0.9975274 -2.466440e-03 0.0147991180
## [7,] 0.9990890 -9.102821e-04 0.0063714981
## [8,] 0.9996643 -3.356934e-04 0.0026817322
```

```
## [9,] 0.9998765 -1.235008e-04 0.0011105537
## [10,] 0.9999547 -4.529953e-05 0.0004539490
## [11,] 0.9999828 -1.716614e-05 0.0001831055
## [12,] 0.9999943 -5.722046e-06 0.0000743866
## par:1 1 1
##
  [1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
   [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##
              [,1]
                            [,2]
                                         [.3]
##
   [1,] 0.7310585 -1.966119e-01 0.1966118813
   [2,] 0.8807971 -1.049936e-01 0.2099871635
   [3,] 0.9525741 -4.517674e-02 0.1355299950
   [4,] 0.9820137 -1.766276e-02 0.0706508160
## [5,] 0.9933071 -6.648064e-03 0.0332403183
## [6,] 0.9975274 -2.466440e-03 0.0147991180
   [7,] 0.9990890 -9.102821e-04 0.0063714981
## [8,] 0.9996643 -3.356934e-04 0.0026817322
## [9,] 0.9998765 -1.235008e-04 0.0011105537
## [10,] 0.9999547 -4.529953e-05 0.0004539490
## [11,] 0.9999828 -1.716614e-05 0.0001831055
## [12,] 0.9999943 -5.722046e-06 0.0000743866
## par:1 1 1
   [1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
##
    [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
              [,1]
                            [,2]
##
   [1,] 0.7310585 -1.966119e-01 0.1966118813
   [2,] 0.8807971 -1.049936e-01 0.2099871635
  [3,] 0.9525741 -4.517674e-02 0.1355299950
  [4,] 0.9820137 -1.766276e-02 0.0706508160
##
   [5,] 0.9933071 -6.648064e-03 0.0332403183
   [6,] 0.9975274 -2.466440e-03 0.0147991180
  [7,] 0.9990890 -9.102821e-04 0.0063714981
## [8,] 0.9996643 -3.356934e-04 0.0026817322
   [9,] 0.9998765 -1.235008e-04 0.0011105537
## [10,] 0.9999547 -4.529953e-05 0.0004539490
## [11,] 0.9999828 -1.716614e-05 0.0001831055
## [12,] 0.9999943 -5.722046e-06 0.0000743866
## par:1 1 1
   [1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
   [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
                            [,2]
##
              [,1]
                                         [.3]
##
   [1,] 0.7310585 -1.966119e-01 0.1966118813
  [2,] 0.8807971 -1.049936e-01 0.2099871635
   [3,] 0.9525741 -4.517674e-02 0.1355299950
   [4,] 0.9820137 -1.766276e-02 0.0706508160
  [5,] 0.9933071 -6.648064e-03 0.0332403183
## [6,] 0.9975274 -2.466440e-03 0.0147991180
   [7,] 0.9990890 -9.102821e-04 0.0063714981
## [8,] 0.9996643 -3.356934e-04 0.0026817322
## [9,] 0.9998765 -1.235008e-04 0.0011105537
## [10,] 0.9999547 -4.529953e-05 0.0004539490
## [11,] 0.9999828 -1.716614e-05 0.0001831055
```

```
## [12,] 0.9999943 -5.722046e-06 0.0000743866
## par:1 1 1
   [1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
    [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##
                                         [,3]
              [,1]
                            [,2]
   [1,] 0.7310585 -1.966119e-01 0.1966118813
##
   [2,] 0.8807971 -1.049936e-01 0.2099871635
   [3,] 0.9525741 -4.517674e-02 0.1355299950
  [4,] 0.9820137 -1.766276e-02 0.0706508160
   [5,] 0.9933071 -6.648064e-03 0.0332403183
   [6,] 0.9975274 -2.466440e-03 0.0147991180
## [7,] 0.9990890 -9.102821e-04 0.0063714981
## [8,] 0.9996643 -3.356934e-04 0.0026817322
## [9,] 0.9998765 -1.235008e-04 0.0011105537
## [10,] 0.9999547 -4.529953e-05 0.0004539490
## [11,] 0.9999828 -1.716614e-05 0.0001831055
## [12,] 0.9999943 -5.722046e-06 0.0000743866
## par:1 1 1
##
   [1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
   [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##
              [,1]
                            [,2]
                                         [,3]
##
   [1,] 0.7310585 -1.966119e-01 0.1966118813
  [2,] 0.8807971 -1.049936e-01 0.2099871635
   [3,] 0.9525741 -4.517674e-02 0.1355299950
   [4,] 0.9820137 -1.766276e-02 0.0706508160
##
   [5,] 0.9933071 -6.648064e-03 0.0332403183
  [6,] 0.9975274 -2.466440e-03 0.0147991180
## [7,] 0.9990890 -9.102821e-04 0.0063714981
   [8,] 0.9996643 -3.356934e-04 0.0026817322
  [9,] 0.9998765 -1.235008e-04 0.0011105537
## [10,] 0.9999547 -4.529953e-05 0.0004539490
## [11,] 0.9999828 -1.716614e-05 0.0001831055
## [12,] 0.9999943 -5.722046e-06 0.0000743866
## par:1 1 1
  [1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
   [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##
              [,1]
                            [,2]
                                         [,3]
   [1,] 0.7310585 -1.966119e-01 0.1966118813
   [2,] 0.8807971 -1.049936e-01 0.2099871635
##
   [3,] 0.9525741 -4.517674e-02 0.1355299950
  [4,] 0.9820137 -1.766276e-02 0.0706508160
  [5,] 0.9933071 -6.648064e-03 0.0332403183
   [6,] 0.9975274 -2.466440e-03 0.0147991180
   [7,] 0.9990890 -9.102821e-04 0.0063714981
  [8,] 0.9996643 -3.356934e-04 0.0026817322
## [9,] 0.9998765 -1.235008e-04 0.0011105537
## [10,] 0.9999547 -4.529953e-05 0.0004539490
## [11,] 0.9999828 -1.716614e-05 0.0001831055
## [12,] 0.9999943 -5.722046e-06 0.0000743866
## par:1 1 1
## [1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
```

```
[7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##
              [,1]
                            [,2]
##
  [1,] 0.7310585 -1.966119e-01 0.1966118813
##
   [2,] 0.8807971 -1.049936e-01 0.2099871635
## [3,] 0.9525741 -4.517674e-02 0.1355299950
## [4.] 0.9820137 -1.766276e-02 0.0706508160
## [5,] 0.9933071 -6.648064e-03 0.0332403183
   [6,] 0.9975274 -2.466440e-03 0.0147991180
## [7,] 0.9990890 -9.102821e-04 0.0063714981
## [8,] 0.9996643 -3.356934e-04 0.0026817322
## [9,] 0.9998765 -1.235008e-04 0.0011105537
## [10,] 0.9999547 -4.529953e-05 0.0004539490
## [11,] 0.9999828 -1.716614e-05 0.0001831055
## [12,] 0.9999943 -5.722046e-06 0.0000743866
print(tandeunsc)
## Unit: microseconds
##
                                                                                lq
   andeunsc <- nlsalt::numericDeriv(rexpr, theta, rho = weedenv) 121.747 151.0425
##
        mean median
                                  max neval
                           uq
## 183.7332 177.271 214.3635 306.252
                                       100
## numericDeriv also has central difference option, as well as choice of eps parameter
## Central diff
andeunsc2<-nlsalt::numericDeriv(rexpr, theta, rho=weedenv, central=TRUE)
## [1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
## [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##
              [,1]
                            [,2]
##
  [1,] 0.7310586 -1.966119e-01 1.966119e-01
## [2,] 0.8807971 -1.049936e-01 2.099872e-01
## [3,] 0.9525741 -4.517666e-02 1.355300e-01
   [4,] 0.9820138 -1.766271e-02 7.065082e-02
## [5,] 0.9933071 -6.648057e-03 3.324028e-02
## [6,] 0.9975274 -2.466509e-03 1.479906e-02
## [7,] 0.9990889 -9.102211e-04 6.371548e-03
## [8,] 0.9996647 -3.352378e-04 2.681902e-03
## [9,] 0.9998766 -1.233799e-04 1.110414e-03
## [10,] 0.9999546 -4.539623e-05 4.539581e-04
## [11,] 0.9999833 -1.670090e-05 1.837134e-04
## [12,] 0.9999939 -6.143885e-06 7.372897e-05
print(andeunsc2)
   [1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
   [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##
              [,1]
                            [,2]
                                         [,3]
  [1,] 0.7310586 -1.966119e-01 1.966119e-01
## [2,] 0.8807971 -1.049936e-01 2.099872e-01
## [3,] 0.9525741 -4.517666e-02 1.355300e-01
## [4,] 0.9820138 -1.766271e-02 7.065082e-02
```

```
## [5,] 0.9933071 -6.648057e-03 3.324028e-02
## [6,] 0.9975274 -2.466509e-03 1.479906e-02
## [7,] 0.9990889 -9.102211e-04 6.371548e-03
## [8,] 0.9996647 -3.352378e-04 2.681902e-03
## [9,] 0.9998766 -1.233799e-04 1.110414e-03
## [10,] 0.9999546 -4.539623e-05 4.539581e-04
## [11,] 0.9999833 -1.670090e-05 1.837134e-04
## [12,] 0.9999939 -6.143885e-06 7.372897e-05
print(sum(andeunsc2^2))
## [1] 23520.58
tandeunsc2<-microbenchmark(andeunsc2<-nlsalt::numericDeriv(rexpr, theta, rho=weedenv, central=TRUE))
## par:1 1 1
   [1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
   [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##
                            [,2]
              [,1]
##
    [1,] 0.7310586 -1.966119e-01 1.966119e-01
##
  [2,] 0.8807971 -1.049936e-01 2.099872e-01
  [3,] 0.9525741 -4.517666e-02 1.355300e-01
## [4,] 0.9820138 -1.766271e-02 7.065082e-02
   [5,] 0.9933071 -6.648057e-03 3.324028e-02
## [6,] 0.9975274 -2.466509e-03 1.479906e-02
## [7,] 0.9990889 -9.102211e-04 6.371548e-03
## [8,] 0.9996647 -3.352378e-04 2.681902e-03
## [9,] 0.9998766 -1.233799e-04 1.110414e-03
## [10,] 0.9999546 -4.539623e-05 4.539581e-04
## [11,] 0.9999833 -1.670090e-05 1.837134e-04
## [12,] 0.9999939 -6.143885e-06 7.372897e-05
## par:1 1 1
  [1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
   [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##
              [,1]
                            [,2]
                                         [,3]
##
  [1,] 0.7310586 -1.966119e-01 1.966119e-01
   [2,] 0.8807971 -1.049936e-01 2.099872e-01
   [3,] 0.9525741 -4.517666e-02 1.355300e-01
## [4,] 0.9820138 -1.766271e-02 7.065082e-02
## [5,] 0.9933071 -6.648057e-03 3.324028e-02
## [6,] 0.9975274 -2.466509e-03 1.479906e-02
##
   [7,] 0.9990889 -9.102211e-04 6.371548e-03
## [8,] 0.9996647 -3.352378e-04 2.681902e-03
## [9,] 0.9998766 -1.233799e-04 1.110414e-03
## [10,] 0.9999546 -4.539623e-05 4.539581e-04
## [11,] 0.9999833 -1.670090e-05 1.837134e-04
## [12,] 0.9999939 -6.143885e-06 7.372897e-05
## par:1 1 1
   [1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
   [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
              [,1]
                            [,2]
## [1,] 0.7310586 -1.966119e-01 1.966119e-01
```

```
[2,] 0.8807971 -1.049936e-01 2.099872e-01
##
   [3,] 0.9525741 -4.517666e-02 1.355300e-01
  [4,] 0.9820138 -1.766271e-02 7.065082e-02
  [5,] 0.9933071 -6.648057e-03 3.324028e-02
##
   [6,] 0.9975274 -2.466509e-03 1.479906e-02
##
  [7,] 0.9990889 -9.102211e-04 6.371548e-03
## [8,] 0.9996647 -3.352378e-04 2.681902e-03
## [9,] 0.9998766 -1.233799e-04 1.110414e-03
## [10,] 0.9999546 -4.539623e-05 4.539581e-04
## [11,] 0.9999833 -1.670090e-05 1.837134e-04
## [12,] 0.9999939 -6.143885e-06 7.372897e-05
## par:1 1 1
##
   [1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
   [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##
              [,1]
                            [,2]
##
   [1,] 0.7310586 -1.966119e-01 1.966119e-01
   [2,] 0.8807971 -1.049936e-01 2.099872e-01
   [3,] 0.9525741 -4.517666e-02 1.355300e-01
   [4,] 0.9820138 -1.766271e-02 7.065082e-02
## [5,] 0.9933071 -6.648057e-03 3.324028e-02
## [6,] 0.9975274 -2.466509e-03 1.479906e-02
## [7,] 0.9990889 -9.102211e-04 6.371548e-03
   [8,] 0.9996647 -3.352378e-04 2.681902e-03
## [9,] 0.9998766 -1.233799e-04 1.110414e-03
## [10,] 0.9999546 -4.539623e-05 4.539581e-04
## [11,] 0.9999833 -1.670090e-05 1.837134e-04
## [12,] 0.9999939 -6.143885e-06 7.372897e-05
## par:1 1 1
   [1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
##
    [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##
              [,1]
                            [,2]
##
   [1,] 0.7310586 -1.966119e-01 1.966119e-01
    [2,] 0.8807971 -1.049936e-01 2.099872e-01
##
  [3,] 0.9525741 -4.517666e-02 1.355300e-01
## [4,] 0.9820138 -1.766271e-02 7.065082e-02
## [5,] 0.9933071 -6.648057e-03 3.324028e-02
   [6,] 0.9975274 -2.466509e-03 1.479906e-02
## [7,] 0.9990889 -9.102211e-04 6.371548e-03
## [8,] 0.9996647 -3.352378e-04 2.681902e-03
## [9,] 0.9998766 -1.233799e-04 1.110414e-03
## [10,] 0.9999546 -4.539623e-05 4.539581e-04
## [11,] 0.9999833 -1.670090e-05 1.837134e-04
## [12,] 0.9999939 -6.143885e-06 7.372897e-05
## par:1 1 1
##
   [1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
   [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##
                            [,2]
              [,1]
##
   [1,] 0.7310586 -1.966119e-01 1.966119e-01
## [2,] 0.8807971 -1.049936e-01 2.099872e-01
## [3,] 0.9525741 -4.517666e-02 1.355300e-01
## [4,] 0.9820138 -1.766271e-02 7.065082e-02
```

```
[5,] 0.9933071 -6.648057e-03 3.324028e-02
##
  [6,] 0.9975274 -2.466509e-03 1.479906e-02
  [7,] 0.9990889 -9.102211e-04 6.371548e-03
## [8,] 0.9996647 -3.352378e-04 2.681902e-03
## [9,] 0.9998766 -1.233799e-04 1.110414e-03
## [10,] 0.9999546 -4.539623e-05 4.539581e-04
## [11,] 0.9999833 -1.670090e-05 1.837134e-04
## [12,] 0.9999939 -6.143885e-06 7.372897e-05
## par:1 1 1
   [1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
   [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
##
  attr(,"gradient")
##
              [,1]
                                         [,3]
                            [,2]
   [1,] 0.7310586 -1.966119e-01 1.966119e-01
##
##
   [2,] 0.8807971 -1.049936e-01 2.099872e-01
   [3,] 0.9525741 -4.517666e-02 1.355300e-01
##
  [4,] 0.9820138 -1.766271e-02 7.065082e-02
  [5,] 0.9933071 -6.648057e-03 3.324028e-02
## [6,] 0.9975274 -2.466509e-03 1.479906e-02
   [7,] 0.9990889 -9.102211e-04 6.371548e-03
## [8,] 0.9996647 -3.352378e-04 2.681902e-03
## [9,] 0.9998766 -1.233799e-04 1.110414e-03
## [10,] 0.9999546 -4.539623e-05 4.539581e-04
## [11,] 0.9999833 -1.670090e-05 1.837134e-04
## [12,] 0.9999939 -6.143885e-06 7.372897e-05
## par:1 1 1
     \begin{bmatrix} 1 \end{bmatrix} \quad -4.576941 \quad -6.359203 \quad -8.685426 \quad -11.883986 \quad -16.075693 \quad -22.194473 
##
    ## attr(,"gradient")
##
              [,1]
                            [,2]
                                         [,3]
##
    [1,] 0.7310586 -1.966119e-01 1.966119e-01
   [2,] 0.8807971 -1.049936e-01 2.099872e-01
  [3,] 0.9525741 -4.517666e-02 1.355300e-01
  [4,] 0.9820138 -1.766271e-02 7.065082e-02
   [5,] 0.9933071 -6.648057e-03 3.324028e-02
## [6,] 0.9975274 -2.466509e-03 1.479906e-02
## [7,] 0.9990889 -9.102211e-04 6.371548e-03
## [8,] 0.9996647 -3.352378e-04 2.681902e-03
## [9,] 0.9998766 -1.233799e-04 1.110414e-03
## [10,] 0.9999546 -4.539623e-05 4.539581e-04
## [11,] 0.9999833 -1.670090e-05 1.837134e-04
## [12,] 0.9999939 -6.143885e-06 7.372897e-05
## par:1 1 1
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## attr(,"gradient")
##
              [,1]
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##
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  [2,] 0.8807971 -1.049936e-01 2.099872e-01
   [3,] 0.9525741 -4.517666e-02 1.355300e-01
## [4,] 0.9820138 -1.766271e-02 7.065082e-02
## [5,] 0.9933071 -6.648057e-03 3.324028e-02
## [6,] 0.9975274 -2.466509e-03 1.479906e-02
## [7,] 0.9990889 -9.102211e-04 6.371548e-03
```

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## [8,] 0.9996647 -3.352378e-04 2.681902e-03
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   [3,] 0.9525741 -4.517666e-02 1.355300e-01
   [4,] 0.9820138 -1.766271e-02 7.065082e-02
##
  [5,] 0.9933071 -6.648057e-03 3.324028e-02
  [6,] 0.9975274 -2.466509e-03 1.479906e-02
  [7,] 0.9990889 -9.102211e-04 6.371548e-03
   [8,] 0.9996647 -3.352378e-04 2.681902e-03
## [9,] 0.9998766 -1.233799e-04 1.110414e-03
## [10,] 0.9999546 -4.539623e-05 4.539581e-04
## [11,] 0.9999833 -1.670090e-05 1.837134e-04
## [12,] 0.9999939 -6.143885e-06 7.372897e-05
## par:1 1 1
   [1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
    [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##
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                            [,2]
##
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##
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   [3,] 0.9525741 -4.517666e-02 1.355300e-01
##
  [4,] 0.9820138 -1.766271e-02 7.065082e-02
  [5,] 0.9933071 -6.648057e-03 3.324028e-02
    [6,] 0.9975274 -2.466509e-03 1.479906e-02
## [7,] 0.9990889 -9.102211e-04 6.371548e-03
## [8,] 0.9996647 -3.352378e-04 2.681902e-03
## [9,] 0.9998766 -1.233799e-04 1.110414e-03
## [10,] 0.9999546 -4.539623e-05 4.539581e-04
## [11,] 0.9999833 -1.670090e-05 1.837134e-04
## [12,] 0.9999939 -6.143885e-06 7.372897e-05
## par:1 1 1
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   [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##
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                            [,2]
                                         [,3]
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   [3,] 0.9525741 -4.517666e-02 1.355300e-01
##
   [4,] 0.9820138 -1.766271e-02 7.065082e-02
## [5,] 0.9933071 -6.648057e-03 3.324028e-02
## [6,] 0.9975274 -2.466509e-03 1.479906e-02
## [7,] 0.9990889 -9.102211e-04 6.371548e-03
## [8,] 0.9996647 -3.352378e-04 2.681902e-03
```

```
## [9,] 0.9998766 -1.233799e-04 1.110414e-03
## [10,] 0.9999546 -4.539623e-05 4.539581e-04
## [11,] 0.9999833 -1.670090e-05 1.837134e-04
## [12,] 0.9999939 -6.143885e-06 7.372897e-05
## par:1 1 1
##
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## attr(,"gradient")
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              [,1]
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                                         [,3]
##
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   [3,] 0.9525741 -4.517666e-02 1.355300e-01
## [4,] 0.9820138 -1.766271e-02 7.065082e-02
## [5,] 0.9933071 -6.648057e-03 3.324028e-02
## [6,] 0.9975274 -2.466509e-03 1.479906e-02
   [7,] 0.9990889 -9.102211e-04 6.371548e-03
## [8,] 0.9996647 -3.352378e-04 2.681902e-03
## [9,] 0.9998766 -1.233799e-04 1.110414e-03
## [10,] 0.9999546 -4.539623e-05 4.539581e-04
## [11,] 0.9999833 -1.670090e-05 1.837134e-04
## [12,] 0.9999939 -6.143885e-06 7.372897e-05
## par:1 1 1
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##
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## attr(,"gradient")
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                            [,2]
                                         [,3]
##
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   [2,] 0.8807971 -1.049936e-01 2.099872e-01
  [3,] 0.9525741 -4.517666e-02 1.355300e-01
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##
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## [6,] 0.9975274 -2.466509e-03 1.479906e-02
## [7,] 0.9990889 -9.102211e-04 6.371548e-03
## [8,] 0.9996647 -3.352378e-04 2.681902e-03
   [9,] 0.9998766 -1.233799e-04 1.110414e-03
## [10,] 0.9999546 -4.539623e-05 4.539581e-04
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## [12,] 0.9999939 -6.143885e-06 7.372897e-05
## par:1 1 1
  [1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
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##
              [,1]
                                         [,3]
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  [2,] 0.8807971 -1.049936e-01 2.099872e-01
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  [5,] 0.9933071 -6.648057e-03 3.324028e-02
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## [9,] 0.9998766 -1.233799e-04 1.110414e-03
## [10,] 0.9999546 -4.539623e-05 4.539581e-04
## [11,] 0.9999833 -1.670090e-05 1.837134e-04
```

```
## [12,] 0.9999939 -6.143885e-06 7.372897e-05
## par:1 1 1
   [1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
   [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##
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                            [,2]
                                         [.3]
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   [2,] 0.8807971 -1.049936e-01 2.099872e-01
   [3,] 0.9525741 -4.517666e-02 1.355300e-01
  [4,] 0.9820138 -1.766271e-02 7.065082e-02
  [5,] 0.9933071 -6.648057e-03 3.324028e-02
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## [10,] 0.9999546 -4.539623e-05 4.539581e-04
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##
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##
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  [3,] 0.9525741 -4.517666e-02 1.355300e-01
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##
   [5,] 0.9933071 -6.648057e-03 3.324028e-02
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## [12,] 0.9999939 -6.143885e-06 7.372897e-05
## par:1 1 1
## [1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
```

```
[7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##
              [,1]
                             [,2]
                                          [,3]
   [1,] 0.7310586 -1.966119e-01 1.966119e-01
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##
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##
  [3,] 0.9525741 -4.517666e-02 1.355300e-01
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## [5,] 0.9933071 -6.648057e-03 3.324028e-02
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   [1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
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## [10,] 0.9999546 -4.539623e-05 4.539581e-04
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## par:1 1 1
     \begin{bmatrix} 1 \end{bmatrix} \quad -4.576941 \quad -6.359203 \quad -8.685426 \quad -11.883986 \quad -16.075693 \quad -22.194473 
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## attr(,"gradient")
##
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                             [,2]
##
  [1,] 0.7310586 -1.966119e-01 1.966119e-01
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  [3,] 0.9525741 -4.517666e-02 1.355300e-01
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## [7,] 0.9990889 -9.102211e-04 6.371548e-03
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## [10,] 0.9999546 -4.539623e-05 4.539581e-04
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## par:1 1 1
## [1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
## [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##
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                                          [,3]
              [,1]
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```
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  [4,] 0.9820138 -1.766271e-02 7.065082e-02
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                            [,2]
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    [1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
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## attr(,"gradient")
##
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```

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[4,] 0.9820138 -1.766271e-02 7.065082e-02
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## [7,] 0.9990889 -9.102211e-04 6.371548e-03
   [8,] 0.9996647 -3.352378e-04 2.681902e-03
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## [12,] 0.9999939 -6.143885e-06 7.372897e-05
## par:1 1 1
   \begin{bmatrix} 1 \end{bmatrix} -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
    [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##
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                           [,2]
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##
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## attr(,"gradient")
##
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                           [,2]
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## [10,] 0.9999546 -4.539623e-05 4.539581e-04
## [11,] 0.9999833 -1.670090e-05 1.837134e-04
## [12,] 0.9999939 -6.143885e-06 7.372897e-05
## par:1 1 1
   [1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
   [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##
             [,1]
                           [,2]
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   [2,] 0.8807971 -1.049936e-01 2.099872e-01
## [3,] 0.9525741 -4.517666e-02 1.355300e-01
## [4,] 0.9820138 -1.766271e-02 7.065082e-02
## [5,] 0.9933071 -6.648057e-03 3.324028e-02
## [6,] 0.9975274 -2.466509e-03 1.479906e-02
```

```
[7,] 0.9990889 -9.102211e-04 6.371548e-03
## [8,] 0.9996647 -3.352378e-04 2.681902e-03
## [9,] 0.9998766 -1.233799e-04 1.110414e-03
## [10,] 0.9999546 -4.539623e-05 4.539581e-04
## [11,] 0.9999833 -1.670090e-05 1.837134e-04
## [12,] 0.9999939 -6.143885e-06 7.372897e-05
## par:1 1 1
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    [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
  attr(,"gradient")
              [,1]
                            [,2]
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##
   [2,] 0.8807971 -1.049936e-01 2.099872e-01
  [3,] 0.9525741 -4.517666e-02 1.355300e-01
## [4,] 0.9820138 -1.766271e-02 7.065082e-02
   [5,] 0.9933071 -6.648057e-03 3.324028e-02
## [6,] 0.9975274 -2.466509e-03 1.479906e-02
## [7,] 0.9990889 -9.102211e-04 6.371548e-03
## [8,] 0.9996647 -3.352378e-04 2.681902e-03
   [9,] 0.9998766 -1.233799e-04 1.110414e-03
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## [11,] 0.9999833 -1.670090e-05 1.837134e-04
## [12,] 0.9999939 -6.143885e-06 7.372897e-05
## par:1 1 1
   [1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
   [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##
              [,1]
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                                         [,3]
##
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## [2,] 0.8807971 -1.049936e-01 2.099872e-01
   [3,] 0.9525741 -4.517666e-02 1.355300e-01
## [4,] 0.9820138 -1.766271e-02 7.065082e-02
  [5,] 0.9933071 -6.648057e-03 3.324028e-02
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## [10,] 0.9999546 -4.539623e-05 4.539581e-04
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## [12,] 0.9999939 -6.143885e-06 7.372897e-05
## par:1 1 1
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##
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## attr(,"gradient")
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##
##
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  [4,] 0.9820138 -1.766271e-02 7.065082e-02
   [5,] 0.9933071 -6.648057e-03 3.324028e-02
## [6,] 0.9975274 -2.466509e-03 1.479906e-02
## [7,] 0.9990889 -9.102211e-04 6.371548e-03
## [8,] 0.9996647 -3.352378e-04 2.681902e-03
## [9,] 0.9998766 -1.233799e-04 1.110414e-03
```

```
## [10,] 0.9999546 -4.539623e-05 4.539581e-04
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## [12,] 0.9999939 -6.143885e-06 7.372897e-05
## par:1 1 1
##
   [1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
   [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
##
## attr(,"gradient")
##
              [,1]
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##
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   [2,] 0.8807971 -1.049936e-01 2.099872e-01
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   [4,] 0.9820138 -1.766271e-02 7.065082e-02
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                            [,2]
                                         [,3]
   [1,] 0.7310586 -1.966119e-01 1.966119e-01
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```

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## par:1 1 1
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##
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```

```
## attr(,"gradient")
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                            [,2]
##
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##
    [1,] 0.7310586 -1.966119e-01 1.966119e-01
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   [6,] 0.9975274 -2.466509e-03 1.479906e-02
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## par:1 1 1
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    [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
##
  attr(, "gradient")
##
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                            [,2]
                                          [,3]
##
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##
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   [4,] 0.9820138 -1.766271e-02 7.065082e-02
   [5,] 0.9933071 -6.648057e-03 3.324028e-02
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  [6,] 0.9975274 -2.466509e-03 1.479906e-02
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## par:1 1 1
    \begin{bmatrix} 1 \end{bmatrix} -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
    [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
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##
              [,1]
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   [2,] 0.8807971 -1.049936e-01 2.099872e-01
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   [3,] 0.9525741 -4.517666e-02 1.355300e-01
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  [4,] 0.9820138 -1.766271e-02 7.065082e-02
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## par:1 1 1
        -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
    [1]
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## attr(,"gradient")
##
              [,1]
                            [,2]
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  [1,] 0.7310586 -1.966119e-01 1.966119e-01
```

```
[2,] 0.8807971 -1.049936e-01 2.099872e-01
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## attr(,"gradient")
##
                            [,2]
              [,1]
##
   [1,] 0.7310586 -1.966119e-01 1.966119e-01
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```
[5,] 0.9933071 -6.648057e-03 3.324028e-02
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  attr(, "gradient")
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              [,1]
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   [1,] 0.7310586 -1.966119e-01 1.966119e-01
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   [3,] 0.9525741 -4.517666e-02 1.355300e-01
##
  [4,] 0.9820138 -1.766271e-02 7.065082e-02
  [5,] 0.9933071 -6.648057e-03 3.324028e-02
## [6,] 0.9975274 -2.466509e-03 1.479906e-02
   [7,] 0.9990889 -9.102211e-04 6.371548e-03
## [8,] 0.9996647 -3.352378e-04 2.681902e-03
## [9,] 0.9998766 -1.233799e-04 1.110414e-03
## [10,] 0.9999546 -4.539623e-05 4.539581e-04
## [11,] 0.9999833 -1.670090e-05 1.837134e-04
## [12,] 0.9999939 -6.143885e-06 7.372897e-05
## par:1 1 1
     \begin{bmatrix} 1 \end{bmatrix} \quad -4.576941 \quad -6.359203 \quad -8.685426 \quad -11.883986 \quad -16.075693 \quad -22.194473 
##
    ## attr(,"gradient")
##
              [,1]
                            [,2]
                                         [,3]
##
    [1,] 0.7310586 -1.966119e-01 1.966119e-01
   [2,] 0.8807971 -1.049936e-01 2.099872e-01
  [3,] 0.9525741 -4.517666e-02 1.355300e-01
  [4,] 0.9820138 -1.766271e-02 7.065082e-02
   [5,] 0.9933071 -6.648057e-03 3.324028e-02
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## [8,] 0.9996647 -3.352378e-04 2.681902e-03
## [9,] 0.9998766 -1.233799e-04 1.110414e-03
## [10,] 0.9999546 -4.539623e-05 4.539581e-04
## [11,] 0.9999833 -1.670090e-05 1.837134e-04
## [12,] 0.9999939 -6.143885e-06 7.372897e-05
## par:1 1 1
   [1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
   [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##
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                                         [,3]
##
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## [7,] 0.9990889 -9.102211e-04 6.371548e-03
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## [8,] 0.9996647 -3.352378e-04 2.681902e-03
## [9,] 0.9998766 -1.233799e-04 1.110414e-03
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## [12,] 0.9999939 -6.143885e-06 7.372897e-05
## par:1 1 1
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##
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## [12,] 0.9999939 -6.143885e-06 7.372897e-05
## par:1 1 1
    \begin{bmatrix} 1 \end{bmatrix} -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
##
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                                         [.3]
##
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[1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
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[,1]
                           [,2]
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##
             [,1]
                           [,2]
##
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##
  [2,] 0.8807971 -1.049936e-01 2.099872e-01
## [3,] 0.9525741 -4.517666e-02 1.355300e-01
## [4,] 0.9820138 -1.766271e-02 7.065082e-02
## [5,] 0.9933071 -6.648057e-03 3.324028e-02
```

```
[6,] 0.9975274 -2.466509e-03 1.479906e-02
##
   [7,] 0.9990889 -9.102211e-04 6.371548e-03
  [8,] 0.9996647 -3.352378e-04 2.681902e-03
## [9,] 0.9998766 -1.233799e-04 1.110414e-03
## [10,] 0.9999546 -4.539623e-05 4.539581e-04
## [11,] 0.9999833 -1.670090e-05 1.837134e-04
## [12,] 0.9999939 -6.143885e-06 7.372897e-05
## par:1 1 1
##
   [1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
   [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##
              [,1]
                            [,2]
                                         [,3]
##
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   [2,] 0.8807971 -1.049936e-01 2.099872e-01
   [3,] 0.9525741 -4.517666e-02 1.355300e-01
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   [5,] 0.9933071 -6.648057e-03 3.324028e-02
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   [8,] 0.9996647 -3.352378e-04 2.681902e-03
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## [10,] 0.9999546 -4.539623e-05 4.539581e-04
## [11,] 0.9999833 -1.670090e-05 1.837134e-04
## [12,] 0.9999939 -6.143885e-06 7.372897e-05
## par:1 1 1
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    [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
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                            [,2]
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##
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   [3,] 0.9525741 -4.517666e-02 1.355300e-01
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  [4,] 0.9820138 -1.766271e-02 7.065082e-02
  [5,] 0.9933071 -6.648057e-03 3.324028e-02
    [6,] 0.9975274 -2.466509e-03 1.479906e-02
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## [9,] 0.9998766 -1.233799e-04 1.110414e-03
## [10,] 0.9999546 -4.539623e-05 4.539581e-04
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##
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## [5,] 0.9933071 -6.648057e-03 3.324028e-02
## [6,] 0.9975274 -2.466509e-03 1.479906e-02
## [7,] 0.9990889 -9.102211e-04 6.371548e-03
## [8,] 0.9996647 -3.352378e-04 2.681902e-03
```

```
## [9,] 0.9998766 -1.233799e-04 1.110414e-03
## [10,] 0.9999546 -4.539623e-05 4.539581e-04
## [11,] 0.9999833 -1.670090e-05 1.837134e-04
## [12,] 0.9999939 -6.143885e-06 7.372897e-05
## par:1 1 1
##
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##
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   [3,] 0.9525741 -4.517666e-02 1.355300e-01
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```
## [12,] 0.9999939 -6.143885e-06 7.372897e-05
## par:1 1 1
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##
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```

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[7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
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   [1,] 0.7310586 -1.966119e-01 1.966119e-01
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##
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   [1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
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## [10,] 0.9999546 -4.539623e-05 4.539581e-04
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    \begin{bmatrix} 1 \end{bmatrix} -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
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## [1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
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## attr(,"gradient")
##
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                                         [,3]
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```
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[4,] 0.9820138 -1.766271e-02 7.065082e-02
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## [7,] 0.9990889 -9.102211e-04 6.371548e-03
## [8,] 0.9996647 -3.352378e-04 2.681902e-03
## [9,] 0.9998766 -1.233799e-04 1.110414e-03
## [10,] 0.9999546 -4.539623e-05 4.539581e-04
## [11,] 0.9999833 -1.670090e-05 1.837134e-04
## [12,] 0.9999939 -6.143885e-06 7.372897e-05
## par:1 1 1
   [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##
              [,1]
                           [,2]
##
   [1,] 0.7310586 -1.966119e-01 1.966119e-01
  [2,] 0.8807971 -1.049936e-01 2.099872e-01
  [3,] 0.9525741 -4.517666e-02 1.355300e-01
   [4,] 0.9820138 -1.766271e-02 7.065082e-02
## [5,] 0.9933071 -6.648057e-03 3.324028e-02
## [6,] 0.9975274 -2.466509e-03 1.479906e-02
## [7,] 0.9990889 -9.102211e-04 6.371548e-03
   [8,] 0.9996647 -3.352378e-04 2.681902e-03
## [9,] 0.9998766 -1.233799e-04 1.110414e-03
## [10,] 0.9999546 -4.539623e-05 4.539581e-04
## [11,] 0.9999833 -1.670090e-05 1.837134e-04
## [12,] 0.9999939 -6.143885e-06 7.372897e-05
## par:1 1 1
   [1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
   [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##
             [,1]
                           [,2]
   [1,] 0.7310586 -1.966119e-01 1.966119e-01
   [2,] 0.8807971 -1.049936e-01 2.099872e-01
## [3,] 0.9525741 -4.517666e-02 1.355300e-01
## [4,] 0.9820138 -1.766271e-02 7.065082e-02
## [5,] 0.9933071 -6.648057e-03 3.324028e-02
## [6,] 0.9975274 -2.466509e-03 1.479906e-02
```

```
## [7,] 0.9990889 -9.102211e-04 6.371548e-03
## [8,] 0.9996647 -3.352378e-04 2.681902e-03
## [9,] 0.9998766 -1.233799e-04 1.110414e-03
## [10,] 0.9999546 -4.539623e-05 4.539581e-04
## [11,] 0.9999833 -1.670090e-05 1.837134e-04
## [12,] 0.9999939 -6.143885e-06 7.372897e-05
## par:1 1 1
   [1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
   [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
             [,1]
                            [,2]
   [1,] 0.7310586 -1.966119e-01 1.966119e-01
##
   [2,] 0.8807971 -1.049936e-01 2.099872e-01
## [3,] 0.9525741 -4.517666e-02 1.355300e-01
## [4,] 0.9820138 -1.766271e-02 7.065082e-02
   [5,] 0.9933071 -6.648057e-03 3.324028e-02
## [6,] 0.9975274 -2.466509e-03 1.479906e-02
## [7,] 0.9990889 -9.102211e-04 6.371548e-03
## [8,] 0.9996647 -3.352378e-04 2.681902e-03
   [9,] 0.9998766 -1.233799e-04 1.110414e-03
## [10,] 0.9999546 -4.539623e-05 4.539581e-04
## [11,] 0.9999833 -1.670090e-05 1.837134e-04
## [12,] 0.9999939 -6.143885e-06 7.372897e-05
## par:1 1 1
   [1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
   [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##
             [,1]
                            [,2]
                                        [,3]
##
  [1,] 0.7310586 -1.966119e-01 1.966119e-01
## [2,] 0.8807971 -1.049936e-01 2.099872e-01
   [3,] 0.9525741 -4.517666e-02 1.355300e-01
## [4,] 0.9820138 -1.766271e-02 7.065082e-02
## [5,] 0.9933071 -6.648057e-03 3.324028e-02
## [6,] 0.9975274 -2.466509e-03 1.479906e-02
   [7,] 0.9990889 -9.102211e-04 6.371548e-03
## [8,] 0.9996647 -3.352378e-04 2.681902e-03
## [9,] 0.9998766 -1.233799e-04 1.110414e-03
## [10,] 0.9999546 -4.539623e-05 4.539581e-04
## [11,] 0.9999833 -1.670090e-05 1.837134e-04
## [12,] 0.9999939 -6.143885e-06 7.372897e-05
print(tandeunsc2)
## Unit: microseconds
##
                                                                                  expr
##
   andeunsc2 <- nlsalt::numericDeriv(rexpr, theta, rho = weedenv,</pre>
                                                                       central = TRUE)
                lq
                    mean median uq
                                               max neval
## 132.513 156.784 185.0685 188.26 212.8665 264.66
## Forward diff with smaller eps
andeunscx<-nlsalt::numericDeriv(rexpr, theta, rho=weedenv, eps=1e-10)
## par:1 1 1
## [1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
## [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
```

```
## attr(,"gradient")
##
              [,1]
                            [,2]
                                         [.3]
##
    [1,] 0.7310597 -0.1966160568 0.1966071750
   [2,] 0.8807977 -0.1049915710 0.2099920238
##
   [3,] 0.9525714 -0.0451905180 0.1355182633
##
  [4,] 0.9820056 -0.0176747506 0.0706457115
## [5,] 0.9933032 -0.0066435746 0.0332534000
## [6,] 0.9975309 -0.0024513724 0.0148148160
   [7,] 0.9990941 -0.0009237056 0.0063593575
  [8,] 0.9996626 -0.0003552714 0.0026290081
## [9,] 0.9998757 -0.0001421085 0.0011368684
## [10,] 0.9999468 0.000000000 0.0004973799
## [11,] 0.9998757 -0.0001421085 0.0001421085
## [12,] 1.0000178 0.000000000 0.0001421085
print(andeunscx)
    \begin{bmatrix} 1 \end{bmatrix} -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
    [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
   attr(,"gradient")
##
##
                            [,2]
              [,1]
##
    [1,] 0.7310597 -0.1966160568 0.1966071750
##
   [2,] 0.8807977 -0.1049915710 0.2099920238
  [3,] 0.9525714 -0.0451905180 0.1355182633
  [4,] 0.9820056 -0.0176747506 0.0706457115
##
   [5,] 0.9933032 -0.0066435746 0.0332534000
## [6,] 0.9975309 -0.0024513724 0.0148148160
## [7,] 0.9990941 -0.0009237056 0.0063593575
## [8,] 0.9996626 -0.0003552714 0.0026290081
## [9,] 0.9998757 -0.0001421085 0.0011368684
## [10,] 0.9999468 0.000000000 0.0004973799
## [11,] 0.9998757 -0.0001421085 0.0001421085
## [12,] 1.0000178 0.000000000 0.0001421085
print(sum(andeunscx^2))
## [1] 23520.58
tandeunscx<-microbenchmark(andeunscx2<-nlsalt::numericDeriv(rexpr, theta, rho=weedenv, eps=1e-10))
## par:1 1 1
   [1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
    [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
  attr(,"gradient")
              [,1]
                            [,2]
##
    [1,] 0.7310597 -0.1966160568 0.1966071750
##
   [2,] 0.8807977 -0.1049915710 0.2099920238
  [3,] 0.9525714 -0.0451905180 0.1355182633
## [4,] 0.9820056 -0.0176747506 0.0706457115
   [5,] 0.9933032 -0.0066435746 0.0332534000
## [6,] 0.9975309 -0.0024513724 0.0148148160
## [7,] 0.9990941 -0.0009237056 0.0063593575
## [8,] 0.9996626 -0.0003552714 0.0026290081
   [9,] 0.9998757 -0.0001421085 0.0011368684
## [10,] 0.9999468 0.000000000 0.0004973799
## [11,] 0.9998757 -0.0001421085 0.0001421085
```

```
## [12,] 1.0000178 0.000000000 0.0001421085
## par:1 1 1
   [1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
    [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##
                            [,2]
                                         [,3]
              [,1]
   [1.] 0.7310597 -0.1966160568 0.1966071750
##
   [2,] 0.8807977 -0.1049915710 0.2099920238
   [3,] 0.9525714 -0.0451905180 0.1355182633
  [4,] 0.9820056 -0.0176747506 0.0706457115
   [5,] 0.9933032 -0.0066435746 0.0332534000
   [6,] 0.9975309 -0.0024513724 0.0148148160
  [7,] 0.9990941 -0.0009237056 0.0063593575
## [8,] 0.9996626 -0.0003552714 0.0026290081
## [9,] 0.9998757 -0.0001421085 0.0011368684
## [10,] 0.9999468 0.000000000 0.0004973799
## [11,] 0.9998757 -0.0001421085 0.0001421085
## [12,] 1.0000178 0.000000000 0.0001421085
## par:1 1 1
##
   [1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
   [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##
              [,1]
                            [,2]
                                         [,3]
##
   [1,] 0.7310597 -0.1966160568 0.1966071750
   [2,] 0.8807977 -0.1049915710 0.2099920238
   [3,] 0.9525714 -0.0451905180 0.1355182633
   [4,] 0.9820056 -0.0176747506 0.0706457115
##
   [5,] 0.9933032 -0.0066435746 0.0332534000
  [6,] 0.9975309 -0.0024513724 0.0148148160
## [7,] 0.9990941 -0.0009237056 0.0063593575
   [8,] 0.9996626 -0.0003552714 0.0026290081
  [9,] 0.9998757 -0.0001421085 0.0011368684
## [10,] 0.9999468 0.000000000 0.0004973799
## [11,] 0.9998757 -0.0001421085 0.0001421085
## [12,] 1.0000178 0.000000000 0.0001421085
## par:1 1 1
   [1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
   [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##
              [,1]
                            [,2]
                                         [,3]
   [1,] 0.7310597 -0.1966160568 0.1966071750
   [2,] 0.8807977 -0.1049915710 0.2099920238
##
   [3,] 0.9525714 -0.0451905180 0.1355182633
  [4,] 0.9820056 -0.0176747506 0.0706457115
  [5,] 0.9933032 -0.0066435746 0.0332534000
   [6,] 0.9975309 -0.0024513724 0.0148148160
   [7,] 0.9990941 -0.0009237056 0.0063593575
  [8,] 0.9996626 -0.0003552714 0.0026290081
## [9,] 0.9998757 -0.0001421085 0.0011368684
## [10,] 0.9999468 0.000000000 0.0004973799
## [11,] 0.9998757 -0.0001421085 0.0001421085
## [12,] 1.0000178 0.000000000 0.0001421085
## par:1 1 1
## [1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
```

```
[7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
                                        [,3]
##
              [,1]
                           [,2]
   [1,] 0.7310597 -0.1966160568 0.1966071750
##
   [2,] 0.8807977 -0.1049915710 0.2099920238
##
  [3,] 0.9525714 -0.0451905180 0.1355182633
  [4,] 0.9820056 -0.0176747506 0.0706457115
## [5,] 0.9933032 -0.0066435746 0.0332534000
   [6,] 0.9975309 -0.0024513724 0.0148148160
  [7,] 0.9990941 -0.0009237056 0.0063593575
## [8,] 0.9996626 -0.0003552714 0.0026290081
## [9,] 0.9998757 -0.0001421085 0.0011368684
## [10,] 0.9999468 0.000000000 0.0004973799
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## [12,] 1.0000178 0.000000000 0.0001421085
## par:1 1 1
   [1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
   [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##
              [,1]
                            [,2]
##
   [1,] 0.7310597 -0.1966160568 0.1966071750
  [2,] 0.8807977 -0.1049915710 0.2099920238
##
  [3,] 0.9525714 -0.0451905180 0.1355182633
   [4,] 0.9820056 -0.0176747506 0.0706457115
  [5,] 0.9933032 -0.0066435746 0.0332534000
## [6,] 0.9975309 -0.0024513724 0.0148148160
## [7,] 0.9990941 -0.0009237056 0.0063593575
## [8,] 0.9996626 -0.0003552714 0.0026290081
## [9,] 0.9998757 -0.0001421085 0.0011368684
## [10,] 0.9999468 0.000000000 0.0004973799
## [11,] 0.9998757 -0.0001421085 0.0001421085
## [12,] 1.0000178 0.000000000 0.0001421085
  par:1 1 1
    [1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
    [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##
              [,1]
                            [,2]
                                         [,3]
   [1,] 0.7310597 -0.1966160568 0.1966071750
##
   [2,] 0.8807977 -0.1049915710 0.2099920238
  [3,] 0.9525714 -0.0451905180 0.1355182633
  [4,] 0.9820056 -0.0176747506 0.0706457115
  [5,] 0.9933032 -0.0066435746 0.0332534000
## [6,] 0.9975309 -0.0024513724 0.0148148160
## [7,] 0.9990941 -0.0009237056 0.0063593575
## [8,] 0.9996626 -0.0003552714 0.0026290081
## [9,] 0.9998757 -0.0001421085 0.0011368684
## [10,] 0.9999468 0.000000000 0.0004973799
## [11,] 0.9998757 -0.0001421085 0.0001421085
## [12,] 1.0000178 0.000000000 0.0001421085
## par:1 1 1
   [1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
## [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##
                           [,2]
                                        [,3]
              [,1]
```

```
[1,] 0.7310597 -0.1966160568 0.1966071750
   [2,] 0.8807977 -0.1049915710 0.2099920238
##
   [3,] 0.9525714 -0.0451905180 0.1355182633
   [4,] 0.9820056 -0.0176747506 0.0706457115
   [5,] 0.9933032 -0.0066435746 0.0332534000
##
##
  [6,] 0.9975309 -0.0024513724 0.0148148160
  [7,] 0.9990941 -0.0009237056 0.0063593575
   [8,] 0.9996626 -0.0003552714 0.0026290081
   [9,] 0.9998757 -0.0001421085 0.0011368684
## [10,] 0.9999468 0.000000000 0.0004973799
  [11,] 0.9998757 -0.0001421085 0.0001421085
## [12,] 1.0000178 0.000000000 0.0001421085
  par:1 1 1
##
   [1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
   [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
  attr(, "gradient")
                           [,2]
##
             [,1]
                                       [,3]
   [1,] 0.7310597 -0.1966160568 0.1966071750
##
   [2,] 0.8807977 -0.1049915710 0.2099920238
##
   [3,] 0.9525714 -0.0451905180 0.1355182633
##
  [4,] 0.9820056 -0.0176747506 0.0706457115
  [5,] 0.9933032 -0.0066435746 0.0332534000
##
  [6,] 0.9975309 -0.0024513724 0.0148148160
   [7,] 0.9990941 -0.0009237056 0.0063593575
  [8,] 0.9996626 -0.0003552714 0.0026290081
  [9,] 0.9998757 -0.0001421085 0.0011368684
## [10,] 0.9999468 0.000000000 0.0004973799
## [11,] 0.9998757 -0.0001421085 0.0001421085
## [12,] 1.0000178 0.000000000 0.0001421085
## par:1 1 1
    [1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
    [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
  attr(,"gradient")
##
             [,1]
                           [,2]
    [1,] 0.7310597 -0.1966160568 0.1966071750
##
   [2,] 0.8807977 -0.1049915710 0.2099920238
##
  [3,] 0.9525714 -0.0451905180 0.1355182633
  [4,] 0.9820056 -0.0176747506 0.0706457115
##
   [5,] 0.9933032 -0.0066435746 0.0332534000
##
  [6,] 0.9975309 -0.0024513724 0.0148148160
  [7,] 0.9990941 -0.0009237056 0.0063593575
  [8,] 0.9996626 -0.0003552714 0.0026290081
   [9,] 0.9998757 -0.0001421085 0.0011368684
## [10,] 0.9999468 0.000000000 0.0004973799
## [11,] 0.9998757 -0.0001421085 0.0001421085
## [12,] 1.0000178 0.000000000 0.0001421085
##
  par:1 1 1
    [1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
   ## attr(,"gradient")
                           [,2]
##
             [,1]
                                       [,3]
##
  [1,] 0.7310597 -0.1966160568 0.1966071750
## [2,] 0.8807977 -0.1049915710 0.2099920238
## [3,] 0.9525714 -0.0451905180 0.1355182633
```

```
[4,] 0.9820056 -0.0176747506 0.0706457115
  [5,] 0.9933032 -0.0066435746 0.0332534000
  [6,] 0.9975309 -0.0024513724 0.0148148160
## [7,] 0.9990941 -0.0009237056 0.0063593575
   [8,] 0.9996626 -0.0003552714 0.0026290081
## [9,] 0.9998757 -0.0001421085 0.0011368684
## [10,] 0.9999468 0.000000000 0.0004973799
## [11,] 0.9998757 -0.0001421085 0.0001421085
## [12,] 1.0000178 0.000000000 0.0001421085
  par:1 1 1
    [1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
    [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
  attr(,"gradient")
##
              [,1]
                            [,2]
                                         [,3]
##
   [1,] 0.7310597 -0.1966160568 0.1966071750
   [2,] 0.8807977 -0.1049915710 0.2099920238
   [3,] 0.9525714 -0.0451905180 0.1355182633
  [4,] 0.9820056 -0.0176747506 0.0706457115
  [5,] 0.9933032 -0.0066435746 0.0332534000
   [6,] 0.9975309 -0.0024513724 0.0148148160
## [7,] 0.9990941 -0.0009237056 0.0063593575
## [8,] 0.9996626 -0.0003552714 0.0026290081
## [9,] 0.9998757 -0.0001421085 0.0011368684
## [10,] 0.9999468 0.000000000 0.0004973799
## [11,] 0.9998757 -0.0001421085 0.0001421085
## [12,] 1.0000178 0.000000000 0.0001421085
## par:1 1 1
   [1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
   [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##
              [,1]
                            [,2]
##
  [1,] 0.7310597 -0.1966160568 0.1966071750
  [2,] 0.8807977 -0.1049915710 0.2099920238
  [3,] 0.9525714 -0.0451905180 0.1355182633
   [4,] 0.9820056 -0.0176747506 0.0706457115
  [5,] 0.9933032 -0.0066435746 0.0332534000
## [6,] 0.9975309 -0.0024513724 0.0148148160
## [7,] 0.9990941 -0.0009237056 0.0063593575
   [8,] 0.9996626 -0.0003552714 0.0026290081
  [9,] 0.9998757 -0.0001421085 0.0011368684
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## [11,] 0.9998757 -0.0001421085 0.0001421085
## [12,] 1.0000178 0.000000000 0.0001421085
## par:1 1 1
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    [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
##
  attr(,"gradient")
##
              [,1]
                            [,2]
   [1,] 0.7310597 -0.1966160568 0.1966071750
   [2,] 0.8807977 -0.1049915710 0.2099920238
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   [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##
              [,1]
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##
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  [2,] 0.8807977 -0.1049915710 0.2099920238
   [3,] 0.9525714 -0.0451905180 0.1355182633
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## [6,] 0.9975309 -0.0024513724 0.0148148160
## [7,] 0.9990941 -0.0009237056 0.0063593575
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## [9,] 0.9998757 -0.0001421085 0.0011368684
## [10,] 0.9999468 0.000000000 0.0004973799
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## [9,] 0.9998757 -0.0001421085 0.0011368684
## [10,] 0.9999468 0.000000000 0.0004973799
## [11,] 0.9998757 -0.0001421085 0.0001421085
## [12,] 1.0000178 0.000000000 0.0001421085
## par:1 1 1
##
   [1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
   [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##
              [,1]
                            [,2]
                                         [.3]
##
    [1,] 0.7310597 -0.1966160568 0.1966071750
   [2,] 0.8807977 -0.1049915710 0.2099920238
   [3,] 0.9525714 -0.0451905180 0.1355182633
   [4,] 0.9820056 -0.0176747506 0.0706457115
  [5,] 0.9933032 -0.0066435746 0.0332534000
  [6,] 0.9975309 -0.0024513724 0.0148148160
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   [7,] 0.9990941 -0.0009237056 0.0063593575
  [8,] 0.9996626 -0.0003552714 0.0026290081
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    \begin{bmatrix} 1 \end{bmatrix} -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
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   [8,] 0.9996626 -0.0003552714 0.0026290081
  [9,] 0.9998757 -0.0001421085 0.0011368684
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## par:1 1 1
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##
   [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##
              [,1]
                            [,2]
                                         [,3]
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   [2,] 0.8807977 -0.1049915710 0.2099920238
   [3,] 0.9525714 -0.0451905180 0.1355182633
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                                          [.3]
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## par:1 1 1
   [1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##
              [,1]
                            [,2]
                                         [.3]
##
   [1,] 0.7310597 -0.1966160568 0.1966071750
   [2,] 0.8807977 -0.1049915710 0.2099920238
   [3,] 0.9525714 -0.0451905180 0.1355182633
   [4,] 0.9820056 -0.0176747506 0.0706457115
  [5,] 0.9933032 -0.0066435746 0.0332534000
  [6,] 0.9975309 -0.0024513724 0.0148148160
   [7,] 0.9990941 -0.0009237056 0.0063593575
   [8,] 0.9996626 -0.0003552714 0.0026290081
  [9,] 0.9998757 -0.0001421085 0.0011368684
## [10,] 0.9999468 0.000000000 0.0004973799
## [11,] 0.9998757 -0.0001421085 0.0001421085
## [12,] 1.0000178 0.000000000 0.0001421085
## par:1 1 1
    \begin{bmatrix} 1 \end{bmatrix} -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
    [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
  attr(,"gradient")
##
              [,1]
                            [,2]
   [1,] 0.7310597 -0.1966160568 0.1966071750
##
   [2,] 0.8807977 -0.1049915710 0.2099920238
  [3,] 0.9525714 -0.0451905180 0.1355182633
  [4,] 0.9820056 -0.0176747506 0.0706457115
   [5,] 0.9933032 -0.0066435746 0.0332534000
   [6,] 0.9975309 -0.0024513724 0.0148148160
  [7,] 0.9990941 -0.0009237056 0.0063593575
## [8,] 0.9996626 -0.0003552714 0.0026290081
   [9,] 0.9998757 -0.0001421085 0.0011368684
## [10,] 0.9999468 0.000000000 0.0004973799
## [11,] 0.9998757 -0.0001421085 0.0001421085
## [12,] 1.0000178 0.000000000 0.0001421085
## par:1 1 1
```

```
[1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
   [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##
                            [,2]
              [,1]
                                         [,3]
##
   [1,] 0.7310597 -0.1966160568 0.1966071750
   [2,] 0.8807977 -0.1049915710 0.2099920238
  [3,] 0.9525714 -0.0451905180 0.1355182633
## [4,] 0.9820056 -0.0176747506 0.0706457115
   [5,] 0.9933032 -0.0066435746 0.0332534000
  [6,] 0.9975309 -0.0024513724 0.0148148160
## [7,] 0.9990941 -0.0009237056 0.0063593575
## [8,] 0.9996626 -0.0003552714 0.0026290081
## [9,] 0.9998757 -0.0001421085 0.0011368684
## [10,] 0.9999468 0.000000000 0.0004973799
## [11,] 0.9998757 -0.0001421085 0.0001421085
## [12,] 1.0000178 0.000000000 0.0001421085
## par:1 1 1
   [1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
   [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##
              [,1]
                            [,2]
                                        [,3]
   [1,] 0.7310597 -0.1966160568 0.1966071750
  [2,] 0.8807977 -0.1049915710 0.2099920238
   [3,] 0.9525714 -0.0451905180 0.1355182633
  [4,] 0.9820056 -0.0176747506 0.0706457115
## [5,] 0.9933032 -0.0066435746 0.0332534000
## [6,] 0.9975309 -0.0024513724 0.0148148160
## [7,] 0.9990941 -0.0009237056 0.0063593575
## [8,] 0.9996626 -0.0003552714 0.0026290081
## [9,] 0.9998757 -0.0001421085 0.0011368684
## [10,] 0.9999468 0.000000000 0.0004973799
## [11,] 0.9998757 -0.0001421085 0.0001421085
## [12,] 1.0000178 0.000000000 0.0001421085
## par:1 1 1
    [1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
   [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
##
## attr(,"gradient")
##
                            [,2]
              [,1]
   [1,] 0.7310597 -0.1966160568 0.1966071750
##
  [2,] 0.8807977 -0.1049915710 0.2099920238
##
  [3,] 0.9525714 -0.0451905180 0.1355182633
  [4,] 0.9820056 -0.0176747506 0.0706457115
   [5,] 0.9933032 -0.0066435746 0.0332534000
## [6,] 0.9975309 -0.0024513724 0.0148148160
## [7,] 0.9990941 -0.0009237056 0.0063593575
## [8,] 0.9996626 -0.0003552714 0.0026290081
## [9,] 0.9998757 -0.0001421085 0.0011368684
## [10,] 0.9999468 0.000000000 0.0004973799
## [11,] 0.9998757 -0.0001421085 0.0001421085
## [12,] 1.0000178 0.000000000 0.0001421085
## par:1 1 1
## [1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
## [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
```

```
[,1]
                             [,2]
##
    [1,] 0.7310597 -0.1966160568 0.1966071750
##
    [2,] 0.8807977 -0.1049915710 0.2099920238
   [3,] 0.9525714 -0.0451905180 0.1355182633
##
    [4,] 0.9820056 -0.0176747506 0.0706457115
##
   [5,] 0.9933032 -0.0066435746 0.0332534000
   [6,] 0.9975309 -0.0024513724 0.0148148160
##
   [7,] 0.9990941 -0.0009237056 0.0063593575
    [8,] 0.9996626 -0.0003552714 0.0026290081
   [9,] 0.9998757 -0.0001421085 0.0011368684
## [10,] 0.9999468 0.000000000 0.0004973799
   [11,] 0.9998757 -0.0001421085 0.0001421085
   [12,] 1.0000178 0.000000000 0.0001421085
   par:1 1 1
    \begin{bmatrix} 1 \end{bmatrix} -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
    [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
   attr(,"gradient")
##
               [,1]
                             [,2]
    [1,] 0.7310597 -0.1966160568 0.1966071750
##
    [2,] 0.8807977 -0.1049915710 0.2099920238
##
   [3,] 0.9525714 -0.0451905180 0.1355182633
  [4,] 0.9820056 -0.0176747506 0.0706457115
##
   [5,] 0.9933032 -0.0066435746 0.0332534000
    [6,] 0.9975309 -0.0024513724 0.0148148160
   [7,] 0.9990941 -0.0009237056 0.0063593575
   [8,] 0.9996626 -0.0003552714 0.0026290081
   [9,] 0.9998757 -0.0001421085 0.0011368684
## [10,] 0.9999468 0.000000000 0.0004973799
## [11,] 0.9998757 -0.0001421085 0.0001421085
## [12,] 1.0000178 0.000000000 0.0001421085
  par:1 1 1
##
     \begin{bmatrix} 1 \end{bmatrix} \quad -4.576941 \quad -6.359203 \quad -8.685426 \quad -11.883986 \quad -16.075693 \quad -22.194473 
    [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
   attr(,"gradient")
##
                             [,2]
##
               [,1]
##
    [1,] 0.7310597 -0.1966160568 0.1966071750
   [2,] 0.8807977 -0.1049915710 0.2099920238
   [3,] 0.9525714 -0.0451905180 0.1355182633
    [4,] 0.9820056 -0.0176747506 0.0706457115
##
   [5,] 0.9933032 -0.0066435746 0.0332534000
##
   [6,] 0.9975309 -0.0024513724 0.0148148160
   [7,] 0.9990941 -0.0009237056 0.0063593575
##
   [8,] 0.9996626 -0.0003552714 0.0026290081
  [9,] 0.9998757 -0.0001421085 0.0011368684
## [10,] 0.9999468 0.000000000 0.0004973799
## [11,] 0.9998757 -0.0001421085 0.0001421085
## [12,] 1.0000178 0.000000000 0.0001421085
   par:1 1 1
    \begin{bmatrix} 1 \end{bmatrix} -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
    [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##
                             [,2]
##
   [1,] 0.7310597 -0.1966160568 0.1966071750
   [2,] 0.8807977 -0.1049915710 0.2099920238
```

```
[3,] 0.9525714 -0.0451905180 0.1355182633
  [4,] 0.9820056 -0.0176747506 0.0706457115
##
  [5,] 0.9933032 -0.0066435746 0.0332534000
## [6,] 0.9975309 -0.0024513724 0.0148148160
   [7,] 0.9990941 -0.0009237056 0.0063593575
## [8,] 0.9996626 -0.0003552714 0.0026290081
## [9,] 0.9998757 -0.0001421085 0.0011368684
## [10,] 0.9999468 0.000000000 0.0004973799
## [11,] 0.9998757 -0.0001421085 0.0001421085
## [12,] 1.0000178 0.000000000 0.0001421085
print(tandeunscx)
## Unit: microseconds
##
                                                                                 expr
##
                                                                         eps = 1e-10)
   andeunscx2 <- nlsalt::numericDeriv(rexpr, theta, rho = weedenv,</pre>
##
                     mean
                            {\tt median}
                                      uq
                                                  max neval
  118.233 142.57 173.1304 172.1375 198.9305 270.593
## Central diff with smaller eps
andeunscx2<-nlsalt::numericDeriv(rexpr, theta, rho-weedenv, central=TRUE, eps=1e-10)
## par:1 1 1
  [1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
   [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##
              [,1]
                            [,2]
                                         [.3]
  [1,] 0.7310597 -1.966116e-01 1.966116e-01
   [2,] 0.8807977 -1.049916e-01 2.099876e-01
##
## [3,] 0.9525714 -4.518164e-02 1.355271e-01
  [4,] 0.9820145 -1.766587e-02 7.065459e-02
## [5,] 0.9933032 -6.643575e-03 3.325340e-02
## [6,] 0.9975309 -2.451372e-03 1.481482e-02
## [7,] 0.9990941 -9.059420e-04 6.359357e-03
## [8,] 0.9996981 -3.197442e-04 2.664535e-03
## [9,] 0.9998757 -1.421085e-04 1.136868e-03
## [10,] 0.9999468 -3.552714e-05 4.618528e-04
## [11,] 0.9999468 -7.105427e-05 2.131628e-04
## [12,] 1.0000178 0.000000e+00 7.105427e-05
print(andeunscx2)
    [1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
   [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
##
## attr(,"gradient")
##
              [,1]
                            [,2]
                                         [,3]
##
   [1,] 0.7310597 -1.966116e-01 1.966116e-01
##
   [2,] 0.8807977 -1.049916e-01 2.099876e-01
  [3,] 0.9525714 -4.518164e-02 1.355271e-01
  [4,] 0.9820145 -1.766587e-02 7.065459e-02
##
   [5,] 0.9933032 -6.643575e-03 3.325340e-02
##
  [6,] 0.9975309 -2.451372e-03 1.481482e-02
  [7,] 0.9990941 -9.059420e-04 6.359357e-03
  [8,] 0.9996981 -3.197442e-04 2.664535e-03
  [9,] 0.9998757 -1.421085e-04 1.136868e-03
## [10,] 0.9999468 -3.552714e-05 4.618528e-04
```

```
## [11,] 0.9999468 -7.105427e-05 2.131628e-04
## [12,] 1.0000178 0.000000e+00 7.105427e-05
print(sum(andeunscx2^2))
## [1] 23520.58
tandeunscx2<-microbenchmark(andeunscx2<-nlsalt::numericDeriv(rexpr, theta, rho=weedenv, central=TRUE, e
## par:1 1 1
## [1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##
              [,1]
                            [,2]
                                         [.3]
##
  [1,] 0.7310597 -1.966116e-01 1.966116e-01
   [2,] 0.8807977 -1.049916e-01 2.099876e-01
##
   [3,] 0.9525714 -4.518164e-02 1.355271e-01
## [4,] 0.9820145 -1.766587e-02 7.065459e-02
## [5,] 0.9933032 -6.643575e-03 3.325340e-02
## [6,] 0.9975309 -2.451372e-03 1.481482e-02
   [7,] 0.9990941 -9.059420e-04 6.359357e-03
## [8,] 0.9996981 -3.197442e-04 2.664535e-03
## [9,] 0.9998757 -1.421085e-04 1.136868e-03
## [10,] 0.9999468 -3.552714e-05 4.618528e-04
## [11,] 0.9999468 -7.105427e-05 2.131628e-04
## [12,] 1.0000178 0.000000e+00 7.105427e-05
## par:1 1 1
   [1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
   [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
##
## attr(,"gradient")
              [,1]
                            [,2]
                                         [.3]
  [1,] 0.7310597 -1.966116e-01 1.966116e-01
##
   [2,] 0.8807977 -1.049916e-01 2.099876e-01
## [3,] 0.9525714 -4.518164e-02 1.355271e-01
## [4,] 0.9820145 -1.766587e-02 7.065459e-02
   [5,] 0.9933032 -6.643575e-03 3.325340e-02
   [6,] 0.9975309 -2.451372e-03 1.481482e-02
## [7,] 0.9990941 -9.059420e-04 6.359357e-03
## [8,] 0.9996981 -3.197442e-04 2.664535e-03
   [9,] 0.9998757 -1.421085e-04 1.136868e-03
## [10,] 0.9999468 -3.552714e-05 4.618528e-04
## [11,] 0.9999468 -7.105427e-05 2.131628e-04
## [12,] 1.0000178 0.000000e+00 7.105427e-05
## par:1 1 1
## [1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##
              [,1]
                            [,2]
                                         [,3]
  [1,] 0.7310597 -1.966116e-01 1.966116e-01
##
## [2,] 0.8807977 -1.049916e-01 2.099876e-01
##
   [3,] 0.9525714 -4.518164e-02 1.355271e-01
   [4,] 0.9820145 -1.766587e-02 7.065459e-02
##
## [5,] 0.9933032 -6.643575e-03 3.325340e-02
## [6,] 0.9975309 -2.451372e-03 1.481482e-02
## [7,] 0.9990941 -9.059420e-04 6.359357e-03
```

```
## [8,] 0.9996981 -3.197442e-04 2.664535e-03
## [9,] 0.9998757 -1.421085e-04 1.136868e-03
## [10,] 0.9999468 -3.552714e-05 4.618528e-04
## [11,] 0.9999468 -7.105427e-05 2.131628e-04
## [12,] 1.0000178 0.000000e+00 7.105427e-05
## par:1 1 1
  [1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
   [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##
              [,1]
                            [,2]
                                         [,3]
   [1,] 0.7310597 -1.966116e-01 1.966116e-01
##
   [2,] 0.8807977 -1.049916e-01 2.099876e-01
   [3,] 0.9525714 -4.518164e-02 1.355271e-01
## [4,] 0.9820145 -1.766587e-02 7.065459e-02
## [5,] 0.9933032 -6.643575e-03 3.325340e-02
   [6,] 0.9975309 -2.451372e-03 1.481482e-02
## [7,] 0.9990941 -9.059420e-04 6.359357e-03
## [8,] 0.9996981 -3.197442e-04 2.664535e-03
## [9,] 0.9998757 -1.421085e-04 1.136868e-03
## [10,] 0.9999468 -3.552714e-05 4.618528e-04
## [11,] 0.9999468 -7.105427e-05 2.131628e-04
## [12,] 1.0000178 0.000000e+00 7.105427e-05
## par:1 1 1
##
   [1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
   [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##
              [,1]
                            [,2]
                                         [,3]
   [1,] 0.7310597 -1.966116e-01 1.966116e-01
  [2,] 0.8807977 -1.049916e-01 2.099876e-01
## [3,] 0.9525714 -4.518164e-02 1.355271e-01
##
   [4,] 0.9820145 -1.766587e-02 7.065459e-02
   [5,] 0.9933032 -6.643575e-03 3.325340e-02
  [6,] 0.9975309 -2.451372e-03 1.481482e-02
## [7,] 0.9990941 -9.059420e-04 6.359357e-03
   [8,] 0.9996981 -3.197442e-04 2.664535e-03
## [9,] 0.9998757 -1.421085e-04 1.136868e-03
## [10,] 0.9999468 -3.552714e-05 4.618528e-04
## [11,] 0.9999468 -7.105427e-05 2.131628e-04
## [12,] 1.0000178 0.000000e+00 7.105427e-05
## par:1 1 1
   [1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
   [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##
              [,1]
                            [,2]
                                         [,3]
   [1,] 0.7310597 -1.966116e-01 1.966116e-01
   [2,] 0.8807977 -1.049916e-01 2.099876e-01
##
   [3,] 0.9525714 -4.518164e-02 1.355271e-01
  [4,] 0.9820145 -1.766587e-02 7.065459e-02
## [5,] 0.9933032 -6.643575e-03 3.325340e-02
   [6,] 0.9975309 -2.451372e-03 1.481482e-02
## [7,] 0.9990941 -9.059420e-04 6.359357e-03
## [8,] 0.9996981 -3.197442e-04 2.664535e-03
## [9,] 0.9998757 -1.421085e-04 1.136868e-03
## [10,] 0.9999468 -3.552714e-05 4.618528e-04
```

```
## [11,] 0.9999468 -7.105427e-05 2.131628e-04
## [12,] 1.0000178 0.000000e+00 7.105427e-05
## par:1 1 1
   [1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
    [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
              [,1]
                            [,2]
                                         [.3]
##
   [1,] 0.7310597 -1.966116e-01 1.966116e-01
   [2,] 0.8807977 -1.049916e-01 2.099876e-01
  [3,] 0.9525714 -4.518164e-02 1.355271e-01
  [4,] 0.9820145 -1.766587e-02 7.065459e-02
   [5,] 0.9933032 -6.643575e-03 3.325340e-02
## [6,] 0.9975309 -2.451372e-03 1.481482e-02
## [7,] 0.9990941 -9.059420e-04 6.359357e-03
## [8,] 0.9996981 -3.197442e-04 2.664535e-03
   [9,] 0.9998757 -1.421085e-04 1.136868e-03
## [10,] 0.9999468 -3.552714e-05 4.618528e-04
## [11,] 0.9999468 -7.105427e-05 2.131628e-04
## [12,] 1.0000178 0.000000e+00 7.105427e-05
## par:1 1 1
## [1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##
              [,1]
                            [,2]
                                         [.3]
##
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  [2,] 0.8807977 -1.049916e-01 2.099876e-01
   [3,] 0.9525714 -4.518164e-02 1.355271e-01
##
   [4,] 0.9820145 -1.766587e-02 7.065459e-02
  [5,] 0.9933032 -6.643575e-03 3.325340e-02
## [6,] 0.9975309 -2.451372e-03 1.481482e-02
##
   [7,] 0.9990941 -9.059420e-04 6.359357e-03
## [8,] 0.9996981 -3.197442e-04 2.664535e-03
  [9,] 0.9998757 -1.421085e-04 1.136868e-03
## [10,] 0.9999468 -3.552714e-05 4.618528e-04
## [11,] 0.9999468 -7.105427e-05 2.131628e-04
## [12,] 1.0000178 0.000000e+00 7.105427e-05
## par:1 1 1
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    [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
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##
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```

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## [12,] 1.0000178 0.000000e+00 7.105427e-05
## par:1 1 1
    \begin{bmatrix} 1 \end{bmatrix} -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
   [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
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##
              [,1]
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[,1]
                           [,2]
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   [1,] 0.7310597 -1.966116e-01 1.966116e-01
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```
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```
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##
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```

```
## [12,] 1.0000178 0.000000e+00 7.105427e-05
## par:1 1 1
   [1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
   [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##
                                         [,3]
              [,1]
                            [,2]
   [1,] 0.7310597 -1.966116e-01 1.966116e-01
##
   [2,] 0.8807977 -1.049916e-01 2.099876e-01
   [3,] 0.9525714 -4.518164e-02 1.355271e-01
  [4,] 0.9820145 -1.766587e-02 7.065459e-02
  [5,] 0.9933032 -6.643575e-03 3.325340e-02
   [6,] 0.9975309 -2.451372e-03 1.481482e-02
## [7,] 0.9990941 -9.059420e-04 6.359357e-03
## [8,] 0.9996981 -3.197442e-04 2.664535e-03
## [9,] 0.9998757 -1.421085e-04 1.136868e-03
## [10,] 0.9999468 -3.552714e-05 4.618528e-04
## [11,] 0.9999468 -7.105427e-05 2.131628e-04
## [12,] 1.0000178 0.000000e+00 7.105427e-05
## par:1 1 1
##
   [1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
   [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##
              [,1]
                            [,2]
                                         [,3]
##
   [1,] 0.7310597 -1.966116e-01 1.966116e-01
  [2,] 0.8807977 -1.049916e-01 2.099876e-01
  [3,] 0.9525714 -4.518164e-02 1.355271e-01
   [4,] 0.9820145 -1.766587e-02 7.065459e-02
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   [5,] 0.9933032 -6.643575e-03 3.325340e-02
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## [7,] 0.9990941 -9.059420e-04 6.359357e-03
   [8,] 0.9996981 -3.197442e-04 2.664535e-03
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  [1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
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## attr(,"gradient")
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  [1,] 0.7310597 -1.966116e-01 1.966116e-01
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  [4,] 0.9820145 -1.766587e-02 7.065459e-02
## [5,] 0.9933032 -6.643575e-03 3.325340e-02
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   [6,] 0.9975309 -2.451372e-03 1.481482e-02
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## [11,] 0.9999468 -7.105427e-05 2.131628e-04
## [12,] 1.0000178 0.000000e+00 7.105427e-05
## par:1 1 1
## [1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
```

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[7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##
              [,1]
                            [,2]
                                         [,3]
   [1,] 0.7310597 -1.966116e-01 1.966116e-01
##
##
   [2,] 0.8807977 -1.049916e-01 2.099876e-01
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  [3,] 0.9525714 -4.518164e-02 1.355271e-01
## [4,] 0.9820145 -1.766587e-02 7.065459e-02
## [5,] 0.9933032 -6.643575e-03 3.325340e-02
   [6,] 0.9975309 -2.451372e-03 1.481482e-02
  [7,] 0.9990941 -9.059420e-04 6.359357e-03
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## par:1 1 1
    \begin{bmatrix} 1 \end{bmatrix} -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
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## attr(,"gradient")
##
              [,1]
                            [,2]
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##
  [1,] 0.7310597 -1.966116e-01 1.966116e-01
   [2,] 0.8807977 -1.049916e-01 2.099876e-01
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## attr(,"gradient")
##
                            [,2]
                                         [,3]
              [,1]
```

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[1,] 0.7310597 -1.966116e-01 1.966116e-01
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## [7,] 0.9990941 -9.059420e-04 6.359357e-03
## [8,] 0.9996981 -3.197442e-04 2.664535e-03
## [9,] 0.9998757 -1.421085e-04 1.136868e-03
## [10,] 0.9999468 -3.552714e-05 4.618528e-04
## [11,] 0.9999468 -7.105427e-05 2.131628e-04
## [12,] 1.0000178 0.000000e+00 7.105427e-05
## par:1 1 1
   [1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
##
   [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##
              [,1]
                            [,2]
                                         [,3]
##
   [1,] 0.7310597 -1.966116e-01 1.966116e-01
##
  [2,] 0.8807977 -1.049916e-01 2.099876e-01
  [3,] 0.9525714 -4.518164e-02 1.355271e-01
##
   [4,] 0.9820145 -1.766587e-02 7.065459e-02
   [5,] 0.9933032 -6.643575e-03 3.325340e-02
  [6,] 0.9975309 -2.451372e-03 1.481482e-02
## [7,] 0.9990941 -9.059420e-04 6.359357e-03
   [8,] 0.9996981 -3.197442e-04 2.664535e-03
## [9,] 0.9998757 -1.421085e-04 1.136868e-03
## [10,] 0.9999468 -3.552714e-05 4.618528e-04
## [11,] 0.9999468 -7.105427e-05 2.131628e-04
## [12,] 1.0000178 0.000000e+00 7.105427e-05
```

```
## par:1 1 1
   [1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
   [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##
              [,1]
                            [,2]
                                         [,3]
##
   [1,] 0.7310597 -1.966116e-01 1.966116e-01
  [2,] 0.8807977 -1.049916e-01 2.099876e-01
   [3,] 0.9525714 -4.518164e-02 1.355271e-01
   [4,] 0.9820145 -1.766587e-02 7.065459e-02
  [5,] 0.9933032 -6.643575e-03 3.325340e-02
  [6,] 0.9975309 -2.451372e-03 1.481482e-02
   [7,] 0.9990941 -9.059420e-04 6.359357e-03
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## [9,] 0.9998757 -1.421085e-04 1.136868e-03
## [10,] 0.9999468 -3.552714e-05 4.618528e-04
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## [12,] 1.0000178 0.000000e+00 7.105427e-05
## par:1 1 1
   [1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
    [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##
              [,1]
                            [,2]
##
   [1,] 0.7310597 -1.966116e-01 1.966116e-01
##
   [2,] 0.8807977 -1.049916e-01 2.099876e-01
##
  [3,] 0.9525714 -4.518164e-02 1.355271e-01
  [4,] 0.9820145 -1.766587e-02 7.065459e-02
##
   [5,] 0.9933032 -6.643575e-03 3.325340e-02
   [6,] 0.9975309 -2.451372e-03 1.481482e-02
## [7,] 0.9990941 -9.059420e-04 6.359357e-03
## [8,] 0.9996981 -3.197442e-04 2.664535e-03
## [9,] 0.9998757 -1.421085e-04 1.136868e-03
## [10,] 0.9999468 -3.552714e-05 4.618528e-04
## [11,] 0.9999468 -7.105427e-05 2.131628e-04
## [12,] 1.0000178 0.000000e+00 7.105427e-05
## par:1 1 1
   [1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
##
  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##
              [,1]
                            [,2]
                                         [,3]
##
  [1,] 0.7310597 -1.966116e-01 1.966116e-01
  [2,] 0.8807977 -1.049916e-01 2.099876e-01
   [3,] 0.9525714 -4.518164e-02 1.355271e-01
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   [4,] 0.9820145 -1.766587e-02 7.065459e-02
##
  [5,] 0.9933032 -6.643575e-03 3.325340e-02
## [6,] 0.9975309 -2.451372e-03 1.481482e-02
   [7,] 0.9990941 -9.059420e-04 6.359357e-03
   [8,] 0.9996981 -3.197442e-04 2.664535e-03
  [9,] 0.9998757 -1.421085e-04 1.136868e-03
## [10,] 0.9999468 -3.552714e-05 4.618528e-04
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## [12,] 1.0000178 0.000000e+00 7.105427e-05
## [1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
## [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
```

```
## attr(,"gradient")
                            [,2]
##
                                          [,3]
              [,1]
##
    [1,] 0.7310597 -1.966116e-01 1.966116e-01
   [2,] 0.8807977 -1.049916e-01 2.099876e-01
##
   [3,] 0.9525714 -4.518164e-02 1.355271e-01
##
  [4,] 0.9820145 -1.766587e-02 7.065459e-02
  [5,] 0.9933032 -6.643575e-03 3.325340e-02
##
   [6,] 0.9975309 -2.451372e-03 1.481482e-02
   [7,] 0.9990941 -9.059420e-04 6.359357e-03
  [8,] 0.9996981 -3.197442e-04 2.664535e-03
  [9,] 0.9998757 -1.421085e-04 1.136868e-03
## [10,] 0.9999468 -3.552714e-05 4.618528e-04
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## [12,] 1.0000178 0.000000e+00 7.105427e-05
## par:1 1 1
    [1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
    [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
##
  attr(, "gradient")
##
              [,1]
                            [,2]
                                          [,3]
##
    [1,] 0.7310597 -1.966116e-01 1.966116e-01
##
   [2,] 0.8807977 -1.049916e-01 2.099876e-01
  [3,] 0.9525714 -4.518164e-02 1.355271e-01
##
   [4,] 0.9820145 -1.766587e-02 7.065459e-02
##
   [5,] 0.9933032 -6.643575e-03 3.325340e-02
##
  [6,] 0.9975309 -2.451372e-03 1.481482e-02
  [7,] 0.9990941 -9.059420e-04 6.359357e-03
   [8,] 0.9996981 -3.197442e-04 2.664535e-03
## [9,] 0.9998757 -1.421085e-04 1.136868e-03
## [10,] 0.9999468 -3.552714e-05 4.618528e-04
## [11,] 0.9999468 -7.105427e-05 2.131628e-04
## [12,] 1.0000178 0.000000e+00 7.105427e-05
## par:1 1 1
    \begin{bmatrix} 1 \end{bmatrix} -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
    [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
   attr(, "gradient")
##
                            [,2]
##
              [,1]
                                          [,3]
##
   [1,] 0.7310597 -1.966116e-01 1.966116e-01
   [2,] 0.8807977 -1.049916e-01 2.099876e-01
##
##
   [3,] 0.9525714 -4.518164e-02 1.355271e-01
##
  [4,] 0.9820145 -1.766587e-02 7.065459e-02
  [5,] 0.9933032 -6.643575e-03 3.325340e-02
  [6,] 0.9975309 -2.451372e-03 1.481482e-02
##
   [7,] 0.9990941 -9.059420e-04 6.359357e-03
  [8,] 0.9996981 -3.197442e-04 2.664535e-03
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## [10,] 0.9999468 -3.552714e-05 4.618528e-04
## [11,] 0.9999468 -7.105427e-05 2.131628e-04
## [12,] 1.0000178 0.000000e+00 7.105427e-05
## par:1 1 1
        -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
    [1]
   [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##
              [,1]
                            [,2]
                                          [,3]
  [1,] 0.7310597 -1.966116e-01 1.966116e-01
```

```
[2,] 0.8807977 -1.049916e-01 2.099876e-01
##
   [3,] 0.9525714 -4.518164e-02 1.355271e-01
  [4,] 0.9820145 -1.766587e-02 7.065459e-02
  [5,] 0.9933032 -6.643575e-03 3.325340e-02
   [6,] 0.9975309 -2.451372e-03 1.481482e-02
##
  [7,] 0.9990941 -9.059420e-04 6.359357e-03
  [8,] 0.9996981 -3.197442e-04 2.664535e-03
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## [10,] 0.9999468 -3.552714e-05 4.618528e-04
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## [12,] 1.0000178 0.000000e+00 7.105427e-05
## par:1 1 1
##
   [1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
   [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##
              [,1]
                            [,2]
##
   [1,] 0.7310597 -1.966116e-01 1.966116e-01
   [2,] 0.8807977 -1.049916e-01 2.099876e-01
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## [7,] 0.9990941 -9.059420e-04 6.359357e-03
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## [10,] 0.9999468 -3.552714e-05 4.618528e-04
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   [1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
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## attr(,"gradient")
##
                            [,2]
              [,1]
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   [1,] 0.7310597 -1.966116e-01 1.966116e-01
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## [5,] 0.9933032 -6.643575e-03 3.325340e-02
   [6,] 0.9975309 -2.451372e-03 1.481482e-02
## [7,] 0.9990941 -9.059420e-04 6.359357e-03
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## [10,] 0.9999468 -3.552714e-05 4.618528e-04
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## par:1 1 1
##
   [1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
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## attr(,"gradient")
##
                            [,2]
              [,1]
##
   [1,] 0.7310597 -1.966116e-01 1.966116e-01
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```

```
[5,] 0.9933032 -6.643575e-03 3.325340e-02
##
  [6,] 0.9975309 -2.451372e-03 1.481482e-02
  [7,] 0.9990941 -9.059420e-04 6.359357e-03
## [8,] 0.9996981 -3.197442e-04 2.664535e-03
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## [10,] 0.9999468 -3.552714e-05 4.618528e-04
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## [12,] 1.0000178 0.000000e+00 7.105427e-05
## par:1 1 1
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##
  attr(,"gradient")
##
             [,1]
                           [,2]
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   [1,] 0.7310597 -1.966116e-01 1.966116e-01
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   [3,] 0.9525714 -4.518164e-02 1.355271e-01
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## par:1 1 1
    [1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
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##
             [,1]
                           [,2]
                                        [,3]
##
    [1,] 0.7310597 -1.966116e-01 1.966116e-01
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##
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  [1,] 0.7310597 -1.966116e-01 1.966116e-01
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```

```
## [8,] 0.9996981 -3.197442e-04 2.664535e-03
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## attr(,"gradient")
##
              [,1]
                            [,2]
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   [1,] 0.7310597 -1.966116e-01 1.966116e-01
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## [10,] 0.9999468 -3.552714e-05 4.618528e-04
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##
   [1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
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##
              [,1]
                            [,2]
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   [1,] 0.7310597 -1.966116e-01 1.966116e-01
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## par:1 1 1
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##
              [,1]
                            [,2]
                                         [,3]
  [1,] 0.7310597 -1.966116e-01 1.966116e-01
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   [3,] 0.9525714 -4.518164e-02 1.355271e-01
  [4,] 0.9820145 -1.766587e-02 7.065459e-02
## [5,] 0.9933032 -6.643575e-03 3.325340e-02
   [6,] 0.9975309 -2.451372e-03 1.481482e-02
## [7,] 0.9990941 -9.059420e-04 6.359357e-03
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```

```
## [11,] 0.9999468 -7.105427e-05 2.131628e-04
## [12,] 1.0000178 0.000000e+00 7.105427e-05
## par:1 1 1
   [1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
    [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
              [,1]
                            [,2]
                                         [.3]
##
   [1,] 0.7310597 -1.966116e-01 1.966116e-01
   [2,] 0.8807977 -1.049916e-01 2.099876e-01
  [3,] 0.9525714 -4.518164e-02 1.355271e-01
  [4,] 0.9820145 -1.766587e-02 7.065459e-02
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## [6,] 0.9975309 -2.451372e-03 1.481482e-02
## [7,] 0.9990941 -9.059420e-04 6.359357e-03
## [8,] 0.9996981 -3.197442e-04 2.664535e-03
   [9,] 0.9998757 -1.421085e-04 1.136868e-03
## [10,] 0.9999468 -3.552714e-05 4.618528e-04
## [11,] 0.9999468 -7.105427e-05 2.131628e-04
## [12,] 1.0000178 0.000000e+00 7.105427e-05
## par:1 1 1
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  [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##
              [,1]
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   [3,] 0.9525714 -4.518164e-02 1.355271e-01
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## [12,] 1.0000178 0.000000e+00 7.105427e-05
## par:1 1 1
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## par:1 1 1
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[,1]
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##
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##
   [1,] 0.7310597 -1.966116e-01 1.966116e-01
  [2,] 0.8807977 -1.049916e-01 2.099876e-01
##
   [3,] 0.9525714 -4.518164e-02 1.355271e-01
##
   [4,] 0.9820145 -1.766587e-02 7.065459e-02
  [5,] 0.9933032 -6.643575e-03 3.325340e-02
## [6,] 0.9975309 -2.451372e-03 1.481482e-02
   [7,] 0.9990941 -9.059420e-04 6.359357e-03
## [8,] 0.9996981 -3.197442e-04 2.664535e-03
## [9,] 0.9998757 -1.421085e-04 1.136868e-03
## [10,] 0.9999468 -3.552714e-05 4.618528e-04
## [11,] 0.9999468 -7.105427e-05 2.131628e-04
```

```
## [12,] 1.0000178 0.000000e+00 7.105427e-05
## par:1 1 1
   [1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
    [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##
                                         [,3]
              [,1]
                            [,2]
   [1,] 0.7310597 -1.966116e-01 1.966116e-01
##
   [2,] 0.8807977 -1.049916e-01 2.099876e-01
##
   [3,] 0.9525714 -4.518164e-02 1.355271e-01
  [4,] 0.9820145 -1.766587e-02 7.065459e-02
  [5,] 0.9933032 -6.643575e-03 3.325340e-02
   [6,] 0.9975309 -2.451372e-03 1.481482e-02
## [7,] 0.9990941 -9.059420e-04 6.359357e-03
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## [9,] 0.9998757 -1.421085e-04 1.136868e-03
## [10,] 0.9999468 -3.552714e-05 4.618528e-04
## [11,] 0.9999468 -7.105427e-05 2.131628e-04
## [12,] 1.0000178 0.000000e+00 7.105427e-05
## par:1 1 1
##
   [1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
   [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##
              [,1]
                            [,2]
                                         [,3]
##
   [1,] 0.7310597 -1.966116e-01 1.966116e-01
  [2,] 0.8807977 -1.049916e-01 2.099876e-01
  [3,] 0.9525714 -4.518164e-02 1.355271e-01
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   [5,] 0.9933032 -6.643575e-03 3.325340e-02
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## [12,] 1.0000178 0.000000e+00 7.105427e-05
## par:1 1 1
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## attr(,"gradient")
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  [4,] 0.9820145 -1.766587e-02 7.065459e-02
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## [11,] 0.9999468 -7.105427e-05 2.131628e-04
## [12,] 1.0000178 0.000000e+00 7.105427e-05
## par:1 1 1
## [1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
```

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[7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##
              [,1]
                            [,2]
                                         [,3]
   [1,] 0.7310597 -1.966116e-01 1.966116e-01
##
##
   [2,] 0.8807977 -1.049916e-01 2.099876e-01
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  [7,] 0.9990941 -9.059420e-04 6.359357e-03
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   [1,] 0.7310597 -1.966116e-01 1.966116e-01
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              [,1]
                            [,2]
                                         [,3]
##
  [1,] 0.7310597 -1.966116e-01 1.966116e-01
   [2,] 0.8807977 -1.049916e-01 2.099876e-01
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## attr(,"gradient")
##
                            [,2]
                                         [,3]
              [,1]
```

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[1,] 0.7310597 -1.966116e-01 1.966116e-01
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```
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  [1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
##
   [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##
              [,1]
                            [,2]
                                         [,3]
   [1,] 0.7310597 -1.966116e-01 1.966116e-01
   [2,] 0.8807977 -1.049916e-01 2.099876e-01
   [3,] 0.9525714 -4.518164e-02 1.355271e-01
  [4,] 0.9820145 -1.766587e-02 7.065459e-02
## [5,] 0.9933032 -6.643575e-03 3.325340e-02
##
   [6,] 0.9975309 -2.451372e-03 1.481482e-02
   [7,] 0.9990941 -9.059420e-04 6.359357e-03
  [8,] 0.9996981 -3.197442e-04 2.664535e-03
## [9,] 0.9998757 -1.421085e-04 1.136868e-03
## [10,] 0.9999468 -3.552714e-05 4.618528e-04
## [11,] 0.9999468 -7.105427e-05 2.131628e-04
## [12,] 1.0000178 0.000000e+00 7.105427e-05
## par:1 1 1
   [1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
##
   [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##
              [,1]
                            [,2]
                                         [,3]
##
   [1,] 0.7310597 -1.966116e-01 1.966116e-01
##
  [2,] 0.8807977 -1.049916e-01 2.099876e-01
  [3,] 0.9525714 -4.518164e-02 1.355271e-01
##
   [4,] 0.9820145 -1.766587e-02 7.065459e-02
##
   [5,] 0.9933032 -6.643575e-03 3.325340e-02
  [6,] 0.9975309 -2.451372e-03 1.481482e-02
## [7,] 0.9990941 -9.059420e-04 6.359357e-03
   [8,] 0.9996981 -3.197442e-04 2.664535e-03
## [9,] 0.9998757 -1.421085e-04 1.136868e-03
## [10,] 0.9999468 -3.552714e-05 4.618528e-04
## [11,] 0.9999468 -7.105427e-05 2.131628e-04
## [12,] 1.0000178 0.000000e+00 7.105427e-05
```

```
[3,] 0.9525714 -4.518164e-02 1.355271e-01
##
    [4,] 0.9820145 -1.766587e-02 7.065459e-02
##
   [5,] 0.9933032 -6.643575e-03 3.325340e-02
   [6,] 0.9975309 -2.451372e-03 1.481482e-02
##
    [7,] 0.9990941 -9.059420e-04 6.359357e-03
    [8,] 0.9996981 -3.197442e-04 2.664535e-03
   [9,] 0.9998757 -1.421085e-04 1.136868e-03
## [10,] 0.9999468 -3.552714e-05 4.618528e-04
   [11,] 0.9999468 -7.105427e-05 2.131628e-04
   [12,] 1.0000178 0.000000e+00 7.105427e-05
  par:1 1 1
        -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
##
    [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
##
  attr(, "gradient")
##
              [,1]
                            [,2]
##
    [1,] 0.7310597 -1.966116e-01 1.966116e-01
##
    [2,] 0.8807977 -1.049916e-01 2.099876e-01
##
   [3,] 0.9525714 -4.518164e-02 1.355271e-01
   [4,] 0.9820145 -1.766587e-02 7.065459e-02
##
    [5,] 0.9933032 -6.643575e-03 3.325340e-02
   [6,] 0.9975309 -2.451372e-03 1.481482e-02
##
   [7,] 0.9990941 -9.059420e-04 6.359357e-03
   [8,] 0.9996981 -3.197442e-04 2.664535e-03
   [9,] 0.9998757 -1.421085e-04 1.136868e-03
## [10,] 0.9999468 -3.552714e-05 4.618528e-04
## [11,] 0.9999468 -7.105427e-05 2.131628e-04
## [12,] 1.0000178 0.000000e+00 7.105427e-05
print(tandeunscx2)
## Unit: microseconds
##
##
   andeunscx2 <- nlsalt::numericDeriv(rexpr, theta, rho = weedenv,</pre>
                                                                          central = TRUE, eps = 1e-10)
##
                         mean median
                                             uq
                                                    max neval
   129.589 159.8595 187.3172 187.055 218.9775 258.036
                                                          100
```

-4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473 [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006

[,3]

[,2]

[1,] 0.7310597 -1.966116e-01 1.966116e-01

[2,] 0.8807977 -1.049916e-01 2.099876e-01

par:1 1 1

attr(, "gradient")

[,1]

##

##

##

##

##

The dir parameter allows us to use a backward difference for the derivative. This appears in nlsModel() for the case where a parameter is on an upper bound for the case algorithm="port". It does not check for nearness to the bound, and for the lower bound assumes that we are stepping AWAY from the bound in the default direction (dir=+1). None of the code addresses the issue where bounds are closer together than the step used for the finite difference, so there are situations where we could crash the code. Nor does the code check if the central difference is specified when near a bound.

- In the case of lower bounds, a central difference can overstep the bound when a parameter is "close" or
- In the case of an upper bound, changing the dir will not change the derivative expression and steps in both forward and backward directions of the parameter are taken.

Symbolic methods from nlsr

The package nlsr has a function model2rjfun() that converts an expression describing how the residual functions are computed into an R function that computes the residuals at a particular set of parameters and sets the attribute "gradient" of the vector of residual values to the Jacobian at the particular set of parameters.

```
# nlsr has function model2rjfun. We can evaluate just the residuals
res0<-model2rjfun(eunsc, start1, data=weeddata1, jacobian=FALSE)
res0(start1)
   [1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
   [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
# or the residuals and jacobian
## nlsr::model2rjfun forms a function with gradient (jacobian) attribute
funsc <- model2rjfun(eunsc, start1, data=weeddata1) # from nlsr: creates a function
tmodel2rjfun <- microbenchmark(model2rjfun(eunsc, start1, data=weeddata1))</pre>
print(tmodel2rjfun)
## Unit: microseconds
##
                                                             lq
                                                                    mean median
                                             expr
                                                     min
    model2rjfun(eunsc, start1, data = weeddata1) 82.721 83.88 88.22254 84.6825
##
               max neval
    86.612 185.774
                     100
print(funsc)
## function(prm) {
##
           if (is.null(names(prm)))
##
        names(prm) <- names(pvec)</pre>
##
      localdata <- list2env(as.list(prm), parent = data)</pre>
##
      eval(residexpr, envir = localdata)
##
           # Saves Jacobian matrix as "gradient" attribute (consistent with deriv())
##
## <bytecode: 0x5634f384edc8>
## <environment: 0x5634f2fe0200>
print(funsc(start1))
##
    \begin{bmatrix} 1 \end{bmatrix} -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
    [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##
                b1
                               b2
   [1,] 0.7310586 -1.966119e-01 1.966119e-01
##
  [2,] 0.8807971 -1.049936e-01 2.099872e-01
## [3,] 0.9525741 -4.517666e-02 1.355300e-01
    [4,] 0.9820138 -1.766271e-02 7.065082e-02
  [5,] 0.9933071 -6.648057e-03 3.324028e-02
  [6,] 0.9975274 -2.466509e-03 1.479906e-02
##
  [7,] 0.9990889 -9.102212e-04 6.371548e-03
## [8,] 0.9996646 -3.352377e-04 2.681901e-03
## [9,] 0.9998766 -1.233793e-04 1.110414e-03
## [10,] 0.9999546 -4.539581e-05 4.539581e-04
## [11,] 0.9999833 -1.670114e-05 1.837126e-04
## [12,] 0.9999939 -6.144137e-06 7.372964e-05
```

```
print(environment(funsc))
## <environment: 0x5634f2fe0200>
print(ls.str(environment(funsc)))
## data : <environment: 0x5634f2fd35f8>
## jacobian : logi TRUE
## modelformula : Class 'formula' language y ~ b1/(1 + b2 * exp(-b3 * tt))
## pvec : Named num [1:3] 1 1 1
                expression({ .expr3 <- exp(-b3 * tt) .expr5 <- 1 + b2 * .expr3 .expr10 <- .expr5^2}
## residexpr :
## rjfun : function (prm)
## testresult : logi TRUE
print(ls(environment(funsc)$data))
## [1] "tt" "y"
eval(eunsc, environment(funsc))
## y \sim b1/(1 + b2 * exp(-b3 * tt))
vfunsc<-funsc(start1)</pre>
print(vfunsc)
   [1] -4.576941 -6.359203 -8.685426 -11.883986 -16.075693 -22.194473
   [7] -30.443911 -37.558335 -49.156123 -61.948045 -74.995017 -90.972006
## attr(,"gradient")
##
                b1
                              h2
                                           b3
## [1,] 0.7310586 -1.966119e-01 1.966119e-01
## [2,] 0.8807971 -1.049936e-01 2.099872e-01
## [3,] 0.9525741 -4.517666e-02 1.355300e-01
## [4,] 0.9820138 -1.766271e-02 7.065082e-02
## [5,] 0.9933071 -6.648057e-03 3.324028e-02
## [6,] 0.9975274 -2.466509e-03 1.479906e-02
## [7,] 0.9990889 -9.102212e-04 6.371548e-03
## [8,] 0.9996646 -3.352377e-04 2.681901e-03
## [9,] 0.9998766 -1.233793e-04 1.110414e-03
## [10,] 0.9999546 -4.539581e-05 4.539581e-04
## [11,] 0.9999833 -1.670114e-05 1.837126e-04
## [12,] 0.9999939 -6.144137e-06 7.372964e-05
tfunsc<-microbenchmark(funsc(start1))</pre>
print(tfunsc)
## Unit: microseconds
##
                    \min
             expr
                              lq
                                     mean median
                                                    uq
  funsc(start1) 13.474 13.7725 14.99547 14.0115 14.4 54.323
```

numDeriv package

The package numDeriv includes a function jacobian() that acts on a user function resid() to produce the Jacobian at a set of parameters by several choices of approximation.

```
# We use the residual function (without gradient attribute) from nlsr
jeunsc<-jacobian(res0, start1)
jeunsc
```

```
##
               [,1]
                             [,2]
    [1,] 0.7310586 -1.966119e-01 1.966119e-01
##
##
    [2,] 0.8807971 -1.049936e-01 2.099872e-01
    [3,] 0.9525741 -4.517666e-02 1.355300e-01
##
##
    [4,] 0.9820138 -1.766271e-02 7.065082e-02
##
    [5,] 0.9933071 -6.648057e-03 3.324028e-02
    [6,] 0.9975274 -2.466509e-03 1.479906e-02
##
    [7,] 0.9990889 -9.102212e-04 6.371548e-03
    [8,] 0.9996647 -3.352378e-04 2.681902e-03
   [9,] 0.9998766 -1.233791e-04 1.110414e-03
## [10,] 0.9999546 -4.539572e-05 4.539580e-04
## [11,] 0.9999833 -1.670116e-05 1.837129e-04
## [12,] 0.9999939 -6.144205e-06 7.373002e-05
# Timings of the analytic jacobian calculations
tjeunsc<-microbenchmark(jeunsc<-jacobian(res0, start1))</pre>
print(tjeunsc)
## Unit: microseconds
##
                                         min
                                                           mean median
                                 expr
                                                   lq
                                                                             uq
    jeunsc <- jacobian(res0, start1) 354.27 358.2145 365.4824 361.045 364.675
##
##
        max neval
##
    542.011
              100
```

Note that the manual pages for numDeriv offer many options for the functions in the package. At 2021-5-27 we have yet to explore these.

Comparisons

In the following, we are comparing to vfunsc, which is the evaluated residual vector at start1=c(1,1,1) with "gradient" attribute (jacobian) included, as developed using package nlsr. This is taken as the "correct" result, even though it is possible that the generated order of calculations may introduce inaccuracies in the supposedly analytic derivatives.

numericDeriv computes a similar structure (residuals with "gradient" attribute): ndeunsc: the forward difference result with default eps (1e-07 according to manual) ndeunsc2: Central difference with default eps ndeunscx: Forward difference with smaller eps=1e-10 ndeunscx2: Central difference with smaller eps=1e-10

```
jeunsc: numDeriv::jacobian() result with default settings.
## Matrix comparisons
attr(ndeunsc, "gradient")-attr(vfunsc, "gradient")
##
                    b1
                                  b2
##
    [1,] -4.066995e-08 -7.619266e-09 -5.198538e-08
##
    [2,]
         1.016833e-08 3.631656e-09 -7.263312e-09
##
   [3,]
         7.050552e-09 -8.473015e-08 1.577186e-08
   [4,] -8.764533e-08 -5.738229e-08 -8.889419e-09
##
    [5,] -3.542825e-08 -6.988878e-09
                                      3.494439e-08
##
   [6,] -1.592723e-08 6.909055e-08 6.229383e-08
##
   [7,] 5.380365e-08 -6.095489e-08 -5.015294e-08
   [8,] -3.432289e-07 -4.556886e-07 -1.691883e-07
   [9,] -1.062480e-07 -1.214742e-07 1.395936e-07
## [10,] 9.833867e-08 9.627771e-08 -9.102750e-09
  [11,] -4.647158e-07 -4.649948e-07 -6.071033e-07
```

[12,] 4.221287e-07 4.220910e-07 6.569545e-07

```
attr(ndeunsc2, "gradient")-attr(vfunsc, "gradient")
                                 b2
                   b1
##
   [1,] -5.513268e-11 1.371020e-11
                                    5.962686e-11
   [2,] 6.850076e-13 -3.831403e-11 3.291006e-12
    [3,] -2.144829e-11 -1.208666e-10 6.925160e-11
   [4,] -2.665634e-11 4.123477e-11 -1.826495e-11
  [5,] 2.175706e-10 -7.825720e-11 9.793778e-11
  [6,] -2.416068e-10 -1.666460e-10 -1.735172e-10
   [7,] -8.350343e-11 5.415257e-11 -8.571976e-11
## [8,] 3.255913e-10 -9.864836e-11 2.024904e-10
## [9,] 1.379652e-11 -5.685668e-10 -1.631673e-10
## [10,] -9.296786e-11 -4.173983e-10 6.710748e-11
## [11,] -4.266865e-11 2.415372e-10 8.632699e-10
## [12,] -2.738492e-10 2.515432e-10 -6.717320e-10
attr(ndeunscx, "gradient")-attr(vfunsc, "gradient")
                                 b2
                   b1
##
    [1,] 1.078625e-06 -4.123528e-06 -4.758257e-06
   [2,] 5.790545e-07 2.014411e-06 4.852963e-06
   [3,] -2.771694e-06 -1.385826e-05 -1.171591e-05
   [4,] -8.202081e-06 -1.204434e-05 -5.113350e-06
   [5,] -3.931620e-06 4.482091e-06 1.311668e-05
  [6,] 3.569890e-06 1.513685e-05 1.576029e-05
  [7,] 5.191947e-06 -1.348438e-05 -1.219078e-05
## [8,] -2.074929e-06 -2.003370e-05 -5.289324e-05
   [9,] -8.676624e-07 -1.872920e-05 2.645423e-05
## [10,] -7.810096e-06 4.539581e-05 4.342184e-05
## [11,] -1.075608e-04 -1.254074e-04 -4.160402e-05
## [12,] 2.399048e-05 6.144137e-06 6.837890e-05
attr(ndeunscx2, "gradient")-attr(vfunsc, "gradient")
##
                   b1
                                 b2
                                               h3
   [1,] 1.078625e-06 3.173646e-07 -3.173646e-07
   [2,] 5.790545e-07 2.014411e-06 4.120706e-07
   [3,] -2.771694e-06 -4.976479e-06 -2.834131e-06
  [4,] 6.797035e-07 -3.162555e-06 3.768434e-06
  [5,] -3.931620e-06 4.482091e-06 1.311668e-05
   [6,] 3.569890e-06 1.513685e-05 1.576029e-05
   [7,] 5.191947e-06 4.279192e-06 -1.219078e-05
  [8,] 3.345221e-05 1.549344e-05 -1.736611e-05
  [9,] -8.676624e-07 -1.872920e-05 2.645423e-05
## [10,] -7.810096e-06 9.868671e-06 7.894701e-06
## [11,] -3.650654e-05 -5.435313e-05 2.945025e-05
## [12,] 2.399048e-05 6.144137e-06 -2.675369e-06
jeunsc-attr(vfunsc, "gradient")
                   b1
                                 b2
   [1,] -2.239464e-11 7.806283e-12 -1.156686e-12
##
  [2,] -2.267631e-12 2.974312e-11 2.957756e-11
  [3,] -8.948509e-12 3.630193e-11 -1.256267e-11
## [4,] -1.649125e-12 1.182179e-13 6.369841e-11
## [5,] -4.272493e-11 -1.109757e-11 3.501116e-11
```

```
[6,] 1.867381e-10 1.793287e-11 3.319552e-11
    [7,] 1.090728e-11 1.840947e-11 9.946462e-12
##
    [8,] 2.035664e-10 -1.520996e-10 1.911593e-10
   [9,] -3.582228e-10 2.039028e-10 -1.905254e-10
## [10,] 3.202474e-10 8.291927e-11 -6.263366e-11
## [11,] 5.931922e-12 -1.779933e-11 3.682277e-10
## [12.] 4.132902e-10 -6.831839e-11 3.817154e-10
## Summary comparisons
max(abs(attr(ndeunsc, "gradient")-attr(vfunsc, "gradient")))
## [1] 6.569545e-07
max(abs(attr(ndeunsc2, "gradient")-attr(vfunsc, "gradient")))
## [1] 8.632699e-10
max(abs(attr(ndeunscx, "gradient")-attr(vfunsc, "gradient")))
## [1] 0.0001254074
max(abs(attr(ndeunscx2, "gradient")-attr(vfunsc, "gradient")))
## [1] 5.435313e-05
max(abs(jeunsc-attr(vfunsc, "gradient")))
## [1] 4.132902e-10
```

Performance results for different computing environments

Here we present tables of the results, preceded by identified descriptions of the machines we used. We use ideas and functions from the document MachineSummary to provide a characterization and identity for each machine used.

M21-LM20.1

?? still to be run

?? What machines provide a range of possibilities.

Discussion of derivative computation for nonlinear least squares

In no particular order, we comment on some issues relating to the Jacobian calculations in nonlinear least squares.

Nomenclature

R is not in step with many other areas of numerical computation when labelling different objects in the nonlinear least squares problem. In particular, R uses the term "gradient" when the object of interest is the Jacobian matrix. In that it is useful in performing iterations of the Gauss-Newton or related equations to have the Jacobian associated with the residuals, and the rows of the Jacobian matrix are "gradients" of the respective residuals, we can accept the attribute name "gradient" to select the required information. Moreover, as in package nlsr it is very useful to have the Jacobian matrix as an attribute of the residual vector, since the main solver function, in this case nlsr::nlfb(), can be called with the same input for the arguments res and jac. These are the functions required to compute the residual and the Jacobian, and using the same function for both is very convenient, but needs some way to return both the residual vector and Jacobian matrix in a coherent fashion.

Numerical approximation near constraints

As far as we are aware there is no software that implements a fully safeguarded system to compute numerical approximation of the Jacobian (or gradients in general optimization) near constraints. The same statement applies even in the case of the much simpler bounds constraints. Users have a perverse tendency to devise ways to foil our best efforts. For example, they may decide that a good way to specify fixed (i.e., masked) parameters that they do not want to vary during a particular calculation is to specify the lower and upper bound of a parameter at the same value. Later runs may want the parameter constraints relaxed.

In nlsr::nlxb(), users may, in fact, specify masked parameters this way. This is a case of "if you can't beat them, join them," but it does provide an easily understood way for users to fix values.

More tricky is dealing with constraints that are close together. Note that these may arise from, for example, two linear (planar) constraints that approach at a narrow angle. In the apex where these constraints intersect, we will have tight bounds on parameters. If the constraint is not one that is imposed by the nature of the residual or objective function, for example, a log() or square root near zero, then we can generally proceed and allow the derivative approximator to evaluate outside the constraints. Things are decidedly nastier if we do have inadmissible values.

The issue of constraints and the need for a step in parameter values for derivative approximations was one of the motivations for trying to find analytic derivatives in package nlsr and the continuing effort to bring them into other R tools.

Appendix 1: Base R numericDeriv code

This code is in two files, nls.R and nls.c and is extracted here.

From nls.R

```
numericDeriv <- function(expr, theta, rho = parent.frame(), dir = 1,</pre>
                 eps = .Machine$double.eps ^ (1/if(central) 3 else 2), central = FALSE)
## Note: this expr must be set up as a call to work properly according to JN??
## ?? we set eps conditional on central. But central set AFTER eps. Is this OK.
    cat("numericDeriv-Alt\n")
    dir <- rep_len(dir, length(theta))</pre>
    stopifnot(is.finite(eps), eps > 0)
   rho1 <- new.env(FALSE, rho, 0)
    if (!is.character(theta) ) {stop("'theta' should be of type character")}
    if (is.null(rho)) {
            stop("use of NULL environment is defunct")
                     rho <- R_BaseEnv;</pre>
   } else {
          if(! is.environment(rho)) {stop("'rho' should be an environment")}
               int nprot = 3;
    if( ! ((length(dir) == length(theta) ) & (is.numeric(dir) ) )
              {stop("'dir' is not a numeric vector of the correct length") }
    if(is.na(central)) { stop("'central' is NA, but must be TRUE or FALSE") }
   res0 <- eval(expr, rho) # the base residuals. ?? C has a check for REAL ANS=res0
    if (any(is.infinite(res0)) ) {stop("residuals cannot be evaluated at base point")}
   ## CHECK_FN_VAL(res, ans); ?? how to do this. Is it necessary?
   nt <- length(theta) # number of parameters</pre>
   mr <- length(res0) # number of residuals</pre>
    JJ <- matrix(NA, nrow=mr, ncol=nt) # Initialize the Jacobian
   for (j in 1:nt){
```

```
origPar<-get(theta[j],rho)</pre>
       xx <- abs(origPar)</pre>
       delta <- if (xx == 0.0) {eps} else { xx*eps }
       ## JN: I prefer eps*(xx + eps) which is simpler ?? Should we suggest / use a control switch
       prmx<-origPar+delta*dir[j]</pre>
       assign(theta[j],prmx,rho)
       res1 <- eval(expr, rho) # new residuals (forward step)</pre>
       if (central) { # compute backward step resids for central diff
          prmb <- origPar - dir[j]*delta</pre>
          assign(theta[j], prmb, envir=rho) # may be able to make more efficient later??
          resb <- eval(expr, rho)</pre>
          JJ[, j] <- dir[j]*(res1-resb)/(2*delta) # vectorized</pre>
       } else { ## forward diff
          JJ[,j] <- dir[j]*(res1-res0)/delta</pre>
       } # end forward diff
    } # end loop over the parameters
    attr(res0, "gradient") <- JJ
    return(res0)
}
From nls.c
#include <stdlib.h>
#include <string.h>
#include <math.h>
#include <float.h>
#include <R.h>
#include <Rinternals.h>
#include "nls.h"
#include "internals.h"
#ifndef MIN
#define MIN(a,b) (((a)<(b))?(a):(b))
#endif
/*
 * call to numeric_deriv from R -
 * .Call("numeric_deriv", expr, theta, rho, dir = 1., eps = .Machine$double.eps, central=FALSE)
 * Returns: ans
 */
SEXP
numeric deriv(SEXP expr, SEXP theta, SEXP rho, SEXP dir, SEXP eps, SEXP centr,
              SEXP rho1)
{
    if(!isString(theta))
    error(_("'theta' should be of type character"));
    if (isNull(rho)) {
    error(_("use of NULL environment is defunct"));
    rho = R_BaseEnv;
    } else
    if(!isEnvironment(rho))
        error(_("'rho' should be an environment"));
    int nprot = 3;
    if(TYPEOF(dir) != REALSXP) {
```

```
PROTECT(dir = coerceVector(dir, REALSXP)); nprot++;
   }
    if(LENGTH(dir) != LENGTH(theta))
    error(_("'dir' is not a numeric vector of the correct length"));
   Rboolean central = asLogical(centr);
    if(central == NA LOGICAL)
    error( ("'central' is NA, but must be TRUE or FALSE"));
      SEXP rho1 = PROTECT(R NewEnv(rho, FALSE, 0));
//
//
     nprot++;
   SEXP
   pars = PROTECT(allocVector(VECSXP, LENGTH(theta))),
        ans = PROTECT(duplicate(eval(expr, rho1)));
   double *rDir = REAL(dir), *res = NULL; // -Wall
#define CHECK_FN_VAL(_r_, _ANS_) do {
    if(!isReal(_ANS_)) {
    SEXP temp = coerceVector(_ANS_, REALSXP);
   UNPROTECT(1);/*: _ANS_ *must* have been the last PROTECT() ! */ \
   PROTECT(_ANS_ = temp);
    _{r_{}} = REAL(_{ANS_{}});
   for(int i = 0; i < LENGTH(_ANS_); i++) {</pre>
    if (!R_FINITE(_r_[i]))
        error(_("Missing value or an infinity produced when evaluating the model")); \
} while(0)
   CHECK_FN_VAL(res, ans);
    const void *vmax = vmaxget();
    int lengthTheta = 0;
    for(int i = 0; i < LENGTH(theta); i++) {</pre>
    const char *name = translateChar(STRING_ELT(theta, i));
   SEXP s_name = install(name);
   SEXP temp = findVar(s_name, rho1);
    if(isInteger(temp))
        error(_("variable '%s' is integer, not numeric"), name);
    if(!isReal(temp))
        error(_("variable '%s' is not numeric"), name);
   // We'll be modifying the variable, so need to make a copy PR#15849
   defineVar(s_name, temp = duplicate(temp), rho1);
   MARK NOT MUTABLE(temp);
   SET VECTOR ELT(pars, i, temp);
   lengthTheta += LENGTH(VECTOR_ELT(pars, i));
   }
    vmaxset(vmax);
   SEXP gradient = PROTECT(allocMatrix(REALSXP, LENGTH(ans), lengthTheta));
   double *grad = REAL(gradient);
    double eps = asReal(eps_); // was hardcoded sqrt(DOUBLE_EPS) { ~= 1.49e-08, typically}
   for(int start = 0, i = 0; i < LENGTH(theta); i++) {</pre>
    double *pars_i = REAL(VECTOR_ELT(pars, i));
    for(int j = 0; j < LENGTH(VECTOR_ELT(pars, i)); j++, start += LENGTH(ans)) {</pre>
        double
        origPar = pars_i[j],
        xx = fabs(origPar),
```

```
delta = (xx == 0) ? eps : xx*eps;
        pars_i[j] += rDir[i] * delta;
        SEXP ans del = PROTECT(eval(expr, rho1));
        double *rDel = NULL;
        CHECK_FN_VAL(rDel, ans_del);
        if(central) {
        pars_i[j] = origPar - rDir[i] * delta;
        SEXP ans_de2 = PROTECT(eval(expr, rho1));
        double *rD2 = NULL;
        CHECK_FN_VAL(rD2, ans_de2);
        for(int k = 0; k < LENGTH(ans); k++) {</pre>
            grad[start + k] = rDir[i] * (rDel[k] - rD2[k])/(2 * delta);
        }
        } else { // forward difference (previously hardwired):
        for(int k = 0; k < LENGTH(ans); k++) {</pre>
            grad[start + k] = rDir[i] * (rDel[k] - res[k])/delta;
        }
        }
        UNPROTECT(central ? 2 : 1); // ansDel & possibly ans
        pars_i[j] = origPar;
    }
    setAttrib(ans, install("gradient"), gradient);
    UNPROTECT(nprot);
    return ans;
}
```

Appendix 2: numericDeriv() from nlsalt package (all in R)

```
# File src/library/stats/R/nlsnd.R
 Part of the modified R package, https://www.R-project.org
# Copyright (C) 2000-2020 The R Core Team
  Copyright (C) 1999-1999 Saikat DebRoy, Douglas M. Bates, Jose C. Pinheiro
# J C Nash 2021
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# GNU General Public License for more details.
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# https://www.R-project.org/Licenses/
###
###
              numeric Jacobian for Nonlinear least squares for R
###
```

```
numericDeriv <- function(expr, theta, rho = parent.frame(), dir = 1,</pre>
                 eps = .Machine$double.eps ^ (1/if(central) 3 else 2), central = FALSE)
## Note: this expr must be set up as a call to work properly according to JN??
## ?? we set eps conditional on central. But central set AFTER eps. Is this OK.
   ndtrace<-TRUE
     if(ndtrace) cat("numericDeriv-Alt\n")
    dir <- rep len(dir, length(theta))</pre>
    stopifnot(is.finite(eps), eps > 0)
    rho1 <- new.env(FALSE, rho, 0)
    if (!is.character(theta) ) {stop("'theta' should be of type character")}
    if (is.null(rho)) {
            stop("use of NULL environment is defunct")
                     rho <- R_BaseEnv;</pre>
    } else {
          if(! is.environment(rho)) {stop("'rho' should be an environment")}
               int nprot = 3;
    if( ! ((length(dir) == length(theta) ) & (is.numeric(dir) ) ) )
              {stop("'dir' is not a numeric vector of the correct length") }
    if(is.na(central)) { stop("'central' is NA, but must be TRUE or FALSE") }
    res0 <- eval(expr, rho) # the base residuals. ?? C has a check for REAL ANS=res0
    if (any(is.infinite(res0)) ) {stop("residuals cannot be evaluated at base point")}
    ## CHECK_FN_VAL(res, ans); ?? how to do this. Is it necessary?
    nt <- length(theta) # number of parameters</pre>
    mr <- length(res0) # number of residuals</pre>
    JJ <- matrix(NA, nrow=mr, ncol=nt) # Initialize the Jacobian
    for (j in 1:nt){
       origPar<-get(theta[j],rho)</pre>
       xx <- abs(origPar)</pre>
       delta <- if (xx == 0.0) {eps} else { xx*eps }
       ## JN: I prefer eps*(xx + eps) which is simpler ?? Should we suggest / use a control switch
       prmx<-origPar+delta*dir[j]</pre>
       assign(theta[j],prmx,rho)
       res1 <- eval(expr, rho) # new residuals (forward step)</pre>
       if (central) { # compute backward step resids for central diff
          prmb <- origPar - dir[j]*delta</pre>
          assign(theta[j], prmb, envir=rho) # may be able to make more efficient later??
          resb <- eval(expr, rho)</pre>
          JJ[, j] <- dir[j]*(res1-resb)/(2*delta) # vectorized</pre>
       } else { ## forward diff
          JJ[,j] <- dir[j]*(res1-res0)/delta</pre>
       } # end forward diff
       assign(theta[j],origPar,rho) # restore the parameter value !! IMPORTANT
    } # end loop over the parameters
    attr(res0, "gradient") <- JJ
    if (ndtrace){
       cat("par:")
       for (j in 1:nt){ cat(get(theta[j],rho)," ") }
       cat("\n")
       print(res0)
    return(res0)
}
```

Nash, John C. 1979. Compact Numerical Methods for Computers : Linear Algebra and Function Minimisation. Book. Hilger: Bristol.