# Improvement to nls() : Solution to Tests(Medium)

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## Theory :

The logistic sigmoid growth curve,here, is given by -

$$y = f(t) = \frac{a}{1 + be^{-ct}} \dots (\Delta)$$

Here, the growth $y = f(t)$ is a function of time $t$.
$b$ : the scale parameter of the sigmoid.
$a$ : the curve's maximum value.
$c$ : the logistic growth rate or steepness of the curve.

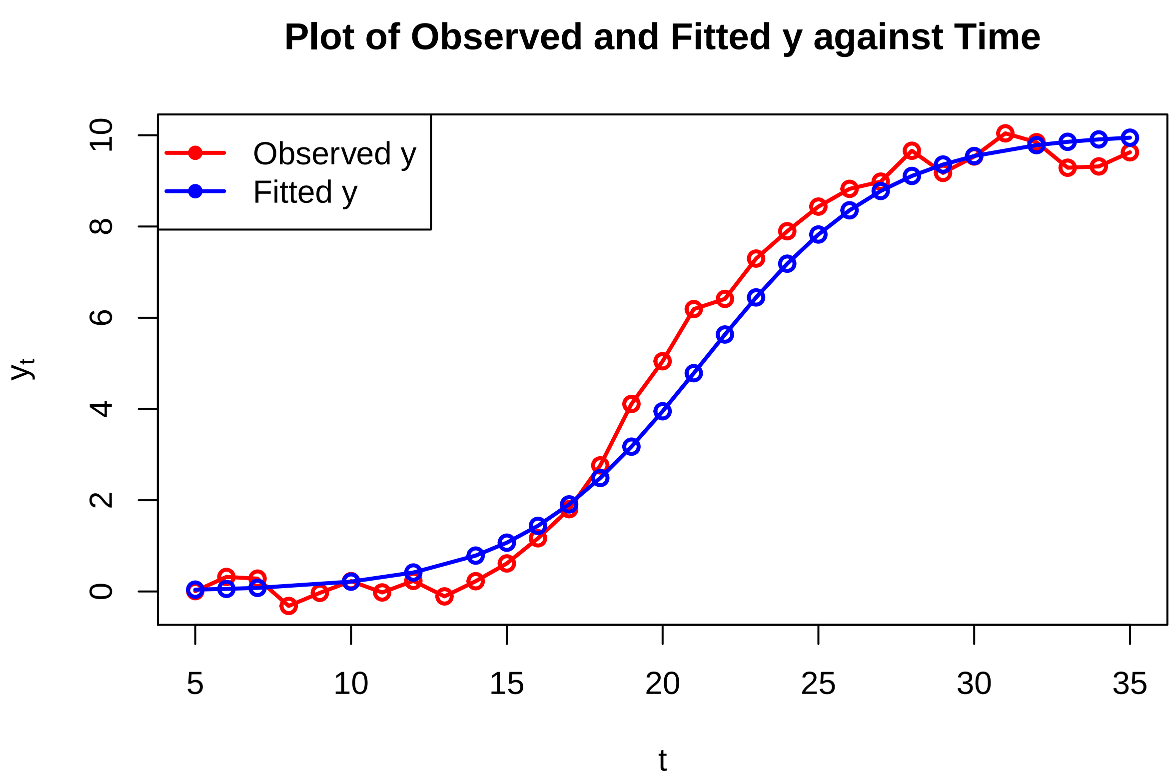
Now, given the test data, we need to estimate a logistic sigmoid growth curve of the form as in Eq. $\Delta$.

## R Code :

```r
### "Improvements to nls()" - GSOC'21
## Medium

# test data

time <- c( 5,  6,  7,  8,  9, 10, 11, 12, 13, 14, 15, 16,
 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29,
 30, 31, 32, 33, 34, 35)
y <- c( 0.0074203,  0.3188325,  0.2815891, -0.3171173, -0.0305409,  0.2266773,
  -0.0216102,  0.2319695, -0.1082007,  0.2246899,  0.6144181,  1.1655192,
  1.8038330,  2.7644418,  4.1104270,  5.0470456,  6.1896092,  6.4128618,
  7.2974793,  7.8965245,  8.4364991,  8.8252770,  8.9836204, 9.6607736,
  9.1746182,  9.5348823, 10.0421165,  9.8477874,  9.2886090,  9.3169916,
  9.6270209 )

# Plot observed y vs. time
plot(time,y,xlab="t",ylab=expression(y[t]),
main="Plot of Observed and Fitted y against Time",
lwd=2,col="red",type="o")
```

```r
# clean the data
time <- time[y>=0]
y <- y[y>=0]

## Approximate estimation of a,b,c
# approximate value of a
a<-max(y)

# approximate value of b
b <- a/y[1]-1

# approximate value of c
time<-time[y!=a]
y<-y[y!=a]
c<-log( (b*y)/(a-y) )/time
c<-mean(c)

# plot of fitted y values
lines(time, a/(1+b*exp(-c*time)),type="o",lwd=2,col="blue")
legend("topleft",legend=c("Observed y","Fitted y"),
col=c("red","blue"),lwd=2,pch=16)

# print a,b,c
cat("The estimated value of a is : ",a,"\n")
cat("The estimated value of b is : ",b,"\n")
cat("The estimated value of c is : ",c,"\n")
```

**Output :**




```
## The estimated value of a is :  10.04212

## The estimated value of b is :  1352.33

## The estimated value of c is :  0.3388275
```

## Explanation :

I first plot the graph to know how the given $y's$ look as a function of time. Since, the logistic function is strictly positive, I first discard the negative y values from the data and the corresponding time points.
Now, $a$ being the maximum value of the growth curve, I take $a$ to be the maximum of the given $y's$.
Here,the scale parameter $b$ is actually $b = e^{cx_0}$(on comparing with the form given in Eq $\Delta$ of the "Easy" file). Unlike $x_0$, $b$ itself involves two parameters $c$ and $x_0$; so, estimating $b$ becomes difficult as we cannot estimate $c$ and $x0$ explictly using Eq.$\Delta$(Note that this Eq. $\Delta$ does not contain any $x_0$ term). Further, the parametrization in Eq. $\Delta$ have easier interpretation that actually help in choosing approximate estimates of parameters (in particular $x_0$). So, I note that $f(0) = \frac{a}{1+b}$ and the logistic growth function is an increasng function. Hence, I estimate $b$ by $b = a/y_{min} - 1$.
Finally, given $y$, $time$ and estimated $a$ , $b$, I compute $c's$ by plugging these values in Eq. $\Delta$ and then take the mean of the resulting vector of $c's$. In this process, I compute $c$ by $c = \frac{log(by/(a-y))}{time}$ ; so, I discard that value of $y$ and corresponding value of $time$ for which $a = y$.

Using the above procedure, my estimated logistic growth function is -

$$\hat{y} = \hat{f}(t) = \frac{10.04212}{1 + 1352.33e^{-0.3388275t}}$$